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Unsteady flow of dusty visco-elastic liquid between two oscillating plates

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Abstract

The unsteady flow of dusty visco-elastic liquid (Kuvshiniski type) between two oscillating plates is considered in view of its growing importance in various technical problems. The analytical expressions for the velocities of liquid and dust particles are obtained by the technique of Laplace transform. The effects of elastic elements in the liquid, the mass concentration and the relaxation time of dust particle on the velocity fields, the skin friction at the lower plate and the volume flow in between the plates are presented numerically/graphically. It is found that both skin friction at the lower plate wall and volume flow in between the plates increase in the presence of the elastic element in the liquid.

Key words: Dusty gas, laminar flows, Kuvshiniski type visco-elastic liquid, Laplace transform, elastic element, skin friction.

1. Introduction

Interest in problems of flow of a dusty gas *i.e.* a mixed system of fluid and particles have increased enormously in recent years. A model equation describing the motion of such mixed system has been given by Saffman[1]. Based upon Saffman's model, numerous authors[2,3,4] investigated a number of dusty gas flow problems in different situations.

There is another class of flow problems which concerns with the study of the flow of dusty non-Newtonian fluids such as latex particles in emulsion paints, reinforcing particles in polymer melts and rock crystals in molten lava. However, the study on this class of problems and rheological aspects of such flows have not received much attention although this has some bearing on the problems of petroleum industry and chemical engineering interest. Srivastava[5], Sharma and Dube[6], Bagchi and Maiti[7], Mukherjee *et al*[8] have made an attempt to make an entry in this field.

In the present paper, we consider the unsteady laminar flow of visco-elastic (Kuvshiniski type) liquid containing uniformly small solid particles between two infinitely extended parallel plates when the lower plate is at rest and the upper one begins oscillating harmonically in its own plane. The velocity fields for the dusty liquid and dust particles are obtained explicitly by using the technique of Laplace transform. The effect of elastic element in the liquid, the mass concentration and the relaxation time of dust particle on the velocity profiles are studied graphically. The skin friction at the lower plate wall and the total volume flow in between the plates are also obtained.

This problem is likely to have some industrial and chemical engineering applications on the problems of transport of solid particles suspended in visco-elastic fluids through channels.

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2. Basic equations and their solutions

We suppose that the dusty visco-elastic liquid fills the space between two infinite parallel flat plates at a distance h apart. The lower plate is kept at rest and the upper one begins to perform harmonic oscillations with a frequency w in its own plane. In our analysis we take a co-ordinate system such that the x-axis coincides with the lower fixed plate and the z-axis is perpendicular to it. The dust particles are assumed to be spherical in shape and uniform in size and the number density of dust particles is taken as constant throughout the flow and let it be N_0. Since the plates are infinite, the velocities will depend on z and time t only.

For the constitutive equation we adopt Kuvshiniski type liquid[9], given by

$$p_{ik} = -p\,\delta_{ik} + p'_{ik},$$

$$(1+\lambda_0 \frac{D}{Dt})\; p'_{ik} = 2\,\mu\, e_{ik},$$

$$\frac{D}{Dt}\; p'_{ik} = \frac{\partial p'_{ik}}{\partial t} + u_m \frac{\partial p'_{ik}}{\partial x_m}$$

$$e_{ik} = \tfrac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right), \tag{1}$$

where p_{ik} is stress tensor and p'_{ik} the deviatoric stress tensor, $(\frac{D}{Dt})$ is the convective time derivative following a liquid element, and u_i is the velocity of liquid particle. Further λ_0 and μ denote the elastic coefficient and viscocity of the liquid respectively.

Following Saffman[1] and using equation (1) we get the equation of motion of dusty visco-elastic liquid as (dropping dashes)

$$(1+\alpha \frac{\partial}{\partial t})\, \frac{\partial u}{\partial t} = R\, \frac{\partial^2 u}{\partial z^2} + \frac{f}{\tau}\, (1+\alpha \frac{\partial}{\partial t})\, (v-u), \tag{2}$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}\, (u-v), \tag{3}$$

where v is the velocity of dust particles and $u' = \dfrac{u}{hw}$,

$$v' = \frac{v}{hw},\quad t' = tw,\quad z' = \frac{z}{h},\quad \alpha = \lambda_0 w,\quad R = \frac{\gamma}{h^2 w}$$

$$f\,(\text{mas concentration}) = \frac{m N_0}{\rho},$$

$$\tau\,(\text{relaxation time}) = \frac{mw}{k}.$$

The initial and boundary conditions in non-dimensional form are

$t \leq 0$:

$$u = \frac{\partial u}{\partial t} = 0 \text{ for all } z, \tag{4}$$

$t > 0$:

$$\begin{aligned} u &= a \sin t \ at \ z = 1 \\ u &= 0 \qquad \text{at } z = 0. \end{aligned} \tag{5}$$

Taking Laplace transforms of (2) and (3), using (4) and (5), we get

$$p(1 + \alpha p)\,\bar{u} = R\,\frac{d^2 \bar{u}}{dz^2} + \frac{f}{\tau}(1 + \alpha p)(\bar{v} - \bar{u}), \tag{6}$$

$$\bar{v} = \frac{1}{1 + p\tau} \cdot \bar{u}\,, \tag{7}$$

where $\bar{u} = \int_0^\infty u\, e^{-pt}\, dt, \quad \bar{v} = \int_0^\infty v\, e^{-pt}\, dt.$

Transformed boundary conditions are

$$\begin{aligned} \bar{u} &= \frac{a}{1 + p^2} \quad \text{at } z = 1 \\ \bar{u} &= 0 \qquad \text{at } z = 0 \end{aligned} \tag{8}$$

Substituting (7) into (6) we get

$$R\,\frac{d^2 u}{dz^2} - \bar{u}\left[\frac{F + p\tau}{p\tau + 1}\right] p(1 + \alpha p) = 0$$

where $f + l = F$.

Thus the solution of (9) is

$$\bar{u} = A \cosh Mz + B \sinh Mz \tag{10}$$

where $M^2 = \dfrac{(F + p\tau)p(1 + \alpha p)}{R(p\tau + 1)}$

On using boundary conditions in (8) and taking inverse transform, we get

$$u = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{a}{1+p^2} \frac{\sinh Mz}{\sinh M} e^{pt}\, dp, \tag{11}$$

where γ is greater than the real part of the singularities of the integrand in (11). The integrand is an integral function of p and has singularities at $p = \pm i$ and at the zeros of sinh M. Calculating the residues and simplifying further, we obtain the expression for u as

$$u = \frac{a}{E_3^2 + E_4^2} [(E_1 E_4 - E_2 E_3)\cos t + (E_1 E_3 + E_2 E_4)\sin t]$$
$$+ 2R\pi \sum_{n=0}^{\infty} \sum_{i=1}^{3} (-1)^{n+1} \frac{n}{1+p_n^{i^2}} Q(p_n^i) \sin(n\pi z)\, e^{p_n^i t}, \tag{12}$$

where $E_1 = \sin N_2 Z \cosh N_1 Z$,
$E_2 = \cos N_2 Z \sinh N_1 Z$,
$E_3 = \sin N_2 \cosh N_1$,
$E_4 = \cos N_2 \sinh N_1$.

$$N_1, N_2 = \tfrac{1}{2} \Big[[\{ (F - \tau\alpha) + \tau(\tau + F\alpha) \}^2 + \{ \tau(F - \tau\alpha) - (\tau + F\alpha)^2 \}]^{1/2}$$
$$\pm \{ \tau(F - \tau\alpha) - (F\alpha + \tau) \} \Big]^{1/2}$$
$$\times \frac{1}{\sqrt{R(1+\tau^2)}},$$

and p_n^is are those roots of the cubic equation

$$p_n^{i3} \cdot \tau\alpha + p_n^{i2} (\tau + \alpha F) + p_n^i (F + n^2 \pi^2 R \tau) + R n^2 \pi^2 = 0 \tag{13}$$

which are having negative real parts,

$$Q(p_n^i) = (1 + p_n^i \tau)^2 / [\{ p_n^i \tau (1 + \alpha p_n^i) + (1 + 2\alpha p_n^i)$$
$$\times (F + \tau p_n^i) \} - \tau p_n^i (F + \tau p_n^i)(1 + \alpha p_n^i)]. \tag{14}$$

By the convolution theorem, we obtain from (7)

$$v = \frac{a}{E_3^2 + E_4^2} \frac{\tau}{\tau^2 + 1} [(E_1 E_3 + E_2 E_4) \{ (1/\tau \sin t - \cos t) + e^{-t/\tau} \}$$
$$+ (E_1 E_4 - E_2 E_3) \{ (\sin t + 1/\tau \cos t) - 1/\tau\, e^{-t/\tau} \}]$$
$$+ 2R\pi \sum_{n=0}^{\infty} \sum_{i+1}^{3} \Big[\frac{(-1)^{n+1}}{p_n^i \tau + 1} \frac{n}{p_n^{i^2} + 1} Q(p_n^i) \sin(n\pi z)$$
$$x\, (e^{p_n^i t} - e^{-t/\tau})]. \tag{15}$$

The dimensionless shearing stress τ_p at the lower plate due to the dusty visco-elastic liquid is

$$\tau_p = [(1-\alpha \frac{\delta}{\partial t}) \frac{\partial u}{\partial z}]_{z \to 0}$$

$$= \frac{a}{E_3^2 + E_4^2} [\cos t \{(E_1 E_4 - E_2 E_3) - \alpha (N_2 E_3 + N_1 E_4)\}$$

$$+ \sin t \{(E_1 E_3 + E_2 E_4) + \alpha (N_2 E_4 - N_1 E_3)\}]$$

$$+ 2\pi R \sum_{n=0}^{\infty} \sum_{i=1}^{3} (-1)^{n+1} \frac{n^2 \pi}{(1+p_n^{i2})} Q(p_n^i) e^{p_n^i t} (1 - \alpha p_n^i) \qquad (16)$$

The volume flow of dusty visco-elastic liquid discharged per unit breadth of the plate is given by

$$Q_v = 2 \int_0^1 u dz = \frac{2a}{(N_1^2 + N_2^2)(E_3^2 + E_4^2)} [(E_4 \cos t + E_3 \sin t)$$

$$\times (N_1 \sin N_2 \sinh N_1 - N_2 \cos N_2 \cosh N_1 + 2N_2)$$

$$+ (E_4 \sin t - E_3 \cos t)(N_1 \cos N_2 \sinh N_1 + N_2 \sin N_2 \cosh N_1)]$$

$$- 2R\pi \cdot \sum_{n} \sum_{i=1}^{3} \frac{1}{(1 + p_n^{i2})\pi} \cdot Q(p_n^i) \cdot e^{p_n^i t}$$

where $n = 1, 3, 5, \ldots$. (17)

3. Discussion

The present analysis reveals that the solution contains three pertinent non-dimensional parameters *viz* α (elastic parameter of the liquid particle), τ (relaxation time of dust particle) and $F(=(f+1), f$ is mass concentration of dust particle). The behaviour of these parameters, therefore, yields a physical insight into the problem. Numerical computation is made to observe the effects of these parameters on velocity profiles, skin friction at the lower plate and volume flow in between the plates.

Figures 1 and 2 depict the velocity profiles of liquid and dust particles against z for different values of mass concentration and elastic parameter when τ is fixed. It is interesting to note that both u and v increase with the increase in f in the case of Newtonian fluid ($\alpha = 0$) but behave in a reverse fashion when the fluid is visco-elastic. It is also to be noted that velocity profile of both dusty fluid and dust particles decrease with the increase in elastic parameters. Figure 3 reveals that u decreases with the increase in relaxation time τ (with F fixed) when $\alpha = 0$ but in the presence of elastic element u increases with the increase in τ. Figure 4 shows that the effect of relaxation time is to increase the velocity of dust particles irrespective of whether the fluid is Newtonian or non-Newtonian. It is also to be remarked

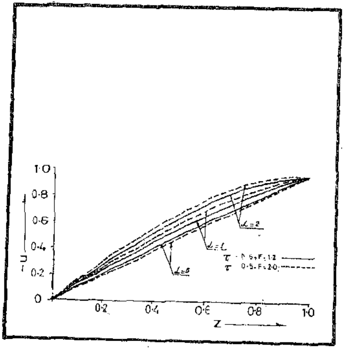

FIG. 1. Velocity profile of dusty fluid against z at $t=5$ and $\tau=0.5$.

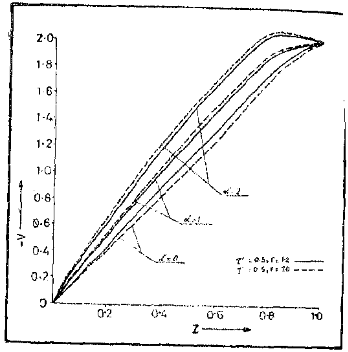

FIG. 2. Velocity profile of dust particles against z at $t=5$ and $\tau=0.5$.

that the influence of relaxation time (τ) is more on the velocity of dust particles than that of dusty liquid. But the mass concentration has very little effect on both u and v irrespective of whether the fluid is Newtonian or non-Newtonian.

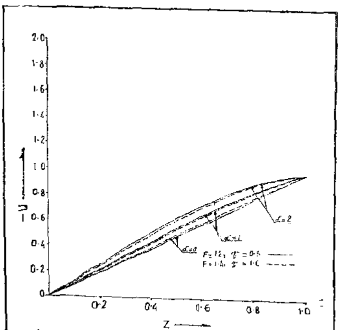

FIG. 3. Velocity profile of dusty fluid against z at $t=5$ and $F=1.2$.

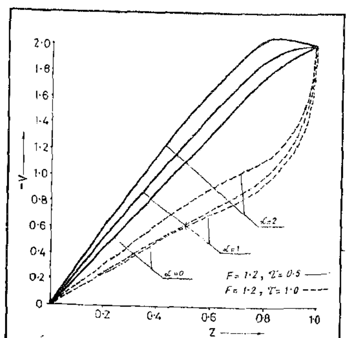

FIG. 4. Velocity profile of dust particles against z at $t=5$ and $F=1.2$.

Table I

	τ_p			Q_v		
	$t = 5$	$t = 10$	$t = 15$	$t = 5$	$t = 10$	$t = 15$
$\alpha = 1$	−1.2426252	0.2952193	1.4101103	−0.4795342	−0.2721468	0.3251387
$\alpha = 2$	−1.5263597	1.1345457	2.1700154	−0.4795486	−0.2722571	0.3252040
$\alpha = 2.25$	−1.5834031	1.3425213	2.3920024	−0.4795531	−0.2722841	0.3252201

Finally, for some representative values of F and τ, skin friction at the lower plate and the volume flow in between the plate walls are calculated numerically for different values of α. Table I reveals that the magnitude of shear stress and total flux increase with the increase in the value of elastic parameter.

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