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Short Communication

Vibrations of orthotropic polygonal plates

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Abstract

Vibrations of clamped edged orthotropic polygonal plates have been investigated following complex variable theory. The frequency equation of different plates is obtained and the numerical results have been presented in tabular form.

Key words: Vibrations, orthotropic material, polygonal plates, irregular-shaped plates.

1. Introduction

Investigations of stresses of irregular-shaped plates are of great interest to design engineers as they frequently occur in modern designs.

If the boundary of a plate is a curve natural to any of the common co-ordinates, the governing equations can be solved in terms of known functions. For more 'exotic' boundaries, the natural co-ordinates must first be determined and, after this is done, the solution would inevitably involve some unfamiliar functions. The determination of natural frequencies in this case will then be very complicated. Therefore, a common co-ordinate system and its associated function are advantageous for plates having complicated boundaries. If the given domain can be conformally mapped on to a simpler one, *i.e.*, the unit circle, the problem then reduces to the solution of the transformed differential system.

The conformal mapping technique has been used by Laura[1,2] Laura and Shahady[3] and Datta[4,5] to elastic stability problems of thin plates with 'exotic' boundaries. Laura and Faulstich[6], Shahady *et al*[7] and Munakata[8] have applied the complex variable theory to the linear vibrations of thin isotropic elastic plates. Laura and co-workers applied conformal mapping technique to get their solutions which are approximate as they were obtained by error minimising method. The deflections of such irregular plates under uniform load have also been studied by Mansfield[9]. It is believed that no work has been reported on the vibrations of orthotropic plates of irregular shape.

The object of the present paper is to study the vibrations of polygonal plates of orthotropic materials. The well known Heaber's approximation[10] has been employed. The frequency equations have been deduced with the help of conformal mapping technique. The solutions thus obtained are given in Table I. The results of square orthotropic plate have been compared with the known results.

The frequencies of different polygonal plates of isotropic materials have also been deduced from the present study and are found to be in excellent agreement with the known results.

Table I

Mapping function co-efficients[3]

$Z = f(\xi) \simeq L\xi$

Polygons of side '2a'	Co-efficient L
Equilateral triangle	1.1353 a
Square	1.08 a
Pentagon	1.0526 a
Hexagon	1.0376 a
Heptagon	1.0279 a
Octagon	1 0219 a
Circle of radius 'a'	a

2. Differential equations and method of solutions

Consider a clamped edge orthotropic plate of thickness 'h'. Following Timoshenko and Woinowsky-Krieger[10] the differential equation for the vibration of an orthotropic plate can be written in rectangular co-ordinates as

$$D_x \frac{\partial^4 W(x,y,t)}{\partial x^4} + 2H \frac{\partial^4 W(x,y,t)}{\partial x^2\, \partial y^2} + D_y \frac{\partial^4 W(x,y,t)}{\partial y^4} + \rho h \frac{\partial^2 W(x,y,t)}{\partial t^2} = 0 \tag{1}$$

where

$$D_x = \frac{E'_x h^3}{12}, \quad H = \frac{E'' h^3}{12} + \frac{G h^3}{6}, \quad D_y = \frac{E'_y h^3}{12}$$

E'_x, E'_y, E'' and G are elastic constants of the material as defined in Timoshenko and Woinowsky-Krieger[10] (pp. 364-365, $W(x, y, t)$ is the deflection of the plate of thickness 'h' and is given by

$$W(x,y,t) = e^{iwt}\, W(x,y) \tag{2}$$

with this form of $W(x,y,t)$, equation (1) reduces to

$$D_x \frac{\partial^4 W}{\partial x^4} + 2H\frac{\partial^4 W}{\partial x^2\,\partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} - \rho h \omega^2 W = 0 \tag{3}$$

Taking $H^2 = D_x D_y$ [Ref. 10. p. 366], equation (3) reduces to

$$16 \frac{\partial^4 W}{\partial z_1^2 \partial \bar{z}_1^2} - \frac{\rho h \omega^2}{D_x} W = 0 \tag{4}$$

with the substitution

$$Z_1 = x + \rho_1 y \tag{5}$$

where ρ_1 is the root of the equation

$D_y \rho_1^4 + 2 H \rho_1^2 + D_x = 0$. Clearly $\rho_1 = i/\beta$ where

$$\beta^2 = \sqrt{D_x}/D_y \tag{6}$$

From equation (4) we have

$$\left(\frac{\partial^2}{\partial z_1\,\partial \bar{z}_1} - \frac{K^2}{4}\right)\left(\frac{\partial^2}{\partial z_1\,\partial \bar{z}_1} + \frac{K^2}{4}\right) W = 0 \tag{7}$$

where $K^4 = \dfrac{\rho h \omega^2}{D_x}$

The solution of equation (7) will be of the form

$$W = A I_0 \left(K \sqrt{z_1 \bar{z}_1} \right) + B J_0 \left(K \sqrt{z_1 \bar{z}_1} \right) \tag{8}$$

where I_0 and J_0 are Bessel functions of zeroth order, A and B are constants to be evaluated from the boundary conditions of clamped edged plates

$$\left.\begin{aligned} W &= 0 \\ \text{and}\quad \partial W/\partial \bar{z} &= 0 \end{aligned}\right\} \text{ on the boundary} \tag{9}$$

Let

$$Z = f(\xi) \simeq L\xi \tag{10}$$

be the mapping function which maps the domain on to a unit circle where $\xi = \gamma e^{i\theta}$

Now with the boundary condition for clamped edged plate as given by (9), one can have, from equation (8) and (10) equating the co-efficients of term independent of ξ on both the sides, the required frequency equation

$$\begin{vmatrix} I_0(K') & J_0(K') \\ I_1(K') & -J_1(K') \end{vmatrix} = 0 \tag{11}$$

where

$$K'^2 = 1/2\ \omega \sqrt{\rho h / D_x}\quad L^2 (1+\beta^2) \tag{12}$$

The solution of equation (11) is $K' = 3.2$. Hence the frequency of a polygonal orthotropic clamped plate is

$$\omega = 2\ (\frac{3.2}{L})^2 \cdot \frac{1}{1+\beta^2}\quad \sqrt{D_x / \rho h} \tag{13}$$

Table II

Fundamental frequency co-efficients for several clamped polygonal plates

Polygon of side '2a'	Isotropic material ($\beta^2 = 1$)		Orthotropic material ($\beta^2 = 1.826$)	
	$w\sqrt{\rho h / D}\, a^2$ Present study	$w\sqrt{\rho h / D}\, a^2$ Known results	$w\sqrt{\rho h / D_x}\, a^2$ Present study	$w\sqrt{\rho h / D_x}\, a^2$ Known results
Equilateral triangle	7.95	8.01 *	5.62	—
Square	8.78	8.85 *	6.22	6.64 ****
Pentagon	9 24	9.32 *	6.54	—
Hexagon	9.51	9.59 *	6.73	—
Heptagon	9.70	9.77**	6.86	—
Octagon	9.81	9.85**	6.94	—
Circle of radius 'a'	10.24	10.22***	7.25	—

* Ref 11, ** Ref. 7, *** Ref. 12, **** Ref. 13.

3. Numerical calculations and discussion

The frequency for free vibrations of orthotropic clamped plates of different shapes are calculated and are given in Table II. The frequency for clamped orthotropic square plate has been compared with the known result. The corresponding frequencies of clamped isotropic polygonal plates have been deduced from the present study and compared with the known results.

From Table II, it is observed that the results thus obtained by the present study using only the first term of conformal transformation series to represent polygons having a small number of sides are in good agreement with the known results from the engineering point of view.

A single equation (13) can be used with a good accuracy for predicting vibration of plates of any shape with less computational labour.

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Refereces

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