REVIEWS

Quantum computing with trapped ions

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Abstract | Quantum information encoded in single trapped ions provides a promising avenue towards a scalable quantum computer. This contribution describes most of the necessary building blocks for such a device. Particular emphasis is given to the implementation of single-qubit and multi-qubit gate operations.

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1. Introduction

Quantum information combines views from quantum mechanics and information theory to further both fields. For instance, by processing quantum information, we can carry out quantum simulations which could help to understand physical systems ranging from from molecules to condensed matter systems (Feynman, 1982). With a device working only with fifty quantum bits (qubits), physical situations could be investigated which are currently intractable with classical computers. Furthermore, a quantum computer could be also used to perform mathematical tasks such as factorizing large numbers while outperforming any classical computer significantly (Shor, 1994).

Examples for physical carriers of qubits are the electron's spin in a magnetic field, two levels of an atom or a Josephson junction devices. A simple quantum computation initializes the qubits, manipulates them and finally reads out the final state of the quantum register. Any physical implementation of quantum computation must be able to perform these tasks. Thus the physical system must provide the following (DiVincenzo, 2001):

1. Well-characterized qubits.
2. The qubits must have much longer coherence times than the time scales required for the fundamental operations.
3. A universal set of quantum gates.
4. A qubit-specific measurement.

Additionally DiVincenzo requires:

5. The ability to interconvert stationary and flying qubits.
6. The ability to faithfully transmit flying qubits between specified locations.

For practical applications, the DiVincenzo criteria have not only to be fulfilled, but also the fidelity and speed of the implementations have to be considered. Furthermore, it is highly desirable to implement all operations as parallel as possible.

The previous listed requirements can be fulfilled with a number of physical approaches. Using nuclear magnetic resonance, quite a number of impressive demonstration experiments have been performed(Gershenfeld and Chuang, 1997; Vandersypen et al., 2001). Usually, the state of the quantum register (molecules) can only be poorly initialized, making the scaling properties of NMR quantum computation not very promising (Warren, 1997; Jones et al., 2000; Linden and Popescu, 2001). Recently, quantum information based on Josephson junctions has achieved some experimental breakthroughs (Clarke and Wilhelm, 2008), like coupling of distant qubits (Schoelkopf and Girvin, 2008; Wallraff et al., 2004) and the generation of Bell states (Steffen et al., 2006; Leek et al., 2008). Methods using linear optics have also been proposed (Knill et al., 2001; Prevedel et al., 2007; Walmesley, 2008) and were used for quantum information processing (Walther et al., 2005; Lanyon et al., 2008).

So far one of the most successful approaches to process quantum information is to store the carriers of quantum information in the electronic states of individual ions (Häffner et al., 2008). Trapping the ions with electromagnetic forces in vacuum isolates them almost perfectly from their environment and thus permits extremely long storage times of the fragile quantum information (exceeding 10 min in some cases (Bollinger et al., 1991; Fisk et al., 1997)). Furthermore, the internal states can be initialized and measured with extremely high accuracy (Hume et al., 2007; Myerson et al., 2008). Finally, laser pulses "compute" on this quantum register by manipulating the electronic and motional states of the ion string.

The history of experimental quantum information begins essentially with a proposal by Ignacio Cirac and Peter Zoller on how to employ the ion trap technology to process quantum information (Cirac and Zoller, 1995). The key idea of the proposal is to use laser pulses to mediate an effective interaction between the electronic states of individual ions. Within a year, the ion trapping group at National Institute of Standards and Technology, Boulder, demonstrated the central operation of such an ion-trap quantum computer (Monroe et al., 1995a): a controlled bit flip on a single ion. On the theoretical side, quantum information continues to impact our understanding of quantum mechanics and of how a quantum computer might operate. For quantum computation, the maybe most relevant implication is the discovery of quantum error correction protocols by Peter Shor (Shor, 1995) and by Andrew Steane (Steane, 1996). These protocols allow for the implementation of arbitrary long quantum algorithms without perfect control.

2. Ion trap quantum computing

A detailed account of the fundamental issues of ion trap quantum computing is given by Wineland et al. (1998) and by Šašura and Bužek (2002). Furthermore, Leibfried et al. (2003a) review the manipulation and control of single ions. The most recent advances in quantum computation with trapped ions were summarized by Häffner et al. (2008), and the generation and applications of entangled ions by Blatt and Wineland (2008).

In a typical experiment, a string of ions is trapped inside a so-called linear trap (Raizen et al., 1992; Drees and Paul, 1964). Laser cooling reduces the temperature so that the ions form a crystal. The ions arrange themselves in a linear configuration because the potential along one of three trap axes is much weaker then along the other two. Such a configuration allows for exquisite quantum control of all relevant degrees-of-freedom. For instance, the states of the trapped ions can be initialized with nearly perfect fidelity in a particular electronic state via optical pumping techniques (Happer, 1972; Weber, 1977; Wineland et al., 1980). Furthermore, the coherence of the electronic states can be preserved for extremely long periods. Coherence times of more than 10 minutes have been observed with $^{9}Be^{+}$ ions (Bollinger et al., 1991) and $^{171}Yb^{+}$ ions (Fisk et al., 1997). These examples demonstrate that the electronic states of the ions can serve as almost ideal qubits. The qubits are manipulated with laser pulses and/or microwaves with thigh accuracy. The biggest challenge, however, is to induce conditional operations between the qubits. The most popular schemes employ the motional degree of freedom of the ion string as the effective mediator. Section 5 discusses the most important approaches to two-qubit gate operations. Finally, the electronic state of individual ions can be detected using the so-called shelving technique (Dehmelt, 1975; Nagourney et al., 1986; Sauter et al., 1986; Bergquist et al., 1986). All these operations have been demonstrated with fidelities exceeding 0.99.

In the following, we use $^{40}Ca^{+}$ ions to illustrate the procedures. However, we note that similar experiments have been carried out also with $^{9}Be^{+}$, Cd^{+}, Mg^{+}, Sr^{+}, and Yb^{+}. A typical ion trap experiment begins with laser cooling the ion string close to the Doppler limit $E_{\text{Doppler}} = \hbar\Gamma/2$. Assuming a few MHz trap frequency and a spectral width of $\Gamma = 20$ MHz of the $S_{1/2} \to P_{1/2}$ dipole transition (see Fig. 1), the oscillator modes i of the ions string are left in average quantum numbers $\bar{n}_i$ of less than 10. In the next step, more advanced cooling techniques, such as sideband cooling (Leibfried et al., 2003a; Diedrich et al., 1989; Monroe et al., 1995b; Roos et al., 1999; Peik, 1999) and electromagnetic induced transparency cooling (Roos, 2000; Morigi et al., 2000), can be used to prepare one, several or all motional modes in the ground state of the trap.

For sideband cooling, the ion string is irradiated on a motional sideband of the $S_{1/2} \leftrightarrow D_{5/2}$ transition (see Fig. 2). Thus the ion is transferred to the $D_{5/2}$ level while one motional quantum is removed from the addressed motional mode. The narrowness of the $S_{1/2} \leftrightarrow D_{5/2}$ transition provides an excellent frequency selectivity so that heating which might arise from other excitation paths can be neglected during this step. The lifetime of the metastable $D_{5/2}$ level of approximately 1.2 s (Barton et al., 2000; Kreuter et al., 2004, 2005) is artificially shortened by a laser coupling the $D_{5/2}$ level to the $P_{3/2}$ level which in turn decays within 7 ns back to the $S_{1/2}$ state. In order to suppress heating during the excitation on the $D_{5/2} \leftrightarrow P_{3/2}$ transition and

Figure 1: Level scheme of $^{40}Ca^{+}$, with Zeeman substructure and required laser wavelengths for manipulation of the calcium ions. The life time of the metastable D-levels is on the order of a second, thus allowing for long coherence times of the qubit.

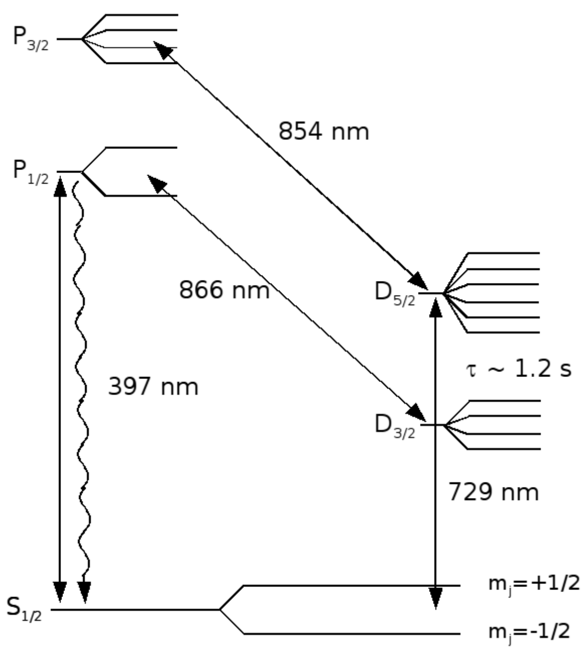
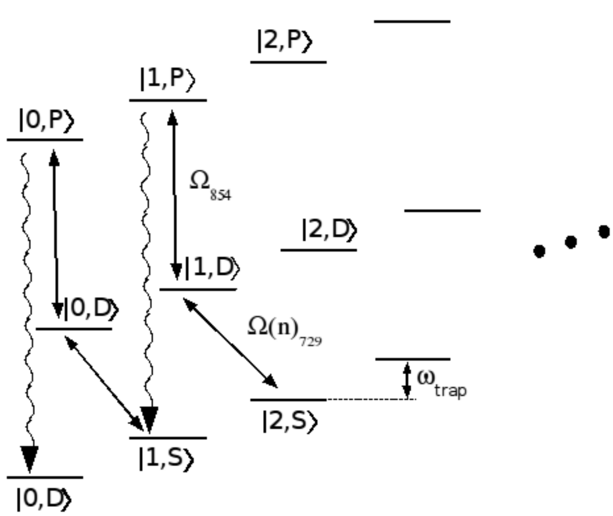


Figure 2: Sideband cooling of a $^{40}Ca^{+}$ ion. The level scheme shows only the most important levels of the ion and of a single motional mode described by a harmonic oscillation with frequency ω_{trap}. Furthermore, it is assumed that the excitation on the $D_{5/2} \leftrightarrow P_{3/2}$ and the subsequent decay do not change the motional state which is only justified in the Lamb-Dicke regime.



the subsequent decay process, it is advantageous that the ion string is already in the Lamb-Dicke limit (see Sec. 3) (Wineland et al., 1998; Morigi et al., 1999).

After this procedure, laser pulses manipulate the internal electronic states. Circularly polarized light pumps the ion into a well-defined electronic state, effectively preparing a pure quantum state. Afterwards, well tailored laser pulses manipulate the quantum information on the $S_{1/2} \leftrightarrow D_{5/2}$

transition, and thus implement the quantum algorithm. Sections 4 and 5 detail this.

Finally, the quantum state of the quantum register has to be read out. Referring to Fig. 1, we see that the ion fluoresces when irradiated on the $S_{1/2} \leftrightarrow P_{1/2}$ transition only when it is projected into the $S_{1/2}$ state. To avoid pumping into the metastable $D_{3/2}$ level, one uses additionally light on the $D_{3/2} \leftrightarrow P_{1/2}$ transition. Note that due to selection rules, the $P_{1/2}$ level does not decay into the $D_{5/2}$ level. If the ion was projected into the $D_{5/2}$ level, it remains dark. In this way millions of photons can be scattered before the population is transferred to the other qubit level. Indeed, tens of photons/ms can be detected and indicate projection into the $S_{1/2}$ state. The absence of photon detection events signals projection in the $D_{5/2}$ level.

Recently Myerson et al. (2008) have demonstrated a quantum state detection fidelity of 0.9999 with an average detection time of 145 μs. For these experiments, not only the total number of collected photons where taken into account, but also their arrival times (see also Langer (2006); Gambetta et al. (2007)). Knowing the arrival times of the photons allows one to take into account the decay of the $D_{5/2}$ level which can happen during the detection time, feigning a prior projection into the $S_{1/2}$ level. Collecting the photons late during the detection window indicates that indeed the ion has been projected originally into the $D_{5/2}$ and not into the $S_{1/2}$ level. Additional efficiency is gained in these experiments by terminating the detection procedure when the estimated error probability for each detection event is below a certain threshold.

3. Hamiltonian of trapped ions

The relevant physics of an ultra-cold string of trapped ions can often be described by the following simple model: each ion is approximated by a two level system, while the ion string motion is modelled as a collection of harmonic oscillators, each representing a normal mode of the ion string. Often a single mode is sufficient for an accurate description, in particular when the involved time scales are much slower then the trap oscillation period. In this case, we essentially deal with the interaction of two-level systems with a quantized harmonic oscillator via laser light. For more detailed discussions, we refer to Refs. (Wineland et al., 1998) and (Leibfried et al., 2003a). The basic level scheme of this system is displayed in Fig. 3.

The Hamiltonian for a single trapped ion interacting with near resonant laser light is (Leibfried et al., 2003a):

$$H = \hbar\Omega\sigma_+ e^{-i(\Delta\, t-\varphi)} \times \exp\left(i\eta\left[ae^{-i\omega_t t} + a^\dagger e^{i\omega_t t}\right]\right) + \text{h.c.}. \quad (1)$$

Here, $\sigma_\pm$ is either the atomic raising or the atomic lowering operator, while $a^\dagger$ and a denote the creation and annihilation operator for a motional quantum, respectively. Ω characterizes the strength of the laser field in terms of the so-called Rabi frequency, φ denotes the phase of the field with respect to the atomic polarization and Δ is the laser-atom detuning. ω_t denotes the trap frequency, $\eta = k_x x_0$ is the Lamb-Dicke parameter, with k_x being the projection of the laser field's wavevector along the x direction, and $x_0 = \sqrt{\hbar/(2m\omega_t)}$ is the spatial extension of the ion's ground state wave function in the harmonic oscillator (m is here the ion's mass). We mention also that the rotating wave approximation has been applied which assumes that both the laser detuning and the Rabi frequency are much smaller than optical frequencies. A similar treatment can be carried out for qubits based on Raman-transitions by eliminating the virtual level through which the two qubits are coupled. We note that in our definition the Rabi frequency measures the frequency with which the population exchanges in contrast to the definition used by Wineland et al. (1998) and Leibfried et al. (2003a).

Using the Lamb-Dicke approximation ($\eta\sqrt{\langle(a+a^\dagger)^2\rangle} \ll 1$), we can rewrite Eq. 1 by expanding the exponential (Leibfried et al., 2003a; Jonathan et al., 2000):

$$\begin{aligned} H = \hbar\Omega\Big\{ & \sigma_+ e^{-i(\Delta\, t-\varphi)} + \sigma_- e^{i(\Delta\, t-\varphi)} \\ & + i\eta(\sigma_+ e^{-i(\Delta\, t-\varphi)} - \sigma_- e^{i(\Delta\, t-\varphi)}) \\ & \left(ae^{-i\omega_t t} + a^\dagger e^{i\omega_t t}\right)\Big\}. \end{aligned} \quad (2)$$

Three cases of the laser detuning Δ are of particular interest (see Fig. 3): $\Delta = 0$ and $\Delta = \pm\omega_t$. This becomes apparent if a second rotating wave approximation is carried out where time dependent terms for the three cases above are discarded:

1. $\Delta = 0$:

   $$H_{\text{car}} = \hbar\Omega\,(\sigma_+ e^{i\varphi} + \sigma_- e^{-i\varphi}). \quad (3)$$

   Here only the electronic states $|g\rangle$ and $|e\rangle$ of the ion are changed (carrier transitions).
2. $\Delta = \omega_t$:

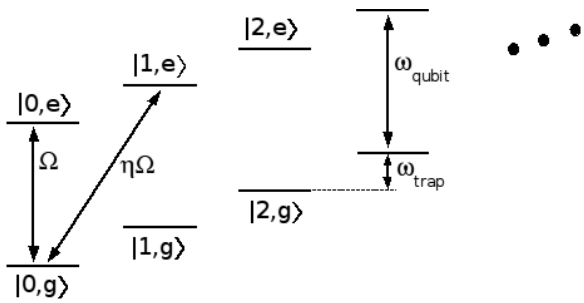
   $$H_+ = i\hbar\Omega\eta(\sigma_+ a^\dagger e^{i\varphi} - \sigma_- a e^{-i\varphi}). \quad (4)$$

   The electronic state of the ion and the motional degree of freedom are excited at the same time. Within this two-level system, Rabi flopping with Rabi frequency

   $$\Omega_{n,n+1} = \sqrt{n+1}\,\eta\,\Omega \quad (5)$$

   occurs, where n describes the number of motional quanta (phonons).

Figure 3: Energy level scheme of a single trapped ion with a ground ($|g\rangle$) and an excited ($|e\rangle$) level in a harmonic trap (oscillator states are labeled $|0\rangle, |1\rangle, |2\rangle, \cdots$). Ω denotes the carrier Rabi frequency. The Rabi frequency on the blue sideband transition $|0,e\rangle \leftrightarrow |1,g\rangle$ transition is reduced by the Lamb-Dicke factor η as compared to the carrier transition (see Eq. 5). The symbols ω_{qubit} and ω_{t} denote the qubit and the trap frequency, respectively.



3. $\Delta = -\omega_{\text{t}}$:

$$H_- = i\hbar\Omega\eta(\sigma_- a^\dagger e^{-i\varphi} + \sigma_+ a e^{i\varphi}). \qquad (6)$$

Simultaneously to exciting the electronic state, here a phonon is destroyed and Rabi flopping with Rabi frequency

$$\Omega_{n,n-1} = \sqrt{n}\eta\,\Omega \qquad (7)$$

takes place.

Section 4 details how single qubit operations can be implemented with the Hamiltonian in Eq. 3 whereas Sec. 5 discusses implementations of two-qubit gates which use the Hamiltonian in Eqs. 5 and 7.

4. Single qubit operations

It can be shown that all quantum algorithms can be implemented as a sequence of single-qubit operations plus one specific two-qubit operation, thus forming a so-called universal set of quantum gates Deutsch (1989).

Using the Hamiltonian for Eq. 3, we directly find single qubit operations:

$$R(\theta,\phi) = \exp\left(i\theta/2\left(e^{i\varphi}\sigma_+ + e^{-i\varphi}\sigma_-\right)\right). \qquad (8)$$

In a Bloch sphere picture, the angle φ specifies the axis of rotation in the equatorial plane and θ the size of the rotation. Rotations around the z axis can be either decomposed into rotations around the x and the y axis or a far detuned laser beam can shift the energies due to an AC–Stark effect to achieve the required phase shift.

The relevant control parameters in the ion trap experiments are the pulse area θ given by the product Ωt of the Rabi frequency Ω and the pulse length t and the phase of the laser field φ. These parameters can be controlled using an acousto-optical modulator in double-pass configuration.

It is useful to visualize single-qubit operations on the Bloch-sphere (see Fig. 5). In the following discussion, we will identify the north pole with the ground state (logical $|1\rangle$) and the south pole with $|0\rangle$ (the excited state). Resonant excitation drives Rabi-oscillations between these two states (see Fig. 4). Fig 6 illustrate the interplay of different single qubit operations. In Fig 6a) first the Bloch vector is rotated from the north to the south pole around the x axis, then the Bloch vector is rotated by π around the y axis back to the north pole and then again around the x axis. As can be seen changing the phase of the single qubit operation has no effect. This is in contrast to Fig. 6b): changing the phase after a rotation by $3\pi/2$ around the x axis, leads to a completely different behavior. The projection onto the z axis by the measurement does not show any evolution of the probabilities until the phase is switched again.

This behavior can be understood intuitively. Suppose we start with an ion in the electronic ground state. As we irradiate the ion resonantly, an atomic polarization which has a well defined phase relation to the laser field builds up. From this picture, it is now straightforward to understand Figs. 6a and 6b. In the first example the electric field of the laser builds up a quadrupole moment that is oscillating in phase at the laser frequency corresponding to a superposition of the $S_{1/2}$ and the $D_{5/2}$-state. When all the population is transferred to the excited $D_{5/2}$-level, the phase reference is lost, and changing the phase of the excitation field has no effect. In the second example, the phase of the laser field is switched when the atom is in a superposition. After switching, the phase of electric field is shifted

Figure 4: Carrier Rabi oscillations of a single $^{40}Ca^{+}$. Each data point represents the average result of one thousand of the following experiments: preparation of the ion in the $S_{1/2}$ state, excitation for a given time on the $S_{1/2} \leftrightarrow D_{5/2}$ transition, and measurement of the population of the $D_{5/2}$ state.

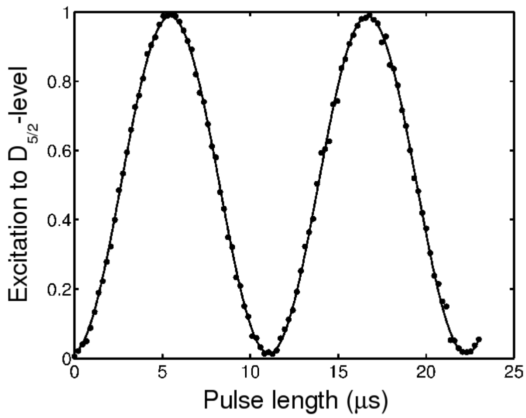
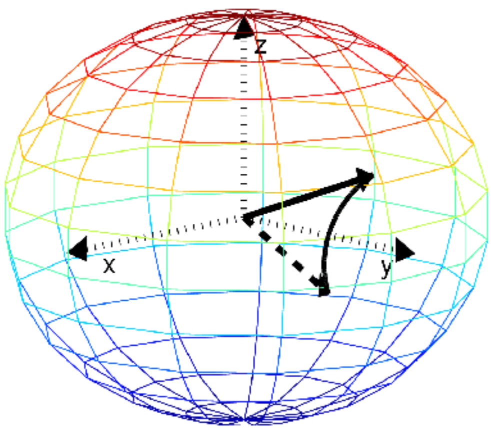


Figure 5: Rotation around the y axis visualized on the Bloch-sphere.



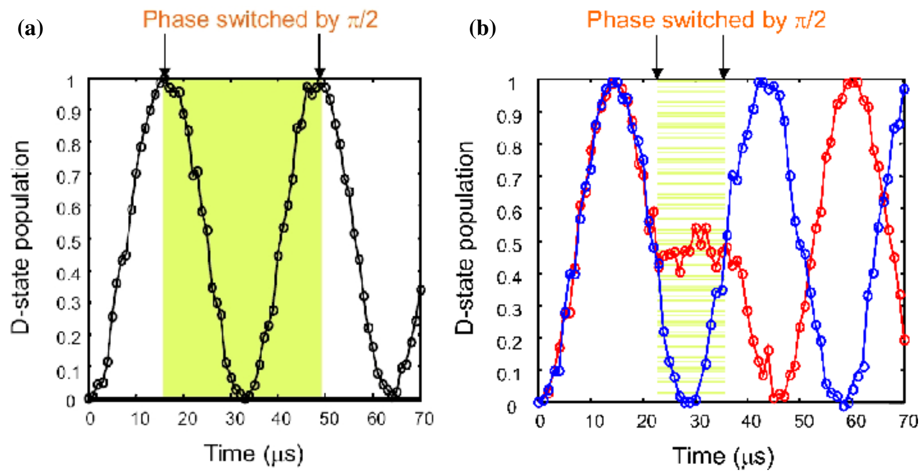
additionally by a phase of $\pi/2$, so that no further excitation takes place as it is the case for a harmonic oscillator.

Single qubit manipulations are carried out routinely with fidelities exceeding 0.99 (see for instance Knill et al. (2008)). The fidelities are usually limited by intensity fluctuations.

5. Two-qubit gates

In ion trap quantum computing, the implementation of suitable two-qubit operations is the most challenging task. The interest in two-qubit operations is documented by a vast number of proposals (Cirac and Zoller, 1995; Mølmer and Sørensen, 1999; Solano et al., 1999; Sørensen and Mølmer, 1999; Milburn, 1999; Sørensen and Mølmer, 2000; Milburn et al., 2000; Childs and Chuang, 2000; Jonathan et al., 2000; Shi-Biao et al., 2000; Wang et al., 2001; DeMarco et al., 2002; Staanum and Drewsen, 2002; Leibfried et al., 2003b; García-Ripoll et al., 2003; Staanum and Drewsen, 2002; Duan, 2004; Schmidt-Kaler et al., 2004; Šašura and Steane, 2004, 2005; Zhu et al., 2006b,a; Leibfried et al., 2007; Aolita et al., 2007a; Roos, 2008; Kim et al., 2008; Ospelkaus et al., 2008; Mueller et al., 2008; Monz et al., 2008; Maunz et al., 2009). Common to all of them (with the exception of (Mueller et al., 2008; Maunz et al., 2009)) is that they use the motional degree of freedom to achieve conditional operations. From these, the gate proposed by Cirac and Zoller (1995); Childs and Chuang (2000) has been realized by Schmidt-Kaler et al. (2003b,c); Riebe et al. (2006), the gate proposed by Mølmer and Sørensen (1999);

Figure 6: Single qubit rotations: In a) the black dots show the population evolution during the pulse sequence $R(\pi,0)R(2\pi,\pi/2)R(\pi,0)$. In b) the blue dots show the evolution for ordinary Rabi oscillations while the red points show the evolution for the pulse sequence $R(3\pi/2,0)R(\pi,\pi/2)R(3\pi/2,0)$.



Sørensen and Mølmer (2000); Milburn et al. (2000); Roos (2008) has been realized by Sackett et al. (2000); Haljan et al. (2005); Home et al. (2006); Friedenauer et al. (2008); Benhelm et al. (2008b); Kirchmair et al. (2009a,b), whereas DeMarco et al. (2002); Leibfried et al. (2003b); Schmidt-Kaler et al. (2004); Monz et al. (2008); Maunz et al. (2009) report both the proposal and the implementation in the same publication.

Below we will present the three most prominent approaches: the Cirac&Zoller gate (Cirac and Zoller, 1995; Schmidt-Kaler et al., 2003b), the geometric phase gate (Leibfried et al., 2003b) and the Mølmer-Sørensen gate (Sørensen and Mølmer, 1999; Mølmer and Sørensen, 1999; Sørensen and Mølmer, 2000; Roos, 2008; Sackett et al., 2000). Common to all of them is that they use the motion of the ion crystal to couple the ions to each other. The motion of an ion crystal with N ions can be efficiently described by $3N$ normal modes as detailed by James (1998). Typically only one of these modes is used as the quantum bus, and the other modes are therefore called spectator modes. Modes used for the experiments discussed below are either the center-of-mass mode or the breathing mode where both ions oscillate out of phase.

5.1. *The Cirac&Zoller-approach*

We discuss first the gate proposed by Cirac and Zoller (1995) in which the motional mode acts directly as a qubit transmitting quantum information: The idea is

1. to map the internal state of one ion to the motion of an ion string,
2. to flip the state of the target ion conditioned on the motion of the ion string,
3. to map the motion of the ion string back onto the original ion.

The operations which modify individual qubits and connect a qubit to the bus (typically the center of mass mode) are performed by applying laser pulses on the carrier (Eqs. 3 and 8) and on the blue sideband 4 (see Fig. 3) of the $S_{1/2} \leftrightarrow D_{5/2}$ transition.

The mapping and re-mapping procedure between the electronic control bit in state $\alpha|g\rangle+\beta|e\rangle$ and the bus mode in the motional ground state $|0\rangle$ is carried out in the following way:

$$(\alpha|g\rangle+\beta|e\rangle)\otimes|0\rangle \xrightarrow{R^{+}(\pi,0)} |e\rangle\otimes(\alpha|0\rangle+\beta|1\rangle). \quad (9)$$

Here $R^{+}(\pi,0)$ is defined analogous to single-qubit operations (Eq. 8) on the $|S,0\rangle \leftrightarrow |D,1\rangle$ transition, using the Hamiltonian from Eq. 4:

$$R^{+}(\theta,\phi)=\exp\left(i\theta/2\left(e^{i\varphi}\sigma_{+}a^{\dagger}+e^{-i\varphi}\sigma_{-}a\right)\right). \quad (10)$$

Next, one applies the CNOT operation U_{CNOT} to the target ion where the information in the bus mode is the control bit. Finally, the bus mode and the control ion are reset to their initial states by another π-pulse, $R^{+}(\pi,\pi)$, on the blue sideband.

We now address the problem of performing a CNOT on a single ion with the motion as a control bit. For this we first realize a controlled phase gate operation by driving an effective 2π-pulse on the two two-level systems $\{|S,0\rangle,\ |D,1\rangle\}$ and $\{|S,1\rangle,\ |D,2\rangle\}$ which changes the sign of all computational basis states $\{|D,0\rangle,\ |S,0\rangle,\ |D,1\rangle,\ |S,1\rangle\}$ except for $|D,0\rangle$. The Rabi frequency depends on the motional quantum number n (see Eq. 5). To avoid complications due to this, Cirac and Zoller (1995) proposed to employ an auxiliary level and this procedure was demonstrated by Monroe et al. (1995a). In the implementations by Schmidt-Kaler et al. (2003a,b), use of the auxiliary level was avoided by employing the following composite-pulse sequence (Childs and Chuang, 2000):

$$U_{\text{phase}} = R^{+}(\pi\sqrt{n+1},0)\, R^{+}\left(\pi\sqrt{\tfrac{n+1}{2}},\pi/2\right)$$
$$R^{+}(\pi\sqrt{n+1},0)\, R^{+}\left(\pi\sqrt{\tfrac{n+1}{2}},\pi/2\right)$$
$$= \left(\begin{array}{c|cccc} & |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \\ \hline |D,0\rangle & 1 & 0 & 0 & 0 \\ |S,0\rangle & 0 & -1 & 0 & 0 \\ |D,1\rangle & 0 & 0 & -1 & 0 \\ |S,1\rangle & 0 & 0 & 0 & -1 \end{array}\right), \quad (11)$$

where we used the definition for the blue-sideband Rabi frequency in Eq. 10.

Enclosing the phase gate operation U_{phase} by a $R^{C}(\frac{\pi}{2},0)$ and another $R^{C}(\frac{\pi}{2},\pi)$ pulse turns the phase gate into a CNOT-gate. Thus, the full pulse sequence is:

$$\begin{aligned} U_{\text{CNOT}} = {} & R_{\text{c}}^{+}(\pi,\pi) \\ & R_{\text{t}}^{C}\left(\frac{\pi}{2},\pi\right) \\ & R_{\text{t}}^{+}\left(\pi\sqrt{n+1},0\right) R_{\text{t}}^{+}\left(\pi\sqrt{\tfrac{n+1}{2}},\pi/2\right) \\ & R_{\text{t}}^{+}\left(\pi\sqrt{n+1},0\right) R_{\text{t}}^{+}\left(\pi\sqrt{\tfrac{n+1}{2}},\pi/2\right) \\ & R_{\text{t}}^{C}\left(\frac{\pi}{2},0\right) \\ & R_{\text{c}}^{+}(\pi,0)\,, \end{aligned} \quad (12)$$

where the subscripts $_{\text{c}}$ and $_{\text{t}}$ label the control and target ion, respectively. Process fidelities of up to 0.92 and entangled states with a fidelity of 0.95 have been demonstrated with this approach (Riebe et al., 2006).

The gate fidelity are well understood in terms of a collection of experimental imperfections. For instance, loss of qubit coherence causes a reduction of the fidelity which can be countered with faster gates. However, this implies larger Rabi frequencies, which in turn spoil the fidelity by uncontrolled AC Stark shifts (Steane et al., 2000; Häffner et al., 2003). Additional errors exist do due to addressing imperfections and residual thermal excitations of the bus and the spectator modes, as well as due to laser intensity fluctuations.

5.2. *Geometrical phase gate*

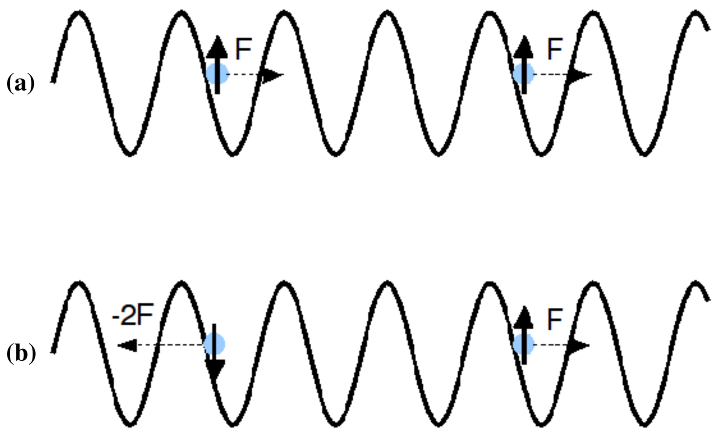
A very promising two qubit-gate is the one realized by the Boulder-group (Leibfried et al., 2003b). Interfering a pair of laser beams at the ion positions induces a state dependend force on the ions. In contrast to the Cirac&Zoller gate, all ions are illuminated simultaneously. Furthermore, the trap frequency is adjusted so that the ion-ion distance is a multiple of the optical lattice constant (see Fig. 7). Due to a frequency difference between the two beams, the optical lattice is moving, and a force oscillating with this frequency difference is acting onto each ion. If the two ions are in a different electronic state, the breathing mode is excited and the wavefunction picks up a phase. If there is no differential force, the breathing-mode is not excited. The frequency difference between the two laser fields is chosen to be detuned by δ from the frequency of the breathing mode $\omega_{\text{breathing}}$ with $\delta \ll (\omega_{\text{breathing}} - \omega_{\text{COM}})$ in order to speed-up the phase evolution and at the same time to reduce the dynamics due to the center-of-mass motion at frequency ω_{COM}. Due to the detuning, the ion string motion returns to the original motional state after the time $t_{\text{gate}} = 1/\delta$. Finally, we pick a light intensity such that we acquire a phase of $\pi/2$ in case the two ions are in different electronic states, and no phase in case the ions are in the same state. Thus, we get the following unitary operator:

$$U'_{\Phi} = \left(\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 00 & 1 & 0 & 0 & 0 \\ 01 & 0 & e^{i\pi/2} & 0 & 0 \\ 10 & 0 & 0 & e^{i\pi/2} & 0 \\ 11 & 0 & 0 & 0 & 1 \end{array}\right)$$
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\pi/2} & 0 & 0 \\ 0 & 0 & e^{i\pi/2} & 0 \\ 0 & 0 & 0 & e^{-i\pi} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\pi} \end{pmatrix} \quad (13)$$

Verifying the above equality, shows that indeed U'_{Φ} is equivalent to a single qubit Z-gate with length $\pi/2$ on both ions plus a controlled phase gate on $|11\rangle$.

Leibfried et al. (2003b) achieved fidelities of up to 0.97 at gate operation times as fast as 10 μs.

Figure 7: Force on two ions in a standing wave of two laser fields. a) both ions spin up—both ions experience the same force; b) one ion spin down, the other spin up. The laser is tuned so that each qubit level experiences a different light shift (in the realization by Leibfried et al. (2003b) with $^9Be^+$ ions their ratio is -2). By tuning the frequency difference of the two laser fields close to the breathing mode frequency, in a) the motion of the ion string cannot be excited efficiently, while in b) the breathing mode is excited.

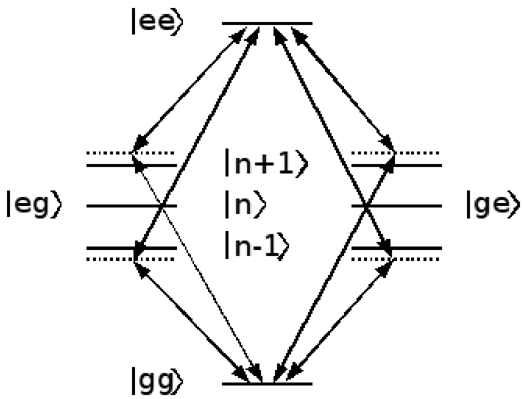


5.3. *Mølmer-Sørensen gate*

A gate very closely related to the geometric phase gate is the Mølmer-Sørensen gate (Sørensen and Mølmer, 1999; Mølmer and Sørensen, 1999; Sørensen and Mølmer, 2000; Roos, 2008). The main idea is to drive collective spin flips of the involved ions. For this, all ions are illuminated with two laser fields, one of which is tuned close to the red sideband ($\omega_{\text{qubit}} - \omega_{\text{trap}} - \delta$), whereas the other one is tuned to the blue sideband ($\omega_{\text{qubit}} + \omega_{\text{trap}} + \delta$), $\delta \ll \omega_{\text{trap}}$. For simplicity, we assume first $\delta \gg \eta\Omega$ where $\eta\Omega$ is the Rabi frequency of both beams on the respective sideband. In this case, we drive the two-photon transitions on the $|gg, n\rangle \leftrightarrow |ee, n\rangle$ manifold (see Fig. 8) as well as on the $|ge, n\rangle \leftrightarrow |ge, n\rangle$ manifold where $|n\rangle$ indicates again the motional degree of freedom. Stopping halfway in this transition, we see that the gate entangles the ions, i.e. we arrive at $|gg, n\rangle + i|ee, n\rangle$ and at $|eg, n\rangle + i|eg, n\rangle$, respectively. Furthermore, it can be shown that this gate is universal. In the limit $\delta \gg \eta\Omega$, the motional degree of freedom is never really excited. Roos (2008) shows that the gate works also if δ approaches Ω under the condition that the ratio Ω/δ is chosen such that after the desired gate time, the motional excitation returns to its original state. The Mølmer-Sørensen gate has been implemented first by Sackett et al. (2000). Haljan et al. (2005) apply this gate directly to clock states which is not easily achievable for the geometric phase gate (Blinov et al., 2004; Langer, 2006; Aolita et al., 2007b). Recently Benhelm et al. (2008b) demonstrated Mølmer-Sørensen gate operations with exceptionally high fidelities of 0.99 which exceeds the threshold for fault tolerant quantum computing as found by Knill (2005). Additionally, this gate has been proven to work with a fidelity of 0.97 even when the mode used for coupling the qubits was cooled only to the Doppler temperature of the $S_{1/2} \leftrightarrow P_{1/2}$ transition corresponding to a mean phonon number of 20 (Kirchmair et al., 2009a).

All three discussed multi-qubit gates have their merits and drawbacks. For instance, the Cirac&Zoller gate relies on addressing the ions individually. This requires additional effort and makes the gate susceptible to addressing imperfections, but on the other hand it allows easy incorporation into a set-up which uses tightly focused laser beams for single qubit operations. The Mølmer-Sørensen gate and the geometric phase gate on the other side require either segmented traps (see Sec. 6) or other strategies to be combined with single-qubit operations to allow for universal quantum computing. For the Mølmer-Sørensen gate the main problem is that it is experimentally very difficult to keep an interferometric stable configuration for all laser beams used for the single and multi-qubit operations. Furthermore, it is not very convenient to construct quantum algorithms from the globally acting entangling operations and single qubit gates. Nevertheless this is possible, for instance, via optimal control techniques. Nebendahl et al. (2008) show how to implement the building blocks for quantum computation (including a quantum error correction protocol) from a Mølmer-Sørensen gate acting on the whole ion string, global single qubit X gates, and local Z-gates.

Figure 8: Energy-level diagram of two trapped ions illustrating the principle of the Sørensen and Mølmer gate. The bus mode is populated with n phonons. Two laser beams tuned close to the blue and red sideband, respectively, drive the system via the dashed virtual levels between the $|n, gg\rangle$ and $|n, ee\rangle$ state. A similar process takes place if the ion string is either in the $|n, eg\rangle$ or in the $|n, ge\rangle$ state.



6. Achievements in ion trap quantum computing, scalability

Based on the previously discussed tools many fundamental concepts in quantum information processing have been demonstrated with trapped ions. We mention here the first deterministic generation of entanglement (Turchette et al., 1998), a test of a Bell inequality (Rowe et al., 2001), four particle entanglement (Sackett et al., 2000), demonstration of a decoherence-free subspace (Kielpinski et al., 2001), the first fully quantum implementation of an algorithm (Gulde et al., 2003), quantum state and process tomography (Roos et al., 2004a,b), an implementation of a quantum eraser (Roos et al., 2004b), the first deterministic teleportations of qubits (Riebe et al., 2004; Barrett et al., 2004), implementations of quantum error correction (Chiaverini et al., 2004) and the semi-classical Fourier transform (Chiaverini et al., 2005), scalable entanglement of up to eight qubits (Leibfried et al., 2005; Häffner et al., 2005), implementations of entanglement purification (Reichle et al., 2006b), the Toffoli gate (Monz et al., 2008), entanglement swapping (Riebe et al., 2008), as well as applications in quantum simulations (Leibfried et al., 2002; Friedenauer et al., 2008), and quantum metrology (Leibfried et al., 2004; Roos et al., 2006).

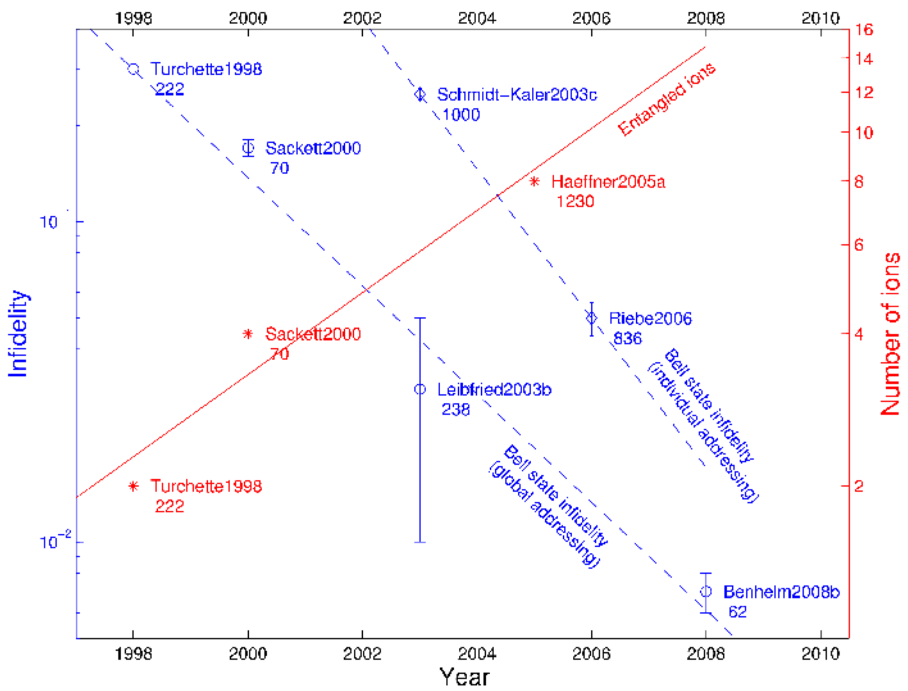
As we increase the number of ions in the trap, it gets more and more difficult to kick the ion string with a single photon (or in the Raman approach—with two photons). In our mathematical description the Lamb-Dicke parameter gets smaller (for the center of mass mode: $\eta \sim \sqrt{N}$, where N is the number of ions). This slows down the operations on the sideband as can be seen in Eq. 5. Further problems arise due to the more complex normal mode spectrum and a decreasing ion-ion spacing with increasing ion number.

Mathematically, these restrictions do not change exponentially with the number of qubits, nevertheless they prohibit scaling to large number of ions for practical reasons. There are at least five ideas to overcome those roadblocks:

1. Split up the ion string in small portions and move the ions around (Kielpinski et al., 2002).
2. Couple the ions via cavities and photons (Cirac et al., 1997).
3. Prepare heralded entanglement via joint fluorescence photon detection and use this entanglement as a resource for teleporting the quantum information between different traps (Gottesman and Chuang, 1999; Maunz et al., 2009).
4. Wire up ion traps and use the image charges induced by the ion motions to couple the ions in different traps (Tian et al., 2004; Daniilidis et al., 2009).
5. Use the radial modes of the ion string (Zhu et al., 2006b; Lin et al., 2009).

The currently most promising approach is to split up the ion string with segmented traps (Kielpinski et al., 2002). In a segmented trap ions can be moved by changing the voltages on the trap electrodes. Furthermore the ion strings can be merged and split. In this way the quantum register size can be tailored to the actual need. These procedures have been successfully demonstrated by Rowe et al. (2002); Barrett et al. (2004). Another requirement for the proposal by Kielpinski et al. (2002) is the transport through junctions.

Figure 9: Progress in reducing the error rate of two-qubit gates (taken from Ref. (Benhelm, 2008)) and in increasing the number of entangled ions. Open circles represent experiments using two-qubit gate operations with global addressing, while diamonds show results based on individual addressing. The performance is measured in terms of the infidelity of produced Bell states. The stars mark the largest number of entangled ions obtained at that time. Numbers below the reference indicate the number of trap cycles required for the operation. Dashed and dotted lines indicate the trends.



First experiments were carried out by Pearson et al. (2006) and Hensinger et al. (2006). The linear transport of ions was studied within the framework of quantum mechanics (Reichle et al., 2006a), while the non-adiabatic transport was investigated theoretically by Schulz et al. (2006) and experimentally by Huber et al. (2008). Furthermore, Hucul et al. (2008) analyzed the transport through various junction geometries quantum mechanically. Finally, Blakestad et al. (2009) have investigated the transport of trapped ions through four-way crosses. They found a negligible loss rate both of the ions themselves and of the coherence of the stored quantum information. Furthermore, they observe only a small energy gain on the order of a few motional quanta during the transport. Those results indicate that this approach to scalable quantum computing is indeed viable.

7. Future challenges and prospects for ion trap quantum computing

In order to achieve universal quantum computing, the algorithms have to be implemented in a fault-tolerant way. It is commonly accepted that this requires quantum error correction. Therefore, one of the most important goals currently is to implement quantum error correction repeatedly with high fidelity to prolong coherence times and to correct for errors induced by the gate operations. The largest obstacle to perform a successful quantum error correction protocol seems to be the limited fidelity of the operations. The current state of the art for the control in ion trap quantum computing can be summarized as follows:

- The qubit coherence times are one or two orders of magnitude longer than the basic (gate) operations. In specific cases, coherence times longer then the gate time by more than five orders of magnitude have been demonstrated (Langer et al., 2005; Olmschenk et al., 2007; Lucas et al., 2007; Benhelm et al., 2008a). Motional decoherence can be strongly suppressed by cooling down the trap electrodes to cryogenic temperatures (Deslauriers et al., 2006; Labaziewicz et al., 2008a,b).

- Initialization accuracies are on the order of 0.999 and can be improved further if necessary.
- Single qubit operation can be carried out with fidelities exceeding 0.995 (Knill et al., 2008). If needed, further improvements are possible with more stable laser fields at the ion positions.
- Implementations of two-qubit gate operations achieve fidelities of about 0.9–0.99 (Benhelm et al., 2008b). Depending on the gate type, various sources limit the fidelity. Errors are caused by off-resonant scattering, imperfect addressing of individual qubits, insufficient cooling, and laser frequency and intensity noise.
- The read-out of a single qubit can be performed with a fidelity of up to 0.9999 (Myerson et al., 2008).
- Ion strings can be shuttled, split and merged (see Sec. 6) with high fidelity and small decoherence (Blakestad et al., 2009).

Overall, two-qubit gate operations seem to be the main limiting factor. Figure 9 shows the progress of the fidelity achieved in the last decade. Benhelm et al. (2008b) demonstrate two-qubit gate fidelities high enough to allow in principle fault tolerant quantum computation according to the scheme proposed by Knill (2005).

Knill (2005) published numerical results which indicate that error rates on the order of 10^{-2} per operation are permitted, however with a huge overhead on the order of 10^6 qubits for one logical qubit. Both, analytical and numerical results, indicate that a more realistic benchmark is a fidelity exceeding 0.9999/operation, provided that certain other criteria can be met, too (Steane, 2004). Specific errors, error propagation, the allowed overhead, specific requirements and the amount of possible parallelization, amongst others, have to be considered to get a full grasp on the situation at hand. Thus, the concept of thresholds is oversimplifying the situation. Furthermore, it seems reasonable that every operation in a quantum computer should be implemented as perfect as possible to achieve fault tolerance while keeping the overhead as small as possible.

In summary, we have shown that all basic requirements for a general purpose quantum computing device have been demonstrated in various experiments with trapped ions. Future efforts will aim to meet all requirements in the same apparatus, improve the fidelity of the operations and scale to more ions.

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