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A comparison of three different techniques for optimal allocation of tolerances and clearances in four-bar function generators*

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Abstract

The paper presents a comparative study of the techniques of continuous dynamic programming (CDP), discrete dynamic programming (DDP) and parametric programming (PP) as applied to the solution of the problem of optimal allocation of tolerances and clearances in four-bar function-generator linkages. For this purpose three different mechanisms generating the functions $y = \sin x$, $y = \log_{10} x$ and $y = x^2$ have been considered. The allocation problem has been solved using the CDP, DDP and PP techniques for both $\pm 1°$ and $\pm 0.5°$ limits on mechanical error considering both normal and uniform distributions for the location of pin centres in the respective clearance circles. The results show that the parametric programming technique yields strictest tolerance and clearance values but takes minimum computation time while discrete dynamic programming technique yields comparatively liberal tolerance and clearance values and takes moderate computation time.

Key words: Continuous dynamic programming (CDP), discrete dynamic programming (DDP), parametric programming (PP), optimal allocation, computation time, structural error.

1. Introduction

A large number of analytical techniques are available in literature to aid the designer in dimensional synthesis of function generator mechanisms for minimum structural error. In practical mechanisms one has to contend with the additional mechanical error caused by the inevitable presence of tolerances on link lengths and clearances in pin joints. The techniques available for analysing the effect of tolerances and clearances or for optimal allocation of tolerances and clearances are relatively fewer[1-12]. Allocation of optimal values for tolerances and clearances is necessary as high values result in low manufacturing cost but cause large error in output while low values result in high manufacturing cost but reduce error in output. Garret and Hall[3] were the first to present a statistical treatment of the effect of tolerances and clearances. They presented the '3-σ mobility bands' for the four-bar mechanisms synthesized for minimum structural error in

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Garret and Hall[2] for generating the functions $y = x^2$, $y = \sin x$ and $y = \log x$ assigning flat tolerances of ± 0.005 inch and clearances of 0.001–0.002 inch. The 3-σ mobility band is a band of varying width around the desired output function in which (in the long run) 99.7 per cent of the mechanisms will operate. Dhande and Chakraborty[6] presented a stochastic model of the four-bar linkage considering the effect of tolerances and clearances and formulated the problem of determining suitable tolerances and clearances for a prescribed limit on output error as an optimization problem. They solved this problem in the case of the four-bar mechanism taken from Garret and Hall[2] for generating the sine function by the application of discrete dynamic programming technique. This study[6] demonstrated the superiority of the stochastic approach over the mobility band approach[3]. Continuing on the lines of Dhande and Chakraborty[6], Chakraborty[7] formulated the problem as a parametric programming problem and solved it using the interior penalty function method incorporating the Powell's conjugate direction method for unconstrained minimization. He considered the mechanisms from Garret and Hall[2] for generating $y = x^2$, $y = \sin x$ and $y = \log_{10} x$ for both $\pm 1°$ and $\pm 0.5°$ error in output. Also both normal and uniform distributions were considered for the location of pin centres in the corresponding clearance circles. Tolerances and clearances obtained by the method were almost identical to those obtained in Dhande and Chakraborty[6]. Rao[8] presented an iterative method in which tolerances and clearances are prescribed earlier for synthesis of a mechanism considering structural and mechanical errors. Rao and Reddy[9] employed the technique of chance-constrained programming to synthesize a mechanism for minimum structural and mechanical error while Rao and Hati[10] used the game theory approach for the purpose. Recently Sharfi and Smith[12] presented a method for allocation of tolerances and clearances which is based on a sensitivity analysis of the output variable to small changes in the link proportions. The clearance values obtained are considerably lower than those obtained[6,7].

A study of relevant literature shows that the stochastic approach employed by Dhande and Chakraborty[6] and Chakraborty[7] is probably the most suitable one for allocation of tolerances and clearances in linkages. However, which particular technique of solution yields the best results in terms of reduced computation time and liberal tolerances and clearances for prescribed error in output is still not very clear. This is because one of the possible techniques *viz.*, continuous dynamic programming has not been explored. Also though more examples have been considered in support of the approach by Chakraborty[7] no information on computation time is presented. Hence with a view to get a clearer picture the present work makes a comparative study of continuous dynamic programming, discrete dynamic programming and parametric programming techniques as applied to the problem of optimal allocation of tolerances and clearances in four-bar function generating mechanisms. All the three functions, *viz.*, $y = \sin x$, $y = \log_{10} x$ and $y = x^2$ have been considered for both $\pm 1°$ and $\pm 0.5°$ permissible deviation in output. Also both normal and uniform distributions have been considered for the location of pin centres in the corresponding clearance circles. The results obtained are presented and discussed.

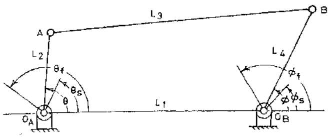

FIG. 1. Four-bar function generator linkage without clearances and tolerances.

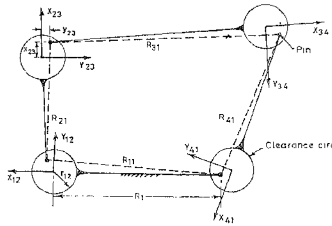

FIG. 2. Four-bar linkage with clearances and tolerances.

2. Formulation of the stochastic model

The stochastic model of the four-bar linkage can be formulated following the procedure indicated in references 6, 7. Figure 1 shows the ideal linkage with no tolerance on link lengths and no clearance in the pin joints. Here $l_i (i = 1, 2, 3, 4)$ represent the nominal link lengths, θ and ϕ represent the orientations of input and output links respectively, the subscript 's' denoting the starting position and 'f' denoting the final position. Figure 2 shows the schematic representation of the practical linkage in which tolerances on link lengths and clearances in pin joints are present. Here $R_i (i = 1, 2, 3, 4)$ represent the actual link lengths given by

$$R_i = l_i + \epsilon l_i \tag{1}$$

where ϵ is the tolerance per unit nominal length, R_i are obviously random variables. Each link i is assumed to carry a pin at one end and a race at the other end in which the pin attached to link j moves. Hence the pin centre of link j can lie anywhere in the clearance circle (of radius r_{ij}) at the end of link i. Let it lie at a point whose coordinates are (x_{ij}, y_{ij}) referred to a rectangular coordinate system (X_{ij}, Y_{ij}) with origin at the centre of the clearance circle and X_{ij} axis along the centreline of link i. Then the equivalent length R_{ij} of link is given by

$$R_{il}^2 = (R_i + x_{ij})^2 + y_{ij}^2 . \tag{2}$$

Since y_{ij} is small compared to $(R_i + x_{ij})$ we can write

$$R_{il} \simeq R_i + x_{ij} .$$

The equivalent linkage is shown by dotted lines in fig. 2. The displacement equation for the equivalent four-bar linkage is given by Garret and Hall[2].

$$\phi = 2 \tan^{-1} \left[\frac{A \pm D}{B + C} \right] \tag{3}$$

where

$$A = \sin\theta$$

$$B = \cos\theta - \frac{R_{11}}{R_{21}}$$

$$C = \frac{R_{11}^2 + R_{21}^2 - R_{31}^2 + R_{41}^2}{2R_{21}R_{41}} - \frac{R_{11}}{R_{41}}\cos\theta$$

$$D = +\sqrt{A^2 + B^2 - C^2}$$

Hence we can write

$$\phi = \phi(\theta, V_i) \tag{4}$$

where $V_i (i = 1, 2, \ldots, 8)$ given by

$$V_1 = R_1,\ V_2 = R_2,\ V_3 = R_3,\ V_4 = R_4,\ V_5 = x_{12},\ V_6 = x_{23},\ V_7 = x_{34},\ \text{and}\ V_8 = x_{41}$$

are all random variables. The mean and variance of the output angle ϕ can be calculated if we know the mean and variances of the random variables V_1 to V_8. Since the manufactured link lengths are known to follow a normal distribution the mean and variance of $V_i (i = 1, 2, 3, 4)$ are given by

$$M[V_i] = l_i \tag{5a}$$

$$D[V_i] = (\epsilon l_i/3)^2 . \tag{5b}$$

As far as the location of the pin centre in the respective clearance circle is considered we can consider two types of distributions, namely, normal and uniform distributions. For normal distribution,

$$M[V_i;\ i = 5, \ldots, 8] = 0 \tag{6a}$$

and

$$D[V_5] = (r_{12}/3)^2$$

$$D[V_6] = (r_{23}/3)^2$$

$$D[V_7] = (r_{34}/3)^2$$

$$D[V_8] = (r_{41}/3)^2. \tag{6b}$$

For uniform distribution

$$M[V_i;\ i = 5, \ldots, 8] = 0 \tag{7a}$$

and

$$D[V_5] = \frac{4}{3\pi} r_{12}^2$$

$$D[V_6] = \frac{4}{3\pi} r_{23}^2$$

$$D[V_7] = \frac{4}{3\pi} r_{34}^2$$

$$D[V_8] = \frac{4}{3\pi} r_{41}^2 . \qquad (7b)$$

Expanding the right hand side of eqn. (4) by Taylor series about the mean of the random variables $V_i (i = 1, 2, \ldots, 8)$ and neglecting higher order terms we get,

$$\phi \simeq \phi(\theta, M[V_i]; i = 1, 2, \ldots, 8) + \sum_{i=1}^{8} (\partial\phi/\partial V_i)_M (V_i - M[V_i]) \qquad (8)$$

As all the V_is are uncorrelated, the mean and variance of ϕ is given by

$$M[\phi] = \phi(\theta, M[V_i]; i = 1, 2, \ldots, 8) \qquad (9a)$$

$$D(\phi) = \sum_{i=1}^{8} (\partial\phi/\partial V_i)_m^2 D[V_i] . \qquad (9b)$$

The partial derivatives $\partial\phi/\partial V_i$ for $i = 1, 2, \ldots, 8$ can be evaluated by differentiating eqn. (3).

3. The allocation problem

In practice one would like to employ as large a tolerance and clearance as possible without increasing the resultant mechanical error beyond a prescribed value. Hence the problem of optimal allocation of tolerances and clearances can be formulated as

$$\min Z = \sum_{i=1}^{8} \frac{1}{D[V_i]} \qquad (10)$$

subject to $\sum_{i=1}^{8} (\partial\phi/\partial V_i)_m^2 D[V_i] \leq D[\phi]$

where $D[\phi]$ is the prescribed variance in the output angle. Once the variances $D[V_i]$ are known the tolerances and clearances can be determined from eqns. (5b) and (6b) or (7b).

4. Techniques for solving the allocation problem

The optimal allocation problem represented in eqn. (10) can be solved by application of the dynamic programming technique or the parametric programming technique.

4.1 Dynamic programming

Letting $x_i = D[V_i]$, $(\partial\phi/\partial V_i)_m^2 = a_i$ and $D[\phi] = b$ we can state the allocation problem of eqn. (10) as

$$\min Z = \sum_{i=1}^{8} f(x_i) \tag{11}$$

$$\text{subject to } \sum_{i=1}^{8} a_i x_i \leq b.$$

This is the standard dynamic programming formulation of a serial multistage decision problem. Here allocation of each tolerance or clearance is treated as a single stage decision problem, $x_i = D[V_i]$ being the stage decision variable and $b = D[\phi]$ being the state variable. Dynamic programming technique[13] decomposes a multistage decision problem into a sequence of single stage decision problems. Since the decision variable x_i can take on any value in the range permitted it can be treated as a continuous variable and the continuous dynamic programming technique can be used to solve the problem by employing any standard optimization algorithm to arrive at the stage optimum. The problem can also be converted to standard discrete dynamic programming type by assuming that the decision variable x_i can take on only values such that $x_i = m\Delta x$ where $m = 0, 1, 2, \ldots, y_i$. The problem then becomes

$$\min Z = \sum_{i=1}^{8} f(y_i \Delta x) \tag{12}$$

$$\text{subject to } \sum_{i=1}^{8} (a_i \Delta x) y_i \leq b$$

where y_i are integers greater than zero. The stage optimum is determined by a simple enumeration. The proper value of a_i is to be chosen by trial and error. In both continuous and dynamic programming as the coefficients a_i are functions of the input angle θ the allocation is valid only for the particular position. However, it is generally found that if the allocation is done for the position of maximum mechanical error it is valid for other positions also *i.e.*, the mechanical error in other positions does not exceed the prescribed value. For this purpose the allocation is carried out first in some arbitrary position. The mechanism is then analysed to find the position of maximum mechanical error and the allocation is carried out at this position to get the final values of clearances and tolerances.

In the present work computer programs in FORTRAN have been developed to implement both the continuous dynamic programming and discrete dynamic programming techniques. The former incorporates the complex method of Box[14] for stage optimization. A value of 75 for y_i is found to give satisfactory results in the discrete programming technique. For both the techniques the range of the state variable 'b' is divided into 100 equal parts.

4.2 Parametric programming

Since the constraints in eqn. (10) will have to be satisfied for all positions they can be expressed with the input angle $\theta(\theta_s < \theta < \theta_f$ as shown in fig. 1) as a parameter. Hence the problem can be stated as

$$\min Z = \sum_{i=1}^{8} \frac{1}{x_i} \tag{13}$$

subject to

$$\text{(i)}\quad g(x, \theta) = \sum_{i=1}^{8} (\partial\phi/\partial V_i)_{m}^{2} x_i - D[\phi] \geq 0$$

and the non-negativity constraint,

$$\text{(ii)}\quad g_i(x) = -x_i \leq 0, \text{ where } x_i = D[V_i].$$

The problem is now formulated as a parametric programming problem. It can be solved by the interior penalty function method of constrained optimization. The penalty function is taken as

$$\phi(x, r_k) = \sum_{i=1}^{8} \frac{1}{x_i} - r_k \left[\int_{\theta_s}^{\theta_f} \frac{1}{g(x, \theta)}\, d\theta + \sum_{i=1}^{8} \frac{1}{g_i(x)} \right] \tag{14}$$

where r_k is the penalty parameter. The minimization of $\phi(x, r_k)$ can be carried out utilizing any standard technique of unconstrained minimization.

In the present work a FORTRAN programme has been developed to solve the parametric programming problem employing the Davidon–Fletcher–Powell method[15] of unconstrained minimization.

The minimization process begins with a feasible set of x_is so that no constraint is violated. The choice of penalty parameter is arbitrary but has to be reduced in successive steps. As it is possible to achieve quicker convergence by assuming a moderate value for r_k, it is assumed to be 1.0 to start with. The choice of proper convergence criterion is important. Theoretically, $\sqrt{\sum_{i=1}^{8} (\phi/dx_i)^2}$ should be equal to zero at the optimum point. As this is not possible in practice, the algorithm is terminated when the value of this function attains a predetermined small number. This number is taken as 0.001 in the present case.

5. Illustrative examples and discussion of results

To illustrate the application of continuous dynamic programming (CDP), discrete dynamic programming (DDP) and parametric programming (PP) techniques for optimal allocation of tolerances and clearances in four-bar function generators examples of linkages presented in Garret and Hall[2] for generating the functions $y = \sin x$, $y = \log_{10} x$ and $y = x^2$ have been considered (Table I). The allocation problem has been solved for (i) $\pm 1°$ and (ii) $\pm 0.5°$ limit on mechanical error considering both normal (N) and uniform (U) distributions for the location of pin centre in the respective clearance circles. Initially the allocation is done in an arbitrary position and then the mechanism is analysed to locate the position of maximum mechanical error. The final allocation is carried out at the position of maximum mechanical error. The positions of maximum mechanical error for the mechanisms generating the functions $\sin x$, $\log_{10} x$ and x^2 were found to be at values of input angle θ equal to 180.211, 8.602 and 179.001° respectively. The results of final allocation along with the computation times required for solution of each allocation problem on DEC 1090 computer are listed in Tables II–IV.

To illustrate the allocation procedure in a typical case, allocation by continuous dynamic programming at an arbitrary position of 196.211° for the mechanism generating the function $y = \sin x$ with $\pm 1°$ error and normal distribution for location of pin centre in the respective clearance circles gave the tolerances and clearances in units of 10^{-3} in as,

tolerances: $l_1 - 3.059$, $l_2 - 2.993$, $l_3 - 2.927$, $l_4 - 3.621$,

clearances: $r_{12} - 3.164$, $r_{23} - 2.881$, $r_{34} - 3.238$, and $r_{41} - 3.641$.

Figure 3 shows the 3σ band of mechanical and total error for the mechanism with these tolerances and clearances. It is found from the figure that the maximum mechanical error of 1.05° (which is greater than the prescribed limit of $\pm 1°$) occurs at an angle of 64° from the starting position *i.e.*, at $\theta = 116.211 + 64 = 180.211°$. The optimal allocation is now carried out at this position and the results of this allocation are listed in the first column of Table II. The 3σ bands of mechanical and total error of the mechanism with these

Table I
Optimal link lengths with zero clearances and tolerances[2]

Function	$y = \sin x$	$y = \log x$	$y = x^2$
Range of x	0–90	1–10	0–1
Range of θ (deg)	90	90	90
Range of ϕ (deg)	90	90	90
θ_s (deg)	116.211	8.602	147.002
ϕ_s (deg)	72.852	109.681	233.353
1	1	1	1
2	2.06980	1.34034	2.70687
3	2.42146	2.47267	3.50957
4	0.74613	2.52348	0.56650

Table II
Results of allocation for the mechanism to generate $y = \sin x$ with permissible mechanical error of $\pm 1°$ and $\pm 0.5°$

		Results of the present work						Results from literature				
		CDP		DDP		PP		Ref. 6		Ref. 7		Ref. 12
		N	U	N	U	N	U	N	U	N	U	
Mechanical error: $\pm 1°$												
Tolerances	l_1	2.609	2.606	2.669	2.669	2.550	2.597	2.628		2.82	2.77	2.47
(Unit: 10^{-3} in)	l_2	2.771	2.769	2.798	2.798	2.549	2.601	2.612		2.52	2.57	2.41
	l_3	2.746	2.744	2.773	2.773	2.533	2.577	2.593		2.37	2.88	2.94
	l_4	3.227	3.220	2.845	2.845	2.824	2.965	2.846		2.92	2.88	2.43
Clearances	r_{12}	2.700	1.390	2.669	1.366	2.550	1.329	2.571		2.82	1.37	1.215
(Unit: 10^{-3} in)	r_{23}	2.597	1.350	2.798	1.365	2.550	1.331	2.556		2.52	1.26	1.205
	r_{34}	2.735	1.443	2.773	1.419	2.534	1.320	2.611		2.37	1.14	1.205
	r_{41}	2.994	1.350	2.845	1.456	2.824	1.520	2.952		2.92	1.40	1.215
CPU time (sec)			101.33	64.95	64.74	22.46	14.05	276 (IBM 7044)				
Mechanical error: $\pm 0.5°$												
Tolerances												
(Unit: 10^{-3} in)	l_1	1.360	1.359	1.194	1.194	1.285	1.279	1.294		1.36		
	l_2	1.356	1.355	1.399	1.399	1.282	1.282	1.286		1.32		
	l_3	1.404	1.403	1.386	1.386	1.261	1.263	1.277		1.13		
	l_4	1.463	1.527	1.423	1.423	1.482	1.483	1.401		1.23		
Clearances	r_{12}	1.404	0.669	1.194	0.683	1.285	0.655	1.266		0.70		
(Unit: 10^{-3} in)	r_{23}	1.358	0.697	1.399	0.683	1.282	0.656	1.259		0.68		
	r_{34}	1.369	0.718	1.386	0.709	1.261	0.646	1.285		0.64		
	r_{41}	1.390	0.705	1.423	0.728	1.482	0.760	1.453		0.83		
CPU time (sec)			112.20	65.30	65.28	11.46	25.63	276 (IBM 7044)				

Table III

Results of allocation for the mechanism to generate $y = \log x$ with permissible mechanical error of $\pm 1°$ and $\pm 0.5°$

		Results of the present work						Results of tolerance			
		CDP		DDP		PP		Ref. 6		Ref. 7	
		N	U	N	U	N	U	N	U	N	U
Mechanical error: $\pm 1°$											
Tolerances	l_1	3.799	3.792	3.799	3.799	3.606	3.607	3.727		3.68	4.99
(Unit: 10^{-3} in)	l_2	4.637	4.357	4.247	4.247	4.132	4.129	4.412		4.85	4.35
	l_3	2.749	2.748	2.876	2.876	2.632	2.634	2.833		2.85	2.33
	l_4	2.850	2.466	2.579	2.579	2.656	2.655	2.860		2.70	2.26
Clearances	r_{12}	3.613	1.865	3.799	1.944	3.607	1.845	3.798		3.68	2.03
(Unit: 10^{-3} in)	r_{23}	4.351	2.056	4.247	2.173	4.132	2.113	4.295		4.85	2.01
	r_{34}	2.855	1.394	2.876	1.472	2.632	1.347	2.829		2.85	1.36
	r_{41}	2.595	1.541	2.579	1.320	2.654	1.358	2.860		2.70	1.65
CPU time (sec)		104.62	98.49	65.37	65.32	23.09	16.66	279 (IBM 7044)			
Mechanical error: $\pm 0.5°$											
Tolerances	l_1	1.997	1.996	1.899	1.899	1.704	1.854				1.66
(Unit: 10^{-3} in)	l_2	2.152	2.151	2.124	2.124	2.357	2.075				1.97
	l_3	1.362	1.361	1.438	1.438	1.281	1.287				1.05
	l_4	1.348	1.304	1.290	1.290	1.315	1.322				1.33
Clearances	r_{12}	1.632	0.982	1.899	0.972	1.704	0.949				1.01
(Unit: 10^{-3} in)	r_{23}	2.009	1.022	2.124	1.087	2.357	1.061				1.37
	r_{34}	1.362	0.723	1.438	0.736	1.285	0.654				0.84
	r_{41}	1.506	0.707	1.290	0.660	1.314	0.675				0.82
CPU time (sec)		104.70	93.19	65.11	64.63	20.19	16.38				—

Table IV
Results of allocation for the mechanism to generate $y = x^2$ with permissible mechanical error of $\pm 1°$ and $\pm 0.5°$

		Results of the present work						Results from literature	
		CDP		DDP		PP		Ref. 7	
		N	U	N	U	N	U	N	U
Mechanical error: $\pm 1°$									
Tolerances	l_1	3.840	3.951	3.671	3.671	3.709	3.704	3.60	3.31
(Unit: 10^{-3} in)	l_2	3.955	3.777	3.681	3.681	3.667	3.661	3.55	2.93
	l_3	4.044	3.685	4.104	4.104	3.661	3.670	3.61	3.04
	l_4	7.177	8.020	6.855	6.855	8.320	8.317	8.81	7.25
Clearances	r_{12}	3.788	1.957	3.671	1.879	3.709	1.901	3.60	2.06
(Unit: 10^{-3} in)	r_{23}	3.939	1.902	3.681	1.883	3.667	1.873	3.55	2.10
	r_{34}	3.683	2.064	4.104	2.100	3.660	1.877	3.61	2.10
	r_{41}	6.852	4.163	6.855	3.508	8.321	4.240	8.81	4.70
CPU time (sec)		96.92	98.39	65.40	65.79	17.2	13.24		
Mechanical error: $\pm 0.5°$									
Tolerances	l_1	1.919	1.917	1.836	1.836	1.850	1.831		1.96
(Unit: 10^{-3} in)	l_2	1.981	1.980	1.840	1.840	1.874	1.754		1.70
	l_3	1.842	1.842	2.052	2.052	1.773	1.793		1.70
	l_4	3.434	3.729	3.428	3.428	3.909	4.649		4.52
Clearances	r_{12}	1.974	0.979	1.836	0.939	1.851	0.937		0.96
(Unit: 10^{-3} in)	r_{23}	2.036	1.013	1.840	0.942	1.887	0.898		0.91
	r_{34}	1.902	0.970	2.052	1.050	1.776	0.946		0.89
	r_{41}	3.485	1.942	3.428	1.754	3.909	2.379		2.15
CPU time (sec)			96.36	64.36	65.09	15.36	23.83		

tolerances and clearances are shown in fig. 4. It can be seen that the mechanical error over the entire range is now constrained to lie within $\pm 1°$.

Comparing the three techniques for optimal allocation of tolerances and clearances it is seen from Tables II to IV that continuous and discrete dynamic programming techniques yield tolerances and clearances of nearly same magnitude while parametric programming technique yields generally stricter tolerance and clearance values. For any particular technique assumption of uniform distribution as against the normal distribution for location of pin centres in the respective clearance circles leads to considerably stricter clearances. As far as the computation times are concerned the continuous dynamic programming technique takes maximum time followed by the discrete dynamic programming and parametric programming techniques. Considering both the magnitude of tolerance and clearance values and the speed of computation the discrete dynamic programming technique appears to be the most suitable for allocation of tolerances and clearances in function-generator linkages.

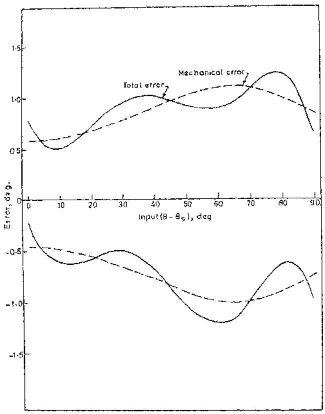

FIG. 3. Error after allocation at an arbitrary position (θ–θ_s = 80°).

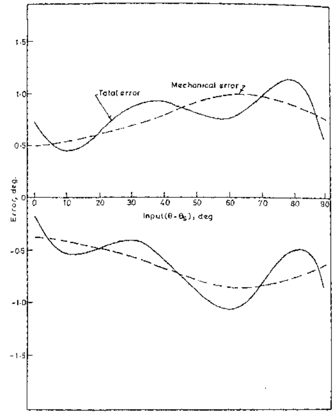

FIG. 4. Error after allocation at maximum mechanical error position θ–θ_s = 64°).

6. Conclusions

The techniques of continuous dynamic, discrete dynamic and parametric programming have been applied to solve the problem of optimal allocation of tolerances and clearances in four-bar function-generator linkages. It is found that the discrete dynamic programming technique takes moderate computation times and yields comparatively liberal tolerances and clearances.

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