Experimental implementation of quantum Ulam's problem in a nuclear magnetic resonance quantum information processor

Avik Mitra AND Anil Kumar [a]

Abstract | The Ulam's problem is a two person game in which one of the player tries to search, in minimum queries, a number thought by the other player. Classically the problem scales polynomially with the size of the number. The quantum version of the Ulam's problem has a query complexity that is independent of the dimension of the search space. The experimental implementation of the quantum Ulam's problem in a Nuclear Magnetic Resonance Information Processor with 3 quantum bits is reported here.

[a]Raja Ramanna Fellow

1. Introduction

Application of quantum laws to classical problems have yielded quite a few interesting results. Quantum Ulam's problem is one such case in which the query complexity of the problem reduces from $\Omega(n)$ (n is the number of qubits) to $\Omega(1)$. "Ulam's Problem"was raised by S.Ulam in the year 1976[1]. He proposed a game between Alice and Bob in which Bob thinks of a number between 0 and 10^6 (slightly less than 2^{20}). Alice's job is to find the number in a minimum number of queries. It had been pointed out by Mancini and Maccone (M&M) that classically the complexity of this problem is $\Omega(n)$ while the quantum algorithm requires just one query to arrive at the desired result and is therefore independent of the size of the search space[2].

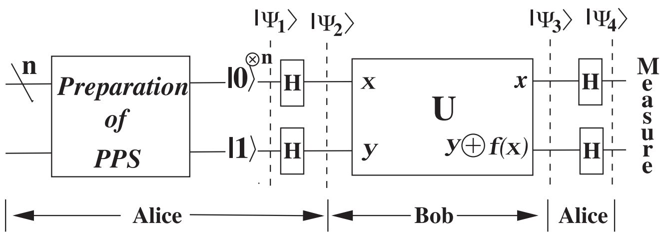
Nuclear Magnetic Resonance (NMR) has been established as a suitable experimental tool for testing quantum algorithms[3–5] and has been successfully applied for the implementation of Deutsch-Jozsa algorithm[6–10], Grover's search algorithm[11,12], Shor's factorization algorithm[13], Quantum games[14,15], Quantum adiabatic algorithms[16–18] and geometric phase algorithms[19,20]. In this article, we report the implementation of the quantum-Ulam's game[2], in a NMR quantum information processor. To the best of our knowledge this is the first experimental implementation of the above game. This article is arranged in the following manner—Section II contains the theoretical description of the quantum version of Ulam problem, section III contains the experimental implementation and section IV deals with results and conclusions.

2. The M&M quantum algorithm for the Ulam Problem

The Ulam's problem describes a game played between two players Alice and Bob. Bob thinks of a number between 0 to 2^n (where n is an integer) that Alice tries to find out by asking yes/no type of queries[1]. The problem is, what would be the minimum number of queries that Alice would need to find the number that Bob has thought. The number of queries that Alice requires in the

NMR Quantum Compuation and Quanum Information Group, Department of Physics and NMR Research Centre, Indian Institute of Science, Bangalore 560012
anilnmr@physics.iisc.ernet.in

Figure 1: A quantum circuit for Ulam's game. The quantum wire with '\' through it represents a set of n qubits. $|\Psi_1\rangle$, $|\Psi_2\rangle$, $|\Psi_3\rangle$ and $|\Psi_4\rangle$ are the states obtained at the various ages of the computation as given in the outline of the algorithm in section II (see text). The boxed 'H's represents Hadamard gates and the symbol '$\oplus$' represents addition modulo 2. The measurement process is collection of the final signal either directly, while observing SQ coherences, or after the application of a z-gradient followed by a flip angle pulse as in the case of observing the populations. The part of the computation performed by Alice and Bob are marked in the lower part of the diagram.



classical case, is $\Omega(n)$, given that there are no error during the transmission. If there are 'ℓ' number of errors then the number of queries needed are $\Omega(n + \ell \log(n))$[2]. Classically, Alice can ask questions like "Is the ith qubit equal to 1". Only 'n' such question will lead Alice to the right answer. However if quantum mechanical resources are used, then, by using superposition Alice can solve the problem in one query.

In the quantum version of the game, one uses two registers. The first register consists of n query qubits and the other register consists of one qubit where Bob store the result of his unitary operation containing information about the number that he had thought. The algorithm starts with Alice creating an equal superposition of all the qubits of both the registers by applying a Hadamard transform on the qubits. Bob then calculates the value of the function f_a by performing a controlled unitary operation. The function f_a is defined as $f_a{:}\mathbb{R}^n \rightarrow \mathbb{R}$ such that $f_a(x) = a \cdot x$ where "$\cdot$"stands for the usual scalar product defined as $a{\cdot}x \equiv \text{mod}_2(\sum_j a_j x_j)$. Alice again interferes the qubits by applying Hadamard gate and then measures the first register to get the result[2]. The outline of the algorithm proposed by M&M is given below. The qubits belonging to the first register is denoted by the subscript A and the qubit belonging to the second register is denoted by the subscript B.

Algorithm

1. Alice first prepares both the registers in a pure state (pseudo-pure state in NMR[3,4]),

$$|\Psi\rangle_1 = |0\rangle_A^{\otimes n}|1\rangle_B,$$

and then performs Hadamard transform on both the registers A and B yielding

$$|\Psi\rangle_2 = \frac{1}{\sqrt{N}} \sum_{x\in\{0,1\}^n} |x\rangle_A \frac{1}{\sqrt{2}}(|0\rangle_B - |1\rangle_B).$$

2. Bob evaluates the function f_a, where 'a' be the number that Bob has thought and codes it in the register B according to the following equation,

$$|\Psi\rangle_3 = \frac{1}{\sqrt{N}} \sum_{x\in\{0,1\}^n} (-1)^{a.x} \times |x\rangle_A \frac{1}{\sqrt{2}}(|0\rangle_B - |1\rangle_B).$$

3. Alice now applies a second Hadamard transform on the both the qubits, yielding

$$|\Psi\rangle_4 = \sum_{y\in\{0,1\}^n} \left[\sum_{x\in\{0,1\}^n} (-1)^{a\cdot x \oplus y \cdot x} \right] \times |y\rangle_A |1\rangle_B,$$

where $\oplus$ is addition modulo 2.

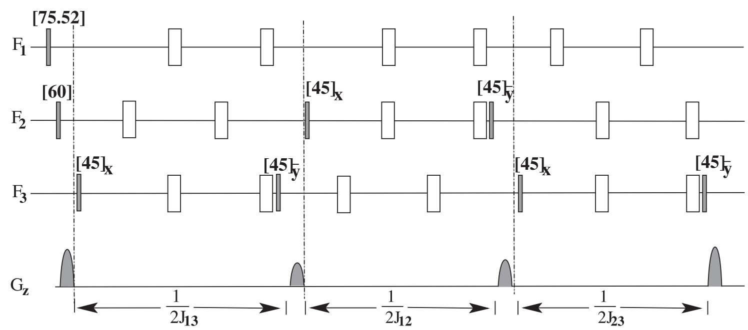
4. Finally Alice measures the qubits of register A in the computational basis.

On measurement, the state of the register A denotes the number that Bob had initially thought. The above algorithm can also be illustrated by a quantum circuit given in Fig. 1.

3. Experimental implementation by NMR

The experimental implementation of the M&M quantum algorithm for solving the Ulam's problem has been carried out in a 3-qubit NMR system. The first two qubits are used as register A and the third qubit is used as register B. This implies that Bob would store the result of his unitary operation in the third qubit and Alice would be required to determine the status of the first two qubits, at the end of the computation.

Figure 2: Pulse programme for preparation of pseudo-pure state (PPS). F1, F2, F3 represent the three fluorine spins which form the three qubits. The broad and unshaded pulses are π pulses. The flip angle and the phase of the other pulses are mentioned on the top of each of them. The pulse programme consists of three J-evolution. During the first J-evolution period $1/2J_{ij}$, the π pulses on F_2 refocusses J_{23} and J_{12} evolutions, while the the π pulses on F_1 and F_3 retain J_{13}. The additional π pulses on F_1 and F_3 just before the $\left[\frac{\pi}{4}\right]_{\bar{y}}$ pulse, regain the sign of the spin operator terms inverted by the π-pulses on F_1 and F_3 in the middle of $1/2J_{13}$ evolution. Similar argument yield the sequence for the J_{12} and J_{23} evolutions.



The system chosen for the experimental implementation is Iodotrifluoroethylene (C_2F_3I) dissolved in acetone-d6. The experiments have been carried out at room temperature in 11.7 Tesla field in AV500 Bruker spectrometer using a triple resonance QXI probe. The Fluorine resonance frequency at this field is 470.5 MHz. The three 'Fluorines' form the three qubits. The NMR Hamiltonian of a weakly coupled 3-qubit system is:

$$\mathcal{H} = -\sum_{i=1}^{3} \omega_i I_z^i + 2\pi \sum_{i<j=1}^{3} J_{ij} I_z^i I_z^j. \tag{1}$$

The first step in the implementation of the algorithm is initialization of the system to the state $|001\rangle$. In liquid state room temperature NMR, since the preparation of a pure state requires extreme conditions, a pseudo-pure state (PPS) is prepared that mimics a pure state[3,4]. The equilibrium deviation density matrix for homonuclear spins under high temperature and high field approximation is proportional to $I_z^1 + I_z^2 + I_z^3$, where the superscript denotes the qubit number and the subscript denotes the magnetization mode[21]. The pulse sequence that creates the $|001\rangle$ PPS starting from this equilibrium density matrix is given in Fig. 2. The pulse sequence use the method of spatial averaging as proposed by Cory *et al*[5]. The 75.52° pulse on qubit 1 and 60° pulse on qubit 2, followed by a crusher gradient creates $\frac{1}{4}I_z^1 + \frac{1}{2}I_z^2 + I_z^3$. The rest of the pulse sequence consists of the basic sequence of

$$U[i,j] = \left[\frac{\pi}{4}\right]_{\phi}^{j} \rightarrow \frac{1}{2J_{ij}} \rightarrow \left[\frac{\pi}{4}\right]_{\phi-90}^{j} \rightarrow G_z, \tag{2}$$

with additional π-pulses during the free evolution period which refocuss the chemical shift evolutions and other J-evolutions, thus making the system evolve only under the desired J coupling. Here the superscript 'j' denotes the qubit number and the subscript 'ϕ' denotes the phase of the $\left[\frac{\pi}{4}\right]$ r.f. pulse. G_z is the crusher gradient which removes all the transverse terms and retains only the longitudinal terms. The operator $U[i,j]$ when applied on equilibrium density matrix $I_z^i + I_z^j$ creates $I_z^i + \frac{1}{2}\left(I_z^j + 2I_z^i I_z^j\right)$. Therefore application of $U[1,2]$, $U[1,3]$ and $U[2,3]$ in the order shown in Fig 2, on the density matrix $\frac{1}{4}I_z^1 + \frac{1}{2}I_z^2 + I_z^3$, creates

$$\frac{1}{4}\left(I_z^1 + I_z^2 + I_z^3 + 2I_z^1 I_z^2 + 2I_z^1 I_z^3 + 2I_z^2 I_z^3 + 4I_z^1 I_z^2 I_z^3\right), \tag{3}$$

which is the spin operator representation for the $|000\rangle$ PPS. A 180° pulse on the third qubit (the first π pulse in Fig. 3) converts it into $|001\rangle$ PPS. This method of creating PPS can be generalized to any number of qubits, however the number of J-evolutions increases to match the number of J-couplings, which increases as $n(n\text{-}1)/2$.

The next step involves the creation of equal superposition by Alice. Equal superposition of the states can be created by $\left[\frac{\pi}{2}\right]_{\bar{y}} \left(= \left[\frac{\pi}{2}\right]_{-y}\right)$ pulse on all the qubits. This pulse acts as a pseudo Hadamard gate[22]. After the equal superposition among all the states has been created, Bob evaluates the function f_a. As mentioned in the previous section, the size of the first register (denoted by the subscript A) is determined by the size of the number that

Figure 3: Pulse sequence for the NMR implementation of the quantum Ulam's game. The narrow filled pulses are 90 degree pulses and the broad unfilled pulses are 180 degree pulses. The phase of each of the pulses are written at their top. Those pulses that do not have any phases mentioned above them can be applied at any phase. The first 180 degree pulse creates $|001\rangle$ PPS from $|000\rangle$ PPS. The $\bar{Y}$ pulse on each qubit implements the pseudo Hadamard gate. The three pulses $Y\bar{X}\bar{Y}$ implement the $\bar{z}$ rotation. The 90 degree Y pulse on qubit 3 along with the evolution under the NMR Hamiltonian with 180 degree pulses in between and the 90 degree X pulse on qubit 3 at the end, implement the CNOT-2,3 gate. Each τ is $1/8J_{23}$, which implies that the total evolution time is $1/2J_{23}$. The composite pulse $XY\bar{X}$ is applied for the $\bar{z}$ rotation. At the last step 90 degree Y pulses are applied on all the qubits which implements the Hermitian conjugate of the pseudo Hadamard gate.

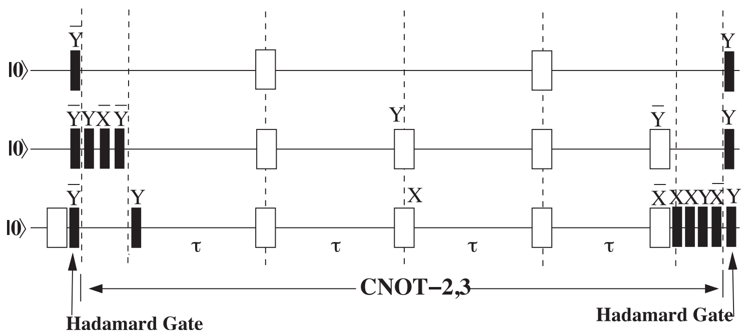
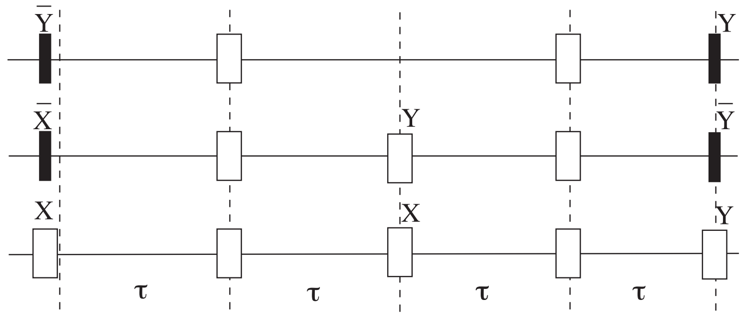


Figure 4: The simplified pulse sequence obtained after the removal of the two redundant 90 degree pulses, one in the beginning and other at the end of the sequence, as well as after the cancelation of pulses that were adjacent to each other and opposite in phase.

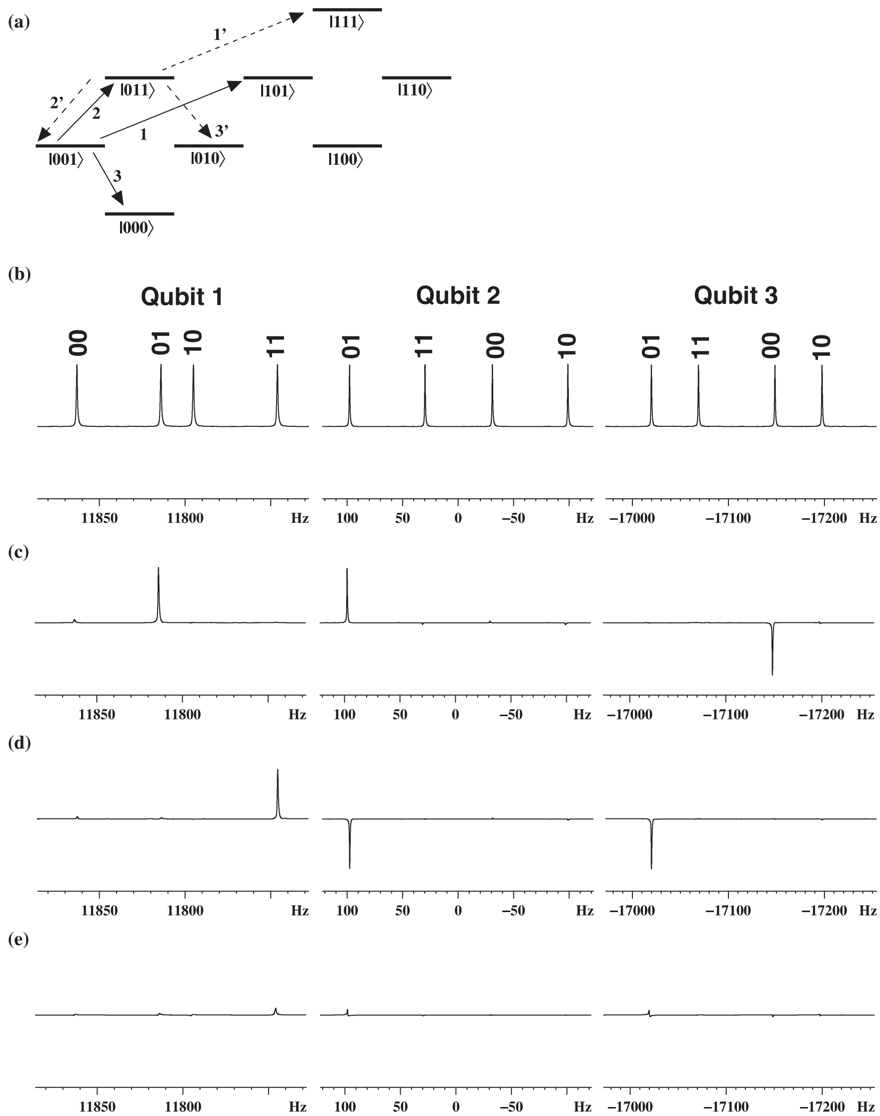


'Bob' thinks. Since we are using two qubits for the first register, it implies that Bob can think of any number among 0, 1, 2, 3. Let us assume that Bob has thought of the number 1 that is 01 in the binary representation. This implies that Bob has to evaluate the function f_{01}. The effect of this unitary transformation is that it takes the third qubit from y to $(-1)^{x.01}y \oplus f_{01}$, while leaving the second qubit unchanged. The above unitary transformation is a CNOT-2,3 gate where the second qubit is the control and the third qubit is the target.

The CNOT-2,3 gate can be implemented by a evolution under the J_{23} coupling (Fig. 3). However, to retain the phases of the coherences after the CNOT gate, a $\left[\frac{\pi}{2}\right]_{\bar{z}}$ ($-\hat{z}$ rotation rotation achieved by a composite pulse of sequence $\left[\frac{\pi}{2}\right]_{\phi}\left[\frac{\pi}{2}\right]_{\phi+\frac{\pi}{2}}\left[\frac{\pi}{2}\right]_{\bar{\phi}}$) is applied on qubit 2 before the J-evolution and on qubit 3 after the J-evolution. The pulse sequence for the implementation of the game is shown in Fig. 3. The phases of the composite pulses that implement the z-rotation are selected in such a way so that the pulses with opposite phases lie adjacent to each other. Two such adjacent $\frac{\pi}{2}$ pulses with opposite phases cancel each other simplifying the pulse programme (Fig 4).

Fig 4 shows the simplified pulse programme that is used for the implementation of the game. All the pulses in this pulse programme are qubit selective soft pulses. For homonuclear spins, the spin or qubit selective soft pulses are long, during which the coherences of the other spins evolve under the chemical shift. This introduces phase factors which

Figure 5: (a) Schematic energy level diagram of a 3-qubit system. The solid and the dashed transition lines represent the single quantum transitions that appear in the spectrum of the three qubits for $|001\rangle$ PPS and $|011\rangle$ output state respectively. The numbers besides these line marks the qubits in whose spectrum the transition would appear. The unprimed and the primed numbers represent the transition for the $|001\rangle$ PPS and the output state $|011\rangle$ respectively. (b) The equilibrium spectra of the three qubits of C_2F_3I. The relative chemical shift of the qubit 1 with respect to the qubit 2 is 11807Hz and that of qubit 3 with respect to qubit 2 is -17114 Hz. The scalar couplings of the sample are J_{12}= 68.1 Hz, J_{13}= 48.9 Hz and J_{23}= -128.8 Hz. (c) The single quantum spectrum obtained for the pseudo pure state $|001\rangle$. Since only $|001\rangle$ is populated in this case, we observe one line of positive amplitude corresponding to $|001\rangle \rightarrow |101\rangle$ transition of qubit 1, one line of positive amplitude corresponding to $|001\rangle \rightarrow |011\rangle$ transition in qubit 2 and one line of negative amplitude corresponding to $|001\rangle \rightarrow |000\rangle$ transition of qubit 3. These transition correspond to the solid lines in Fig 5. (d) The spectrum for population and SQ coherences for the output state $|011\rangle$. In this state also only one level, namely (011), is populated. Hence we observe only one line corresponding to $|011\rangle \rightarrow |111\rangle$ transition of qubit 1, one line corresponding to $|011\rangle \rightarrow |001\rangle$ transition of qubit 2 and one line corresponding to $|011\rangle \rightarrow |010\rangle$ transition of qubit 3. These transitions correspond to the dashed lines shown in Fig 5. (e) The SQ coherence spectrum corresponding to the output state $|011\rangle$. This shows that there are no single quantum coherences present in the output state confirming the expected result.



distort the final result. Therefore, simultaneous π pulses are applied on qubit 2 and 3 along with the π pulse on qubit 1 (Fig. 4). The pulse programme is designed in a symmetric manner so that any phase factors due to unwanted evolutions are canceled. The modulated pulses that simultaneously act on

Table 1: Table containing the flip angles (Θ, Φ) and the phases (α, β, γ) of the pulses used in tomography (Eq. 4). The rightmost column shows the terms of the density matrix whose magnitude is provided by the corresponding pulse sequence

	Θ	Φ	α	β	γ	Terms observed
I	0	0	$\bar{Y}$	-	-	$I_x^i, 2I_x^iI_z^j, 2I_x^iI_z^k, 4I_x^iI_z^jI_z^k$
II	0	0	X	-	-	$I_y^i, 2I_y^iI_z^j, 2I_y^iI_z^k, 4I_y^iI_z^jI_z^k$
III	$\pi/2$	0	$\bar{Y}$	$\bar{Y}$	-	$I_x^i, 2I_x^iI_x^j, 4I_x^iI_x^jI_z^k$
IV	$\pi/2$	0	$\bar{Y}$	X	-	$I_x^i, 2I_x^iI_y^j, 4I_x^iI_y^jI_z^k$
V	$\pi/2$	0	X	$\bar{Y}$	-	$I_y^i, 2I_y^iI_x^j, 4I_y^iI_x^jI_z^k$
VI	$\pi/2$	0	X	X	-	$I_y^i, 2I_y^iI_y^j, 4I_y^iI_y^jI_z^k$
VII	$\pi/2$	$\pi/2$	$\bar{Y}$	$\bar{Y}$	$\bar{Y}$	$I_x^i, 4I_x^iI_x^jI_x^k$
VIII	$\pi/2$	$\pi/2$	X	X	$\bar{Y}$	$I_y^i, 4I_y^iI_y^jI_x^k$
IX	$\pi/2$	$\pi/2$	X	$\bar{Y}$	X	$I_y^i, 4I_y^iI_x^jI_y^k$
X	$\pi/2$	$\pi/2$	$\bar{Y}$	X	X	$I_x^i, 4I_x^iI_y^jI_y^k$
XI	$\pi/2$	$\pi/2$	X	X	X	$I_y^i, 4I_y^iI_y^jI_y^k$
XII	$\pi/2$	$\pi/2$	$\bar{Y}$	$\bar{Y}$	X	$I_x^i, 4I_x^iI_x^jI_y^k$
XIII	$\pi/2$	$\pi/2$	$\bar{Y}$	X	$\bar{Y}$	$I_x^i, 4I_x^iI_y^jI_x^k$
XIV	$\pi/2$	$\pi/2$	X	$\bar{Y}$	$\bar{Y}$	$I_y^i, 4I_y^iI_x^jI_x^k$

all the three qubits with the required phase and amplitude have been designed by the 'offs' option in shaped tool of Bruker software. After designing the shape and duration for the pulses, their powers are calibrated for the desired flip angles. In the present experiments all pulses are designed to be modulated "SEDUCE"pulses[23] of 500 μs duration.

4. Results

We have implemented the above game in which the number thought by 'Bob' is 1 (01 in binary representation). The input state is $|001\rangle$ PPS which is created from the highly mixed equilibrium state. To check whether the input state has been created, a measurement is performed in a separate experiment. The deviation density matrix describing the $|001\rangle$ PPS contains only one diagonal element $|001\rangle\langle 001|$. The measurement is performed by the application of a $\pi/2$ pulse on each of the qubits. The resulting spectrum corresponds to one line each of positive amplitude in the spectra for qubits 1 and 2 and one line of negative amplitude in the spectrum of qubit 3 (Fig. 5(a), the solid transition lines). The experimental spectra of Fig 5(c) confirm the above expected result and the creation of $|001\rangle$ PPS.

'Alice' and 'Bob' perform their unitary transforms on the above state. After the implementation of the unitary transforms by both Alice and Bob, the final state obtained would be $|011\rangle$. On measurement, at the end of the computation, the qubits in the first register should reflect the number thought by 'Bob'(that is $|01\rangle$ in our case) and the qubit in the other register should be restored to its initial state (i.e. $|1\rangle$). To check whether the state has been reached, the population and the single quantum (SQ) coherences of the final state are observed. The population spectrum of the final state is obtained by the application of a gradient G_z, a $\pi/2$ and the measurement of the signal. For the state $|011\rangle$ we expect one line with positive amplitude representing the $|011\rangle \rightarrow |111\rangle$ transition of qubit 1, one line with negative amplitude representing the $|011\rangle \rightarrow |001\rangle$ transition of qubit 2 and one line of positive amplitude representing the $|011\rangle \rightarrow |110\rangle$ transition of qubit 3 as can be seen in 5(a) (dashed transition lines) and confirmed by the population spectrum of Fig. 5(d). To observe the SQ coherences spectrum, the FID was collected without application of any gradient and pulses at the end of the computation. In this case we expect no SQ coherence for any of the three qubits, as confirmed by the spectra in Fig. 5(e).

The populations and the SQ coherences form only a part of the density matrix for the 3-qubit system. The rest of the terms of the density matrix consist of the zero (ZQ), double (DQ) and triple quantum (TQ) coherences which are not directly observable in NMR. To check the amplitude of these coherences, tomography of the full density matrix is performed. The SQ coherences observed by this protocol is used to achieve a uniform scaling for all the terms in the density matrix[17]. The four experiments that were performed are:

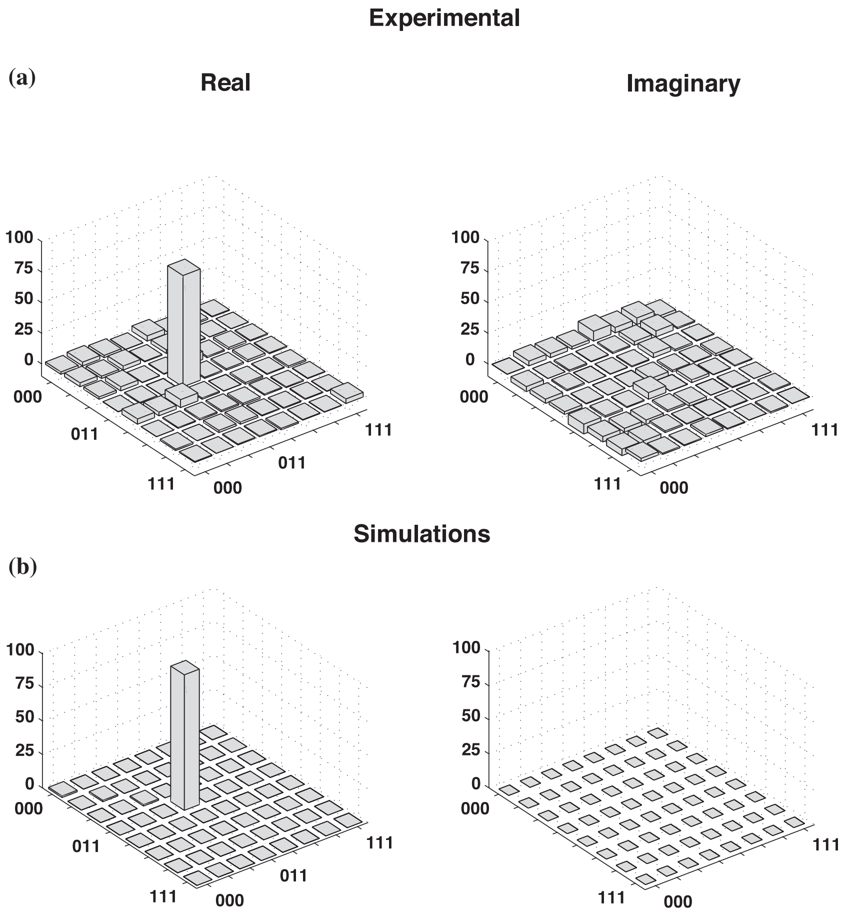
$$\text{A}: \left[\frac{\pi}{2}\right]_\alpha^i [\Theta]_\beta^j [\Phi]_\gamma^k \rightarrow G_z \rightarrow \left[\frac{\pi}{2}\right]_y^i,$$

$$\text{B}: \left[\frac{\pi}{2}\right]_\alpha^i [\Theta]_\beta^j [\Phi]_\gamma^k \rightarrow G_z \rightarrow [\pi]^j \left[\frac{\pi}{2}\right]_y^i,$$

$$\text{C}: \left[\frac{\pi}{2}\right]_\alpha^i [\Theta]_\beta^j [\Phi]_\gamma^k \rightarrow G_z \rightarrow [\pi]^k \left[\frac{\pi}{2}\right]_y^i,$$

$$\text{D}: \left[\frac{\pi}{2}\right]_\alpha^i [\Theta]_\beta^j [\Phi]_\gamma^k \rightarrow G_z \rightarrow [\pi]^{j,k} \left[\frac{\pi}{2}\right]_y^i, \quad (4)$$

Figure 6: The tomographs of the real and the imaginary parts of the (a) experimentally obtained and the (b) theoretically predicted deviation density matrices of the output state $|011\rangle$. It can be seen that only one state $|001\rangle$ is populated whereas all other populations and coherences are nearly zero.



where

$$i, j, k \in \{1, 2, 3\} \text{ and } i \neq j \neq k.$$

The protocol is to selectively convert a given coherence to z-magnetization and then kill all the transverse magnetization by the gradient pulse G_z. Finally the retained z-magnetization is converted to observable SQ coherences. The magnitude of the SQ coherence observed represent the magnitude of the specific coherences. The table 1 lists the various pulse angles, the phases and the observed spin operator terms required for the complete tomography experiment. Fig. 6(a) contains result of the tomograph of the above experiment. A simulation of the expected result is given in Fig. 6. The experimental result agrees qualitatively with the expected result. In order to quantitatively evaluate the experimental result, the " *average absolute deviation* $\langle \Delta X \rangle$"and the " *maximum absolute deviation* $\Delta X_{\max}$"of the experimentally obtained density matrix from the theoretically predicted one was calculated by the formulae

$$\langle \Delta X \rangle = \frac{1}{N^2} \sum_{i,j=1}^{N} |x^T_{i,j} - x^E_{i,j}|,$$
$$\Delta X_{\max} = \text{Max}|x^T_{i,j} - x^E_{i,j}|, \;\; \forall \; i, j \in \{1, N\}, \quad (5)$$

where $x^T_{i,j}$, $x^E_{i,j}$ are the theoretical and experimental elements. The average absolute deviation $\langle \Delta X \rangle$ was found to be 2.6% and the maximum absolute deviation $\Delta X_{\max}$ was found to be 9.2% on the searched element. The errors are mainly due to r.f. inhomogeneity (simulation not shown).

5. Conclusion

We have demonstrated the experimental implementation of the quantum version of the Ulam's problem on a 3-qubit NMR quantum information processor. While scaling to higher number of qubits is straight forward it could pose difficulties due to decoherence which may need special effort to control/reduce errors. Since this game has similarities to some problems in coding theory, the authors believe that the present experiment open up the possibilities of experimental realization of other such quantum version of the established classical problems in this genre[24].

Acknowledgement

We would like to thank Nino Yanoni for suggesting the use of C_2F_3I as a good candidate for a three qubit system. The use of AV-500 NMR spectrometer funded by the Department of Science and Technology (DST), New Delhi, at the NMR Research Centre, Indian Institute of Science, Bangalore, is gratefully acknowledged. A.K. acknowledges DAE for Raja Ramanna Fellowship, DST for a research grant on "Quantum Computing using NMR techniques"and for DST Ramanna Fellowship.

Received 7 May 2009.

References

1. S. Ulam, *Adventures of a Mathematician* Scribner, New york, 1976, p281.
2. S. Mancini, L. Maccone, quant-phys/0508156.
3. D. G. Cory, A.F. Fahmy and T.F. Havel, Proc. Natl. Acad. Sci. USA 94 (1997) 1634.
4. N. Gershenfeld and I.L. Chuang, Science 275 (1997) 350.
5. D. G. Cory, M. D. Price and T.F. Havel, Physica D 120 (1998) 82.
6. I. L.Chuang, L. M. K. Vanderspyen, X. Zhou, D.W. Leung and S. Llyod, Nature 393 (1998) 143.
7. J. A. Jones and M. Mosca, J. Chem. Phys. 109 (1998) 1648.
8. Kavita Dorai, Arvind, Anil Kumar, Phys. Rev. A. 61 (2000) 042306.
9. Ranabir Das and Anil Kumar, Phys. Rev. A. 68 (2003) 032304.
10. T. Gopinath and Anil Kumar, J. Magn. Resonan., 193 (2008) 168–176.
11. I. L. Chuang, N. Gershenfeld and M. Kubinec, Phys .Rev. Lett. 80 (1998) 3408.
12. J. A. Jones, M. Mosca and R. H. Hansen, Nature 393 (1998) 344.
13. L.M.K. Vanderspyen, Matthias Steffen, Gregory Breyta, C.S.Yannoni, M.H. Sherwood and I.L. Chuang, Nature 414 (2001) 883.
14. Jiangfeng Du, Hui Li, Xiaodong Xu, Mingjun Shi, Jihui Wu, Xianyi Zhou, Rongdian Han, Phys. Rev. Lett. 88 (2002) 137902.
15. Avik Mitra, Sivapriya K, Anil Kumar, Jour. Magn. Reson., 87 (2007) 306.
16. M. Steffen, W. van Dam, T. Hogg, G. Bryeta, I. Chuang, Phys. Rev. Lett. 90 (2003) 067903.
17. Avik Mitra, Arindam Ghosh, Ranabir Das, Apoorva Patel, Anil Kumar, J. Magn. Res., 177 (2005) 285–298.
18. Avik Mitra, T.S. Mahesh and Anil Kumar, J. Chem. Phys., 128 (2008) 124110.
19. Arindam Ghosh and Anil Kumar, Physics Letters A 349(2006) 27–36.
20. T. Gopinath and Anil Kumar, Physical Review A 73 (2006) 022326.
21. R. R. Ernst, G. Bodenhausen, A. Wokaun, 'Principles of Nuclear Magnetic Resonance in One and Two Dimensions', Clarendon Press, Oxford, 1994.
22. Ranabir Das, Doctoral Thesis, Indian Institute of Science, Bangalore, Nov 2004.
23. M. A. McCoy, L. Mueller, J. Magn. Res. A, 101 (1993) 122–130.
24. A. Pelc, Theo. Comp. Sci., 270 (2002)71–109.

Anil Kumar, Ramanna Fellow, Department of Physics and NMR Research Centre, Indian Institute of Science, Bangalore; Ph.D., I.I.T. Kanpur (1969) Post-Doc in USA at Georgia Tech. and UNC, Chapel Hill, (1969–72) and at ETH, Zurich, 1973-76 (with Prof. R.R. Ernst, Chemistry Nobel 1991). Joined IISc, Bangalore in 1977, Assistant Prof 1982, and Professor in 1990; Chairman Department of Physics (1994-97) and Convener, NMR Facility (1998–2003). Visiting Prof. at UNC (1989-90) and at ETH Zurich (1979–80) with Prof. Kurt Wuthrich (Chemistry Nobel 2002). Honorary Professor at IISc (2003-2008). Fellow of all three Indian National Science Academies, The Third World Academy of Sciences and the International Society of Magnetic Resonance (ISMAR). He has published over 160 papers and has received several awards. URL sif.iisc.ernet.in/anil

Dr. Avik Mitra, Ph.D. in the area of "Quantum Information Processing by Nuclear Magnetic Resonance" (supervisor Prof. Anil Kumar) I.I.Sc., Bangalore (2007); M.Sc. (Physics), I.I.T. Kanpur (2001); B.Sc., Banaras Hindu University, Varanasi (1998). Presently working with Varian India, Pune.