

## BOOK REVIEWS

**Concise science dictionary.** Oxford University Press, Oxford House, 219, Anna Salai, Post Box 1079, Madras 600 006, 1985, pp. 752, Rs. 190.

This dictionary, one might even say mini encyclopaedia, of science is aimed at the young scientist in his last years of schooling or first year at University. The coverage is catholic and up-to-date: it even includes "megaton weapons" and "brain death". It is refreshingly free from pedantry and its type size and spacing give an open look to the page that positively invites browsing. The level of treatment is consciously kept to that of the audience aimed at, which together with its attractive layout make it also ideal for the inquiring layman who seeks definitions and explanations for the scientific terms and concepts he is increasingly exposed to.

The explanations and definitions are brief and clear and sometimes models of pedagogic exposition. The two-hundred word note on entropy should convey even to the non-specialist some idea of this important concept. The entry on centripetal force can hardly be bettered, though the limitation of the treatment and consequent avoidance of non-inertial frames of reference lead to a nearly summary dismissal of centrifugal force. Not all entries are equally successful, the definition of Reynolds number for instance hardly attempts to convey its meaning or significance. But one should not carp on minor defects in what is really an excellent and enjoyable presentation. One hopes to see an Indian edition at a price that would enable it to be on the shelves of every aspiring young student.

S.S.

**Proceedings of the 31st International Technical Communication Conference** (April 29 -May 2, 1984, Seattle, Washington). Society for Technical Communication, Seattle, Washington, 1985, pp. 576, \$ 45. Distributed by Univelt, Inc., P.O. Box 28130, San Diego, CA 92128.

When a new publication on technical communication joins the massive edifice of books already available on the subject, it is not easy to see where the newcomer fits in and how it is to be used. What makes the Proceedings of the 31st ITCC different from its predecessors is that it is not just a record of a an event but a reference document of considerable educational value to professional communicators. This excellently-produced account does not merely provide answers to questions that technical communicators confront daily; it also offers current perspective on the theoretical, empirical and pedagogical aspects of technical communication.

The 576-page document, a marvel of organisation, vividly reflects the structure of the Conference and preserves its thematic coherence by separating its papers into five specialised stems. Since I cannot hope to do a full review (within a 500-word limit!) I shall briefly

mention the focus of each stem and comment very selectively on some of the papers that engaged my interest.

The Advanced technology applications stem (43 presentations) covers the training, evaluation and application dimensions of word-processing equipment and explores publication data bases, electronic and on-line documentation, document control, graphic support and integrated publications. Papers examining the considerations for selecting a word-processing system are very informative and timely.

The Management and professional development stem (36 presentations) richly recovers the papers on the management of personnel and financial resources in technical communication. Several success stories provide the context for descriptions of advances, trends and tools in communications management. *Hiring a technical editor* offers sound guidelines for recruitment and *Managerial communication in college* uses entrepreneurial group projects to develop communication skills in trainee managers. Two workshop outlines on *The older worker and new technology* and *Making meetings work* captured my interest, and the non-inclusion of more details was frustrating.

In the Research, education and training stem (41 papers), the ones on the relation between authors and editors, readability analysed from a linguistic perspective, format preferences, computerised instructional models and student and course evaluation are valuable. The reports of workshops on proposal writing, oral presentation, and in-house writing improvement programmes, are useful inclusions.

Today's technology demands skill in multichannelled communication and the Visual communication stem (20 papers) explores exciting trends in visual media such as films, slides, video and computer graphics.

The Writing and editing stem (73 papers) is the most substantial section of the book and reaffirms that writing is still the most central preoccupation of the scientific and technical community. The implicit view underlying the papers describing academic writing programmes in American universities (*Sensitizing researchers to publications production*, *Practical writing and editing techniques* and *The scientists guide to obfuscation!*) is that poor communication in science and technology is the result not of poor command of the language, but of inadequate understanding of the communication task. This is a perspective sorely needed in the training of our own researchers in India.

One last word — as a teacher I find the Proceedings a rich source book of ideas for use in the classroom, to enhance students' understanding of the conventions of technical communication and to refine their own strategies and skills.

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**Higher mathematics from an elementary point of view** by H. Rademacher, edited by D. Goldfeld. Birkhauser Verlag, CH-4010, Basel, Switzerland, 1982, pp. 138, S. Fr. 58.

Based on Professor Rademacher's lectures in 1947 at Stanford University, this pleasing account of diverse topics in Elementary Number Theory could fortunately be brought out

recently, thanks to the laudable efforts of Birkhauser. Covering a wide spectrum of results concerning prime numbers, Farey fractions, decimal expansions, rational approximation to real numbers, the exclusion-inclusion principle, Ford circles, the modular group and modular functions and finally linkages, this nice book with the remarkable clarity so characteristic of Professor Rademacher's exposition is bound to leave a lasting impression even on those with no prior contact with the profound yet simple and marvellous subject of Number Theory. The brief but beautiful notes appended by Professor Goldfeld to the various chapters suffice to put the contents in perspective *via* references to more recent developments.

Beginning with Euclid's proposition that "there can be no last prime numbers", chapter 1 provides a rapid survey of the advances (until 1947) in the theory of prime numbers including Dirichlet's theorem on primes in arithmetic progressions, the Prime Number Theorem and Vinogradov's result on all large odd (natural) numbers being expressible as sums of three (odd) primes. A thoughtful note to chapter 1 refers to further developments such as in Sieve Theory, the theorem of Chen that every sufficiently large even number is the sum of a prime and a number with at most two factors and the result of Montgomery and Iwaniec-Jutila on the existence of primes in intervals of the form  $(x, x+x^\alpha)$  for a fixed exponent  $\alpha$  and all large  $x$ . The next three chapters cover the fundamental theorem of arithmetic (on unique factorization into primes), Farey's ordering of fractions and the 'law' behind decimal expansions of common fractions. A recent remarkable application of the Fermat theorem (on the order of prime residue classes modulo an integer) to the construction of codes is neatly explained in a note to chapter 5.

The exclusion-inclusion principle is presented in chapter 6 and the multiplicativity of Euler's  $\phi$ -function derived. In the next two chapters, the reader is treated to a beautiful survey on approximations to irrational numbers by rational numbers including the theorems of Dirichlet, Liouville, Thue and Siegel, solution of Pell's equation, Ford's representation of fractions and finally Hurwitz's theorem. "Not only do algebraic numbers resist (rational) approximation to any order higher than their degree" but, by a remarkable theorem of Roth (as referred to in a note to chapter 7), "no irrational algebraic number is (rationally) approximable to any order greater than 2". This reference fortunately offsets the sentence (on page 55): "However the cubic algebraic numbers may permit an approximation of order  $1/n^3$ ".

Chapters 9 and 10 contain a discussion of the structure of 'linear transformations' of the complex plane, a clear insight into the action of the modular group  $\Gamma$  on the upper half-plane  $H$  (with a proof of the 'discreteness' of  $\Gamma$ ), the construction of a 'fundamental region' for  $\Gamma$  in  $H$  and finally the introduction of the elliptic modular invariant *via* Eisenstein series. The concluding chapter gives an account of what may appear far removed from Number Theory, *viz.*, linkages (mechanisms consisting of rods joined together by hinges enabling the system as a whole to move freely in a plane). Tchebycheff was one of the eminent mathematicians of the last century to attack the fascinating problem of producing straight line motion through a linkage. "As with all first-rate mathematicians, his unsuccessful efforts to solve this problem led to first-rate mathematics" and indeed to the discovery of Tchebycheff polynomials which are polynomial approximations to a straight line. The linkage problem itself was solved by Lipkin, a student of his and more solutions flowed in later!

Those to whom the *Enjoyment of mathematics* by Rademacher and Toeplitz provided immense delight (and also others who may not have heard of that well-known book) are sure to like this pretty 'companion volume' as well.

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**Ergodic theory and semisimple groups** by R.J. Zimmer. Birkhauser Verlag, CH-4010, Basel, Switzerland, 1984, pp. 210, S. Fr. 78.

A class of discrete subgroups of Lie groups, called lattices, play an important role in various branches of mathematics including number theory, geometry, dynamics, ergodic theory, etc. We recall that a discrete subgroup of a Lie group  $G$  is called a lattice if the homogeneous space  $G/\Gamma$  admits a finite Borel measure invariant under the natural action of  $G$  (on the left). For instance, if  $G = \mathbb{R}^n = \{(t_1, \dots, t_n) \mid t_j \in \mathbb{R}, j = 1, \dots, n\}$  then  $\mathbb{Z}\mathbb{Z} = \{(i_1, \dots, i_n) \mid i_j \text{ integers}\}$  is a lattice in  $\mathbb{R}^n$ . More generally, it is well-known that any lattice in  $\mathbb{R}^n$  consists of all integral combinations of  $n$  linearly independent vectors in  $\mathbb{R}^n$ . A similar description of all lattices was given by Malcev for lattices in nilpotent Lie groups; more generally, lattices in solvable groups are also understood quite satisfactorily, thanks to the work of L. Auslander, G.D. Mostow and other authors. For an account of this and various generalities on lattices the reader may refer M.S. Raghunathan's book<sup>1</sup>.

Lattices also arise naturally in various semisimple Lie groups: For instance,  $SL(n, \mathbb{Z})$ , the group of integral unimodular matrices, is a lattice in  $SL(n, \mathbb{R})$ , the special linear group;  $Sp(n, \mathbb{Z})$ , the group of integral symplectic matrices is a lattice in  $Sp(n, \mathbb{R})$ , the group of all symplectic matrices; the special orthogonal group  $SO(Q)$  of a nondegenerate quadratic form  $Q$  with rational coefficients, contains the subgroup of its integral elements as a lattice, etc. More generally, by a theorem of Borel and Harish-Chandra, if  $G$  is the group of real elements of a semisimple algebraic group defined over rationals, then  $G_{\mathbb{Z}}$ , the group of integral elements in  $G$ , is a lattice in  $G$ . On the other hand, any Lie group is essentially a semidirect product of a solvable and a semisimple Lie group and the study of lattices can, to a large extent, be reduced to the two cases separately. Thus the thrust of the study of lattices is reduced to those in semisimple Lie groups.

In the recent years the theory made remarkable strides primarily at the hands of G.A. Margulis. It is obvious that any classification of lattices in semisimple Lie groups must account, apart from the lattices  $G_{\mathbb{Z}}$ , as above, for lattices arising from the modifications suggested by the following observations: a) if  $\Gamma$  is a lattice in  $G$  and  $\Delta$  is a subgroup such that  $\Gamma \cap \Delta$  is of finite index in both  $\Gamma$  and  $\Delta$  (we then say  $\Delta$  is commensurable with  $\Gamma$ ) then  $\Delta$  is also a lattice in  $G$  and b) if  $G$  is the quotient of a Lie group  $H$  by a compact normal subgroup, then the image of any lattice in  $H$  is a lattice  $G$ . The lattices from the enlarged class accounting for these modifications are called arithmetic lattices. It is known that groups, like  $SL(2, \mathbb{R})$ , of  $\mathbb{R}$ -rank 1 (that is, any diagonalisable subgroup is one-dimensional) admit nonarithmetic lattices. On the other hand, Margulis proved the arithmeticity conjecture, due to A. Selberg, asserting that if  $G$  is a (connected) semisimple Lie group with trivial centre, no (nontrivial) compact factors and  $\mathbb{R}$ -rank  $\geq 2$  then any 'irreducible' lattice in  $G$  is arith-

metic. He also proved that if  $\Gamma$  is such a lattice and  $\rho: \Gamma \rightarrow G'$  is a homomorphism of  $\Gamma$  into a simple noncompact Lie group  $G'$  with trivial center, such that  $\rho(\Gamma)$  is Zariski dense in  $G'$ , then  $\rho$  can be extended to a representation of  $G$  into  $G'$ . This property, called superrigidity, together with an assertion about homomorphisms of  $\Gamma$  into  $p$ -adic groups and some technical work yields the arithmeticity theorem.

The book under review gives a lucid account of these results and the outgrowth of the seminal ideas involved in them. We shall discuss more about the contents below. But before that, it may be more enlightening to discuss the relation of these results with ergodic theory in general and the author's pursuits in particular.

In Margulis's scheme the proof of the superrigidity theorem hinges on construction of a measurable  $\Gamma$ -equivariant map  $\phi$  of  $G/P$ , where  $P$  is a minimal parabolic subgroup (e.g.,  $P =$  all upper triangular matrices if  $G = SL(n, \mathbb{R})$ ) into a homogeneous space of the form  $G'/L$ , where  $L$  is a proper 'algebraic' subgroup of  $G'$ , the  $\Gamma$ -action of  $G'/L$  being given by the homomorphism  $\rho: \Gamma \rightarrow G'$  as above and, secondly, on showing that any such equivariant map  $\phi$  is 'algebraic'. Margulis first obtained such an equivariant map using the 'multiplicative ergodic theorem', which is an analogue of the classical Birkhoff ergodic theorem for matrix-valued cocycles. Independently, in his boundary theory for semisimple groups and symmetric spaces, H. Furstenberg introduced another general procedure for obtaining  $\Gamma$ -equivariant maps of the 'boundaries'  $G/P$ , as above, into various  $\Gamma$ -spaces, using certain ergodic theoretic arguments. This provided an alternative (and simpler) approach to the superrigidity theorem. At this stage the author observed that the ideas could be recast into a wider framework to study ergodic finite-measure-preserving actions of semisimple Lie groups, the theory for lattices being particular to transitive actions. In the general framework, homomorphisms (of  $\Gamma$  into  $G'$ ) are replaced by cocycles of ergodic actions and the idea is to show that under suitable conditions they are 'almost constant'. This enabled him to prove the following rigidity property of actions of semisimple groups: Let  $G$  and  $G'$  be two connected semisimple Lie groups with trivial center and no compact factors and suppose that the  $\mathbb{R}$ -rank of  $G$  is at least 2. Let  $S$  and  $S'$  be finite-measure spaces on which  $G$  and  $G'$ , respectively, act essentially freely, ergodically and preserving the measures. Then the actions are orbit equivalent (that is, there exists an isomorphism of the measure spaces taking orbits into orbits) if and only if  $G$  is isomorphic to  $G'$  and, via an isomorphism of the groups, the two actions are isomorphic. This is remarkable since it is in complete contrast with the case of connected amenable (e.g. solvable) Lie groups: any two essentially free, ergodic, finite-measure-preserving actions of two such (possibly different) groups are orbit-equivalent.

These and related results on rigidity are the subject of chapter 5, which, in a sense, is central to the book. The subject unfolds gradually, enabling the reader understand the development of ideas. In the later part the author also presents his general superrigidity theorem for cocycles and various corollaries. Notable among them is an analogous rigidity result for actions of lattices on homogeneous spaces of the ambient groups (not necessarily with a finite invariant measure). In particular, it turns out that  $SL(n, \mathbb{Z})$ -actions on  $\mathbb{R}^n$ ,  $n = 2, 3, \dots$  are all mutually non-orbit-equivalent.

Chapter 6 develops the notion of arithmetic lattices and includes a proof of the arithmeticity theorem (stated earlier). It also contains a proof of Margulis's characterization of arithmetic lattices, as precisely those whose 'commensurator' subgroups are dense. It may be

of interest to note that this result has led to interesting examples in addressing an old question in dynamics about proximal flows (cf. [2]).

Thanks to some of the later work of Margulis and a theorem of D. Kazdan, one knows that if  $\Gamma$  is an irreducible lattice in a connected semisimple Lie group  $G$  with finite center, no compact factors and  $R$ -rank  $\geq 2$  then any normal subgroup  $\Delta$  of  $\Gamma$  is either of finite index or contained in the center of  $G$ . Chapter 8 contains a proof of this in the case when  $\Gamma/\Delta$  is not amenable. If  $\Gamma/\Delta$  is amenable and the  $R$ -rank of each simple factor of  $G$  is at least 2 then the result follows from a theorem of Kazdan asserting that any unitary representation of  $\Gamma$  as above, which admits 'almost invariant vectors' actually admits (non-trivial) invariant vectors. This property, shared by many groups, is called property  $T$  or the Kazdan property. It has many interesting applications. Very recently it was used by Margulis and D. Sullivan, independently, in resolving the classical Banach-Ruziewicz problem about the uniqueness of finitely additive measures invariant under all isometries, on spheres of dimension  $\geq 4$ . (cf. [3] for various details). Chapter 7 of the book gives a no-tears exposition of the Kazdan property and some of its consequences might also benefit many mathematicians not directly concerned with the main theme of the book.

Chapters 9 and 10 contain brief discussions on various current trends in various areas related to the material of the book in a general way.

The first four chapters are preparatory in spirit. Chapter 1 gives a general outline of the book. Chapter 2 is devoted to ergodic theory and, in particular, gives an exposition of C.C. Moore's ergodicity theorems in the case of semisimple Lie groups. This reviewer has a bias for the area and feels that the author should have made the chapter an excuse to give an account in the wider context of ergodic theory. For instance, some of the notions familiar to classical ergodic theorists could have been discussed. Also simpler tools such as the Mautner lemma (cf. [4], Appendix II) could have been highlighted. Even so, it is undeniable that the material covered is available in book form for the first time and is certainly more accessible than the original sources.

Chapter 3 discusses algebraic groups, groups of real and  $p$ -adic points of such groups, their actions on algebraic varieties (resp. their real or  $p$ -adic points when the variety is defined over the appropriate field) and the orbit structure under the induced actions on i) the spaces of probability measures on and ii) the spaces of measurable functions with values in, the algebraic variety (resp. real or  $p$ -adic points). The crucial theme from here relevant in the subsequent chapters is that these actions are 'smooth' which means that all orbits are locally closed and thus the situation is opposite to that for ergodic actions. That the corresponding assertion is true for the actions on the varieties themselves is taken as the starting point (Theorem 3.1.3). It may be worthwhile to note here that Theorem 3.1.3 can be proved without involving Galois cohomology but simply using the inverse function theorem which is available for any local field (cf. [5], Lemma 1.22).

Chapter 4 deals with cocycles and amenable actions. These notions play a crucial role in the transition from Margulis's ideas for the case of lattices to the general framework of the author.

The reviewer would also like to make a couple of minor observations which may help the reader. The proof of Theorem 2.4.2 can be substantially simplified if one observes that  $N$  and

$N^-$ , the unipotent radical of the 'opposite' parabolic, generate  $G$ . Secondly, one of the cases in the proof of Theorem 7.4.4 is redundant since, by Mautner Lemma, in any unitary representation of  $G = SL(n, \mathbb{R}) \times \mathbb{R}^n$  any  $SL(n, \mathbb{R})$ -invariant vector is already  $G$ -invariant.

The book is well-written and makes a pleasant reading. The author often illustrates the results and the various points to be made, for the particular case of  $SL(n, \mathbb{R})$ , which would be very useful to a reader not too comfortable with the theory of general semisimple algebraic groups. The author should be thanked for the work which would go a long way in generating interest in the subject.

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**Perspectives in mathematics—Anniversary of Oberwolfach, 1984** edited by W. Jager, J. Moser and R. Remmert. Birkhauser Verlag, P.O. Box 133, CH-4010 Basel, Switzerland, 1984, pp. 587, S.Fr. 115.

The Mathematical Research Institute in Oberwolfach has been a leading centre of mathematics and its applications in Germany since the fifties. International conferences have been held there regularly to discuss many important topics in mathematics. *Perspectives in mathematics* is a collection of review and research papers written by leading authors in their fields to commemorate the anniversary of the Institute.

The volume contains twenty-two articles. Those in English can be roughly classified into eight in pure mathematics, five in mathematical physics, four on applications of mathematics to fields like computer science and statistics and one on mathematical modelling of epidemics. Only the articles in mathematical physics will be highlighted, inadequacy of the reviewers in other fields merely allows a mention of the contents in the remaining papers.

Non-abelian gauge theories in a Euclidean space  $R^n$  are presented by Donaldson, wherein he shows the relation between the Yang-Mills equations in  $R^4$  and the Yang-Mills-Higgs equations in  $R^3$ . The topology of the solutions are studied and classified through the homotopy groups. Methods of construction of the solutions (instantons and monopoles) are also reviewed. Stern has displayed the interplay between the solutions of self-dual non-

abelian gauge theories on a manifold  $M$ , and the topology of  $M$ . The homology group of  $M$  is shown to restrict the classes of principal bundles (which give a geometric description of gauge theories) on  $M$ . Zehnder has studied the fixed points (which are the initial conditions for periodic solutions of differential equations). Restricting to Hamiltonian systems, results on the fixed points of symplectic maps (in two dimensions) are enunciated using functional analysis and topology, leading to a variational principle for forced oscillations. A very nice review of the philosophy and evolution of the calculus of variations is given by Hildebrandt, along with some interesting problems which merit further research. Simon has posed thirty-two interesting (albeit difficult) problems in mathematical physics covering a wide range of topics. These are fundamental problems which may appeal only to very rigorous mathematical physicist.

A brief look at the articles in pure mathematics show: survey of rigid analytic geometry (Bosch) where the concepts of analytic functions over the field of complex numbers are generalized to that over a general field  $C_p$  (the completion of  $p$ -adic numbers); differential geometric analysis of Gauss maps of immersions of a Riemann surface into a Euclidean space (Eells); the study of the Cauchy problem for linear and nonlinear hyperbolic differential operators (Garding); survey of algebraic independence of transcendental numbers (Waldschmidt); an overview of the progress in the past twenty-five years in the geometry in total absolute curvature theory (Kuiper); exposition of the recent trends in the thinking and approach in mathematics, especially algebra (Roquette); survey of linear algebraic groups (Springer).

Interesting review papers on the applications of mathematics like topology to computer graphics (Banchoff), combinatorial optimization (Grotschel), theoretical statistics (Nielsen and Cos); and the mathematical modelling of epidemics are also presented.

Barring a few typographical errors, this volume gives one a broad outlook of some of the important fields of mathematics, and also gives an insight into the inroads some 'modern' subjects like differential geometry have made into other fields. Though the broad spectrum of subjects covered, and the cost, might make this volume a luxury for individual possession, any conscientious library must possess it.

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**Multivariate approximation theory III** edited by Walter Schempp and Karl Zeller. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1985, pp. 400, S. Fr. 84.

This book brings out the proceedings of a conference on Approximation theory at Oberwolfach. The general problem studied in this meeting can be stated as follows: We are interested in the approximation of functions of several variables. It is assumed that we know the values of the function and/or the values of some derivatives of the function at certain number of points in the domain. The aim is to define the function everywhere in the domain. The articles presented here look at several aspects of this problem.

The problem stated above is quite classical. It is well-known that global polynomial fitting



leads to oscillations and so it is reasonable to look at piece-wise polynomial approximation on a given triangulation of the domain. Some interface conditions are imposed so that there is global smoothness. A typical spline space is defined as follows:

$$S_k^\mu = \{s \in C^\mu(\Omega); s|_T \in P_k \text{ for all triangles } T\}.$$

A finite element spline space. The study of these spaces consists of estimating their dimensions, providing suitable bases, representation of arbitrary function in these bases and error estimates. There are about six articles in these proceedings which deal with this aspect of the problem.

It is extremely important to have basis functions with small supports because this leads to a sparse matrix system. This phenomenon corresponds to vertex splines. They are examined by Chui and Lai.

Since there exist a lot of literature on one dimensional ( $1d$ ) splines, it is natural to construct splines in  $2d, 3d$  based on univariate splines. They are called Blending splines. They have the same interpolation error as compared with the tensor product interpolation but they require less information about the function. Such procedures are carried out by some authors in this volume.

While providing an approximation for a given function  $f$ , it is important to preserve the shape of  $f$ . The function may be convex, monotonic and exhibit peaks and singularities. The complexity of the problem increases with the number of variables. Therefore one looks for more sophisticated methods in higher dimensions to minimize the cost and computer time.

As a measure of the variation of  $f$ , Goodman uses the following expressions in his article:

$$V(f, A) = \int_A (f_x^2 + f_y^2)^{1/2},$$

$$V_1(f, A) = \int_A (f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2)^{1/2},$$

$$V_1(f, l) = \int_l |\text{grad } f_1 - \text{grad } f_2|.$$

It is then natural to study the shape preserving properties of the approximation using these measures. Goodman does this in the case of the so-called Box splines.

Another aspect of the problem is when the experimental values of the function are known up to some noise addition. One way out is to use some probabilistic methods. This has been done by Bozzini and Lenarduzzi. Another approach to tackle this problem would be to try to fit it in a known shape. This is possible if one has a prior knowledge about the shape of the function. The parameters involved in this process can be determined by classical least square type methods. A generalization of this method, suggested by Watson, serves useful even when the exact location of the peaks or singularities are not known.

On the application side, there are two articles about the construction of quadrature schemes which generalize the well-known Gaussian scheme to multi-dimensions; two papers which are devoted to the application of the theory to boundary value problem. Quarteroni establishes the convergence of spectral methods whereas Gurlbeek *et al* suggest a boundary allocation method.

Apart from these, there is no other article which deals with applications. More attention is devoted to approximation in  $2d$  than in  $3d$ . In conclusion, this volume presents the state-of-the-art in the subject and is not an introductory textbook. It will therefore be useful for people who are already familiar with the ideas in the field and doing research.

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**Geometric theory of foliations** by Cesar Camacho and Alcides Lins Neto (Translated by Sue Goodman). Birkhauser Verlag, Basel, Switzerland, 1985, pp. 205, S. Fr. 84.

Consider the trajectories of an ordinary differential equation  $dx/dt = f(x)$  where  $x$  is a point on the plane and  $f$  a nowhere-vanishing function. They divide up the plane into connected one-dimensional submanifolds like noodles spread out (thinly!) on a plate. This is a simple example of a *foliation* i.e., 'a decomposition of a given manifold into a union of connected disjoint submanifolds of the same dimension, called leaves, which pile up locally like pages of a book'.

The subject began with Reeb's thesis in the 1940's in which he constructed a two-dimensional foliation on  $S^3$ , thus answering in the affirmative a question first raised by Hopf. In 1958 Haefliger proved (in his thesis) that there exist no (real) analytic two-dimensional foliations on  $S^3$ . In 1965, the Russian mathematician S.P. Navikov proved that any two-dimensional foliation of  $S^3$  has a compact leaf.

These are the basic results in a subject which has grown a lot since then and became a basic part of 'geometric topology'. Using a little more than general topology, and elementary differential geometry the authors give a detailed introduction to the subject including full proofs of the above classical results. There is also a proof of the result of Lima on the maximum number of commuting pointwise linearly independent vector fields on  $S^3$  (the number is one).

The presentation is very simple (in fact this makes some of the proofs longer than necessary) and, despite an early reference to handle-body decompositions, quite-self contained. There are a large number of carefully worked out constructions, including, of course, a description of the Reeb foliation of  $S^3$ . There is also a set of excellent exercises, and a beginner could not hope for a better introduction to geometry, except, perhaps, for the Milnor classics.

A minor quibble: More recent work could have been summarized in a final chapter, the Godbillon-Vey invariant and its relevance explained somewhere. But let that pass — this is a lovely book, carefully written and elegantly produced, and strongly recommended for any mathematics library.

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**Electric machines** by I.J. Nagrath and D.P. Kothari. Tata McGraw-Hill, Asaf Ali Road, 3rd Floor, New Delhi 110 002, 1985, pp. 672, Rs. 39.

Once again the authors have brought out a book which is highly suitable for the requirements of Indian students and teachers. This book by and large follows the track of *Electric machinery* by A.E. Fitzgerald, C. Kingsley and S.D. Umans, the well-known publication by McGraw-Hill, New York, but is in some sense better suited to Indian readership.

I have always felt depressed that teachers in many engineering colleges in India continue to follow the books on which we were brought up, although power supply, the use and even the analysis of electrical machines have undergone a great change over the last 25 years. Many colleges were reluctant to accept even Fitzgerald and Kingsley's book.

Apart from a general apathy on the part of some of the teachers to update their knowledge, one could detect that there was a distinct need of a book which made a fairly comprehensive treatment of electric machines, realizing that specialized topics such as AC and DC machine designs have to make room for subjects like electronics and computers in today's curriculum.

The important aspects about Indian readership I think are that: (i) a book on electric machines should not be drastically different from the classics on which most teachers have been brought up, (ii) an Indian reader generally prefers a book with a large number of worked-out examples and a number of problems for exercise with answers.

The book by Nagrath and Kothari adequately fulfils both these requirements. To give an example, one may wonder whether circle diagrams serve any purpose these days, since no induction motor designer would have any need for it now. But one cannot rule out the fact that circle diagrams provide a synthetic understanding of induction motors and most teachers would like to teach their students the art and the science of circle diagrams. The authors have very ably emphasized such classical physical concepts throughout their book while they have taken care to cover, for example, elements of power electronics, which must constitute an important topic in today's syllabus of electric machines.

Although I was somewhat critical that the authors have tried to cover too much in an essentially undergraduate course, I revised my opinion after going through the book. A teacher may decide before hand sections out of this book that he should cover and leave out certain portions, without in any way disturbing the continuity or development of the subject. May I suggest that the authors should propose as a model a list of sections that should be covered to meet the syllabi of the IITs or some engineering colleges of the country? That will help the teachers in preparing their teaching schedule.

Finally, I congratulate the authors on writing an excellent book which I have already started recommending to my students.

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**Chemical and metallurgical thermodynamics** Volumes I and II, by M.L. Kapoor. Nem Chand and Bros, Roorkee, 1984, Vol. I - pp. 304, Rs. 22.50 and Vol. II - pp. 567, Rs. 42.50.

As a fundamental science, thermodynamics has extensive applications in all branches of engineering. Today's engineering students are exposed to the atomic thinking of the scientists and 'the matter in bulk thinking' of the engineers. The students of metallurgical engineering in particular have remarkable insight into the microscopic world, know about molecules and energy levels and have acquired the basic quantum concepts. In this context, it will be unrealistic to teach thermodynamics from the classical purely macroscopic approach to the subject. The curriculum in most branches of engineering includes courses in quantum mechanics in which basic ideas of thermodynamics are derived from statistical considerations. In the two volumes on chemical and metallurgical thermodynamics (Volume I devoted to fundamentals and Volume II to applications) Professor M.L. Kapoor has tried to develop the subject matter in a way that retains the generality of purely macroscopic thermodynamics and quantum statistical mechanics. Apart from the well-written section of statistical mechanical interpretation of thermodynamics in the first volume, there are sections where the author has tried to draw upon the student's insight into microscopic matters. For example, there are sections devoted to the statistical thermodynamic interpretation of specific heats, statistical thermodynamic analysis of solution and statistical thermodynamics of chemical reactions.

The application of the theory to chemical and metallurgical situations has been selected from a wide field. Though the work is intended as a text-book for the graduate students of chemical and metallurgical engineering, its usefulness to the students of chemical engineering, is likely to be limited since the majority of examples are taken from metallurgical processes. It should be pointed out that the applications of thermodynamic concepts to topics of special interest to physical and mechanical metallurgies are conspicuously absent.

A novel feature is the inclusion of a short chapter on conventional experimental techniques. The usefulness of this chapter would have been enhanced if a more serious attempt had been made to show how thermodynamic quantities could be estimated or derived from measured data, in each case.

The two volumes are comparable in quality to some of the best books on the subject published abroad and at the same time being low priced, are well within the reach of students in India.

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