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Photoelastic determination of Mode I stress intensity factor

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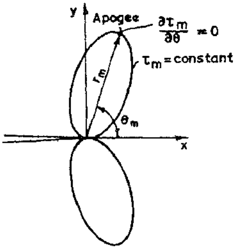
Abstract

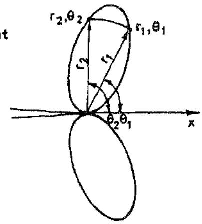
Application of earlier methods of analysis of photoelastic information in a single edge notch specimen subject to tensile loading to determine Mode I stress intensity factor brought into focus the constraints and the limitations of the earlier methods. In this investigation, a new approach called the constant θ method has been put forth. The results obtained by this method are compared along with the values determined by using the earlier approaches against the theoretical values of the boundary collocation method. The method is quite reliable and the results are in fair agreement with the boundary collocation results.

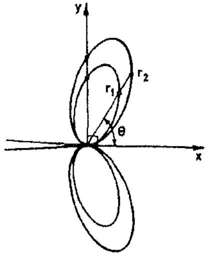
Key words: Stress intensity factor, crack, fracture mechanics, photoelasticity.

1. Introduction

Among the experimental techniques to determine the stress intensity factor the well-known are the method of caustics due to Theocaris[1] and the photoelastic method. The photoelastic method has been very widely used starting from Irwin[3] who used the fringes obtained by Wells and Post[2]. The information gathered from the photoelastic fringes has been analysed by Irwin[3], Schroedl and Smith[4], Bradley and Kobayashi[5], Prabhu[6] and Rakesh[7] for the determination of the Mode I stress intensity factor. Irwin and Rakesh[3,7] used a single fringe loop. Schroedl and Smith, Bradley and Kobayashi and Prabhu[4-6] used two loops. Irwin[3] used the coordinates r_m, θ_m corresponding to the apogee point of any loop (fig. 1), whereas Rakesh[7] used the coordinates (r, θ_1) and (r, θ_2) for any loop (fig. 2). Thus Irwin's[3] method becomes a special case of Rakesh[7]. In like manner, Bradley and Kobayashi[5] applied Irwin's[3] method to two loops. Schroedl and Smith[4] considered the intersections of a line at right angles to the crack axis with two loops and used the photoelastic information using a differencing technique. In all the above methods, measurements are made not at the crack tip but in the vicinity of the crack tip. Again the expression used for the state of stress at any point (r, θ) are the Irwin's modified Westergaard's equations to take into account the far-field stress σ_{ox} which is used to account for the tilt in the fringe loop. Irwin's method is said to be valid in a region very close to the crack tip. Prabhu[6] redefined the state of stress ahead of the crack tip so as to utilise the photoelastic data obtained from a region 'not too near the crack tip'. The formulation was such that the stress distribution is one as defined by stress singularity equations and when one goes far away from the crack tip the stress distribu-







NOTE :- Mean lines of the fringes are drawn.

FIG. 1. Irwin's method

FIG. 2. Rakesh method

FIG. 3. Constant θ method $\theta = 90°$, Schroedl-Smith method

tion corresponds to the far-field stress distribution. Hence, there exists a region (in between the crack tip and the far-field) in which both the crack singularity effect and the far-field stress effect act simultaneously. Thus the difference in the principal stresses was written as

$$\sigma_1 - \sigma_2 = \frac{K}{(r/a)^{1/2}} + \sum_{m=1}^{M} B_m (r/a)^m \tag{1}$$

where

$K = K_1 \text{ Sin } \theta / (2\pi a)^{1/2}$ and $m = 1,2,3 \ldots M$

a = crack length
K_1 = stress intensity factor
r = distance from crack tip.

Along any fixed θ, data at $(M+1)$ points is used to form $(M+1)$ linear simultaneous algebraic equations in K, B_1, B_2, etc. The solution of these equations give the information regarding K and $B_m s$, from which K_1, the SIF, is calculated.

Prabhu *et al*[6] claim the validity of their method up to a value of $r/a \lesssim 1$.

Irwin's method

Irwin[3] modified Westergaard's equation by introducing a second parameter $-\sigma_{ox}$ to the rectangular stress σ_x as,

$$\sigma_x = \frac{K_1}{\sqrt{2\pi r}} \text{ Cos } \frac{\theta}{2} \left(1 - \text{Sin } \frac{\theta}{2} \text{ Sin } \frac{3\theta}{2}\right) - \sigma_{ox}$$

$$\sigma_y = \frac{K_1}{\sqrt{2\pi r}} \text{ Cos } \frac{\theta}{2} \left(1 + \text{Sin } \frac{\theta}{2} \text{ Sin } \frac{3\theta}{2}\right) \tag{2}$$

$$\tau_{xy} = \frac{K_1}{\sqrt{2\pi r}} \text{ Cos } \frac{\theta}{2} \text{ Sin } \frac{\theta}{2} \text{ Cos } \frac{3\theta}{2}$$

The maximum shear stress is expressed as

$$(2\,\tau_m)^2 = (\sigma_y - \sigma_x)^2 + (2\,\tau_{xy})^2 \tag{3}$$

$$= \frac{K_1^2}{2\pi r}\,\mathrm{Sin}^2\theta + \frac{2\sigma_{ox}\,K_1\,\mathrm{Sin}\,\theta\,\mathrm{Sin}\,3\theta/2}{\sqrt{2\pi r}} + \sigma_{ox}^2 \tag{4}$$

Irwin noted from the fringe loops that

$$\frac{\partial\tau_m}{\partial\theta} = 0 \tag{5}$$

at the extreme position on the fringe loop where $r = r_m$ and $\theta = \theta_m$

Differentiating eqn. (4) with respect to θ and using eqn. (5) gives

$$\sigma_{ox} = -\frac{K_1}{\sqrt{2\pi r_m}}\;\frac{\mathrm{Sin}\,\theta_m\,\mathrm{Cos}\,\theta_m}{\left(\mathrm{Cos}\,\theta_m\,\mathrm{Sin}\,\frac{3\theta_m}{2} + \frac{3}{2}\,\mathrm{Sin}\,\theta_m\,\mathrm{Cos}\,\frac{3\theta_m}{2}\right)} \tag{6}$$

The two parameters K_1 and σ_{ox} are determined as

$$\sigma_{ox} = \frac{-2\,\tau_m\,\mathrm{Cos}\,\theta_m}{\mathrm{Cos}\,\frac{3\theta_m}{2}\,\left(\mathrm{Cos}^2\theta_m + \frac{9}{4}\,\mathrm{Sin}^2\theta_m\right)^{1/2}} \tag{7}$$

$$K_1 = \frac{2\,\tau_m\,(2\pi r_m)^{1/2}}{\mathrm{Sin}\,\theta_m}\left[1 + \left(\frac{2}{3\,\tan\theta_m}\right)^2\right]^{-1/2}\left[1 + \frac{2\tan\frac{3\theta_m}{2}}{3\,\tan\theta_m}\right] \tag{8}$$

Schroedl-Smith method

When eqn. (4) is evaluated along a line perpendicular to the crack axis, it reduces to

$$(2\tau_m)^2 = \frac{K_1^2}{2\pi r} + \frac{K_1\,\sigma_{ox}}{\sqrt{\pi r}} + \sigma_{ox}^2 \tag{9}$$

Solving eqn. (9) for K_1 and retaining only the positive root, the quadratic formula gives

$$K_1 = \sqrt{\pi r}\;\left[\,(8\tau_m^2 - \sigma_{ox}^2)^{1/2} - \sigma_{ox}\,\right] \tag{10}$$

Neglecting σ_{ox}^2 compared to $8\ \tau_m^2$.

$$K_1 = \sqrt{\pi r}\ [\ \sqrt{2}\,(2\tau_m) - \sigma_{ox}\ ] \tag{11}$$

By adopting a differencing technique using two loops,

$$K_1 = \sqrt{2\pi r_i}\quad \frac{(2\tau_m)_i - (2\tau_m)_j}{1-(r_i/r_j)^{1/2}} \tag{12}$$

The values of K_1 are obtained from different combinations of loops and are statistically conditioned.

Conclusions from previous work

All the earlier mentioned investigators except Prabhu[6] have used the Irwin's two parameter equation (eqn. 2). However the methods of analysis have been different. Each method appears to be having a constraint in its application and hence could lead to errors in the evaluation of Mode 1 SIF. Etheridge and Dally[8] have made a critical review of the methods for determining SIFs from isochromatic fringes. The following are the salient features of the critical review[8]

"The two parameter methods are all applicable for determining K in the range $73 < \theta_m < 139°$ provided $r_m / a < 0.03$. If no measurement errors are made in r_m or θ_m the two parameter methods will predict K with an accuracy of $\pm$ 5 per cent. With the Schroedl-Smith method prediction of K to $\pm$ 5 per cent is anticipated if the fringe loop radii r_i and r_j are measured without error".

Thus the stipulation of $r / a < 0.03$ of the Irwin's method becomes a severe constraint as the fringes crowd at the crack tip and become indistinguishable. However, this restriction in the value of $r / a < 0.03$ remains an open question.

In regard to the Schroedl-Smith method of making measurements along a 90° line, the fringes in an SEN specimen subjected to pure tensile load show a tilt and hence the stress gradient increases along a 90° line. Even in this case inaccurate measurement of r_i and r_j (values of r on two fringes) might result in erroneous value for the SIF.

In the present investigation, the series method of Prabhu[6] also resulted in the overestimation of the SIF and was dependent on the direction along which the expressions were applied.

Motivation for the present work

It was realised that there was a need for a method of analysis specially applicable to determine Mode 1 SIF

a) which uses photoelastic information in a region not too near the crack tip (where both the singular and far-field are effective, that is $r / a < 1$)

b) which does not impose too much of a constraint on the angle θ

c) for which (r, θ) values can be determined accurately

d) which gives the SIF values in fair agreement with the boundary collocation results.

The above features were to a large extent satisfied by the proposed new method called the constant θ method.

The Schroedl-Smith method is a special case of this method in which $\theta = 90°$.

2. The proposed new method (Constant θ method)

Rearrangement of eqn. (4) gives

$$\left(\frac{K_1}{\sqrt{2\pi r}}\right)^2 + \frac{2K_1 \sigma_{ox}}{\sqrt{2\pi r}} \frac{\text{Sin } 3\theta/2}{\text{Sin } \theta} + \frac{1}{\text{Sin}^2\theta} (\sigma_{ox}^2 - 4\tau_m^2) = 0 \tag{13}$$

Solving the quadratic and neglecting $\sigma_{ox}^2(\text{Sin}^2 \frac{3\theta}{2} - 1)$ as small compared to $4\tau_m^2$

$$\frac{K_1}{\sqrt{2\pi r}} = \frac{1}{\text{Sin } \theta} \left[2\tau_m - \sigma_{ox} \text{ Sin } \frac{3\theta}{2}\right] \tag{14}$$

Applying the above equation to two loops and getting the intersection on a single radiating line at θ at r_1 and r_2 and using the differencing technique, one gets

$$K_1 = \sqrt{2\pi r_1} \left[\frac{N_1 - N_2}{1 - \sqrt{r_1/r_2}} \quad \frac{F_\sigma}{t \text{ Sin } \theta}\right] \tag{15}$$

using the stress optic law, $\sigma_1 - \sigma_2 = 2\tau_m = \dfrac{NF_\sigma}{t}$

and applying it to two loops where N_1 and N_2 are the fringe orders, F_σ= material fringe value, r_1 and r_2 radial distances of the fringes from the crack tip, θ angle made by the radius with the crack axis, t thickness of specimen (fig. 3).

3. Experimental procedure and analysis

Single edge notch specimens of 50 mm width (fig. 4) were made from Araldite sheets of 6 mm thickness obtained from cold casting in the proportion 100:9 by weight of CY 230 resin and HY 951 hardener supplied by Hindustan Ciba-Geigy. The crack had a width and length of 0.2 and 7.5 mm respectively.

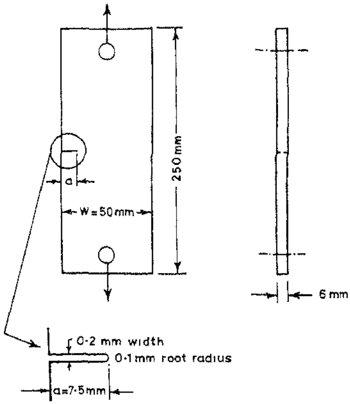


FIG 4. Details of the specimen.

Thus the a/w ratio for the specimens was 0.15. They were subjected to an annealing cycle to relieve them of any possible residual stresses induced during the making of the specimens. A Carl Zeiss diffusion polariscope using sodium vapour lamp for the monochromatic light source was used. With a loading fixture the specimens were loaded in uniaxial tension by using loading pins. Thus an opening mode or Mode I deformation at the crack tip was realised. The loading was of such a magnitude so as to get at least three well-defined fringes in the neighbourhood of the crack tip so that measurements of distances from the crack tip to the fringe contour could be made accurately. This also resulted in an r/a ratio less than unity where r is the distance of any point on the fringe from the crack tip and a is the crack length. Fringe photographs were taken both in the bright and dark fields to facilitate measurements on half and integral order fringes.

Analysis

The fringe photographs were projected with a magnification of ten on a screen. The boundaries of each of the fringes were traced and a meanline for each fringe was drawn. From the crack tip, radial lines were drawn oriented at different inclinations θ_1, θ_2, etc., to the crack axis in increments of 10° commencing from 40° as in fig. 3. For each of the values of θ the distances of the points of intersection of the radial lines with the mean fringe line from the crack tip were determined.

In addition to the above measurements, the values of r_m and θ_m, the polar coordinates of the apogee point (fig. 1) were also determined. With the above readings it was possible to determine the stress intensity factor by the earlier methods[3,4,6] and also by the proposed new method called the constant θ method. Wherever the method required information from two fringes as in the methods of Prabhu, Schroedl & Smith and constant θ method different combinations of the distances, r, two at a time were used to determine the SIF and the results were statistically conditioned. The experimentally obtained values are compared with the theoretical values obtained by the boundary collocation method of Gross *et al*[9] (Table I).

4. Results and conclusions

1. The results obtained by Irwin's method are at considerable variance with the boundary collocation value.
2. Prabhu's method applied to three loops improves the values and brings about a better convergence than the values obtained using two loops. However, even here the results are at considerable variance with the boundary collocation value.
3. The method of Schroedl and Smith gives a single value and that is along a 90° line. The value so obtained is found to be reasonably close to the boundary collocation value.
4. (a) The constant θ method appears to be very promising in that the statistically conditioned values evaluated along any value of θ greater than 40° give a K_1 value in fair agreement with the boundary collocation value.

   (b) The flexibility in the choice of θ makes the constant θ method quite attractive.

   (c) Any value of θ greater than 40° should be all right. However, a line intersecting a fringe squarely appears to be a proper choice for θ.

   (d) For values of θ less than 40° the value for K_1 would be large and erroneous as the line would be more or less tangential to the loops and hence would lead to erroneous values for r.

Table I

SIF in SEN specimens: Statistically conditioned values

θ Deg	Prabhu's method 2-loops	Prabhu's method 3-loops	Schroedl-Smith method*	Irwin's method**	Constant θ method	Boundary collocation value
	kgf cm$^{-3/2}$	kgf cm$^{-3/2}$	kgf cm$^{-3/2}$	kgf cm$^{-3/2}$	kgf cm$^{-3/2}$	kgf cm$^{-3/2}$
90	56.54	55.38	50.85		50.85	48.3
80	58.45	58.29		$N = 4$ $K_1 = 48.67$	49.22	
70	59.61	58.23		$N = 3.5$ $K_1 = 36.060$	47.79	
60	61.66	57.72		Av=42.365	45.86	
50	63.27	60.03			47.9	
45	65.76	57.34			47.59	
40	64 88	62.56			52.71	

* Evaluated along $\theta = 90°$ only as required by the method.

** Evaluated only at the apogee point as required by the method.

(e) Schroedl & Smith method is a special case of constant θ method wherein $\theta = 90°$

5. Irwin's method gives a value in fair agreement with the boundary collocation method provided great care is exercised in the measurements of r_m and θ_m on fringes very close to the crack tip which is rather difficult. In the present investigation measurements on a fourth order fringe loop gave $K_I = 48.67$ kgf cm$^{-3/2}$ whereas measurements on a 3½ order loop gave a K_I value of 36.06 kgf cm$^{-3/2}$.

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