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Impact dampers*

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Abstract

This paper gives an introduction to impact dampers, which have been discussed in literature, and their extension to torsional oscillations, which appears to be new. It extensively surveys literature on impact dampers, and on the fundamental phenomenon of metal-to-metal impact in so far as it is of interest to impact dampers.

Key words: Coefficient of restitution, damping, Hertzian theory, impact, impact damper, MUVIN, resonance, stress wave, St. Venant's theory, torsional impact damper.

1. Introduction

Although mechanical impact dampers have been known for a long time and studied extensively, no mention seems to have been made about similar dampers in torsional oscillations. Study of such torsional impact dampers is being carried out in our lab. This paper gives an account of the research work in progress and planned for the future and a survey of existing literature on linear impact dampers and the phenomenon of impact in so far as it is applicable to the study of impact dampers.

Results of experiments on torsional impact dampers and a theoretical analysis of such dampers will be reported in due course of time.

2. The impact damper

Impact dampers consist of a solid particle free to move in a container with some clearance. When this is attached to a vibrating system, the vibration of the main mass, called the primary mass, causes the particle to move back and forth in the container. The particle hits against the sides of the container, and consequently momentum transfer takes place between the particle and the primary system. The collisions result in some energy dissipation, thereby providing additional damping in the system. It is an

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established fact that such a device is very effective in reducing considerably the vibration of the primary system, especially in the resonant region.

This device compares favourably with Frahm damper and is effective against both deterministic and random excitations in reducing the amplitude of vibration systems near resonance. However, repeated collisions of the particle against the side of the container give rise to large impact forces and excessive noise. The impact forces and the consequent contact stresses, surface wear and noise level can be considerably reduced if one colliding mass is replaced by several small impacting masses. It has been conclusively demonstrated, both analytically and experimentally, that if a single unit impact damper is replaced by a multi-unit one, in which a large number of small particles move in separate containers (one particle in each container) operating in parallel, the new multi-unit damper would be more efficient. Each particle is arranged to move in a separate channel so as to minimise the frictional forces on the particles, because it is known that friction has a detrimental effect on the damper efficiency.

Applications of impact dampers are numerous. They provide uniformly distributed damping forces and hence can be effective in all modes of vibration.

3. Literature survey

Paget[1] was the first to study impact dampers. Lieber and Jensen[2] were among the early investigators to discuss the design, development and some applications of impact dampers for aircraft vibration control. Assuming that the impacts are perfectly plastic (*i.e.* the coefficient of restitution is zero) and that the motion of the primary system is simple harmonic, they obtained a formula for using damping data in the theoretical analysis of aircraft vibration and flutter from work-energy considerations. McGoldrick[3] demonstrated experimentally that the resonant amplitude of a forced vibrating system could be reduced by an impact damper. Vigness[4] gave some theoretical considerations for forced vibration. Grubin[5] considered the problem with a finite coefficient of restitution and presented two methods of solution. The first method is an exact solution satisfying the initial conditions and determining the motion of the system from impact to impact, which is computationally tedious, if not impractical. The second method is to develop a steady-state solution by writing the solution after 'n' collisions and studying the asymptotic behaviour as 'n' goes to infinity. By using the latter method, the inverse problem (of determining the damper characteristics for a prescribed degree of vibration control) also can be numerically solved. The impact damper problem with an undamped primary system was analysed by Arnold[6] introducing an unknown phase angle between the motion and the applied harmonic force and replacing the impact force by a Fourier series. Frequency response curves for the amplitude of vibration were presented, but the experimental results did not agree very well with the theory. Warburton[7] gave a simpler analysis of the same problem. Lieber and Grubin[8, 9], Roitenberg[10], Galaka[11], Erlikh[12] and Kobrinskii[13] also have contributed to the study of impact dampers. Kaper[14] considered the impact damper operating in the vertical direction, thereby introducing a bias due to the weight of the impacting mass. Sadek[15] used the approach of Arnold[6] to obtain a solution in which the two impacts per cycle do not necessarily occur at equal intervals of time. The amplitude *versus* frequency curves obtained indicated a jump phenomenon typical of nonlinear systems.

Most of the studies referred to above were carried out under the assumption of two symmetric impacts per cycle. This assumption results in two possible solutions[5], but to know which, of the two possible motions, actually prevails, it is necessary to turn our attention to the question of stability of motion. Masri and Caughey[16] derived the exact solution for the symmetric two-impacts-per-cycle motion and determined the asymptotically-stable regions. Their stability analysis involved a perturbation of the phase space trajectory of the motion, and showed stable motion if all the eigenvalues of a certain matrix are numerically less than unity. Egle[17] gave a simplified stability analysis and determined the regions of stability. He also presented some experimental evidence to support his theory. However, his analysis is over-simplified. The stability criterion given is only a static one; that is to say, it can only predict a nonoscillatory type of divergence. It is possible that the perturbations diverge in an oscillatory manner even though the theory would classify it as a stable motion. These analyses are of somewhat limited scope in that they do not preclude the possibility of steady-state motion with other patterns of collisions.

Masri[18] presented an exact solution for steady-state motion with any number of impacts per cycle, symmetric or unsymmetric, based on the discontinuous theory of nonlinear oscillations. The governing differential equations between impacts are linear, corresponding to the analytic arcs on the phase plane, while the impact conditions correspond to the discontinuities. The complete periodic solution is constructed and the asymptotic stability of the limit cycle is investigated. The results of the stability analysis were corroborated by the 'experiments' using both analog[19] and digital computers. Masri[20] and Cempel[21] discussed the problem of vibration of a multi-degree of freedom system with impacts. Cempel derived the equations of motion using the theory of distributions and solved the equations of motion using operational calculus. He also discussed questions of existence and stability of the solution.

As stated earlier, the disadvantages of large impact forces, surface wear and excessive noise levels of single unit impact dampers are overcome to some extent if multi-unit impact dampers are used[20, 22]. Masri[20] gave an exact solution for the symmetric two-impacts-per-particle per-cycle motion for a multi-unit impact damper and determined the regions of asymptotic stability. The results were shown to be in good agreement with the motion simulated on a digital computer and with experimental findings. It has been demonstrated experimentally[22] that, for such a multi-unit vibration impact neutralizer, called MUVIN, the efficiency is greater, while the contact stress and noise levels are considerably lower than those for a single unit damper. The general analysis of such MUVIN systems becomes very complicated when the number of particles becomes very large. Cempel[23] showed how for MUVIN the sequence of impulsive forces of impact may be replaced by an equivalent continuous interaction force. This replacement leads to greater simplification of the MUVIN analysis. In another paper[24], Cempel formulated a receptance model of MUVIN and showed the possibility of dynamic analysis and synthesis of the interaction with multi-degree freedom systems and with beams, plates and shells. He also showed both theoretically and experimentally that the MUVIN system can be used to limit resonant structural vibration.

Practical applications of impact dampers also have been discussed in the literature. Some of them are: elimination or reduction of vibration of turbine blades[1], lathe tools[25], airplane ailerons[22, 25], moving electrical contacts of power generators[26], tall buildings and other structures[27, 28], lighting poles along highways[29], helicopter tension rods[22], etc. There have also been discussions on the applications, for example, of impact dampers to continuous systems[30].

The heart of the impact damper is, of course, the collision phenomenon. It is therefore obvious that a fundamental understanding of the impact phenomenon is of great importance in the study of impact dampers.

4. The impact phenomenon

After St. Venant[31] and Hertz[32] published their theories of impact, several investigators have carried out follow-up theoretical work and experiments on collision of bodies. Many of these are concerned with estimating the range of validity of the Hertzian theory of impact (presented in the next section), determining the coefficient of restitution and measurement of impact forces. Vincent[33], Rayleigh[34], Sears[35], Hopkinson[36], Raman[37], Love[38] and Andrews[39–41] are among the early investigators of impact. Subsequently, Mason[42], Zener and Feshbach[43], Bowden and Tabor[44], Tabor[45], Mindlin[46], Davies[47], Poritsky[48], Crook[49], Tillet[50], Hunter[51], Goodier *et al*[52], Goldsmith[53], Goldsmith and Lyman[54], Hoppmann[55], Lubkin[56], Dyson[57], Malyshev[58], Deresiewicz[59], Tsai[60], Schwieger and Spuida[61] and Bokor and Leventhall[62] are some of the others who have contributed to the understanding of the impact phenomenon.

In an important paper[37], Raman discussed the problem of transverse impact of a solid sphere on an infinitely extended elastic plate of finite thickness and calculated the theoretical coefficient of restitution as a function of the elastic constants and densities of the materials, the diameter of the sphere and the thickness of the plate and of the velocity of impact. Experiments confirming the validity of the formula within the limits of its applicability were also reported. Crook[49], by a piezoelectric method, succeeded in measuring continuously the impact forces between metal cylinders and between a hard sphere and metal flats. The experimental results confirm the validity of the impact theories of St. Venant and Hertz within the appropriate ranges of their applicability. Hunter[51] theoretically calculated the vibrational energy in the elastic waves generated by a highly localized transient normal force and applied the results to the problem of collision of a small body on a flat surface of a large one. The result is of interest in justifying the validity of the Hertzian theory for the case of a small body colliding against a massive one. Veluswamy and Crossley[63, 64] studied multiple impacts of a ball between two plates experimentally, and suggested, on the basis of the results, a mathematical model that fits the observed facts. This work is of interest in the context of investigations on impact dampers.

Impacts on non-metals are more complex to study than those on metals, but the former are of minor relevance in the study of impact dampers.

4.1. *The Hertz theory of impact*

Impacts between metals may be plastic or elastic, producing some permanent deformation or none. According to St. Venant[31], the nature of an elastic collision is determined by the elastic waves generated in the colliding bodies by the collision. Hertz[32] assumed that the stresses and the strains in the neighbourhood of the point of collision are the same as if the colliding force were purely static. In other words, the stresses and strains may be determined from the equations of equilibrium of the theory of elasticity. The theory of St. Venant applies to impacts between the end faces of long rods, while that of Hertz applies to impacts between small bodies. These theories are complementary and certain problems can be solved by combining the two theories.

The assumption of quasi-static condition in Hertz's theory neglecting the inertia and stress wave effects, demands equality of approaching and rebounding velocities. This implies that the coefficient of restitution is unity and that the elastic energy of the two bodies acquired during the impact is completely reversible. The condition for the Hertzian theory to be valid is, according to Love[38], "that the duration of impact should be a large multiple of the gravest period of free vibration of either body which involves compression at the place in question". The elastic waves generated in the colliding bodies during the process of collision travel to and fro within the bodies along the directions of compression. If the duration of contact is so large that the elastic waves get enough time to execute several passages to and fro in the meantime, then the assumption of quasi-statical condition of Hertz's theory is justified. Thus, one would suppose that the assumptions of Hertz's theory are not justified when one body is very massive, since the impact is over even before the first reflection begins, whereas the assumed quasi-statical conditions are realised by several wave reflections at boundaries[52, 65]. However, the Love criterion quoted and explained above is too restrictive and the theory can be extended to the case of impacts of a small body with a massive one. It is this kind of metal-to-metal impacts of small bodies with a massive body that we are concerned with in the study of impact dampers.

Experiments[50] on impact of steel balls on steel blocks consistently show a difference of approach and rebound velocities, indicating a coefficient of restitution in the range of 0.90–0.95, compared to the value of unity as demanded by the Hertzian theory. Hunter[51] calculated the vibrational energy in the elastic waves generated by the impact, but found that this is not quite sufficient to account for the loss of kinetic energy. The probable reasons for the difference in the energies are the dissipation of energy in the metal due to the high strain rates involved and welding followed by fracture at the microlevel[51, 52]. However, at small or even moderate velocities of impact (compared to the velocity of propagation of elastic waves in the massive body), the Hertzian theory is shown by Hunter[51] to be justified for the case of collisions of small bodies with a massive one.

Thus, it may be concluded that for such collisions as occur in impact dampers, the theory of Hertzian impact may be used for the calculation of stresses, strains, time of contact and surface compliance. However, if a coefficient of restitution approach is used for writing down the equations of motion of the impacting ball(s) and of the primary system and the dynamical-kinematical conditions of impact, it is also justifiable to use a numerical value of less than unity for the coefficient of restitution and to treat the impacts as instantaneous. The duration of impact is small compared to the periodic time of travel of the ball(s) in the container(s) of the impact damper, although it is still large compared to the time of transit of elastic waves within the ball(s).

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