REVIEWS

Unified modelling theory for qubit representation using quantum field graph models

Vishal Sahni[1], *Dayal Pyari Srivastava*[2] AND *Prem Saran Satsangi*[3]

Abstract | Linear graph theory, a branch of topology, has been applied to such diverse systems as ranging from electrical networks through real physical systems and "conceptual" socio-economic-environmental systems to "esoteric" creational systems. Linear graph theory represents one step towards a systems modelling discipline which coordinates various branches of knowledge into one scientific order.

This paper presents a quantum field graph model which is facilitated by considering a qubit as a basic building block and representing it through an appropriate linear graph. The system graph is in two separate parts corresponding to a qubit representation ($|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$), consisting of "ket 0" and "ket 1" subsystem graphs. Unit "Poynting"-like vectors $|0\rangle$ and $|1\rangle$ behave like quantum across (potential) variables specifying "direction" of information (data)/energy propagation, as it were, while α and β behave like quantum through (flow-rate or "force") variables specifying probability parameters for quanta of information (data)/energy flow-rate (energy flux or power flow, W/m^2). For n independent states, there will be precisely n basis quantum potential vectors (or unit "Poynting" vectors). The model has been successfully applied for several quantum gates as well as applications such as quantum teleportation and has the potential for successfully modelling systems at the high end of complexity scale.

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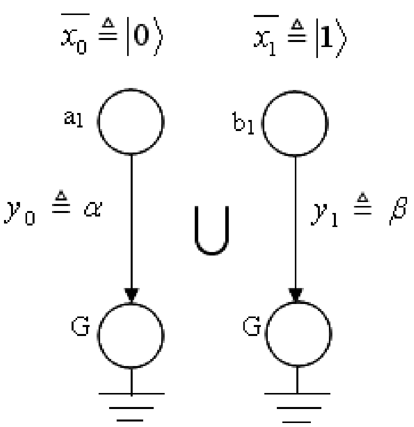
1. Introduction

Quantum Information Science is a broad and rapidly expanding field. The theory of classical information, computation, and communication developed extensively during the twentieth century. Though undeniably useful, this theory cannot fully characterize how information can be used and processed in the physical world — a quantum world. Some achievements of quantum information science can be described as generalizations or extensions of the classical theory that apply when information is represented as a quantum state rather than in terms of classical bits. The well-established theory of classical information and computation is actually a subset of a much larger topic, the emerging theory of quantum information and computation.

In a quantum computer, the fundamental unit of information (called a quantum bit or *qubit*), is not binary but rather more quaternary in nature. This qubit property arises as a direct consequence of its adherence to the laws of quantum mechanics which differ radically from the laws of classical physics. A qubit can exist not only in a state corresponding to the logical state 0 or 1 as in a classical bit, but also in

Figure 1: Single Qubit



states corresponding to a blend or *superposition* of these classical states. In other words, a qubit can exist as a zero, a one, or simultaneously as both 0 and 1, with a numerical coefficient representing the probability for each state. This may seem counterintuitive because everyday phenomenon are governed by classical physics, not quantum mechanics – which takes over at the atomic level.

Topology implies the mathematical study of those properties of geometric forms that remain invariant under certain transformations as bending, stretching, squeezing or any continuous deformation that does not involve tearing or joining. One important branch of topology is the theory of linear graphs. The abstract graph theory has its own elegance, of course, but it is appreciation of its wide ranging application to modelling quantum computing systems which forms the focus of this paper.

Linear graph theory has been applied to such diverse systems as ranging from electrical networks through real physical systems and "conceptual" socio-economic-environmental systems to esoteric systems of consciousness. It is shown that linear graph theory represents one step towards a systems modelling discipline which coordinates various branches of knowledge into one scientific order [5,6,7,8].

A linear graph theoretic model had been developed [2,3] for quantum systems in which each qubit is represented by 2 nodes and entanglements are represented as edges. There is a ground node, connections to which play an important role especially in quantum measurement. However, the technique required use of hyper-edges for depicting entanglement and representation becomes difficult for applications like teleporation.

Secion 2 presents a quantum field graph model by considering a qubit as a basic building block and developing a "ket 0" subgraph and a "ket 1" subgraph". The system graph, corresponding to a qubit representation $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, is in two separate parts, consisting of "ket 0" and "ket 1" subsystem graphs [7,8]. Unit Poynting vectors $|0\rangle$ and $|1\rangle$ behave like quantum across (potential) variables specifying "direction" of information (data)/energy propagation, as it were, while α and β behave like quantum through variables specifying probability parameters for quanta of information/energy flow-rate (energy flux or power flow, W/m^2). For n independent states, there will be precisely n basis quantum potential vectors (or unit "Poynting" vectors) [4,7,8,9]. The conclusions are presented in Section 3.

2. Quantum Field Graph Models

2.1. *Single Qubit*

This is the simplest case of representation involving a single qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ as shown in Figure 1. A single qubit representation is physically realizable as an "infinitesimal" (nanoscale) element of quantum force-field (e.g. electromagnetic force-field).

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where α and β are complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$ to conform with probability interpretation

and where $|\psi\rangle$ is the "Poynting"-like information/energy-flux or power vector (W/m^2)

$|0\rangle$ and $|1\rangle$ are the Orthogonal Unit "Poynting"-like vectors

α—Scalar probability parameter for information/Power flow along $|0\rangle$ unit "Poynting"-like vector

β—Scalar probability parameter for information/Power flow along $|1\rangle$ unit "Poynting"-like vector

$$\overline{x_0} \triangleq |0\rangle \quad \overline{x_1} \triangleq |1\rangle \quad y_0 \triangleq \alpha \quad y_1 \triangleq \beta$$

Y = Classical through variable vector of scalars, $Y = \begin{bmatrix} y_o \\ y_1 \end{bmatrix}$

$\overline{X}$ = Quantum across variable vector of "Poynting"-like vectors $\overline{X} = \begin{bmatrix} \overline{x_o} \\ \overline{x_1} \end{bmatrix}$

$$|\psi\rangle = \text{Information/Power Flow Vector} = Y^T\overline{X} = \begin{bmatrix} y_o & y_1 \end{bmatrix} \begin{bmatrix} |0\rangle \equiv \overline{x_0} \\ |1\rangle \equiv \overline{x_1} \end{bmatrix}$$

$$|\psi\rangle = \langle Y|\,|\overline{X}\rangle = Y^T\overline{X}$$

Figure 2: Superposition of Two Independent Qubits

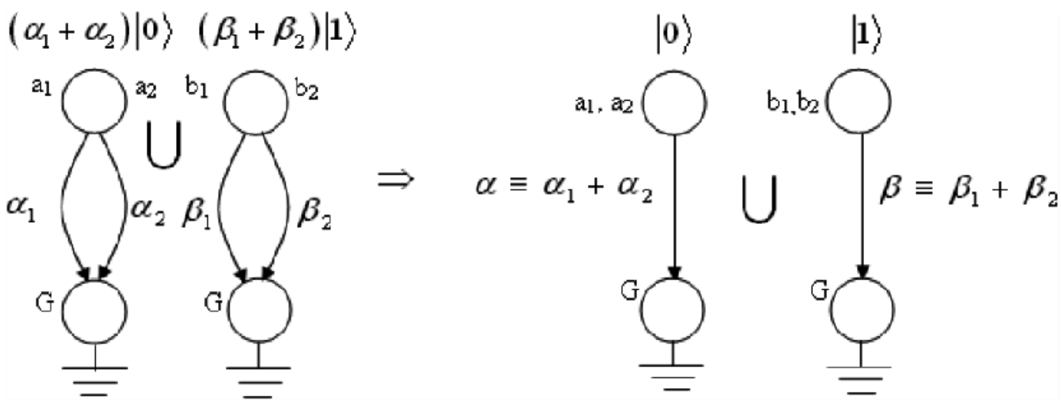
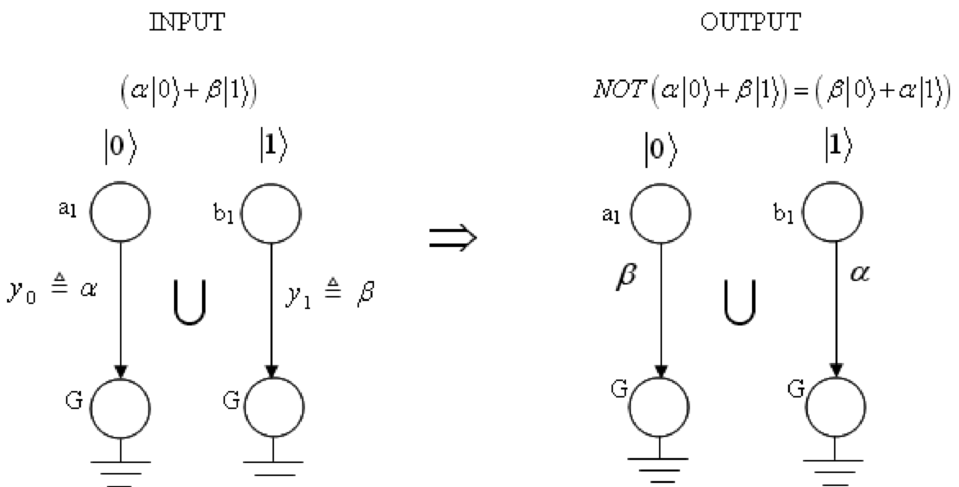


Figure 3: NOT Gate operation



We can generalize Poynting vector in W/m^2 as active and reactive power flow (as in electrical power systems) for completeness, considering α and β as complex numbers.

Y – Classical vector

$\overline{X}$ – Quantum vector

2.2. *Superposition of two independent qubits*

The next case is of two independent qubits which would be mathematically represented as $|\psi_1\rangle \equiv (\alpha_1|0\rangle + \beta_1|1\rangle)$; $|\psi_2\rangle \equiv (\alpha_2|0\rangle + \beta_2|1\rangle)$ and is shown in Figure 2.

$$|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$$
$$= (\alpha_1+\alpha_2)|0\rangle + (\beta_1+\beta_2)|1\rangle \equiv \alpha|0\rangle + \beta|1\rangle$$

where $\alpha = \alpha_1 + \alpha_2$ and $\beta = \beta_1 + \beta_2$

such that $|\alpha_1|^2 + |\beta_1|^2 = 1$ and $|\alpha_2|^2 + |\beta_2|^2 = 1$

$$|\psi_1\rangle = \langle Y_1||\overline{X_1}\rangle = Y_1^T\overline{X_1}$$

where $Y_1 = \begin{bmatrix} y_{10} \\ y_{11} \end{bmatrix}$ $\quad y_{10} = \alpha_1$

$y_{11} = \beta_1 \quad \overline{x_{10}} \triangleq |0\rangle \quad \overline{x_{11}} \triangleq |1\rangle \quad \overline{X_1} = \begin{bmatrix} \overline{x_{10}} \\ \overline{x_{11}} \end{bmatrix}$

$$Y_1^T\overline{X_1} = \begin{bmatrix} y_{10} & y_{11} \end{bmatrix} \begin{bmatrix} |0\rangle \equiv \overline{x_{10}} \\ |1\rangle \equiv \overline{x_{11}} \end{bmatrix}$$

$$|\psi_2\rangle = \langle Y_2||\overline{X_2}\rangle = Y_2^T\overline{X_2}$$

where $Y_2 = \begin{bmatrix} y_{20} \\ y_{21} \end{bmatrix}$ $\quad y_{20} = \alpha_2 \quad y_{21} = \beta_2$

$$\overline{x_{20}} \triangleq |0\rangle \quad \overline{x_{21}} \triangleq |1\rangle \quad \overline{X_2} = \begin{bmatrix} \overline{x_{20}} \\ \overline{x_{21}} \end{bmatrix}$$

$$Y_2^T \overline{X_2} = \begin{bmatrix} y_{20} & y_{21} \end{bmatrix} \begin{bmatrix} |0\rangle \equiv \overline{x_{20}} \\ |1\rangle \equiv \overline{x_{21}} \end{bmatrix}$$

$$|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle = Y_1^T \overline{X}_1 + Y_2^T \overline{X}_2$$

$$= \begin{bmatrix} Y_1^T & Y_2^T \end{bmatrix} \begin{bmatrix} \overline{X_1} \\ \overline{X_2} \end{bmatrix}$$

$$= Y^T \overline{X} \text{ where } Y \triangleq \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad \overline{X} = \begin{bmatrix} \overline{X_1} \\ \overline{X_2} \end{bmatrix}$$

2.3. *NOT Gate Operation*

A NOT gate converts an input of $|0\rangle$to a $|1\rangle$ and vice versa. For the superposition state, $\alpha|0\rangle + \beta|1\rangle$, the output would be $\alpha|1\rangle + \beta|0\rangle$ which is represented using quantum graph in Figure 3.

$$NOT\{\alpha|0\rangle + \beta|1\rangle\} = \{\beta|0\rangle + \alpha|1\rangle\}$$

$$NOT\left\{ Y^T X = \begin{bmatrix} y_0 & y_1 \end{bmatrix} \begin{bmatrix} |0\rangle \equiv \overline{x_0} \\ |1\rangle \equiv \overline{x_1} \end{bmatrix} \right\}$$

$$= \begin{bmatrix} y_1 & y_0 \end{bmatrix} \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix} = Y'^T \overline{X} \text{ where } Y' = \begin{bmatrix} y_1 \\ y_0 \end{bmatrix}$$

2.4. *Hadamard Gate Operation*

This example demonstrates a Hadamard gate operation in which qubit $|0\rangle$ is transformed to $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and $|1\rangle$ to $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$. Figure 4.1 shows the graph representation for both these cases.

$$|\psi_i\rangle = y_i^T \overline{x_i}$$

$$\text{where } y_i = \begin{bmatrix} y_{i0} \\ y_{i1} \end{bmatrix} \quad \overline{x_i} = \begin{bmatrix} \overline{x_{i0}} \\ \overline{x_{i1}} \end{bmatrix}$$

$$= \begin{bmatrix} y_{i0} & y_{i1} \end{bmatrix} \begin{bmatrix} \overline{x_{i0}} \\ \overline{x_{i1}} \end{bmatrix} \triangleq |0\rangle \text{ or } |1\rangle \text{ resp.}$$

$$Y_\phi \triangleq \begin{bmatrix} y_{\phi 0} \\ y_{\phi 1} \end{bmatrix} \quad Y_i \triangleq \begin{bmatrix} y_{i0} \\ y_{i1} \end{bmatrix}$$

Figure 4.1: Hadamard Gate Operation

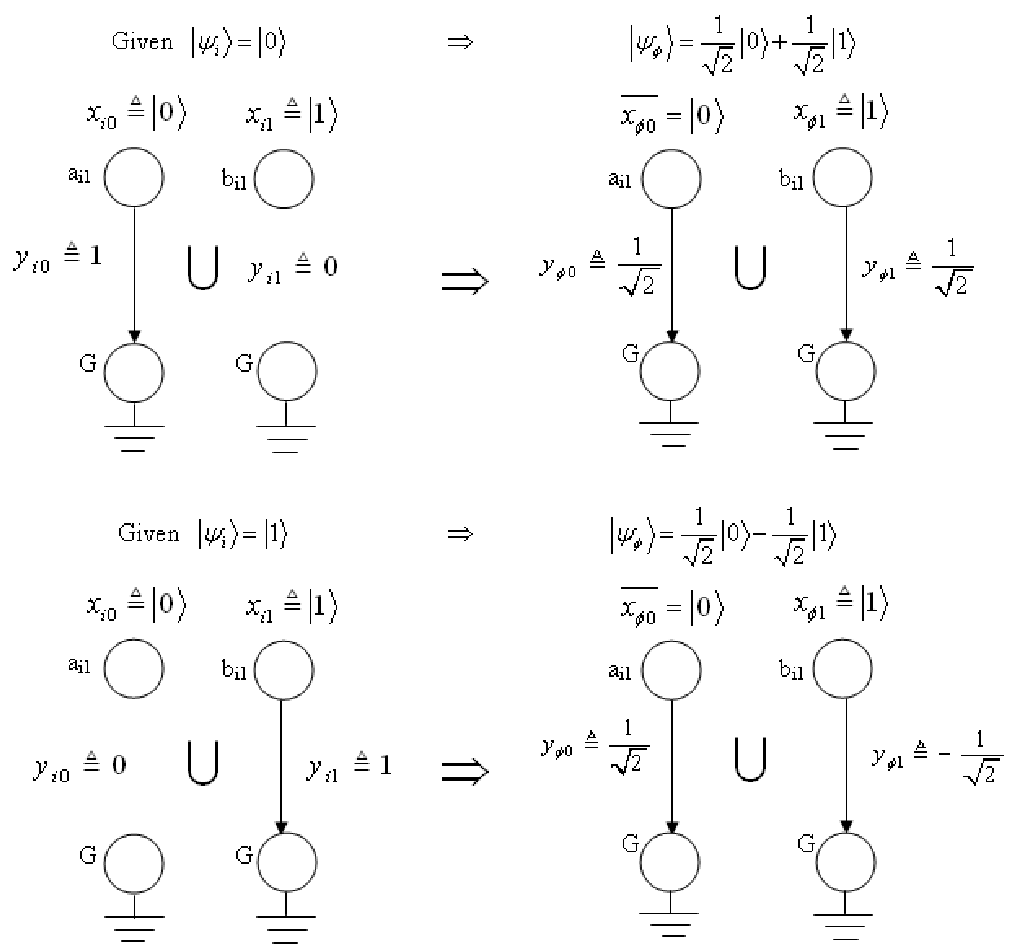
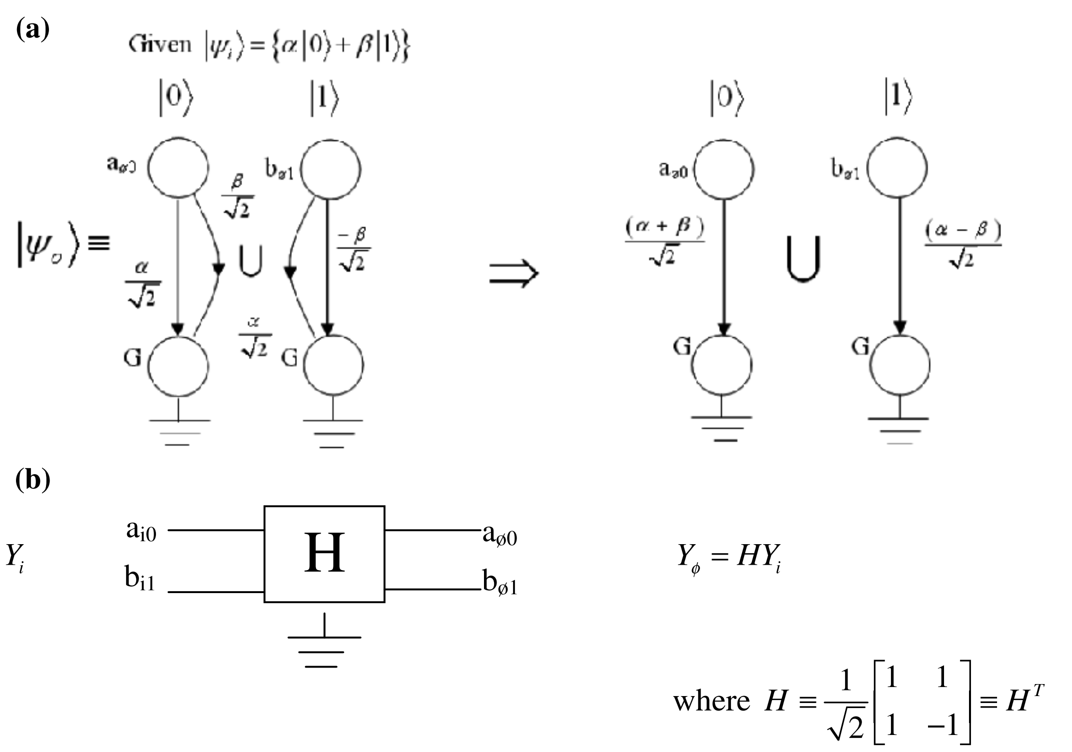


Figure 4.2: (a) Hadamard Gate Operation (b) Transfer function representation



$$Y_\phi = HY_i$$

$$\begin{bmatrix} y_{\phi 0} \\ y_{\phi 1} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} y_{i0} \\ y_{i1} \end{bmatrix}$$

$$|\psi_0\rangle = Y_\phi^T \overline{X_\phi}$$

$$= \begin{bmatrix} y_{\phi 0} & y_{\phi 1} \end{bmatrix}\begin{bmatrix} \overline{x_{\phi 0}} \\ \overline{x_{\phi 1}} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \text{ or } \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \text{ resp.}$$

$$\text{where } Y_\phi = \begin{bmatrix} y_{\phi 0} \\ y_{\phi 1} \end{bmatrix} \quad X_\phi = \begin{bmatrix} x_{\phi 0} \\ x_{\phi 1} \end{bmatrix}$$

Figure 4.2(a) shows the graph representation for superposition of both cases, i.e. $\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha\frac{|0\rangle+|1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{\alpha+\beta}{\sqrt{2}}|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}|1\rangle$ and Fig. 4.2(b) shows transfer function representation of Hadamard gate.

Given $|\psi_i\rangle = Y_i^T \overline{X}_i$

$$= \begin{bmatrix} \alpha & \beta \end{bmatrix}\begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix}$$

$$\Rightarrow |\psi_\phi\rangle = Y_\phi^T \overline{X}_\phi = [HY_i]^T \overline{X}_\phi = Y_i^T H \overline{X}_\phi$$

$$= \begin{bmatrix} Y_{i0} & Y_{i1} \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix}$$

$$= \begin{bmatrix} \alpha & \beta \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}(\alpha+\beta)|0\rangle + \frac{1}{\sqrt{2}}(\alpha-\beta)|1\rangle$$

such that

$$|\alpha|^2 + |\beta|^2 = 1 \triangleq \left(\frac{|\alpha|^2}{2} + \frac{|\beta|^2}{2}\right) + \left(\frac{|\alpha|^2}{2} + \frac{|-\beta|^2}{2}\right)$$

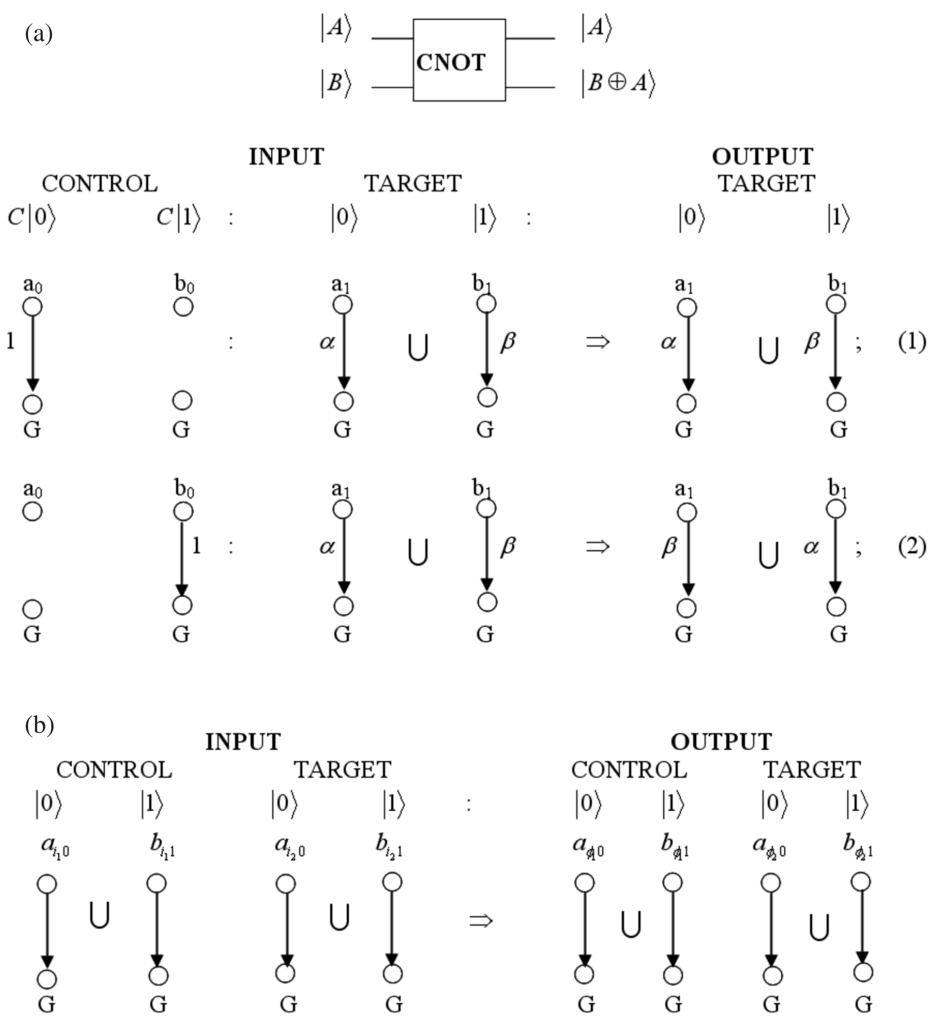
2.5. *Controlled NOT gate*

A Controlled NOT gate has got two qubits, the first one is the control qubit and the second one is the target qubit. If the control qubit is $|0\rangle$, the target qubit is left unchanged. If the control qubit is $|1\rangle$, the target qubit is flipped like a NOT gate. Graph models for both cases are shown in Figure 5(a) and for superposition of states in Figure 5(b).

$$CNOT\{\alpha|0\rangle + \beta|1\rangle\} = \{\alpha|0\rangle + \beta|1\rangle\} \quad \text{if } C = |0\rangle \qquad (1)$$

$$CNOT\{\alpha|0\rangle + \beta|1\rangle\} = \{\beta|0\rangle + \alpha|1\rangle\} \quad \text{if } C = |1\rangle \qquad (2)$$

Figure 5: (a) CNOT Gate Operation; (b) CNOT Gate Operation for superposition of states.



If C = |0⟩

$$CNOT\left\{y^T\overline{X} = \begin{bmatrix} y_0 & y_1 \end{bmatrix}\begin{bmatrix} |0\rangle \triangleq \overline{x_0} \\ |1\rangle \triangleq \overline{x_1} \end{bmatrix}\right\}$$

$$= \begin{bmatrix} y_0 & y_1 \end{bmatrix}\begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix} = Y^T\overline{X}$$

If C = |1⟩

$$CNOT\left\{y^T\overline{X} = \begin{bmatrix} y_0 & y_1 \end{bmatrix}\begin{bmatrix} |0\rangle \triangleq \overline{x_0} \\ |1\rangle \triangleq \overline{x_1} \end{bmatrix}\right\}$$

$$= \begin{bmatrix} y_1 & y_0 \end{bmatrix}\begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix} = Y'^T\overline{X}$$

Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

Truth Table of a CNOT gate

$$Y_\phi = U_{CN}Y_i$$

$$\left\{Y_\phi \triangleq \begin{bmatrix} Y^0_{\phi_1} \\ Y^1_{\phi_1} \\ Y^0_{\phi_2} \\ Y^1_{\phi_2} \end{bmatrix}\right\}$$

$$= \begin{bmatrix} I_{4x4} & 0 & 0 & 0 \\ 0 & I_{4x4} & 0 & 0 \\ 0 & 0 & 0 & I_{4x4} \\ 0 & 0 & I_{4x4} & 0 \end{bmatrix}\left\{\begin{bmatrix} Y^0_{i_1} \\ Y^1_{i_1} \\ Y^0_{i_2} \\ Y^1_{i_2} \end{bmatrix} \triangleq Y_i\right\}$$

Figure 6: Swap Gate Operation

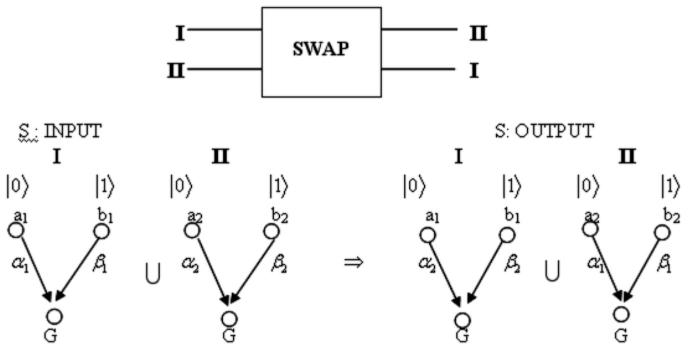


Figure 7: Bell (Entangled States)

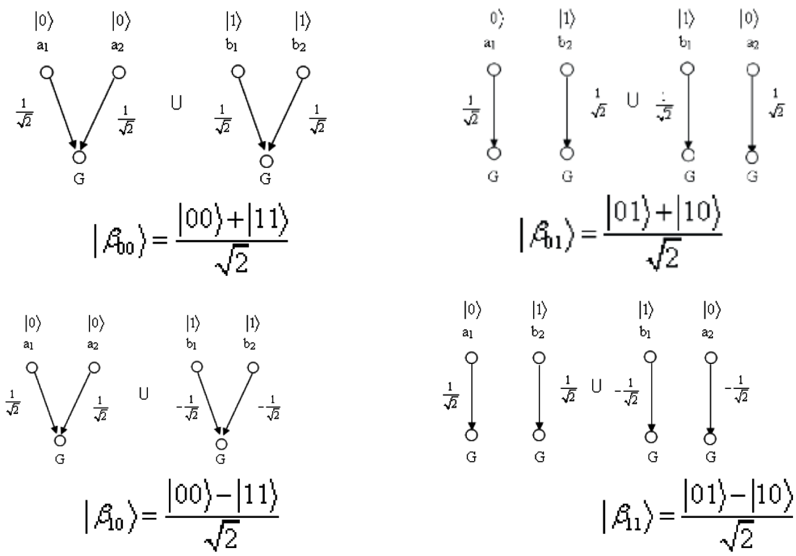
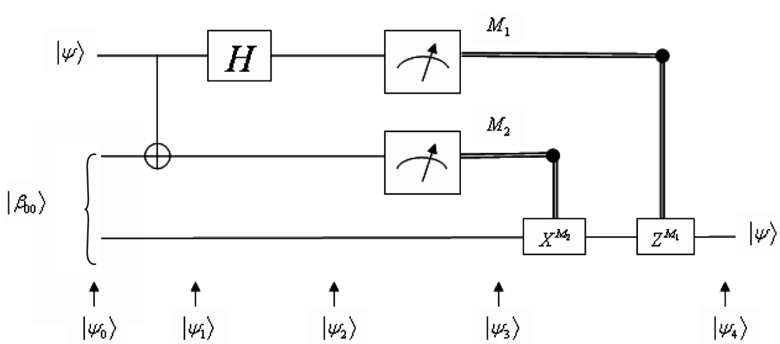


Figure 8: Circuit for teleporting a qubit



$$\text{where } Y_i \triangleq \begin{bmatrix} [1\ 0\ 1\ 0]^T \\ [1\ 0\ 0\ 1]^T \\ [0\ 1\ 1\ 0]^T \\ [0\ 1\ 0\ 1]^T \end{bmatrix}$$

$$\text{and } Y_\phi \triangleq \begin{bmatrix} [1\ 0\ 1\ 0]^T \\ [1\ 0\ 0\ 1]^T \\ [0\ 1\ 0\ 1]^T \\ [0\ 1\ 1\ 0]^T \end{bmatrix}$$

2.6. *Swap Gate*

A swap gate is also a 2-input gate. It takes two qubits as inputs and interchanges them. Thus, it transforms $(\alpha_1|0\rangle + \beta_1|1\rangle);(\alpha_2|0\rangle + \beta_2|1\rangle) \rightarrow (\alpha_2|0\rangle + \beta_2|1\rangle);(\alpha_1|0\rangle + \beta_1|1\rangle)$. Graph model for the same is shown in Figure 6.

$$S\{(\alpha_1|0\rangle + \beta_1|1\rangle);(\alpha_2|0\rangle + \beta_2|1\rangle)\}$$
$$\rightarrow \{(\alpha_2|0\rangle + \beta_2|1\rangle);(\alpha_1|0\rangle + \beta_1|1\rangle)\}$$

2.7. *Bell (Entangled States)*

Bell states are called as *EPR states* or *EPR pairs* after Einstein, Podolsky and Rosen who first pointed out the strange properties of these states which find use in several quantum applications like quantum teleportation. These states are entangled and have the property that their state $|a\rangle \otimes |b\rangle$ cannot be decomposed as $|ab\rangle$. The four Bell states can also be expressed in graph form as shown in Figure 7. The technique improves upon those presented earlier [1,2] in that no hyper-edges are needed to denote entanglement.

2.8. *Quantum Teleportation*

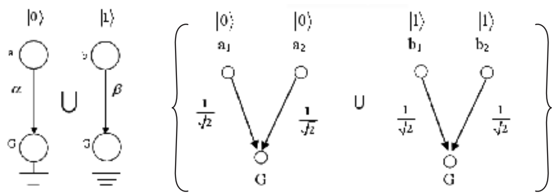
Quantum teleportation is a technique of moving quantum states around, even in the absence of a quantum communications channel linking the sender of the quantum bit to the recipient. Suppose Alice and Bob are two friends who met a long time ago. Before they separated, they generated an EPR pair and each one of them took one qubit of the EPR pair as they went different ways. After several years, Alice wants to send a qubit $|\psi\rangle$ to Bob. However, she can send only classical information to Bob. The solution lies in quantum teleportation with the quantum circuit employed and the graph models of states of their qubits in the three stages shown in Figure 8 and Figure 9 respectively.

Step I

$$|\psi_0\rangle = |\psi\rangle \beta_{00}$$

$$= \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)]$$

Figure 9: Quantum Teleporation – Step I

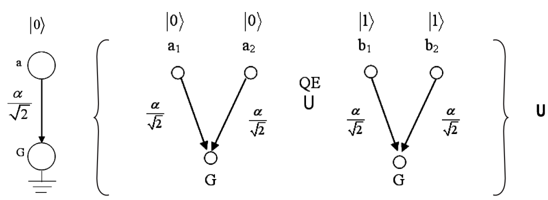


Step II

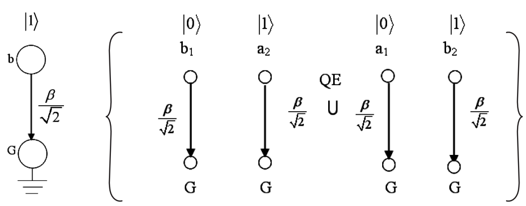
$$|\psi_1\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)]$$

$$\equiv \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|110\rangle + \frac{\beta}{\sqrt{2}}|101\rangle$$

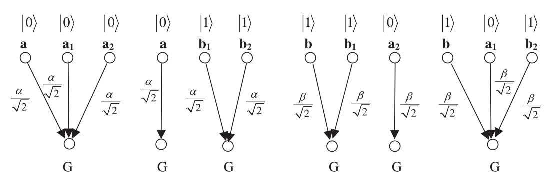
Figure 9: Quantum Teleporation – Step II



where QE stands for Quantum Entanglement Union (or Mutual Coupling)



OR ALTERNATIVE REPRESENTATION

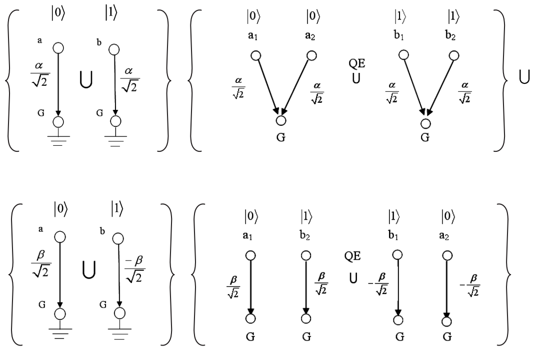


Step III

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\big[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle) \times (|10\rangle + |01\rangle)\big]$$

(depicted in Figure 9: Quantum Teleporation – Step III)

Figure 9: Quantum Teleporation – Step III



$$\equiv \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\alpha}{\sqrt{2}}|100\rangle + \frac{\alpha}{\sqrt{2}}|111\rangle$$
$$+\frac{\beta}{\sqrt{2}}|010\rangle + \frac{\beta}{\sqrt{2}}|001\rangle - \frac{\beta}{\sqrt{2}}|110\rangle - \frac{\beta}{\sqrt{2}}|101\rangle$$

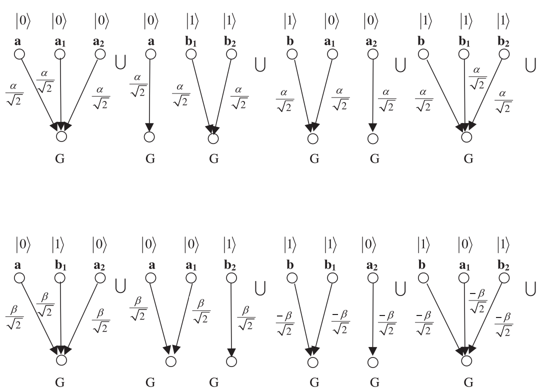
Can also be re-arranged as:

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\big[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\big]$$

OR

$$|\psi_2\rangle = \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\alpha}{\sqrt{2}}|100\rangle + \frac{\alpha}{\sqrt{2}}|111\rangle + \frac{\beta}{\sqrt{2}}|010\rangle + \frac{\beta}{\sqrt{2}}|001\rangle - \frac{\beta}{\sqrt{2}}|110\rangle - \frac{\beta}{\sqrt{2}}|101\rangle$$

Figure 9: Quantum Teleporation – Step III Alternative Representation

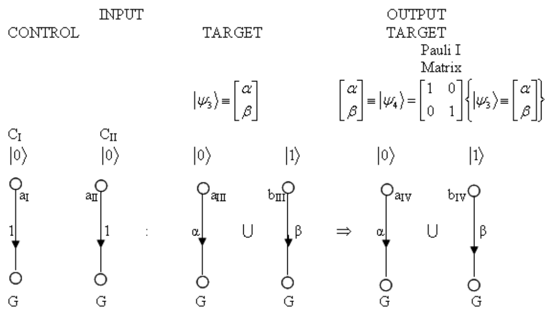


See "Another Alternative Representation": Fig. 9, p. 10.

Figure 10 on p. 11 shows the block diagram for quantum teleportation. After Alice conveys her measurement CC to Bob, there are 4 conditions as described below.

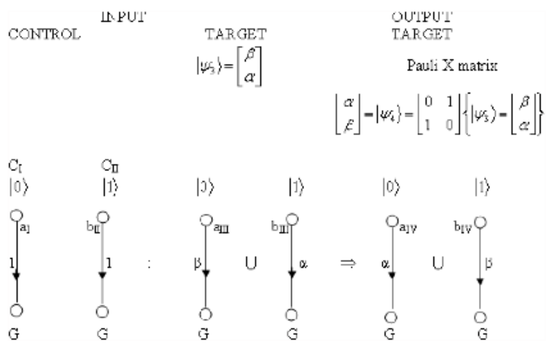
Condition (i)

If CC $= |00\rangle$, apply Pauli I gate



Condition (ii)

If CC $= |01\rangle$, apply Pauli X Gate



Condition (iii)

If CC $= |10\rangle$, apply Pauli Z Gate

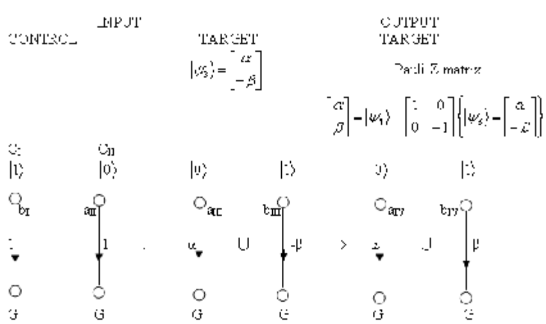


Figure 9: Quantum Teleportation – Step III (Another Alternative Representation)

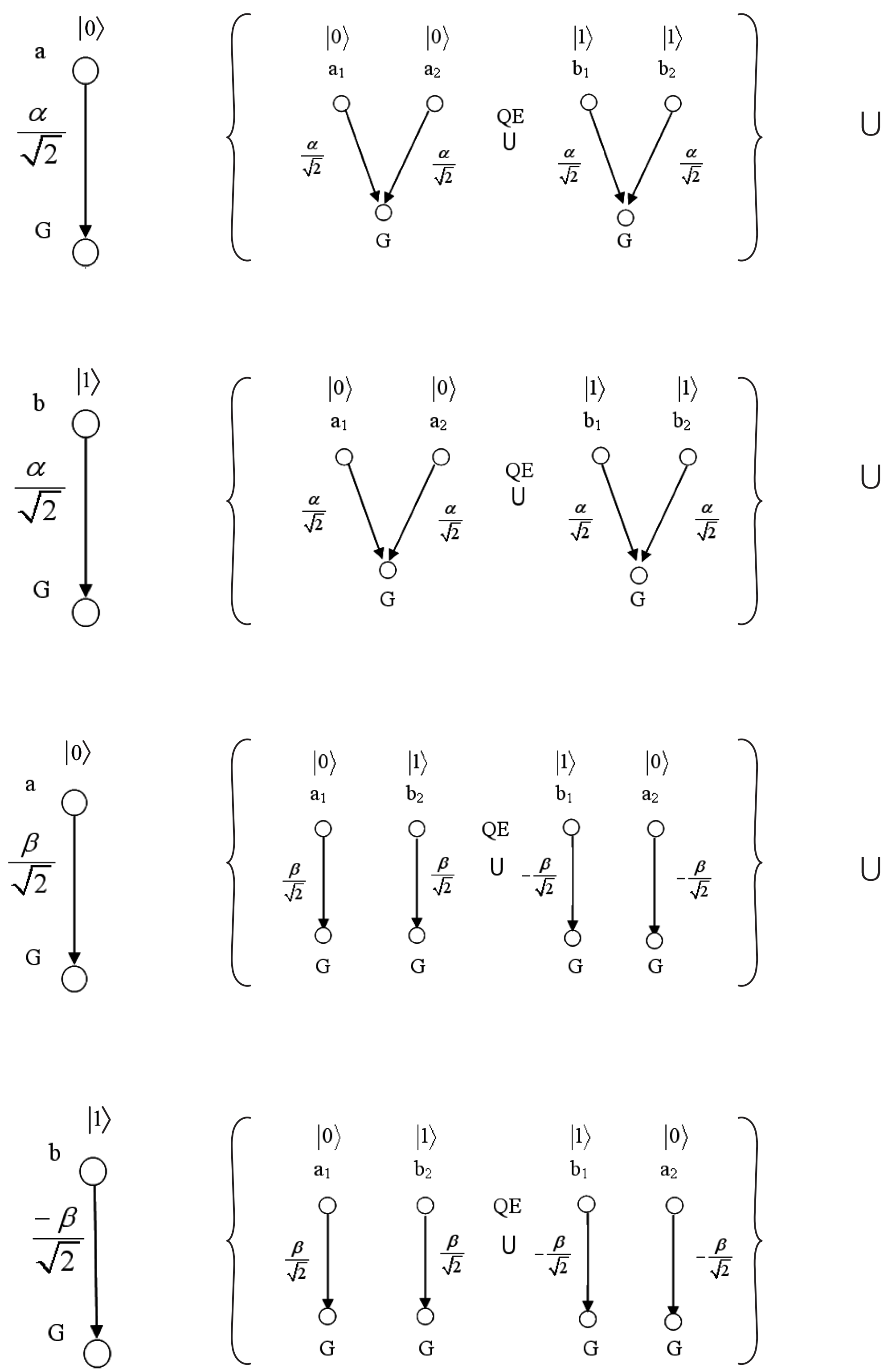
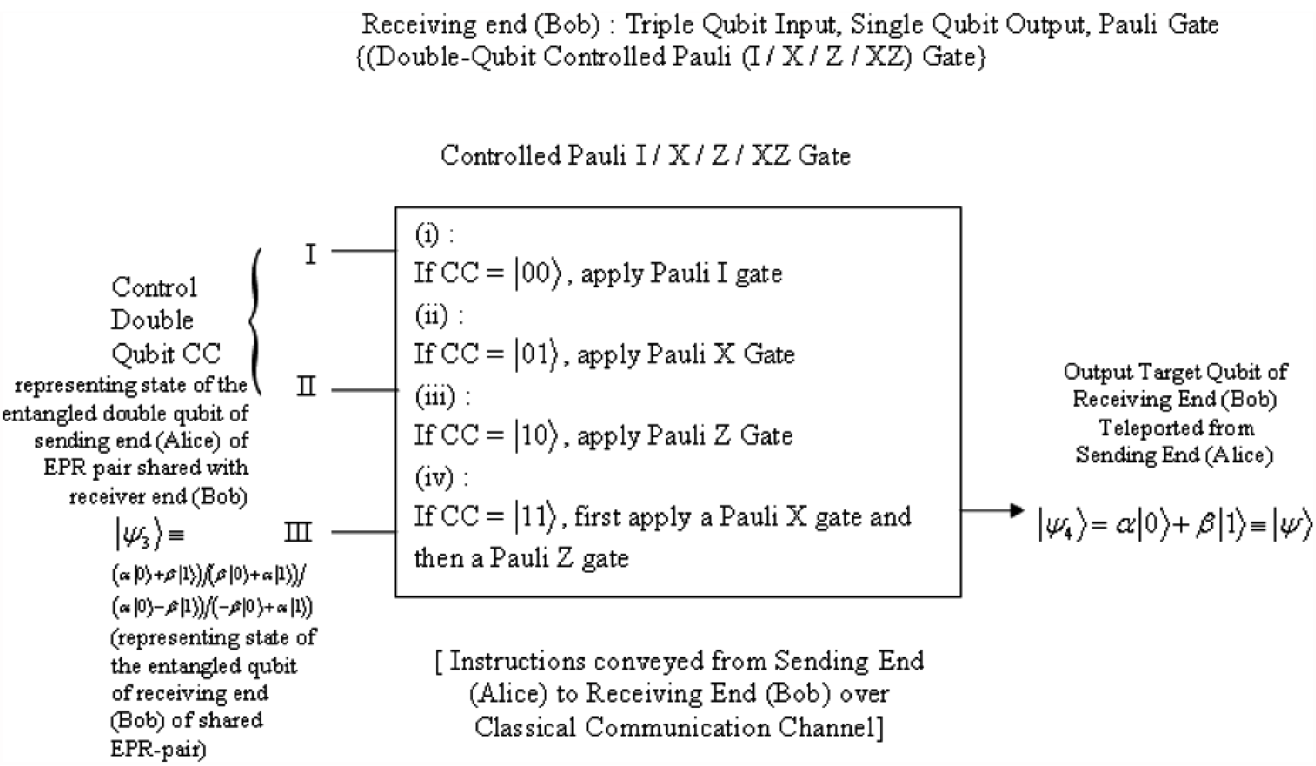
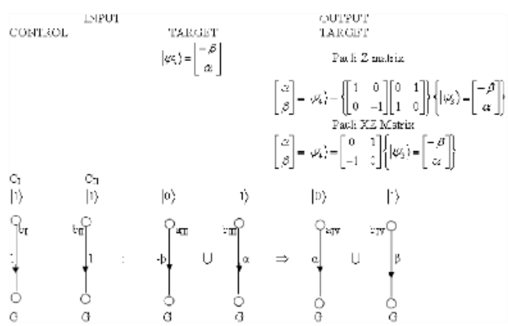


Figure 10: Block Diagram for Quantum Teleportation



Condition (iv)

If CC = |11⟩, first apply Pauli X Gate and then Pauli Z Gate



Conclusion

Quantum information science has arisen in response to a variety of converging scientific challenges. At present, quantum computers and quantum information technology remains in its pioneering stage. Quantum computers will emerge as the superior computational devices at the very least, and perhaps one day make today's modern computer obsolete. Quantum computation has its origins in highly specialized fields of theoretical physics, but its future undoubtedly lies in the profound effect it will have on the lives of all mankind. It may be inferred that a modelling theory founded on linear graph theory and quantum force-fields is indeed capable of modelling a large variety of Quantum Information Processing systems and will have a profound impact on the development of the budding field.

For more than last one century, modern scientists, particularly the modern physicists, have been enamoured of development of 'theory of everything'. Although they had succeeded in unifying the three fundamental forces of nature, gravity has continued to elude them. There is a promise so far as M-string theory is concerned, that it might deliver something on this front, but it is still to be proved by experimentation.

In terms of systems modelling, instead of the system scientists and practitioners getting bogged down with similar dreams of modelling theory of everything, one could perhaps look for 'modelling theory of many things'. There is certainly a possibility of founding such a unified modelling theory of many things, not everything, based on linear graph theory and unified field theory, not just quantum or string theory but perhaps some futuristic M-string theory or whatever that qualifies as the grand unified theory of so-called everything. This 'modelling theory of many things', will then span perhaps all kinds of systems, including natural systems, designed physical systems, designed abstract systems and human-activity systems [8]. Various kinds of systems can give rise to their own peculiar problems, which can be attempted to be resolved to the extent possible. Kristy Kitto [1] prepared a complexity scale for systems ranging from simple (e.g. projectile motion, billiard balls, thermodynamic equilibrium, microeconomics etc. amenable to Newtonian mechanics, thermodynamics, computational complexity, algorithmic information theory etc.) through complicated (e.g. weather dynamics, food webs etc. amenable to chaos / fractals, statistical mechanics, catastrophe theory, network theory etc.) to complex (e.g. bound states, quantum tunneling, electron and photon behaviour, genetic regulatory networks, quark and gluon behaviour, biological development, evolution of mind, language, societies ... amenable to quantum field theory, evolution and natural selection, post modernism etc.) at the high-end. It is at this higher end of complexity scale that the unified quantum field graph theory holds considerable promise and potential for modelling systems successfully [8].

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