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BOOK REVIEWS

Special functions in queuing theory and related stochastic processes by H.M. Srivastava and B.R.K. Kashyap. Academic Press, Orlando, Florida, 1982, pp 308, $ 42.50.

A queuing system consists of one or more 'service stations' where service is rendered to a stream of randomly arriving customers. The mechanism of a queuing system is specified by describing the probabilistic behaviour of the 'input process' representing customers and the 'service mechanism'. Typical specifications include the probability distributions of interarrival times of customers, service times and various independence/conditional independence assumptions. The 'state' of the queuing system at any given time (in the system theoretic sense) is then specified by quantities such as the number of customers in the queue at that time, the amount of time each of them has been waiting, etc. In the simplest prototypes, such as the M/M/1 queue, it is only the former that needs to be specified. The dynamics of the system is then described by the evolution equation for the probability law of the state, a special case of Kolmogorov's forward equation occuring in the theory of Markov processes. For queuing processes, which are discrete-valued in continuous time (of course, queues in discrete time can be and have been studied), these equations are differential-difference equations. A standard approach to handle these is to use transform techniques, which, for the simplified prototypes at least, lead often to closed form solutions expressible in terms of special functions such as the gamma, beta and hypergeometric functions. Classically, these functions were discovered for applications to mathematical physics, but they also turn out to be quite handy in queuing theory for evaluating various parameters of interest of the system, such as mean waiting time, first passage time, etc., which are related to the solution of the above-mentioned evolution equation. The book under review is devoted to this aspect of queuing theory.

The authors strive to give a rather extensive overview of the subject and have done a very good job of it. The book begins with two introductory chapters on the basic concepts of queuing theory and special functions respectively. These are succinctly written and provide the necessary background for what follows. The next chapters proceed to give the details of numerous cases of application of special functions to queuing systems. These have been broadly classified as Poisson queues, queues with variable parameters, queues with Poisson arrivals or service and queues with general arrival and service distributions. Within each chapter, however, it is a compendium of a lot of specific problems which share in common the feature of being amenable to the techniques under study. The last chapter, which contributes the last bit of the title, surveys the application of these techniques to fields other than queuing, such as population genetics, life testing, chance-constrained linear programming, etc. The book concludes with an extensive bibliography.

The authors have succeeded in providing a rather complete 'sourcebook' for the applications of special functions to queuing theory. It is very well-organized, well-written and comprehensive and will no doubt form a valuable component of every queuing theorists's arsenal.

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Classgroups and Hermitian modules by A. Frohlich. Birkhauser Verlag, CH-4010, Basel, Switzerland, 1984, pp. 226, S. Fr. 54.

This book contains an expanded and updated version of a course which the author gave at King's College, London in 1979. It gives a detailed exposition of the research of the author on the Galois module structure of the ring of algebraic integers in tame normal extensions, and its connection with the functional equation of the Artin L-function. It also gives a complete solution of the converse problem of Galois module structure theory: that of expressing the symplectic local and global root numbers and conductors as algebraic invariants. A partial solution of this problem was obtained by the author and, based on his work, a complete solution was given by Ph. Cassou-Nogues and M. Taylor.

Let $\mathcal{O}$ be a Dedekind domain, A a finite dimensional separable algebra over the quotient field F of $\mathcal{O}$, and $\mathscr{A}$ an $\mathcal{O}$-order in A. The clas group $Cl\,[\mathscr{A})$ of $\mathscr{A}$ is then, by definition, the kernel of the rank map $K_0(\mathscr{A}) \to \mathbb{Z}$; where $K_0(\mathscr{A})$ is the Grothendieck group of locally free $\mathscr{A}$-modules. Now we shall describe the original problem of Galois module structure theory for a tame extension N/K of number fields with Galois group Γ. The ring $\mathcal{O}_N$ of integers in N is a locally free module over the group ring $\mathbb{Z}\,\Gamma$. It was conjectured by the author of the book under review, and proved by M. Taylor, that the class $(\mathcal{O}_N)$ in the class group $Cl(\mathbb{Z}\,\Gamma)$ is determined by the values of the Artin root number (the constant in the functional equation) $W(\chi)$ for symplectic characters χ of the Galois group Γ (For this theory see the author's new book: *Galios module structure of algebraic integers,* Ergebnisse der Mathematik (new series, Springer Verlag). The converse *i.e.,* $(\mathcal{O}_N)$ determines the values $W(\chi)$ for symplectic, χ, is false and additional algebraic structure was needed to formulate a meaningful converse problem. The author guessed that this additional structure should come from the trace-form. The conjectural converse in terms of the new concepts of a Hermitian module and its discriminant was then formulated by the author in 1979 and was finally proved by Ph. Cassou-Nogues and M.J. Taylor in 1983. These results are described in the final chapter of the book and the earlier chapters of the book develop the necessary algebraic machinery like the Hom description of classgroups and the notions of generalised Pfaffian. Though a brief description of this theory was given by the author in 1977, the details and full proofs are appearing for the first time in this book.

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Infinite dimensional Lie algebras by V.G. Kac. Birkhauser Verlag, P.O. Box 34, ČH-4010, Basel, Switzerland, 1983, pp. 245, S.Fr. 49.

The book contains an excellent exposition of a class of infinite dimensional Lie algebras called the Kac-Moody Lie algebras. The study of these algebras was started independently by V.G. Kac and R.V. Moody in 1968. The definition of these Lie algebras is a straightforward generalization of the definition of semisimple Lie algebras *via* the Cartan matrix and the Chevalley generators. The theory of Kac-Moody Lie algebras is developing rapidly and has connections with many areas of mathematics and mathematical physics, such as the theory of modular forms and theta functions, Hamiltonian mechanics and quantum field theory.

Let $A=(a_{ij})_{1\le i,j\le n}$ be a generalized Cartan matrix *i.e.* an integral $n\times n$ matrix with $a_{ii}=2$, $a_{ij}\le 0$ and $a_{ij}=0$ implies $a_{ji}=0$. Assume that the matrix A is symmetrizable *i.e.* there exists a diagonal matrix D with positive rational entries such that the matrix DA is symmetric. The associated Kac-Moody Lie algebra $G(A)$ is the complex Lie algebra of $3n$ generators e_i, f_i, h_i $(i=1,\dots,n)$ and the following defining relations:

$$\begin{array}{lll} [h_i,h_j] = 0, & [e_i,f_i] = h_i, & [e_i,f_j] = 0 \text{ if } i\neq j, \\ [h_i,e_j] = a_{ij}e_j & (ad e_i)^{1-a_{ij}} e_j = 0 & \text{if } i\neq j \\ [h_i,f_j] = -a_{ij}f_j & (ad f_i)^{1-a_{ij}} f_j = 0 & \text{if } i\neq j. \end{array}$$

The algebra $G(A)$ is a finite dimensional (semisimple) Lie algebra if and only if the matrix A is positive definite. This was proved in 1966 by Serre.

The first five chapters of the book under review deal with the structure of $G(A)$ for an arbitrary generalized Cartan matrix A. $G(A)$ contains a finite dimensional abelian subalgebra H spanned by the h_i, $i=1,\dots,n$ and one has,

$$G(A) = \dot{+}_{\alpha\in H^*} G_\alpha$$

Where $G_\alpha=\{x\in(A):[h,x]=\alpha(h)x\ h\in H\}$, $G_0=H$ and dim G_α is finite for all $\alpha\in H^*$. Let Δ denote the subset $\{\alpha\in H^*:\ G_\alpha\neq\{0\}\}$. Δ is called the set of roots of $G(A)$. Define functionals $\alpha_i\in H^*$, $i=1,\dots,n$ by $\alpha_i(h_j)=a_{ij}$. Any element α of Δ can be written as an integral linear combination of the α_i, $i=1,\dots,n$, say $\alpha=\sum_{i=1}^{n} n_i\alpha_i$, where the (n_i) are either all non-negative or all non-positive. The Weyl group W of $G(A)$ is defined as the subgroup of Aut H^* generated by the reflections $r_1: H^\circ\to H^*$ defined by $r_i(\lambda)=\lambda-\lambda(h_i)\alpha_i$. W leaves Δ invariant. If $G(A)$ is infinite dimensional W is an infinite Coxeter group, and there exist roots $\alpha\in\Delta$ such that α does not belong to the orbit $W\alpha_i$ for any $i=1,\dots,n$.

Let $\Lambda=(\lambda_1,\dots,\lambda_n)$ be any n-tuple of non-negative integers. An integrable representation of $G(A)$ is an irreducible representation π_Λ of $G(A)$ in a complex vector space $L(\Lambda)$ determined by the property that there exists $0\neq v\in L(\Lambda)$ such that

$$\pi_\Lambda(e_i)v=0 \text{ and } \pi_\Lambda(h_i)v=\lambda_i v\ (i=1,\dots,n).$$

(If $G(A)$ is a finite dimensional simple Lie algebra then π_Λ is an irreducible finite dimensional representation of $G(A)$. Conversely any irreducible finite dimensional representation of $G(A)$ is equivalent to π_Λ for some Λ (Cartan)). A formula generalizing Weyl's character formula is obtained for the formal character of the representations π_Λ. For $\Lambda = 0$, the formula is a generalization of the Weyl denominator identity. These results may be found in chapters 9 and 10.

An important subclass of the Kac-Moody Lie algebras are the affine Lie algebras. The matrix A is called an affine Cartan matrix if there exists an n- tuple δ $(r_1,...,r_n)$ of positive integers such that $A\delta = 0$. The corresponding Lie algebra is called an affine Lie algebra. The affine Lie algebras possess an explicit realization [Chapters 6-8]. In the simplest case $G(A)$ is obtained as a one dimensional central extension of the Lie algebras of polynomial maps from the circle into a simple finite dimensional Lie algebra. The generalized Weyl denominator identity is equivalent to the famous MacDonald identities. The representation theory of the affine Lie algebras has connections with the theory of modular forms and Korteweg de Vries type equations. These aspects are discussed in chapters 11 and 14.

The book is very well organized and is essentially self-contained. Although the treatment in the main text is very abstract the exercises contain a large number of examples. The exercises also highlight the basic differences between the infinite dimensional Lie algebras and the semisimple Lie algebras.

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***CR* submanifolds of Kaehlerian and Sasakian manifolds** by Kentaro Yano and Masahiro Kon (Progress in Mathematics, Vol. 30). Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1983, pp. 208, S. Fr. 44.

The notion of CR submanifold is a generalization of complex analytic and totally real submanifolds and is defined as follows: Let M be a Kaehlerian manifold with almost complex structure J. A submanifold N of M is called a CR submanifold if there is a differentiable distribution $D: x \to D_x$ of N such that

i) D is a holomorphic distribution, that is, $JD_x = D_x$ for each x in N and

ii) the complementary orthogonal distribution $D^\perp$ of D is a totally real distribution, that is $JD_x \subset T_x^\perp N$ for each x in N, where $T_x^\perp N$ is normal space to N at x.

It is clear that a CR submanifold reduces to a complex manifold if $D = TN$ and a totally real submanifold if $D^\perp = TN$.

A study of differential geometry of CR submanifolds of a Kaehlerian manifold was initiated by Bejancu[1,2] in 1978. Since then many papers on the subject have appeared.

Sasakian manifold is an odd-dimensional analogue of Kaehlerian manifold and that these manifolds are known to admit many properties similar to those of Kaehlerian manifolds.

Corresponding to the notion of CR submanifolds of a Kaehlerian manifold, one has contact CR submanifolds of Sasakian manifold. There exist some natural relations between submanifolds of a Sasakian manifold and those of a Kaehlerian manifold *via* Boothby-Wang fibration.

The authors claim that the purpose of the book under review is to gather and arrange the results on CR submanifolds of Kaehlerian and Sasakian manifolds obtained up to 1983. Despite their claim, the monograph reads more like a compendium of results obtained primarily by the authors and their collaborators. It fails to include many other results in this area. For instance, an interesting paper on 'The cohomology of CR submanifolds' by Chen[3] has not been mentioned. Chen proves that if N is a closed submanifold of a Kaehlerian manifold M and the De Rham group $H^{2k}(N, R)=0$ for some $k \leq \dim D$, then either D is not integrable or $D^{\perp}$ is not minimal. Alas, this is not the only omission in the book. Many results, mostly concerning integrability of certain distributions[5,6], have been omitted from the theory of generic submanifolds of Kaehlerian manifolds developed by Chen[3]. These omissions seem all the more significant in a book which purports to give its readers an updated account of the progress made in this area.

The prerequisites for reading of these notes are nothing more than a rudimentary knowledge of Riemannian geometry as well as some elementary facts about the geometry of Kaehler manifolds. Since the original material is widely scattered, the reader who is curious about the sort of the geometric structure discussed in this book and is interested in pursuing this line of research finds it helpful to have all the results assembled in one place.

Most of the background material that is needed to study the book is contained in the first two chapters. More specifically, chapter one is rather a compilation of the basic concepts, definitions and formulas in the theory of Riemannian, Kaehlerian and Sasakian manifolds. Some general results on the so-called f-structure of K-Yano are also given. Chapter two deals with the study of submanifolds of Riemannian space forms, especially those of spheres. First some general formulas on submanifolds have been stated and then various theorems have been proved under some restrictions on submanifolds such as submanifolds are minimal, have parallel mean curvature vector, the normal connection is flat, etc.

Chapter three is devoted to the study of contact CR submanifolds of Sasakian manifolds. In particular, minimal contact CR submanifolds and contact CR submanifolds with the parallel mean curvature vector are discussed.

Chapter four is concerned with a detailed study of CR submanifolds of Kaehlerian manifolds. First some general integrability theorems are given and then CR submanifolds with semi-flat normal connection are studied. Finally, minimal CR submanifolds with parallel mean curvature vector are discussed.

Chapter five describes briefly the relationship between submanifolds of Sasakian manifolds and those of Kaehlerian manifolds by exploiting Boothby-Wang fibration. The sixth and the final chapter is concerned with the real hypersurfaces of complex space forms. Fundamental formulas and results on real hypersurfaces are stated and a theorem on Pseudo-Einstein real hypersurfaces is proved. After establishing some results on minimal submanifolds of complex projective spaces, the chapter ends with characterization of certain kinds of real hyper surfaces by the restriction on Ricci curvature or sectional curvature.

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Large deviation and the Malliavin calculus by Jean Michel Bismut. Birkhauser Verlag, CH-4010, Basel, Switzerland, 1984, pp. 216, S. Fr. 49.

The book under review is more of a long research paper. It presents some original work of the author on small time ($t \downarrow\downarrow 0$) asymptotic expansions for the conditional probability density of a diffusion process on a Riemannian manifold, along with the rederivation of some results by earlier workers (Azencott, Molchanov) using different techniques. The main tool used is the stochastic calculus of variations developed by Malliavin. (See *e.g.*, *Lectures on stochastic differential equations and Malliavin calculus* by S. Watanabe, T.I.F.R. Notes No. 73, 1984).

The monograph consists of five chapters, preceded by an introduction. The latter summarizes the corresponding problem in the finite dimensional setting, thus motivating and giving a nice perspective for the results to come for diffusion processes (a summary of which is also included). The first chapter develops a deterministic version of the Malliavin calculus, in particular the conditions for invertibility of the 'Malliavin covariance matrix', and derives for later use a direct sum decomposition of the Hilbert space of continuous maps on [0,1] with square-integrable derivatives. (This space, endowed with the finitely additive Gauss measure, is more basic for stochastic analysis than the Wiener space obtained by completing it w.r.t. a measurable norm, *viz.* the sup. norm, and 'lifting' the Gauss measure to this completion to obtain Wiener measure. The reason is that the latter choice of measurable norm is not unique). The second chapter starts with the study of the Brownian motion on a Riemannian manifold along the lines of Malliavin and Eells-Elworthy. This is done intrinsically by lifting the Brownian motion to the bundle of orthonormal frames on the given manifold, whence it is given by a globally defined diffusion process. Malliavin calculus of variations is then developed for diffusions on Riemannian manifold using the probabilistic approach developed by the author himself (See *e.g.*, the expository article by Veretennikov, *Russ. Math. Surv*, 1983, **38**(3)). Using this, the conditional law of the diffusion conditioned on a fixed terminal state is studied and the semimartingale property of the conditional diffusion established. The next chapter, which contains some of the most complicated analysis in this book, continues with conditional diffusions and their associated flows, leading to a 'large deviations' result for the same. More precisely, one has an estimate for the probability of the given flow differing from a certain deterministic flow by more than a given quantity (in a certain metric) for small time. This result is the key step in adapting Laplace's method in the finite dimensional case to the present problem. (This point has been driven home rather well in the introduction). The next (fourth) chapter uses it to give an asymptotic expansion for the conditional law for small time. All this development uses intrinsic methods and assumes ellipticity of the second-order operator (the extended generator) associated with the given diffusion. Malliavin's calculus was developed for the more general 'hypoelliptic' case. The author has two unproved conjectures for the hypoelliptic case a discussion of which forms the content of the last chapter.

In summary, this book is a research monograph and as such, is accessible only to specialists. It is tersely written, but still does a very good job of motivating the main results and projecting them in relation with the corresponding known results in simpler situations. Extensive references are given and often the proofs bank upon earlier works of the author and others for details. It is certainly a difficult book, containing some very hard and very good mathematics, and will have its place as a significant work in diffusion theory.

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Least absolute deviations by P. Bloomfield and W.L. Steiger, (Vol. 6 in the series progress in Probability and Statistics). Birkhauser Verlag, CH-4010, Basel, Switzerland, 1983, pp. xiv + 349, S. Fr. 64.

This book is possibly one of the very first comprehensive texts on the approximation in the L_1-sense, and is hence a valuable contribution to the mathematical literature.

The contents offer a rich panorama of a wide range of results with illuminating comments on their limitations and range of applicability. More importantly, many open problems are posed to challenge the serious research worker.

In the brief historical survey dating back to the 18th century, we learn that R.J. Boscovitch was perhaps the first to deal with the problem of finding a line that minimizes the absolute deviation among all lines constrained to pass through the mean of the data on a plane. This is distinct from the method of least squares which is well known for fitting line models.

The subject matter is a unified treatment of the role of Least Absolute Deviation (LAD) techniques, the so-called discrete L_1 analog of least squares, in several domains, neatly organized into three parts: theory, applications and algorithms. Each chapter ends with note containing a review of the literature. Theorems are made transparent to the reader by explaining the significance of the assumptions made and of the results derived.

Chapter 2, dealing with linear regression, contains: (1) a theorem on the limiting error distribution for LAD; (2) an analysis of the robust properties for the regression estimator;

and (3) a comparison with the least squares (and other so-called Huber M-estimators) on the basis of a Monte-Carlo experiment which is well detailed.

Chapter 3 is devoted to stationary k^{th} order linear time series. An expression is derived for the rate of convergence of the LAD estimator to the autoregressive parameters. As in chapter 2, Monte-Carlo experiments are given for comparing LAD to least squares with respect to autoregressions. The conclusion is that the LAD estimators are more efficient than the other estimators.

In chapter 4, applications like additive models for two-way tables and properties (like convergence) of Tukey's median polish techniques are considered.

Chapter 5 discusses the interpretation of the LAD regression as an estimate of the conditional mean of y given x. The requirement that the conditional mean be a linear function of x is then weakened to the requirement that it merely be a smooth function. This leads to the introduction of cubic splines as estimates of the conditional median. The quantile splines are illustrated by examples.

Chapter 6 considers the relationship between linear programming (LP) and LAD fits. The most interesting part of this chapter is section 2, where the result is Theorem 1: Any LAD curve-fit may be expressed as an equivalent bounded feasible LP problem, and conversely. In section 3, some complexity questions are addressed. One of the conclusions is that it is known if there exist exponential problems for any 'reasonable' LAD algorithm or in fact what the worst case behaviour might be.

The concluding chapter (Chapter 7) presents some recent work on exact, finite algorithms for robust estimation. In fact, most LAD algorithms are actually variants of the simplex method on a certain LP problem. Three best LAD algorithms are described, compared and studied for complexity. These are due to Barrodale-Roberts, Bartles-Coun-Sinclair, and Bloomfield-Steiger. Differences in performance are related to features of these algorithms.

Appendix (Chapter 8) contains results of the Monte-Carlo study.

Except for the numbering of sections and equations which require some effort while back-referencing, the book is highly readable. As a whole, the book deserves to be recommended for advanced study in the theory of approximations.

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Advances in Hamiltonian systems edited by J.P. Aubin, A. Ben Soussan and I. Ekeland. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1983, pp. 195, S. Fr. 46.

Dynamical systems describing real physical processes are, as a rule, Hamiltonian in one sense or another, if energy dissipation can be disregarded. The integration of these systems (*i.e.*, finding the solutions) is a nontrivial problem. In the past only those problems that could be solved by means of finitely many algebraic operations and 'quadratures', the computation of integrals of known functions, were regarded as 'soluble' (integrable). However, only in the simplest cases when the system had just one degree of freedom, or could be decomposed into several independent one-dimensional systems, did the integration turn out to be possible, due to the existence of integrals of motion like the energy. For this reason very soon attention was focussed on the qualitative investigation of the motion of Hamiltonian systems. This investigation is usually done in the neighbourhood of a particular solution.

A rigorous investigation of the question of integrability of Hamiltonian systems close to stable positions of equilibrium is due to C. Siegel in a series of papers from 1941 to 1954. After equilibrium points (stationary points), periodic solutions are the simplest objects of study in the qualitative theory of dynamical systems. Nevertheless, even the problem of the existence of periodic solutions is often highly nontrivial. In this context the use of topological methods has been very fruitful in recent years.

It is an amazing and beautiful discovery of the past two decades that Hamiltonian systems are the arena for an interplay of methods from ordinary differential equations, non-linear functional analysis, Morse theory, algebraic geometry, differential geometry and topology. One of the most celebrated results is the so-called KAM (Kolmogorov-Arnold-Moser) theorem on the persistence of conditionally periodic solutions (invariant tori) under small perturbations of the Hamiltonian of a completely integrable system.

Another recent breakthrough is due to A. Weinstein and J. Moser on the existence of periodic solutions and an estimate of their number, ingeniously improving on a classical result of Lyapunov.

The book under review contains papers dealing with the study of the existence of periodic orbits for Hamiltonian systems. The collection consists of seven papers from a conference held at the University of Rome in February 1981. The results presented here are in the realm of the so-called Morse and Lyusternik-Shnirelman theory which combines the calculus of variations with the topology of function spaces.

The first paper by A. Ambrosetti is a survey of recent results in the study of the existence of periodic orbits for Hamiltonian systems. It describes the so-called dual variational method. The following paper by V. Benci is concerned with the existence of periodic solutions when the period is assigned. The third paper by G. Mancini deals with minimal periods, *i.e.* if a T-periodic solution has been found, it investigates whether T is the 'true' period or some T/n, n integer, is the minimal one. Mancini surveys known results trying to answer this difficult question. The fourth paper by I. Ekeland and J.M. Lasry presents an abstract formulation of the dual action principle covering previously known results. The next two papers (in French), by P. Bernhard and J. Blot respectively, deal with Hamiltonian systems from the viewpoint of optimal control theory, and perturbation of Hamiltonian systems using the dual variational method. The last paper by E. Gaussens deals with numerical searches for periodic solutions based on known methods.

The modern language of symplectic manifolds or Poisson manifolds and all that goes along with them is not used. Nevertheless it is a useful collection of papers for those interested in the fascinating world of Hamiltonian dynamics.

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Convexity and its applications edited by Peter M. Gruber and Jorg M. Wills. Birkhauser-Verlag, Basel, 1983, pp. 421, S. Fr. 110.

The book contains sixteen survey articles on convexity. Many of them are extensions of papers presented at the conferences on convexity at the Technische Universität Wien (July 1981) and at the Universität Siegen (July 1982), while some of them have been written specially for the volume at the invitation of the editors. The articles are well written and give an excellent account of many important aspects of convexity.

The first paper by A. Bachem on 'Convexity and optimization in discrete structures' illuminates the importance of convexity for optimization. It discusses the trends in combinatorial optimization with special emphasis on matroid and oriented-matroid theory. The paper by C. Bandle on 'Isoperimetric inequalities' gives the proof of the classical inequality $L^2 \geq 4\pi A$ with the discussion of possible extensions of this inequality for surfaces. The paper by G.D. Chakerian and H. Groemer gives a detailed account of convex bodies of constant width and related concepts and the one by J.H.H. Chalk on 'Algebraic lattices' highlights the geometry of numbers.

In 1849, Arthur Cayley, in a letter to George Salmon, conjectured that a cubic surface would contain only a finite number of lines. Salmon replied that this number is 27. H.S.M. Coxeter traces the history of this problem and sketches some more recent developments.

W. Fenchel takes the reader on a hurried trip of history of convexity. Starting with the definition of a convex arc by Archimedes, the author gives a rapid survey of all important developments in the area of convexity. P.M. Gruber discusses the approximation of convex bodies by polytopes and by special convex bodies such as a simplices and balls. K. Leichtweiss throws light on those results which have both convexity version as well as differential geometry version. The paper by P. McMullen and R. Schneider deals with the investigation of those functions on convex bodies which are valuations.

In a paper on 'Minimal and closest points, nonexpansive and quasi-nonexpansive retractions in real Banach spaces', P.L. Papini discusses the concepts indicated in the title and gives characterizations of Hilbert spaces among real Banach spaces in terms of these concepts.

Ellipsoids, which are the affine transforms of the Euclidean balls, have interesting properties. C.M. Petty gives certain characterizations of ellipsoids and illustrates where they occur. R.R. Phelps surveys in his paper some recent results on the extremal structure of bounded closed convex subsets of Banach spaces.

The paper by R. Schneider and W. Weil deals with zonoids, *i.e.*, those convex bodies in Euclidean space which can be approximated, in the sense of the Hausdorff metric, by finite vector sums of line segments. Th author discusses several equivalent definitions of a zonoid and relates questions on certain topics such as ranges of vector measures, the isometric embedding of Banach spaces in L_1 ($[0,1]$), etc., to the questions on zonoids. The paper by G. Fejes Tóth surveys recent results on packagings and coverings, while the paper by W. Weil surveys some results in stereology, the discipline dealing with problems of determining the characteristic geometric properties of (usually 3-dimensional) objects by investigations of sections, projections and other transformed images.

The last paper in the volume is on 'Semi-platonic manifolds' by J.M. Wills. A semi-platonic manifold is a polyhedral 2-manifold with certain additional algebraic, geometrical and topological properties, closely related to those of five platonic solids. The short paper gives a few results and beautiful diagrams of 16 of the known 19 semi-platonic manifolds.

Each paper is followed by a long list of references. The book will provide a wealth of information to a generalist and will serve as a very useful reference work for a specialist.

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Systems of micro-differential equations by Masaki Kashiwara. Birkhauser Verlag AG, Ringstrasse 39, CH-4016, Therwil, Switzerland, 1983, pp. 159, S. Fr. 42.

Some fifty years ago Dirac obtained the following remarkable equation

$$\frac{1}{x+i0} = vp\ \frac{1}{y} - i\pi\delta(x)$$

which leads to the concept of the Dirac measure being expressed as the difference of boundary values

$$\frac{1}{x+i0} - \frac{1}{x-i0} = -2\pi i\delta(x)$$

Here $\frac{1}{x+i\epsilon}$, $\epsilon \neq 0$ is analytic in the complement of the real axis and the limits are taken from above and below. More generally, Sato[1] showed for the first time that any distribution in an openset V of $\mathbb{R}$ may be realized as the difference of boundary values of analytic functions defined outside of V. Painleve's result says that such a representation is unique modulo analytic functions. These concepts have natural extensions to open sets in $\mathbb{R}^n$ (*via* relative cohomology for open pairs) and one arrives at the category of *hyperfunctions* of Sato, which are more general objects than distributions.

Hyperfunctions enjoy remarkable properties such as they can be extended or 'prolonged' to larger domains (they form a flabby sheaf) and the prolongation works often even for hyperfunction solutions of partial differential equations.

In 1969, Sato[2] constructed a sheaf C over the conormal sphere bundle of a real analytic manifold M, whose sections are called *microfunctions*. Every hyperfunction on M induces a microfunction whose support is, by definition, the *singular support* of the hyperfunction. This concept has turned out to be a very powerful tool using which regularity of hyperfunction solutions may be adequately studied. In fact, the parallel concept of the C^∞ wave front set of Hormander, which plays a crucial role in the C^∞ regularity theory, has largely been attributed to Sato's ideas.

The theory of hyperfunctions of Sato has deep links with questions on holomorphic continuation in several complex variables. Indeed, Martinéau's[3] 'cohomology calculation proof' of the so-called 'Edge of the wedge theorem' reconstructs Sato's theory and conversely, the theorem itself follows directly from Sato's theory.

The theory of hyperfunctions, despite its awesome technical power in handling partial differential equations and other questions of importance in several complex variable theory and physics, is only slowly beginning to gain popularity in recent years. Indeed, it can be safely said that the theory is actively pursued only in Japan to a great extent and in France to a lesser extent. The reasons for this can be many; the principal one, perhaps being that enormous background of algebraic and analytic machinery is needed for an appreciation of the theory.

At the outset it must be pointed out that the book under review is highly inaccessible to any one not already sufficiently exposed to Sato's theory. The author, Masaki Kashiwara, one of the giants in the field, writes (or lectures!) in merciless abstractness and any feeling of what goes on can (perhaps) only be gained from the introduction by Jean-Luc Brylinski.

The book is intended to be a systematic development of the interaction of hyperfunction theory with micro-differential operators which are 'analytic versions' of pseudo-differential operators. Here one studies micro-differential equations in terms of micro functions and symplectic machinery on the conormal sphere bundle. This constitutes *microlocal analysis* in the 'analytic set up'.

The book deals with all the basic questions for micro-differential systems: microlocal invertibility, preparation theorem (without proof) of Weierstrass type, invariance of the ring of micro-differential operators under coordinate and canonical transformations. It is remarkable that quantized contact transformations give isomorphic copies of the ring corresponding to canonical transformations. This situation must be compared with Egorov's theorem (or Fourier conjugation) in the C^∞ theory. Purely algebraic versions of the classical Cauchy-Kovalevska theorem is also given. These results together with some algebraic structure theorems for modules (holonomic systems included) occupy the first three chapters. The next chapter deals with the beautiful theory of prolongation and propagation of holomorphic solutions. These results are applied in chapter 5 to study the local behaviour of hyperfunction solutions of holonomic systems. Chapter 6 gives an index theorem for holonomic systems.

The book is written in the style of a research monograph and can only be recommended to practicing hyperfunction theorists. It appears that Harvey's thesis[4] is the first of its kind to explain some of Sato's ideas in a systematic manner, followed perhaps by Schapira's Springer Notes[5]. These together with the conference lectures in Springer, edited by Komatsu[6], should form background material for a meaningful reading of Kashiwara's book.

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Workshop on nonperturbative quantum chromodynamics edited by K.A. Milton and M.A. Samuel. Birkhauser Verlag AG, Ringstrasse 39, CH-4106, Therwil, Switzerland, 1983, pp. 265, $ 25.88.

This volume is the eighth in the series titled Progress in Physics edited by A. Jaffe *et al.* It represents the proceedings of a workshop held at Stillwater, Oklahoma in March 1983. It includes talks by several prominent workers in the broad area of nonperturbative QCD and is a collection of several papers, giving the general gist of early developments. It starts with problems at the interface between perturbative and nonperturbative QCD by S. Brodsky and moves on to several phenomenological applications for low energy domain using MIT bag model and nonlinear chiral models, etc. There are several useful discussions on how one might extract (nonperturbative) effects buried in QCD using various approximations and gives a summary of the essential successes in the use of the lattice version.

There is an inherent difficulty in bringing out proceedings of the workshop, such as the volume under review; it is inevitable that by the time the book appears in the market, several new advances would have occurred and the volume will therefore appear incomplete. For example, the topological sector of the chiral model, which incorporates the anomalies of QCD has provided a rich structure. There are several exotic phenomena associated with the monopole solutions unavoidable when the Yang-Mills symmetry is broken. It is natural neither of these two topics find a place in the proceedings. However, it is surprising that there is no work related to Q-vacua of QCD represented at the workshop. The publishers announce that 'this compendium' will 'serve as a source for futher study and as a resource book on this inviting yet infant area of particle research'. This reviewer is inclined to agree.

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Observing visual double stars by Paul Couteau, translated by Alan H. Batten. The MIT Press, 28, Carleton Street, Cambridge, Mass. 02142, USA, 1982, pp. 257, $ 8.95.

This book is divided into eight chapters beginning with a historical introduction and ending with a catalogue of 744 double stars. In his foreword, Professor Jean Claude Pecker gives a brief sketch of the life of the author which speaks of the dedication that one would need in order to make a lasting contribution to the field of astronomy.

The book provides an excellent introduction to the subject of visual double stars. Paul Couteau, in his endeavour to lure some observers into this thinning field of double star observations has provided in this book all the necessary information required for a beginner. Chapter two describes the optical concepts essential for any observer of double stars. Chapter three deals with the basic measuring instruments like the filar micrometer and its various modifications. The author also briefly describes the modern techniques like the Labeyrie's speckle interferometry. In cha, er four the author gives some practical advice to the newcomers to this field. The problem of" ' 'ntification and a good introduction to the available catalogues are dealt with in chapter five. The various methods of orbital solution and the derivation of stellar masses are given in chapter six.

In chapter seven, titled 'Voyage to the country of double stars', the author ends with an appeal to the monasteries. I quote below the author to emphasize his feelings towards this neglected field: "An observatory oriented towards the study of double stars can easily abandon this kind of research and turn to some other speciality just because of the personality of an astronomer. If only one could entrust this work to certain observatories by special regulations". As the author points out, astronomy is typically a monastic activity. If some take up the work of observing visual doubles just for the love of observing, it would definitely provide them food for meditation and strengthen their spirituality. Of course, we would have masses for some more of these binaries determined.

I strongly recommend this book for all libraries and all those amateurs who love observing without much-ado about doing advanced scientific research.

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Addendum

The following references are to be read along with Dr. Rahul Pandit's review of the book *Scaling and self-similarity in physics* which appeared in our February 1984 issue (Vol. 65 (B), pp. 62-63).

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