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A numerical model for the simulation of tides in estuarial networks

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Abstract

A one-dimensional numerical model, based on Preissmann scheme for the discretization of shallow water wave equations, has been developed and applied to the Meghna Delta, consisting of 15 branches connected at 8 junctions with 3 sea-ward and 3 river-end boundaries. The model was calibrated by adjusting Manning's roughness coefficient. Considering the simplified schematization of the delta and limitations imposed by insufficient data, the results of model calibration and validation were found to be satisfactory.

Key words: Numerical model, Meghna Delta, tides, esturial networks.

1. Introduction

Bangladesh is situated at the main outfall of the Ganges-Brahmaputra river system, discharging into the Bay of Bengal. The outfall forms a complex estuarial network, covering the south-western zone of Bangladesh. Proper management of this esturial network is of considerable social and economical importance to the country.

In the recent years numerical models have been extensively used for the analysis and management of tidal problems. Commonly available numerical models for the simulation of tides in esturial networks are the System-11 of Danish Hydraulic Institute[1], the NETFLOW model of Delft Hydraulic Laboratory[2], the CARIMA model of the SOGREAH Consulting Engineers[3], the DELTA model of the Hydraulic Research Station[4] and DWOPER model of the U.S National Weather Services[5]. In Bangladesh numerical modelling is at its infancy. A few isolated attempts had been made by Khan and Alam[6], Khan[7], Khan and Ahsan[8] and Chowdhury[9] in the last four years. These models were developed for specific purposes and lack flexibility for general applications.

Presently, Master Plan Organization[10], responsible for the planning of long-term national water plan of Bangladesh, is trying to procure a one-dimensional unsteady flow simulation model from foreign research organizations for analysing the hydraulic characteristics of the rivers of Bangladesh. In the present study an attempt has been made to develop a numerical model for such studies, using the locally available research facilities. This paper outlines the development of the model and its application to the Meghna Delta in Bangladesh.

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2. Mathematical model

2.1 *Governing equations*

Numerical models for the simulation of tides in estuarial networks consist of solving shallow water wave equations subjected to initial and boundary conditions. For one-dimensional models, these equations can be written as:

$$\frac{\partial H}{\partial t} + \frac{1}{B}\frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{2Q}{A}\frac{\partial Q}{\partial x} - \frac{Q^2}{A^2}\frac{\partial A}{\partial x} + gA\frac{\partial H}{\partial x} + gA\frac{Q[Q]}{K^2} = 0 \tag{2}$$

where

H = water surface elevation
Q = discharge across channel section
A = cross-sectional area of channel section
B = top width of the channel section
K = conveyance of the channel section
  $= A R^{2/3}/n$
R = hydraulic radius of channel section
n = Manning's roughness coefficient
t = time coordinate
x = distance coordinate.

2.2 *Boundary conditions*

Generally, two types of boundary conditions, external and internal, are present in tidal models. In the sea-ward model boundaries,tidal stage hydrographs are generally specified as external boundary conditions. Fluvial inflows or reflecting boundary conditions may be used as river-end external boundary conditions. The internal boundary conditions at channel junctions are the equality of water levels and continuity of discharge. These conditions can be written as:

$$H_k = H_l = \ldots = H_m \tag{3}$$

$$\sum_{j=k}^{m} Q_j = S\,\frac{\mathrm{d}H}{\mathrm{d}t} \tag{4}$$

where the subscripts represent channel numbers meeting at a junction and S represents junction storage area.

2.3 *Initial conditions*

In tidal hydraulics initial conditions generally used are horizontal water surface and zero discharge throughout the model domain. These type of initial conditions can be applied when the river bed elevations at all computational points lie below the assumed initial water level. Otherwise a rough estimate of water levels and corresponding discharges on the basis of Manning's formula and continuity equation can provide the initial conditions. Due to the hyperbolic nature of the problem, the computations converge to the true solution after a few cycles.

3. Numerical model

3.1 Finite difference scheme

Equations (1) and (2) in their present forms defy analytical solutions. Therefore, a large number of schemes have been developed for their solutions[11,12]. In the present study, Preissmann four-point implicit scheme has been used. This scheme is given by:

$$f(x,\ t) = \frac{\theta}{2}(f_{i+1}^{n+1}+f_i^{n+1})+\frac{1-\theta}{2}(f_{i+1}^{n}+f_i^{n}) \tag{5}$$

$$\frac{\partial f}{\partial x}=\frac{\theta}{\Delta x}(f_{i+1}^{n+1}+f_i^{n+1})+\frac{1-\theta}{\Delta x}\ (f_{i+1}^{n}+f_i^{n}) \tag{6}$$

$$\frac{\partial f}{\partial t}=\frac{1}{2\Delta t}\ (f_{i+1}^{n+1}-f_{i+1}^{n}+f_i^{n+1}-f_i^{n}) \tag{7}$$

where f represents the dependent variable, θ is a time weighting factor such that $0<\theta\leqslant 1$, the superscript n represents time level and the subscript i grid number in a river reach. This scheme has been widely used for the discretization of shallow water wave equations, as the scheme is consistent and unconditionally stable at all Courant Numbers[11,12] for $\theta > 0.5$. In the present study $\theta = 0.6$ has been used to avoid numerical oscillations[13].

3.2 Discretization of the equations

Using Preissmann scheme, equations (1) and (2) can be discretized to the following pair of equations:

$$C_i\Delta H_i+D_i\Delta H_{i+1}+E_i\Delta Q_i+F_i\Delta Q_{i+1}+G_i = 0 \tag{8}$$

$$C_i'\Delta H_i+D_i'\Delta H_{i+1}+E_i'\Delta Q_i+F_i'\Delta Q_{i+1}+G_i' = 0 \tag{9}$$

where $\Delta f_i = f_i^{n+1}-f_i^n$ and the equations have been linearized by assuming $(\Delta H)^2 = (\Delta Q)^2 = (\Delta H)\ (\Delta Q) = 0$, etc. In equations (8) and (9) the coefficients are functions of the geometry of the river and known water levels and discharges.

The details of discretization and linearization can be found in Cunge *et al*[11] and Liggett and Cunge [13]. Using the known values of H_i^n, Q_i^n, etc., the coefficients C_i, D_i, . . . G_i' can be computed and the linearized system of equations, obtained by writing equations (8) and (9) at all grid-points, can be solved. This furnishes approximate values of unknowns at new time $(n+1)\Delta t$. The coefficients can then be updated and the linear system solved providing again new values of the unknowns. This process can be repeated until the solution converges to acceptable limits. The significant feature of this system of equations is that in most cases the second approximation (first iteration) is so good that there is no need for further iterations. Since the coefficients are expressed as derivatives of the function $f(H, Q)$, the solution benefits from the quadratic convergence characteristics of Newton method[11]. However, when the geometric properties of cross-sections vary rapidly with water levels two or more iterations are necessary. An example of such variation in river geometry is the transition of flow confined within river banks to a very flat flood plain in one computational time step.

3.3 Solution algorithm

Application of equations (8) and (9) to all grid-points in an estuarial network leads to a very large sparse matrix, which can be solved by standard matrix solution techniques, assuming all grid points dynamically linked. The main drawback of this method is the large size of the matrix, requiring excessive computational time. This practically nullifies the advantages of using implicit numerical models. Therefore, the present formulation is based on double sweep algorithm for loop-river system[11]. In this method a system of linear simultaneous equations is developed, with water levels at channel junctions as only unknowns. This system of equations can be written as:

$$[A]\{\Delta H\} = \{B\} \tag{10}$$

where $[A]$ is a coefficient matrix, $\{\Delta H\}$ represents water level changes at junctions, and $\{B\}$ is a column matrix. The coefficients $a_{k,l}$ and b_k of the matrices $[A]$ and $\{B\}$ are formed by sweeping towards the kth junction from junction l, connected to the kth junction, through the corresponding connecting branch. Considering the schematization of the Meghna Delta in fig 2, the coefficient matrix $[A]$ will be of size 8×8 *i.e.* equal to the number of junctions in the river network. The fifth junction is connected to the second, fourth and sixth junctions through the eighth, seventh and ninth branches respectively. The elements of equations (10) for $k = 5$ can be written as:

$$a_{52}\Delta H_2 + a_{54}\Delta H_4 + a_{55}\Delta H_5 + a_{56}\Delta H_6 = b_5. \tag{11}$$

The other coefficients in equation (11) are zero as the fifth junction is not directly connected to other junctions in the network. The process is repeated for each junction in the estuarial network. The form of the matrix $[A]$ depends on the connections of the junctions in the network. In general, the matrix has a diagonal of non-zero elements, with majority of off-diagonal elements being zero. The non-zero off-diagonal elements indicate direct connection between junctions.

The recurring relationships[11] used for expressing the dependence of water levels at junction k on junction l, in the matrix $[A]$, are given by:

$$\Delta Q_{i+l,j} = p_{i,j}\Delta H_{i+i,j} + q_{i,j}\Delta H_i + r_{i,j} \tag{12}$$

$$\Delta Q_{i,j} = p_{i,j}\Delta H_{i,j} + q_{i,j}\Delta H_i + r_{i,j} \tag{13}$$

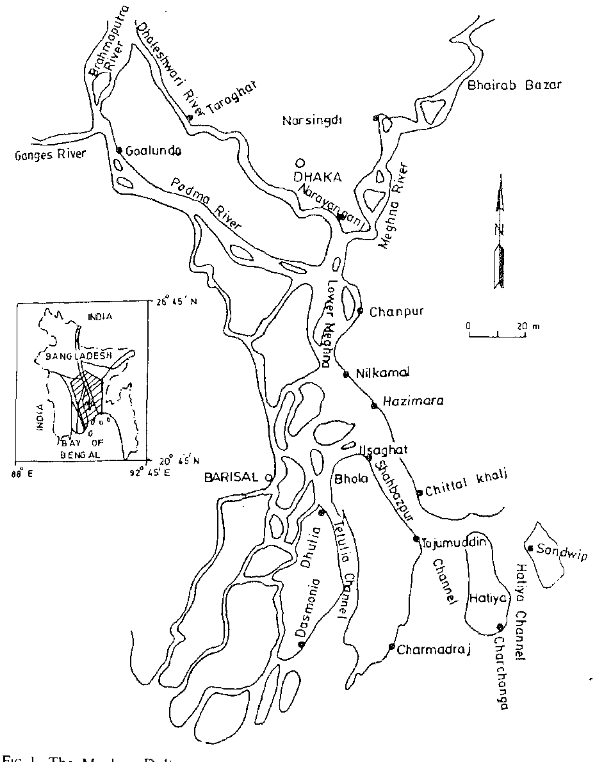


FIG 1. The Meghna Delta.

depending on the direction of sweep. Equation (12) is used for forward sweep, while equation (13) is used for backward sweep. In equations (12) and (13) the subscript j represents branch number connecting junction l to junction k and i is section number in the branch. The subscript j is used to indicate that sweeps are to be made through all branches connected to kth junction. The sweep coefficients $p_{i,j}$, $q_{i,j}$ and $r_{i,j}$ are functions of the coefficients of equations (8) and (9) and the values of the sweep coefficients at previous gridpoints. Equation (12) or (13) for grid-point adjacent to the junction is substituted in equation (4) and simplified using equation (3) to obtain the coefficients of the junction matrix. This solution technique reduces the size of the matrix to the number of junctions in the network, instead of the number of grid-points, thereby reducing the

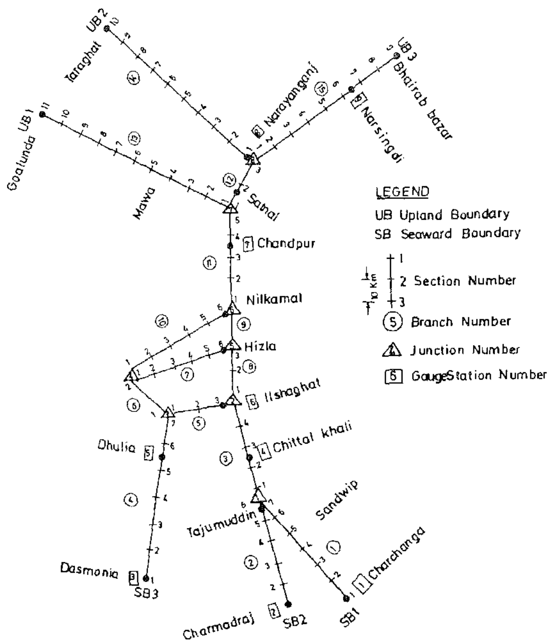


FIG 2. Schematic representation of the Meghna Delta.

computational efforts required for the solution of the matrix significantly. In the present model the junction matrix has been solved by Gauss-Jordan elimination. Since the coefficient matrix $[A]$ is diagonally dominant, pivoting of the matrix at each step of solution is not necessary. This saves considerable computational time.

Once water levels at junctions are known, each branch in the network can be treated independently, using the known water levels at junctions as boundary conditions. New discharges at interior grid-points are computed by equation (12) or (13) and water levels by either of the two following recurring relationships:

$$\Delta H_{i,j} = L_{i,j}\Delta H_{i+1,j} + M_{i,j}\Delta Q_{i+,1j} + N_{i,j} \tag{14}$$

$$\Delta H_{i+1,j} = L_{i,j}\Delta H_{i,j} + M_{i,j}\Delta Q_{i,j} + N_{i,j} \tag{15}$$

depending on the direction of the last sweep along the branch while forming the junction matrix. In equations (14) and (15) the coefficients are functions of the coefficients of equations (8) and (9).

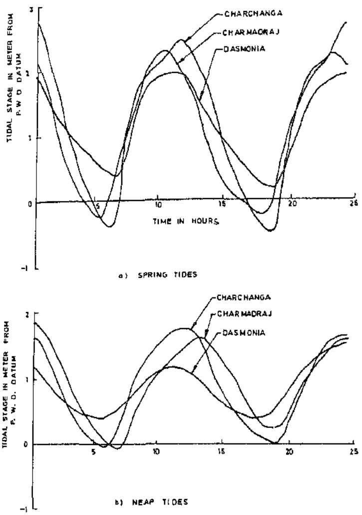


FIG 3. Sea-ward model boundary conditions.

3.4 Computer programme

A computer programme has been developed for the solution of the governing equations using the concept outlined above. The programme has been written in FORTRAN-77 and run on IBM-4331 Model K02 computer at the Computer Center, Bangladesh University of Engineering and Technology, Dhaka. The computer storage required for applying the model to the Meghna Delta was about 120 kilo-bytes. The computational time per grid-point per time step, including processing of the computed water levels and discharges to find maximum and minimum values of water levels, discharges and velocities, tidal excursion, tidal prism, etc., at each grid-point over a tidal cycle was about 0.0181 second. The solution algorithm used in this study is similar to the algorithm of the CARIMA model of the SOGREAH Consulting Engineers[3]. Most of the one-dimensional numerical models for the simulation of unsteady flows in large river networks are based on similar algorithm[11].

4. Application of the model

As a practical application, the model was applied to the Meghna Delta in Bangladesh, because of the availability of geometric and hydrographic data of the river system from a previous study[14]. The study was based on the 'method of cubature', solving equation (1) for discharges at different sections. Water levels at the sections were linearly interpolated from adjacent gauging stations.

In the Meghna Delta (fig. 1), the main channels divide into a number of tributaries and distributaries and have three main outfalls, namely Shahbazpur channel, Hatiya channel and Tetulia channel. The main flow carrying channel[15] is the Shahbazpur channel. Fresh water discharges mainly come from the rivers Padma and the Meghna. The tide is mainly semi-diurnal in character with strong shallow water effects, which distort the shape of the tidal hydrographs considerably[15].

4.1 Schematization of the network

Matin[14] had schematized the Meghna delta by dividing the branches into small reaches of length $\Delta x = 10$ km. The schematized portion of the delta has three upland discharge boundaries at Goalundo, Bhairab Bazar and Taraghat and three sea-ward boundaries at Charmadraj, Charchanga and Dasmonia. This schematization resulted in 15 branches, 8 junctions and 85 computational sections as shown in fig. 2. Actual and schematized lengths of each branch are shown in Table I. The geometric parameters at grid-points were extracted by analysing sounding charts of the rivers, obtained from Bangladesh Inland Water Transport Authority.

The cross-sections at grid-points were represented by equivalent rectangular sections. At each section top-width of the channel and cross-sectional area were available from the previous study. The minimum width to depth ratio[14] of the rivers within the study reach was 34.5:1. Therefore, all the channels may be considered as wide and the use of rectangular section for the simulation of dry season flow appears justified[4]. Junction storage area at all the eight junctions was neglected due to the non-availability of junction storage data. Figure 2 also shows the locations of the gauging stations for which tidal hydrographs were available for the calibration and validation of the model.

4.2 Calibration of the model

The model was calibrated by adjusting Manning's roughness coefficients in different branches of the river system and comparing the computed and observed tidal stages at Chittal Khali, Dhulia, Ilshaghat, Chandpur, Narayanganj and Narsingdi. The tidal hydrographs used at Charchanga, Charmadraj and Dasmonia as boundary conditions are shown in fig.3a. At the river end boundaries, at Goalundo, Taraghat and Bhairab Bazar, constant fluvial inflows of 9170.0, 20.5 and 3070.0 cumecs were specified respectively. These data were also obtained from Matin[14] and correspond to spring tide of April 1977. The results of model calibration are showin in fig. 4. The values of Manning's roughness coefficients for different branches are given in Table I. During the application of the model time step of 1800 seconds was used. To start the computations from a common horizontal initial water level at Charmadraj, initial limos of the tidal hydrographs at Charchanga and Dasmonia were replaced by transitional curves during the first cycle. Moreover, all the three hydrographs were also made cyclic to obtain harmonic

Table I
Channel lengths and roughness coefficients

Branch no.	Name of the river	Channel length (km)*		Roughness coeff.**	
		Actual	Schematized	Manning's	Chezy's
1	Hatya Channel	60.0	60.0	0.015	96
2	Shahbazpur Channel	45.0	50.0	0.015	96
3	Lower Meghna	45.0	40.0	0.015	92
4	Tetulia	56.0	60.0	0.020	68
5	Ganespura	27.5	20.0	0.025	55
6	Tetulia	15.8	10.0	0.025	57
7	Azimpura-Dharmaganj	46.5	50.0	0.025	57
8	Lower Meghna	27.5	20.0	0.020	72
9	Lower Meghna	7.5	10.0	0.020	72
10	Jayanti	56.7	50.0	0.025	57
11	Lower Meghna	38.5	40.0	0.025	57
12	Meghna	21.8	20.0	0.025	57
13	Padma	96.5	100.0	0.030	46
14	Dhaleshwari	86.6	90.0	0.035	36
15	Meghna	80.5	80.0	0.030	49

*Obtained from Matin[14]
**Calibrated values (n in $m^{-1/3}s$ and C in $m^{1/2}s^{-1}$)

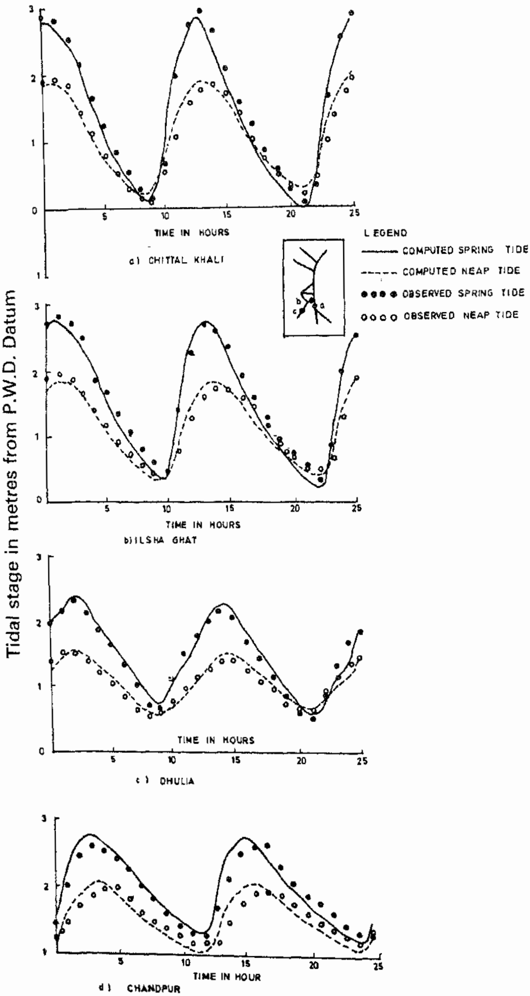
conditions. The model was found to reach harmonic condition after three tidal cycles. After reaching harmonic conditions, subsequent tidal hydrographs at the model boundaries were directly used as obtained from field observations.

4.3 Validation of the model

For the calibrated values of Manning's roughness coefficients the model was used to reproduce neap tide at the interior gauging stations. The sea-ward boundary conditions, corresponding to neap tide on April 1977 as shown in fig. 3b, were used for model validation. The river-end boundary inflows were 9880.0, 23.5 and 2985.0 cumecs at Goalundo, Taraghat and Bhairab Bazar respectively. The results of model validation are also shown in fig. 4.

5. Results and discussion

Matin[14] had faced many problems during collection and preparation of data for his study. Some of them were the non-availability of correlation between local and Public Works Department (PWD) datums for the tidal stage data at Charchanga, Chittal Khali and Dhulia, discharge data at Bhairab Bazar and Taraghat were not available, large segment of missing tidal stage data at Narayanganj, Narsingdi and Chandpur. Moreover, the


Tidal stage in metres from P.W.D. Datum
TIME IN HOURS
a) CHITTAL KHALI
LEGEND
COMPUTED SPRING TIDE
COMPUTED NEAP TIDE
OBSERVED SPRING TIDE
OBSERVED NEAP TIDE
TIME IN HOURS
b) ILSHA GHAT
TIME IN HOURS
c) DHULIA
TIME IN HOUR
d) CHANDPUR

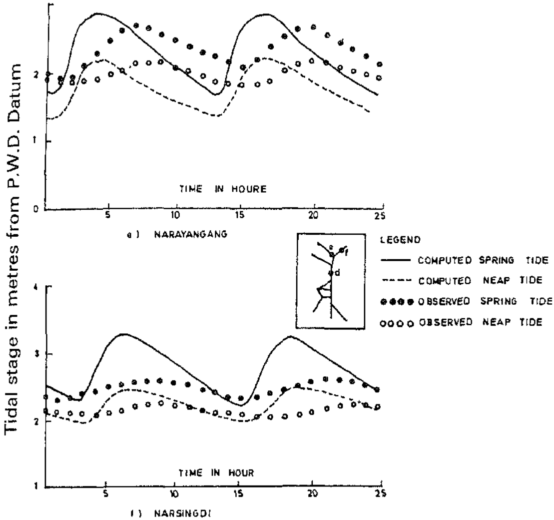


FIG 4. Calibration and validation of the model.

correlations of the datums of the hydrographic charts, within the study area, to a common datum was another problem. Therefore, some adjustments of the available hydrographic and hydraulic data were necessary[14]. The missing tidal stage data were estimated by graphical interpolation. Where the available data are sparse, like that of Narayanganj and Narsingdi, such estimation can give erroneous tidal range. Fresh water discharges of the Meghna and Dhaleshwari rivers at Bhairab Bazar and Taraghat respectively were obtained from old rating curves. The combined discharge of the Meghna and the Dhaleshwari rivers is small compared to that of the Padma at Goalundo. The discharge of the Meghna falls within the standard deviation of the discharge of the Padma[15]. The discharge of the Dhaleshwari is practically negligible, compared to the other two rivers. Therefore, the errors in the estimation of discharges will not have significant effects on the computed results. The correlations between local datums and PWD datum for both tidal stage and hydrographic charts appear approximate. In the absence of sufficient and reliable data such simplifications and approximations are unavoidable.

Figure 3 shows the variations of observed spring and neap tides at Charchanga, Charmadraj and Dasmonia used for model calibration and validation. This figure indicates that the tidal range decrease from east to west along the coast of the Meghna

Delta. Along the coastline, high water and low water levels decrease in the same direction. The distortions in the observed tidal hydrographs at these locations indicate the existence of strong shallow water effects at the outfall of the delta. At the outfalls the rivers are very wide, with widths as high as 30 km. Therefore, one-dimensional representation of these channels is not very consistent with the assumptions of one-dimensional hydrodynamic models[16].

Comparison of the computed and observed tidal hydrographs in fig. 4 indicates that the model has simulated the variations of tidal range and shape within acceptable limits, except at Narayanganj and Narsingdi. At these locations the computed tidal ranges are very high and significantly out of phase with the observed hydrographs. Adjustments of Manning's roughness coefficients, within acceptable limits, did not improve the results appreciably. One of the possibilities of improving the results, without reschematizing the network and further data collection, was the introduction of approximate non-zero junction storage area in the model. From fig. 1 it appears that all the junctions of the channels in the network, specially the confluences of the Meghna and the Padma upstream of Chandpur and the Shahbazpur and Tetulia channels at Ilshaghat, have considerable junction storage areas. However, the schematization of the river system[14] did not include this parameter, probably because of the simplified methodology used in the study. To ascertain its effects, junction storage areas of 1.5×10^8 and 0.5×10^8 m^2 were introduced at the confluences of the Padma and the Meghna rivers and the Dhaleshwari rivers respectively. These values were extracted from 1:200,000 map of the river system of Bangladesh and represent approximate junction storage areas, as the exact locations of the adjacent grid-points were not known. The effects of introducing junction storage on the computed tidal stages at Narayanganj and Narsingdi are shown in fig. 5, for the spring and neap tides in fig. 3 as boundary conditions at the sea-ward limits of the model. This had improved the results considerably. Therefore, it is expected that the model will give better simulation of tides with the availability of relevant data and refinement of the correlations between chart datums and PWD datum. Since hydrographs at Narayanganj and Narsingdi were constructed from limited field data[14], the tidal range indicated in the figure may not represent the actual field conditions. Under these conditions attempts to simulate the field conditions in a better way was not considered justified.

To calibrate the model, covering about 710 km of major channels, hydrographs at only six interior gauging stations were available. The hydrographs at Narayanganj and Narsingdi appear to be of questionable accuracy. As a result, the calibration of the model can, at best, be considered approximate. During the model calibration Manning's roughness coefficient was assumed constant for each branch. The calibrated values of Manning's roughness coefficients of the different branches of the Meghna Delta are shown in Table I. The corresponding values of Chezy's roughness coefficients, assuming the channels very wide, are also given in the table. The values of Manning's roughness coefficients vary from 0.015 to 0.035 $m^{-1/3}s$ and that of Chezy's roughness from 96 to 39 $m^{1/2}s^{-1}$. McDowell and Prandle[17] had used Manning's roughness coefficients from 0.0126 to 0.04 $m^{-1/3}s$ for the Hoogly-Rupnarain river system at the western boundary of the Ganges Delta. For the simulation of tides in the Pussur-Sibsa river system, in the

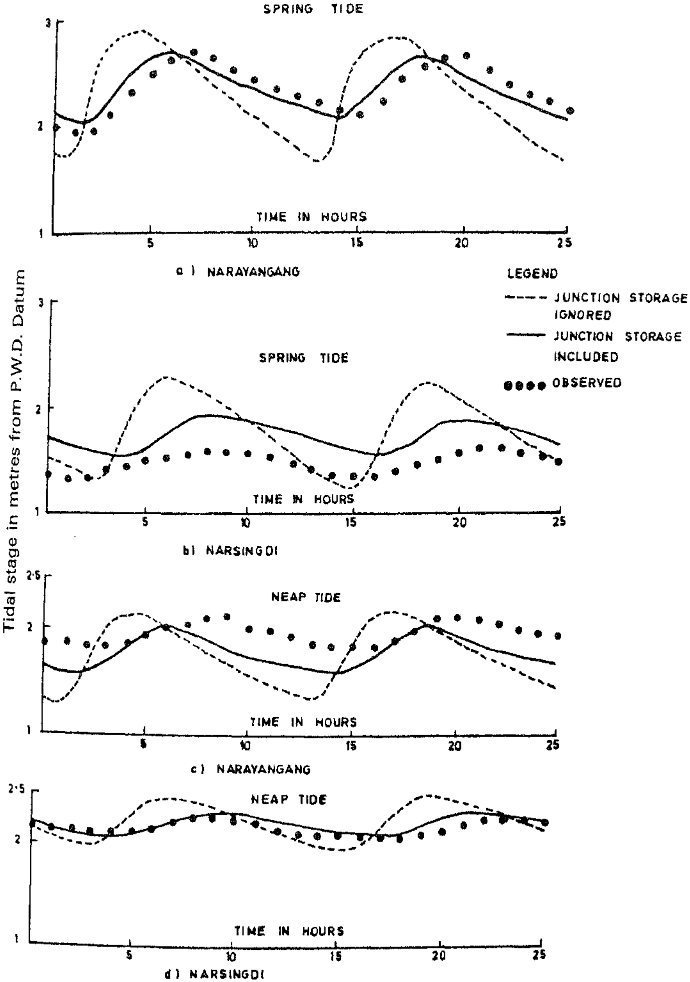


FIG 5. Effects of junction storage on computed tidal stage.

central zone of the Ganges Delta, Chowdhury[9] had used Chezy's roughness coefficients between 100 and 50 $m^{1/2}s^{-1}$. Williams and Chidley[18] had used constant Chezy's roughness coefficient of 40 $m^{1/2}s^{-1}$ for the Burishwar-Bishkhali river system, adjacent to the Meghna Delta. The Meghna Delta is a part of the Ganges Delta, forming eastern boundary. Therefore, the values of roughness coefficients used for the calibration of the model are quite reasonable when compared with the previous studies of similar tidal elements in the Ganges Delta. The wide variations of roughness coefficients among mathematical models of tidal systems with similar hydraulic characteristics suggests that the resistance coefficients in the numerical models are to some extent dependent on the type of numerical scheme[11,12]. This is because of the varying amount of 'non-physical resistance' associated with the difference scheme, which is effected by the dissipative interface used for stabilizing the scheme and discretization of the non-linear coefficients in the governing equations. Moreover, roughness is also dependent on the geometric schematization of the river system[7,9].

6. Conclusions

Considering the simplified schematization of Meghna delta, the numerical model has simulated the tidal characteristics of the estuarial network within reasonable limits. The major junctions of the Meghna Delta have large junction storage areas and cause considerable damping and attenuation of the tidal wave. For better simulation of tides in the Meghna Delta proper schematization of the network, including junction storage, is considered necessary. The discrepancies between the computed and observed stages at Narayanganj and Narsingdi, even after the inclusion of junction storages, upstream of Chandpur, may be attributed to the errors in correlating local datum with the PWD datum, estimation of stage hydrographs from sparse data and the variations of junction storage with tidal stage. Most of the confluences in the network have large bars, with various degrees of submergence with tidal stage, causing large variations in junction storage.

The study shows that numerical models of tidal regimen can be easily developed by using the locally available research facilities and expertise. However, the main difficulty in utilizing the model results is the lack of reliable and sufficient field data for model calibration and validation. There is no alternative to systematic data collection to improve the reliability of model results. Proper coordination between research organizations and data collecting agencies can solve the problem to a great extent.

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