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Designing a serrated trough for an oscillating conveyor*

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Abstract

Oscillating conveyors are used in industries to convey a variety of materials. But conveying becomes difficult when it is required to convey up an incline, to convey rolling particles or to convey the material with low coefficient of friction. A new idea of using a trough with saw-tooth like serrations has been proposed by Konnur and Parameswaran to overcome this difficulty. The present paper suggests a simplified method of determining the dimensions of serrations. The operating parameters under which a serrated trough gives better performance than a conventional plane trough are also indicated.

Key words: Oscillating conveyors, serrations, conveyance.

1. Introduction

Oscillating conveyors are commonly used in industries for conveying a variety of bulk materials such as dry chemical powders, ores, sand, food grains, etc. Earlier studies [1,2] have shown that these conveyors pose difficulties and limitations while conveying rolling particles such as steel balls, while conveying up an incline and while conveying materials with low coefficient of friction.

These difficulties are due to excessive backward relative displacement of the particles. An earlier study[3] conducted by the authors has shown a method of overcoming such difficulties by using a new type of trough with saw-tooth like serrations which help in preventing the backward sliding or rolling of the particles. A generalized analysis of motion of a rolling particle by simulating its motion on a digital computer has also been presented in that study.

In the present paper a simplified method of estimating the dimensions of serrations is described. Only pure rolling and pure sliding types of motion are considered. However, there is a possibility of combined sliding and rotation during the forward relative displacement of the particle. But in order to simplify the estimation this case can be considered similar to the one of pure sliding motion. The error involved will be on the safer side and gives a conservative estimate of the serration step length.

The size of the particle affects the conveyability. A simple analysis shows how to take into account the size of the particle while deciding the dimensions of the serration.

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The performance curves for plane and serrated troughs are superimposed in order to get an idea of the regions of operating parameters under which a serrated trough can be preferred to a plane trough.

2. Analysis of motion of a particle

For the purpose of analysis a spherical particle is considered and only the sliding and rolling frictions are taken into account. It is assumed that the particle is small in comparison with the size of the serration, the particle does not leave the contact with the trough surface until it reaches the end of the serration step and all the impacts between the particles and the walls of the serration are fully plastic.

2.1. *Equations of motion*

A schematic diagram of a rolling particle resting on a sinusoidally oscillating trough is shown in fig. 1. The motion of the trough is governed by

$$\bar{X} = \frac{X}{r} = \sin\tau\cos\alpha \tag{2.1}$$

$$\bar{Y} = \frac{Y}{r} = \sin\tau\cos\alpha \tag{2.2}$$

where $\tau = \omega t$.

Considering the free body diagram of the particle the following equations can be written

$$F = m(g\sin\beta' + \ddot{x}) \tag{2.3}$$

$$N = m(g\cos\beta' + \ddot{y}) \tag{2.4}$$

$$I\ddot{\theta} = -F\frac{D}{2} - kN \tag{2.5}$$

where $I = am\dfrac{D^2}{4}$ = mass moment of inertia and

a = a constant depending upon the geometry of the particle.

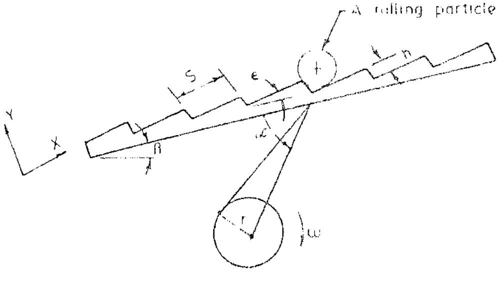
2.2. *Pure rolling of the particle*

When the particle is in contact with the short side of the serration the relative backward movement is positively prevented and the particle sticks to the trough until either rolling or sliding or both are initiated in the forward direction. The rolling begins at the instant

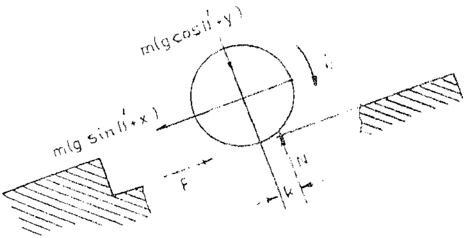
$$\tau = \tau_{R1}$$

when $-m(g\sin\beta' + \ddot{x}) = \dfrac{2k}{D}N$

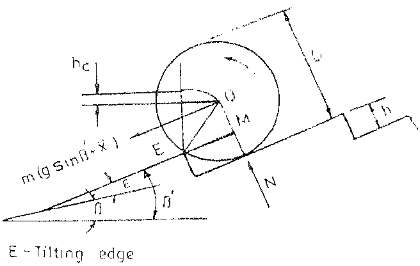
which gives
$$\sin\tau_{R1} = \frac{g}{r\omega^2}\,\frac{\sin\beta' + \dfrac{2k}{D}\cos\beta'}{\cos\alpha' + \dfrac{2k}{D}\sin\alpha'} \tag{2.6}$$



a. Oscillating serrated trough.



b. Free body diagram of the rolling particle.



c. Backward tilting of the particle.

FIG. 1. Conveyance of a rolling particle on a sinusoidally oscillating serrated trough.

Eliminating F from equations (2.3) and (2.5) and re-arranging the terms, the equation of motion can be written as

$$\ddot{x} = A_x + B_x \sin \tau \tag{2.7}$$

where $$A_x = -\frac{1}{1+a}\left(\frac{g}{r\omega^2}\right)\left(\sin \beta' + \frac{2k}{D}\cos \beta'\right) \tag{2.8}$$

and $$B_x = \frac{1}{1+a}\left(\frac{2k}{D}\sin \alpha' - a \cos \alpha'\right). \tag{2.9}$$

This motion will end in two possible ways, *viz*: (1) from pure rolling to combined rotation and slipping and (2) stoppage of rolling leading to zero relative velocity.

The first type of termination of pure rolling occurs at the instant of time τ_{RS} when

$$|F| = \mu_s N$$

which gives $$\sin \tau_{RS} = \frac{g}{r\omega^2}\,\frac{\sin \beta' + \lambda \cos \beta'}{\cos \alpha' + \lambda \sin \alpha'} \tag{2.10}$$

where $$\lambda = (1+a)\mu_s - \operatorname{sgn} \dot{\theta}\left(\frac{2k}{D}\right). \tag{2.11}$$

When the second type of termination occurs at $\tau = \tau_{R2}$, $\dot{x} = \dot{x}_c = \dot{X}$.
Solving equation (2.7) with the end conditions at $\tau=\tau_{R1}$, $\dot{x}=\dot{\bar{X}}$ and τ_{R2}, $\dot{x}=\dot{\bar{X}}$

$$\tau_{R2} + \frac{\cos \tau_{R2}}{\sin \tau_{R1}} = \tau_{R1} + \frac{\cos \tau_{R1}}{\sin \tau_{R1}}. \tag{2.12}$$

From this equation τ_{R2} can be obtained.

If μ_s is very high, $\tau_{R_s} > \tau_{R2}$

and the particle will have only pure rolling until it terminates. In this condition, maximum relative displacement occurs during the interval $\Delta_r = \tau_{R2} - \tau_{R1}$.

Integrating equation (2.7) twice with the initial condition at τ_{R1}, $\dot{x} = \dot{\bar{X}}$ and $\bar{x} = \bar{X}$ and simplifying,

$$\frac{(1+a)\,(\bar{x}_r)_{\max}}{\frac{2k}{D}\sin \alpha' + \cos \alpha'} = \left(1 - \frac{\Delta_r^2}{2}\right)\sin \tau_{R1} + \Delta_r \cos \tau_{R1} - \sin \tau_{R2}. \tag{2.13}$$

Representing $$\frac{(1+a)\,(\bar{x}_r)_{\max}}{\left(\frac{2k}{D}\right)\sin \alpha' + \cos \alpha'} = \eta$$

and $$\frac{1}{\sin \tau_{R1}} = \frac{r\omega^2}{g}\,\frac{\cos \alpha' + \left(\frac{2k}{D}\right)\sin \alpha'}{\sin \beta' + \left(\frac{2k}{D}\right)\cos \beta'} = \xi.$$

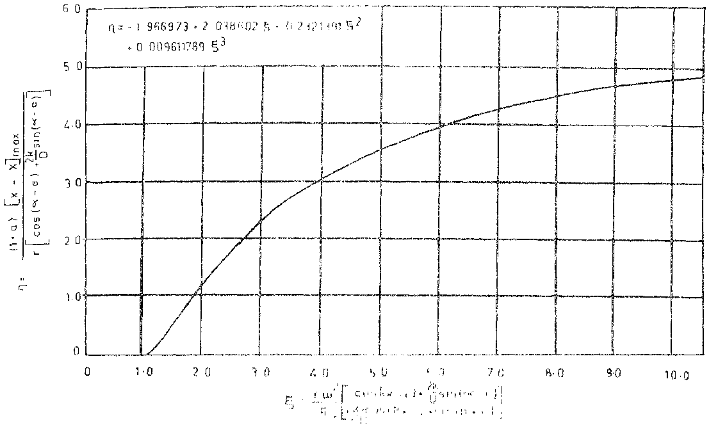


FIG. 2. Influence of ξ on the distance travelled by a rolling particle on the long side of serration during jumpless motion.

The values of η can be computed by taking a set of values of ξ from equation (2.13). The relationship between η and ξ is shown in fig. 2. The empirical equation for this curve can be

$$\eta = -1.966973 + 2.038602\xi - 0.232139\xi^2 + 0.0096117\xi^3. \tag{2.14}$$

2.3. *Pure sliding of the particle*

This motion is similar to that of a non-rolling particle and occurs if the value of k is large, such that

$$\frac{2k}{D} > \mu_s .$$

From the condition of relative rest the forward sliding motion begins at τ_{s1}, given by

$$\sin \tau_{s1} = \frac{g}{r\omega^2} \frac{\sin \beta' + \mu_s \cos \beta'}{\cos \alpha' + \mu_s \sin \alpha'} \tag{2.15}$$

Considering only the equations (2.3) and (2.4) and substituting $F = \mu N$ the equation of motion during sliding can be written as

$$\ddot{x} = A + B \sin \tau \tag{2.16}$$

where $$A = -\frac{g}{r\omega^2}[\sin\beta' + \mu\cos\beta']$$ (2.17)

and $$B = \mu\sin\alpha'.$$ (2.18)

Equation (2.16) is of the same form as that of equation (2.7). Also the end conditions at $\tau = \tau_{s1}$, $\dot{x} = \dot{X}$, $\dot{x} = \dot{X}$ and at $\tau = \tau_{s2}$, $\dot{x} = \dot{X}$ are similar to those considered under pure rolling type of motion. Therefore the equation for maximum relative displacement $(\bar{x}_r)_{max}$ occurring during the interval $\Delta_s = \tau_{s2} - \tau_{s1}$ is also similar to equation (2.13) and by representing

$$\frac{1}{\sin\tau_{s1}} = \frac{r\omega^2}{g}\frac{\cos\alpha' + \mu\sin\alpha'}{\mu\cos\beta' + \sin\beta'} = \xi'$$

and $$\frac{(\bar{x}_r)_{max}}{\mu\sin\alpha' + \cos\alpha'} = \eta'$$

the relationship between ξ' and η' is given by the same curve as shown in fig. 2 and the same empirical relationship as given by equation (2.14) can be used by replacing ξ and η by ξ' and η' respectively.

2.4. *Combined sliding and rotation*

The particle will move with combined rotation and sliding if $\tau_{R2} > \tau_{RS} > \tau_{R1}$.

Table I
Correction factors C for maximum relative displacement
$D = 3$ mm, $r = 4.55$ mm, $\alpha = 40°$,
$k = 0.05$ mm, $\varepsilon = 10°$.

β	Frequency cycles per min.	$\mu = 0.15$	$\mu = 0.2$	$\mu = 0.25$	$\mu = 0.3$	$\mu = 0.35$
0	480	1.0569	1.1179	1.1880	1.26	1.35
0	500	1.0382	1.0927	1.1394	1.213	1.28
0	520	1.0250	1.0734	1.1201	1.176	1.228
0	540	1.0147	1.0564	1.096	1.14	1.187
10	480	1.0618	1.1073	1.162	1.223	1.3
10	500	1.0519	1.0929	1.134	1.183	1.238
10	520	1.0443	1.0795	1.115	1.1535	1.197
10	540	1.036	1.067	1.098	1.129	1.165
20	480	1.002	1.024	1.0663	1.11	1.1775
20	500	1.008	1.034	1.0615	1.094	1.1342
20	520	1.019	1.038	1.0602	1.083	1.1121
20	540	1.024	1.039	1.0597	1.078	1.0985

The exact analysis of motion can be done by simulating the motion on a computer[3]. However, in order to simplify the design procedure an assumption can be made that the major portion of forward relative displacement is due to sliding. With this assumption $(\bar{x}_r)_{\max}$ can be calculated from

$$(\bar{x}_r)_{\max} = C\eta'[\mu \sin \alpha' + \cos \alpha']$$

where C is the correction factor.

A set of values of C obtained by conducting exact analysis for a typical set of operating conditions is given in Table I.

For smaller values of μ, higher values of β and frequencies the correction factor is very small. Therefore, in such cases even if $C = 1$ is taken, one would estimate a smaller and hence a conservative value of the step length of serration.

The particles of most of the commercial bulk materials are not exactly spherical and the values of k are quite large. Due to this the sliding motion will be predominant during their relative displacement. Also in the case of materials with smaller coefficient of friction the sliding type of motion predominates.

Thus, estimating the dimensions of serrations by assuming pure sliding, though approximate, can be justified.

3. Influence of the size of the particle

When the particle size in comparison with that of the serration increases beyond a certain limit, the effectiveness of the serrated trough vanishes. This is because of the probability of backward tilting of the rolling particle and its tendency to climb the edge of the short side of the serration. There are two circumstances during which a rolling particle would experience backward tilting over the edge of the serration. These are: (i) during the positive acceleration of the trough, and (ii) during the impact of the particle with the short side of the serration when the particle relatively rolls or slides down along the long side of the serration.

3.1. Tilting during the acceleration of the trough

Referring to fig. 1c and taking moments about the tilting edge E, the tilting will be possible if

$$\left(\frac{D}{2} - h\right)(g \sin \beta' + r\omega^2 \cos \alpha') \geqslant E_1(g \cos \beta' + r\omega^2 \sin \alpha') \tag{3.1}$$

where $E_1 = \dfrac{D}{2}\sqrt{1 - \left(1 - \dfrac{2h}{D}\right)^2}$

Simplifying and rearranging the terms the tilting will be possible if

$$\frac{2h}{D} \leqslant 1 - \frac{1}{\sqrt{1+z^2}} \tag{3.2}$$

where $z = \dfrac{\sin \beta' + \dfrac{r\omega^2}{g} \cos \alpha'}{\cos \beta' + \dfrac{r\omega^2}{g} \sin \alpha'}$

3.2. Climbing over the edge due to impact

For the purpose of estimation the relative velocity at the time of impact is taken as the velocity of impact. This assumption is justified since the trough mass is very large compared to that of the particle. Also it is assumed that the particle will not rebound after impact, in order to simplify the problem.

The particle tends to rotate about the edge E during impact. Applying the principle of impulse and momentum to the particle the following equation can be written.

$$I\dot{\theta}_1 + mv_1\left(\frac{D}{2} - h\right) + 0 = I\dot{\theta}_2 + mv_2\frac{D}{2} \tag{3.3}$$

$v_1 = \dot{x}_1 - \dot{X}_1$ = relative velocity at the time of impact. Since the particle rotates about E,

$$v_2 = \frac{D}{2}\dot{\theta}_2 .$$

Substituting in equation (3.3)

$$v_2 = \left[1 - \frac{2h}{D(1+a)}\right] v_1 . \tag{3.4}$$

If the particle has to go backwards it has to climb on the edge so that its centre of gravity rises to a height h_c. Assuming that the particle falls on the back step as soon as it reaches a height h_c even if the velocity of its centre of gravity reduces to zero, and applying the principle of conservation of energy backward motion will occur if

$$\tfrac{1}{2} mv_2^2 + \tfrac{1}{2} I\dot{\theta}_2^2 \geqslant mgh_c \tag{3.5}$$

which gives

$$v_2^2 \geqslant \frac{2gh_c}{1+a}. \tag{3.6}$$

Table II
Computed values $(2h/D)_{min}$ for the conveyance of steel balls
$\alpha=40°$, $\mu=0.15$, $r=4.55$ mm,
$k=0.05$ mm, $D=3$ mm.

$\varepsilon(°)$	$\beta(°)$	$(2h/D)_{min}$ for different frequencies (Hz)		
		8	9.33	10
	0	0.71	0.35	0.78
	5	0.82	0.62	0.87
	10	0.96	0.86	0.88
10	15	1.03	1.01	0.93
	20	1.1	1.1	1.06
	25	1.13	1.15	1.1
	30	1.13	1.16	1.16
	0	0.81	0.66	0.42
	5	0.9	0.89	0.73
	10	0.97	1.01	0.93
20	15	1.01	1.05	1.01
	20	1.04	1.08	1.07
	25	1.06	1.1	1.11
	30	1.07	1.13	1.13

From geometrical relationship

$$h_c=\frac{D}{2}\left[1-\sin\left(\beta'+\sin^{-1}\left(1-\frac{2h}{D}\right)\right)\right] \tag{3.7}$$

substituting for v_2 and h_c in equation (3.6) and simplifying the condition for back climbing can be written as

$$v_1^2 \geqslant \frac{gD}{1+a}\left[\frac{1-\sin\left(\beta'+\sin^{-1}\left(1-\frac{2h}{D}\right)\right)}{\left(1-\frac{2h}{D(1+a)}\right)^2}\right] \tag{3.8}$$

The results of a study conducted by taking some typical sets of operating conditions to calculate the minimum ratio of $2h/D$ below which the particle can travel backwards by climbing over the edge are given in Table II.

4. Designing the serrated trough

The design consists of determination of the dimensions S, h and ε of the serrations. Since ε is involved in the equations of the relative displacement it has to be initially assumed. While S is determined from the $(\ddot{x}_r)_{max}$ values, h depends upon the size D of the particle.

4.1. Determination of S

The conveyance will occur if during the forward relative displacement the particle travels up to or beyond the length of step S and falls on the next step. Therefore the condition for conveyance will be

$$S \leqslant (x_r)_{\max}$$

i.e. $$S \leqslant r(\bar{x}_r)_{\max}\,.$$

4.2. Determination of h

Once S is determined after assuming a suitable value of ε,

$$h = S \tan \varepsilon.$$

This value of h has to be such that

$$\frac{2h}{D} \geqslant \left(\frac{2h}{D}\right)_{\min}.$$

Some typical values of $(2h/D)_{\min}$ given in Table II will help to estimate a suitable value under a given set of operating conditions.

5. A comparison between a serrated trough and a plane trough

For the purpose of comparison of the performance of a serrated trough with that of a plane trough the particle is considered to be non-rolling and all the parameters are expressed in non-dimensional groups. However, for the serrated trough, typical values of μ and ε are taken for the study. Designating the non-dimensional groups as:

$$P_1 = \frac{\mu g \cos \beta}{r\omega^2 \cos \alpha}$$

$$P_2 = \frac{\tan \beta}{\mu}$$

$$P_3 = \mu \tan \alpha$$

$$(V_R)_{PL} = \frac{\bar{x}_r}{\cos \alpha} \text{ for plane trough}$$

and $$(V_R)_{SR} = \frac{\bar{x}_r}{\cos \alpha \cos \varepsilon} \text{ for serrated trough.}$$

It can be shown[4] that

$$(V_R)_{PL} = f(P_1, P_2, P_3)$$

and $$(V_R)_{SR} = \phi(P_1, P_2, P_3, \mu, \varepsilon).$$

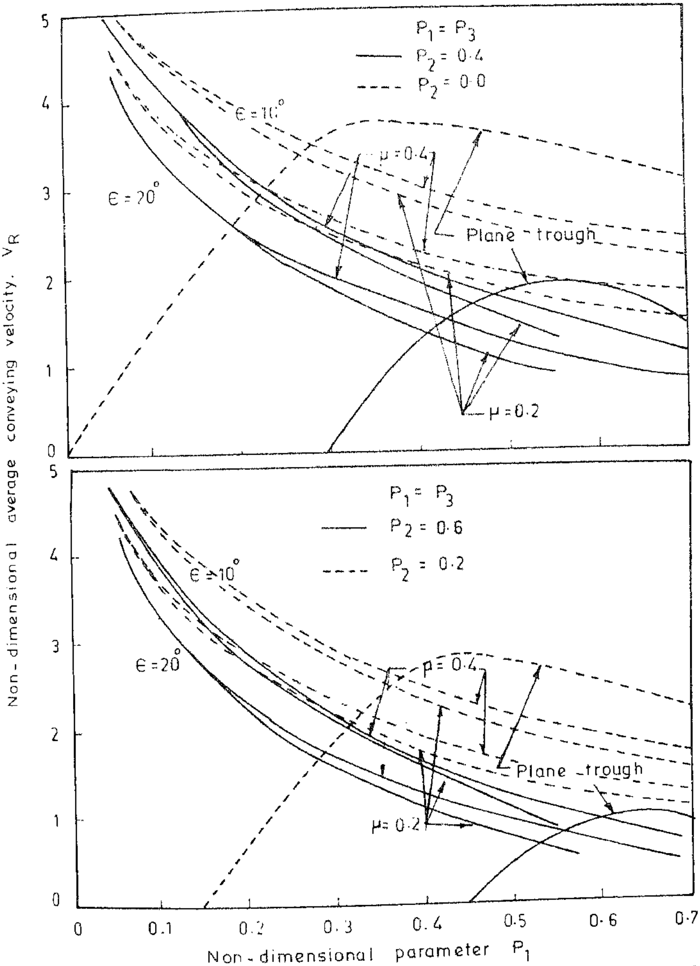


FIG. 3. Conveying velocity of a non-rolling particle on a serrated trough compared with that on a plane trough for $P_1 = P_3$.

It can be shown that the performance of the plane trough reaches its best when $P_1 = P_3$ for any μ and α^4. Therefore, it is possible to compare the performance of serrated trough with the best performance of a plane trough by taking $P_1 = P_3$.

Taking a set of P_1, P_2, μ and ε and using the condition $P_1 = P_3$, the values of $(V_R)_{SR}$ and $(V_R)_{PL}$ are computed and the results are plotted in fig. 3. These figures show the regions of operating conditions which indicate the superiority of one type of trough over the other.

6. Conclusions

The method of estimating the dimensions of the serration is quite simple. Both for purely rolling as well as purely sliding types of motion the same empirical equation as given in (2.14) can be used by using the appropriate variables (*i.e.* ξ and η or ξ' and η') depending on the type of motion. For higher values of β and lower values of μ (say less than 0.25) the estimation of $(\bar{x}_r)_{max}$ can be made by assuming pure sliding type of motion even in the case of combined sliding and rotation, without introducing serious errors. Even if the error is introduced it gives a conservative estimation of the size of the serration. The particle size plays an important role in deciding the height h of the serration. Therefore, the value of $(2h/D)_{min}$ should be properly selected. It is preferable to assume slightly higher values for $(2h/D)_{min}$ in order to be on the safer side. Figure 3 shows that for smaller values of P_1 and larger values of P_2 the serrated trough gives higher conveying velocity than that obtainable from a plane trough.

Nomenclature

a	a multiplying factor
A, A_x	non-dimensional factors
B, B_x	non-dimensional factors
D	diameter of the particle
F	a force acting in the direction parallel to the sliding surface
F_r	tangential force which the trough tries to impose on the particle
g	acceleration due to gravity
h	depth of serration
I	mass moment of inertia of the particle about an axis passing through its centre of gravity
k	lever arm of rolling friction
m	mass of the particle
N	a force acting in the direction normal to the sliding surface
P_1, P_2, P_3	non-dimensional parameters
r	amplitude of oscillation of the trough
S	length of the long side of the serration
t	time
V_R	non-dimensional average conveying velocity

x	displacement of the centre of gravity of a particle parallel to the sliding surface
x_r	displacement of the centre of gravity of a particle with respect to the trough surface along x-axis
X	displacement of the trough in the direction parallel to the sliding surface
Y	displacement of the trough in the direction normal to the sliding surface
α	throw angle
α'	$(\alpha - \varepsilon)$
β	angle between the trough surface and the horizontal
β'	$(\beta + \varepsilon)$
ε	angle of serration
θ	angle of rotation of the particle
μ	kinetic coefficient of friction
μ_s	static coefficient of friction
ξ, η	non-dimensional factors
τ	non-dimensional time
ω	circular frequency of the trough

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