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Optimized driving mechanisms for oscillatory conveyors*

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Abstract

The simplest linkages are investigated as driving mechanisms of constant load-pressure type horizontal oscillatory conveyors. Conveyance is optimized, subject to necessary constraints and permitting limited back-slip. The off-set slider-crank is found to give only marginal improvement over the centric slider-crank and that only when the transmission angle permitted is not high. Prefixing a double-crank with optimized dimensions to a centric slider-crank is found to more than double the conveyance, with good transmission angles. Numerical results are for a particular stroke and friction coefficient. Enough information is provided to enable computation under different conditions.

Key words: Oscillatory conveyors, slider-crank, driving mechanisms.

1. Introduction

The oscillatory conveyor is the oldest member of the family of conveyors using vibrations to convey material. It operates on the sliding principle. In the present paper we confine ourselves to constant-load-pressure type horizontal oscillatory conveyor, where horizontal trough moves in its own plane. The term constant-load-pressure type is used to distinguish it from the type in which the trough moves at an angle to its own plane. The latter is called as 'variable-load-pressure' type oscillatory conveyor. The constant-load-pressure type is still very much in use in mining applications[1]. The reciprocating feeder represents another popular application[2].

The working of this type of conveyor basically depends on the kinematics of the trough motion. Hence optimizing the drive that gives this trough motion ultimately optimizes the conveyance. An effort has been made in the present work to optimize the driving mechanism for maximum conveying velocity subject to some of the practical constraints.

Earlier work on this type of conveyor, while dealing with the question of optimizing the conveyance, has either depended on idealized motion diagrams[3,4] or has directly proceeded from a given type of mechanism[5]. Mueller and Mansour's work[5] is concerned with different modes of material motion and gives an example of optimizing mechanism proportions. The present work deals with obtaining the optimal dimensions and driving speed of the centric and off-set slider crank mechanisms for maximum conveyance and finding out how far the conveyance can be improved by introducing a double-crank in series. This work does not account for bulk material effects. It is found that the off-set

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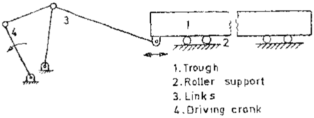

FIG. 1. Constant load-pressure type oscillatory conveyor.

slider-crank allows only marginal improvement over the centric slider-crank, partly due to the transmission angle limitation. The double-crank in series with the centric slider-crank is found to give more than double the conveyance while keeping reasonable transmission angle and proportions. The work by Kopylov[6], referred to in ref. (1), appears also to be concerned with the question of limits of capabilities of individual types of mechanisms.

2. Principle of operation

Consider a unit mass resting on the trough. The trough is made to oscillate in its plane (horizontal in this case), driven mechanically or otherwise (fig. 1). Typical velocity and acceleration curves for trough motion are shown in fig. 2.

If f_0 is the coefficient of static friction between the material and the trough, the maximum attainable particle acceleration is numerically gf_0. If the trough acceleration a exceeds gf_0 or falls below $-gf_0$, the material will slip on the trough. During this separate motion, a constant frictional force (depending on the coefficient of kinetic friction) will be acting on the material.

In fig. 2, positive trough acceleration is always below gf_0, but negative trough acceleration falls below the critical acceleration $(-gf_0)$ at point B on the velocity diagram. Hence the material separates from the trough at point B and executes uniformly decelerated motion as shown by the line BE on the velocity diagram. At point E, the velocities of the trough and the material become equal and joint motion starts again. The shaded area BCE on the velocity diagram indicates the displacement Δ of the particle relative to the trough during a cycle.

If, unlike as in fig. 2, the positive trough acceleration exceeds gf_0, then the material can also slip in the negative direction. In that case the net particle displacement will be

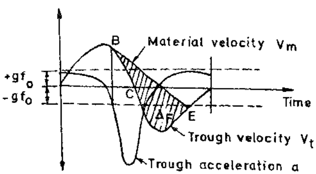

FIG. 2. Typical velocity and acceleration curves.

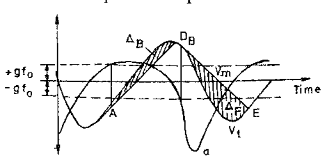

FIG. 3. Velocity and acceleration curves with backward slip.

$$\Delta = \Delta_F - \Delta_B \tag{1}$$

Δ_F = Forward displacement in cm

Δ_B = Backward displacement in cm.

If N is the speed of the driving crank in RPM, then the velocity of conveyance of the material is

$$V_{\mathrm{con}} = \frac{\Delta . N}{60} \mathrm{cm/s}. \tag{2}$$

The backward slip Δ_B refers to the total negative slip (Negative slip can occur more than once in a cycle). In the following analysis, distinction between static and kinetic coefficients of friction is neglected as a reasonable simplification.

3. Backward slip and steady-state condition

Figure 3 shows a case where the backward slip exists. At point A on the velocity diagram, trough acceleration exceeds gf_0 and the material slips on the trough. Velocities of the trough and the material again equalise at point D on the velocity diagram. The shaded area between A and D shows the backward slip Δ_B. The forward slip starts from point B on the velocity diagram, where the trough acceleration falls below $-gf_0$. It continues up to point E. The shaded area between B and E is the forward displacement Δ_F of the material during a cycle. The net particle displacement will be as given by equation (1) and the conveying velocity is given by equation (2).

After point E on the velocity diagram, the same cycle will repeat, provided the time interval between A and E is less than the time period of the trough motion. But if the time interval between points A and E is larger than the time period (or if the crank rotation from A to E is more than 360°), then the next cycle will be different. However, after simulating the particle motion for a few cycles the steady state is practically reached. The computed conveyance is based on this steady-state condition.

Backward slip can occur more than once in a cycle. This fact must be taken into account while computing the conveyance.

4. Effect of variation of crank speed

If the angle θ is used to specify the displacement of the input crank from some reference position and S indicates trough displacement from one of its end positions (positive in the direction of conveyance), then for any crank position $dS/d\theta$ and $d^2S/d\theta^2$ can be found out. These two quantities are independent of the angular speed of the crank ω. But trough velocity and trough acceleration will depend on ω and are given by

$$v = \omega \times dS/d\theta \tag{3}$$

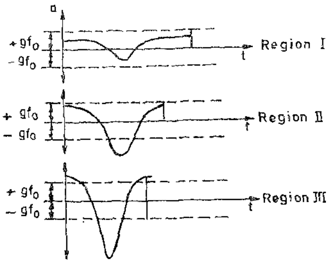

FIG. 4. Effect of ω on acceleration curve.

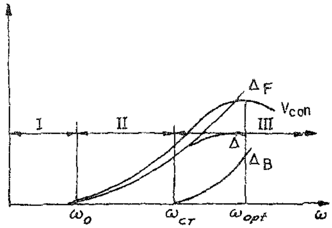

FIG. 5. Variation of conveyance with ω.

$$a = \omega^2 \,.\, \mathrm{d}^2S/\mathrm{d}\theta^2 \,. \tag{4}$$

Thus a change in ω will change the velocity and acceleration curves (fig. 4). Hence the conveyance will also change. Variation of conveyance per cycle with ω is shown in fig. 5. It shows three distinct regions, explained below:

Region I: In this region with $0 < \omega < \omega_0$, the maximum trough acceleration $a_{\max} < gf_0$, and the minimum trough acceleration $a_{\min} > (-gf_0)$. Since there can be no slip in either of the directions, there is no conveyance.

Region II: In this region with $\omega_0 < \omega < \omega_{\mathrm{cr}}$, we have $a_{\max} < gf_0$ and $a_{\min} < -gf_0$*.

Hence forward displacement of the material on the trough exists and backward displacement Δ_B does not exist.

At $\omega = \omega_{\mathrm{cr}}$ (fig. 4), $a_{\max} = gf_0$. For any ω below ω_{cr} (critical speed), backward slip does not exist.

Region III: In this region with $\omega_{\mathrm{cr}} < \omega$ we have $a_{\max} > gf_0$ and $a_{\min} < -gf_0$.

Hence both Δ_F and Δ_B exist. Net slip is the difference between the two.

The three cases shown in fig. 4 can be alternatively represented as in fig. 6. Here θ replaces t and $S'' = \mathrm{d}^2S/\mathrm{d}\theta^2$ replaces the acceleration. Accordingly there is only one S'' curve but different (gf_0/ω^2) limits. This approach is particularly advantageous for computation of conveyance.

The peak conveyance is obtained at $\omega = \omega_{\mathrm{opt}}$ in region III. Some authors, *e.g.*, Gutman[3], limit themselves to $\omega = \omega_{\mathrm{cr}}$ and avoid back-slip. Though this means additional wear of the trough and power, there is no reason why we should not go towards ω_{opt} and get more conveyance. It is a question of a trade-off between trough life and gain in capacity.

* The case of $a_{\max}$ going beyond gf_0 first, before $a_{\min}$ goes below $-gf_0$ represents net backward conveyance.

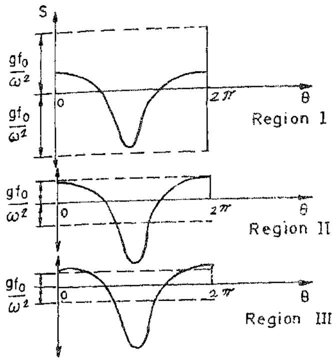

FIG. 6. The three slip conditions.

5. Computation of the conveyance

Conveyance, that can be theoretically obtained from a particular driving mechanism, can be computed using equations (1) and (2). The forward slip Δ_F and the backward slip Δ_B are the areas represented on the velocity-time diagram. These areas can be found without numerical or graphical integration as follows:

Referring to fig. 2, Δ_F, for example, is given by:

$$\Delta_F = \int_B^E (V_m - V_t)\,dt \tag{5}$$

where

V_m = Material velocity,

V_t = Trough velocity.

V_m can be expressed as $V_m = V_B - f_0 g(t - t_B)$ (6)

where

V_B = Trough velocity at point B,

t_B = Time corresponding to point B.

Substituting (6) in (5) and noting that $V_t dt = ds$ (s = trough displacement), we get

$$\Delta_F = (S_B - S_E) + (V_B(t_E - t_B) - \frac{f_0 g}{2}(t_E - t_B)^2 \tag{7}$$

where

S_B = Trough displacement at point B,

S_E = Trough displacement at point E,

t_E = Time corresponding to point E.

A similar equation can be written for Δ_B.

6. Off-set or centric slider-crank mechanism as the drive

The slider-crank mechanism is considered first, as it is the simplest. Both centric and offset slider-cranks are considered. Figure 7 shows the off-set slider-crank and the various parameters involved. r, l and h are the independent dimensions. For a given stroke X, the coupler length l can be calculated for a given set of r and h by the expression

$$l = (X/2)\,[1 + 4h^2/(X^2 - 4r^2)]^{1/2}. \tag{8}$$

The outer dead-centre position can then be laid out using the $(l+r)$ value. Assuming that conveyance to the right is required it should be laid to the right if h is positive and to the left if negative.

Trough velocity and acceleration can be found out, for any position, using the following equations:

$$V_t = \omega \,.\, \mathrm{d}S/\mathrm{d}\theta \tag{9}$$

$$a = \omega^2 \mathrm{d}^2 S/\mathrm{d}\theta^2 \tag{10}$$

where,

$$S = r\,.\cos\theta - \sqrt{(l-r)^2 - h^2} + \sqrt{l^2 - (h - r\sin\theta)^2}$$

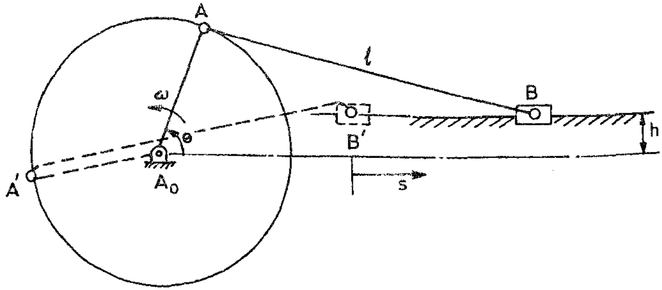

FIG. 7. Off-set slider crank.

With an increase in the stroke, keeping proportions and angular speed the same, the conveyance can be generally expected to increase. One can thus fix up as large a stroke as possible under the circumstances. The following studies are based on a fixed stroke.

An important constraint on the mechanism selection is the minimum transmission angle. From the point of view of inertia stresses, the accelerations were generally found to be low and did not act as a constraint. The free variables for an optimization search are: r, h and ω. Since only three variables are involved a grid search only was used, this providing for a better insight and more information.

A stroke of 20 cm and a coefficient of friction of $f_0 = 0{\cdot}4$ were chosen. For various values of off-set and connecting rod lengths conveyance obtained is plotted (fig. 8). The lines indicating minimum transmission angle are shown. This indicates that if we want to maintain a high value of transmission angle near 60°, the centric slider-crank is the best. At slightly lower values of transmission angle, off-set improves conveyance marginally but at the expense of more space. Hence, practically the best range for design is around the peak point of curve $h/r = 0$. A slightly higher value of l/r, than at the peak will improve the transmission angle and reduce the accelerations.

The optimum working speed of this mechanism is found to be in the range 75–85 RPM, for a coefficient of friction $f_0 = 0{\cdot}4$ and a stroke of 20 cm. Any increase in f_0

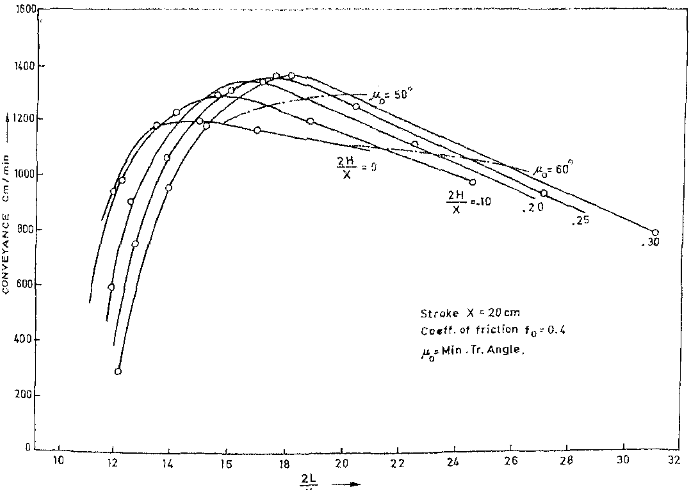

FIG. 8. Optimal conveyance with slider-crank mechanisms

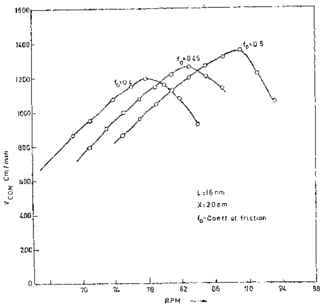

FIG. 9. Conveyance with centric slider crank.

is found to increase the optimum speed and also the conveyance (fig. 9). In practice, it is not possible to fix up the f_0 value accurately, but only a range can be given. Hence the working speed is to be chosen carefully to see that the conveyance does not fall down too much, even under extreme conditions.

7. Slider-crank prefixed with a double-crank

Prefixing a double-crank in series with a slider-crank has been found useful[1,3]. Properly connected, it enhances the asymmetry in the acceleration curve thus increasing the conveyance. The objective of the present section is to demonstrate how far conveyance can be improved in this way by optimizing the dimensions and driving speed. In view of what was found in the previous section regarding the off-set, the centric slider-crank only is considered here.

Figure 10 shows the entire driving mechanism scheme. Since the crank-radius r is directly decided by the stroke, we have the following free parameters to be varied in the optimization process: (a/d), (b/d), (c/d), γ, l and ω. The following constraints must be taken into account regarding the prefixed mechanism:

(i) The conditions for double-crank must be satisfied;
(ii) The minimum transmission angle of the double-crank should be above a specified limit;
(iii) The minimum of the analogue of the transmission angle on the input side should not be too low (This item is included firstly since there is reverse power flow for part of the cycle and secondly to indirectly maintain the dynamic characteristics of the mechanism at a reasonable level).
(iv) Link length ratios should be within reasonable limits (limiting a/d, b/d and c/d is believed to be enough in view of the control introduced over the transmission angles).

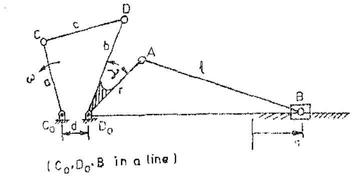

FIG. 10. Slider-crank prefixed with a double crank.

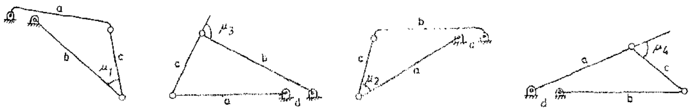

FIG. 11. Extreme transmission angles in the double-crank.

Figure 11 shows the double-crank mechanism C_0CDD_0 (of fig. 10) in the four positions in which the transmission angles of regular and reverse power flow reach their extremes. The angles μ_1, μ_2, μ_3 and μ_4 are self-explanatory. The output side angles μ_1 and μ_3 are preliminarily treated as part of the set of independent variables for the optimization process. Their prescribed limits (taken 30°–90° and 30°–150° in the present work) are maintained by a transformation of variables [*e.g.* $\mu_1 = (90°+\mu_1 \text{ min})/2+(1/2)(90°-\mu_1 \text{ min}) \sin\theta_1$, θ_1 being the new independent variable in place of μ_1; θ_2 replaces μ_3]. *It is to be noted that when μ_1 and μ_3 are both controlled, the mechanism automatically becomes a double-crank, provided (a/d) is greater than* 1. We make (a/d) as the third provisional independent variable and choose it between 1 and a prescribed maximum. This is done by a transformation of variable (third independent variable θ_3 in place of a/d).

Having chosen μ_1, μ_3 and (a/d), the double-crank in itself is fully defined and the link lengths (b/d) and (c/d) can be determined from

$$(b/d)+(c/d)=\sqrt{(Q+P)}$$

and $$(b/d)-(c/d)=\pm\sqrt{(Q-P)},$$

where

$$P=4(a/d)/(\cos\mu_1+\cos\mu_3)$$

and $$Q=P\cos\mu_1+(a/d-1)^2,$$

(a/d) has to be so chosen that $Q-P$ does not become negative. This amounts to choosing (a/d) such that:

$$(a/d)+(d/a)\geqslant 2(2-\cos\mu_1+\cos\mu_3)/(\cos\mu_1+\cos\mu_3).$$

The starting point of the optimization search being chosen to satisfy this constraint, it was found that the constraint was never violated during the subsequent search. Provision was made to deal with the constraint if it is encountered. The pattern search method of Hooke and Jeeves[7,8] was used.

Let us now consider the input side angles μ_2 and μ_4. The following significant result is proved in the appendix:
Denoting the least of μ_1, μ_2, μ_3, μ_4 as μ_{min}, $\mu_1 = \mu_{min}$ when $a < b$ and $\mu_2 = \mu_{min}$ when $b < a$.

Thus, if $b < a$, we have $\mu_2 < \mu_4$. If $a < b$, both μ_2 and μ_4 are $> \mu_1$. With the lowest value of μ_1 allowed being in any case not less than the lowest value allowed for μ_2 or μ_4, it is clear that we need consider only a limit on μ_2 and leave out μ_4. The optimization process is terminated when μ_2 reaches a prescribed lower limit (10° in the present work).

In addition to the three independent variables θ_1, θ_2, θ_3 of the double-crank proper, we have the following three additional variables for the search: γ, l and ω.

Some of the local optima obtained are listed in Table I and are for a stroke of 20 cm and a coefficient of friction of 0·4, as in the previous section on slider-crank. One solution with no limitation on μ_2 is given in the last row. Depending on the starting point either $\mu_{1\,min} (= 30°)$ or $\mu_{2\,min} (= 10°)$ limits the optimum.

It can be seen from Table I that the conveyance can be more than doubled while maintaining the transmission angles, by prefixing a double-crank mechanism.

8. Conclusion

The present work to some extent investigates the type of driving linkage that is best suited for a horizontal constant load-pressure type oscillatory conveyor. It also presents some local-optimal dimensions and speeds for a particular stroke and friction coefficient. Even though based on limited numerical results, it may be safely concluded that a double-crank with optimized proportions should invariably be prefixed to the centric slider-crank since it makes a tremendous difference to the performance of the conveyor.

Table I

Sl No	a/d cm	b/d cm	c/d cm	γ deg.	L cm	RPM	μ_1 deg.	μ_3 deg.	μ_2 deg.	V_{con} cm/min
1	3.36	3.40	1.30	−51.57	14.69	99.93	30.00	49.37	34.25	2862.96
2	3.99	4.02	1.27	−51.57	15.32	99.93	30.12	45.14	33.48	2892.16
3	2.62	2.54	1.24	−71.99	15.48	107.17	31.60	36.01	22.04	3344.84
4	6.00	6.05	5.80	−51.57	18.03	96.98	49.86	107·60	50.64	2481.15
5	4.85	1.25	4.88	−95.11	17.26	107·30	30.02	44.14	02.92	4269.64

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Appendix

To prove that μ_1 is the least of μ_1, μ_2, μ_3, μ_4 when $a<b$ and that μ_2 is the least when $b<a$.

We utilize the following relationships: $\cos\mu_1=[b^2+c^2-(a-1)^2]/(2bc)$, $\cos\mu_2=[a^2+c^2-(b-1)^2]/(2ac)$ and $\cos\mu_3=[(a+1)^2-b^2-c^2]/(2bc)$.

When $\mu_1<\mu_2$, *i.e.* $\cos\mu_1>\cos\mu_2$, we have: $[(b+a-1)^2-c^2](b-a)>0$. The first factor must be positive for a double-crank. Hence the second factor must also be positive. That is, when $\mu_1<\mu_2$, $a<b$ in a double-crank. Similarly, when $\mu_2<\mu_1$, $b<a$ in a double-crank.

Now if $\mu_3<\mu_1$, we obtain $b<a$. However, when $b<a$, μ_2 is critical and not μ_1. Hence we now compare μ_3 with μ_2. If $\mu_3<\mu_2$, we obtain $(a-b+1)^2>c^2$. This is impossible for a double-crank. Thus we see that μ_3 cannot be the minimum in a double-crank. It can be shown analogously that μ_4 cannot also be the minimum.

Combining the results of the above two paragraphs, the conclusion follows.