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A new approach to economic and minimum emission dispatch

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Abstract

In recent years, due to environmental considerations, operation at absolute minimum cost cannot be the only basis for dispatching electric power. A new computational approach is proposed to find economic dispatch as well as minimum emission dispatch and the results are compared. The applicability of the proposed technique is demonstrated on a sample six-generator system.

Key words: Emission dispatch, economic dispatch.

1. Introduction

Economic dispatch is a familiar problem pertaining to the allocation of the amount of power to be generated by different units in the system on an optimum economy basis. The problem has been tackled by many research workers in the past 20 years, starting from Kirchmayer[1]. But recently, the problem which has attracted attention is pollution minimization, due to increasing demand from the public for clean air. Environmental pollution is a direct consequence of industrial advancement. Technology, which has made economic development possible, produces enormous quantities of harmful by-products and wastes. Although the power industry is not only the major cause of atmospheric pollution, because of the high concentration of pollutants it causes, it has been the prime target of attack from ecologists and pollution control agencies.

The combustion of fossil fuels gives rise to particulate material and gaseous pollutants apart from the discharge of heat to water courses. The particulate material does not cause a serious problem in air contamination but the three principal gaseous pollutants, oxides of carbon (CO_x), oxides of sulphur (SO_x) and oxides of nitrogen (NO_x) cause detrimental effects on human beings. However, the usual control practice is to reduce offensive emissions through post-combustion cleaning systems such as electrostatic precipitators, stack gas scrubbers.

Economy was hitherto considered to be the sole criterion in the operation of a power system. But in the context of increasing public awareness of the environmental situation and the plea for clean air, equal attention is now being focussed on minimum emission dispatch.

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Gent[2] has published a paper on minimum emission dispatch wherein a computer programme has been developed for online steam unit dispatch resulting in the minimization of NO_x emission employing the Newton–Raphson convergence for curve fitting. However, this method does not guarantee convergence unless the initial approximation is close to the solution and also requires an evaluation of the derivative of the function. Vertis and Eisenberg[3] have considered all the primary types of pollutants associated with the power generating system and assigned penalty factors to each of the types of a quadratic function of generation. However, they have not illustrated their method by a suitable example. Delson[4] has presented a method of achieving controlled emission dispatch using constant penalty factors, to schedule the generators to meet the given environmental restrictions at minimum operating cost.

Finnigan and Fouad[5] proposed two nonlinear programming solution procedures for economic dispatch with pollution constraints. Zahavi and Eisenberg[6] tackled the economic environmental dispatch by the use of interactive search method, based on the golden section search technique. Kothari and Mittal[7] demonstrated that reduced NO_x emission is possible by systematic scheduling. Kothari *et al*[8] presented a computer-oriented technique for the thermal power generation scheduling which resulted in the minimization of nitrogen oxide (NO_x) emissions.

The economic and minimum emission dispatch problems are nonlinear in nature. Hence, nonlinear programming techniques are to be used to solve them. If the derivatives of the objective function and/or the constraints are not given in explicit terms, additional work in local experimental would be required to determine the gradient in the case of gradient methods. Also, the gradient methods may show a slow zig-zag progress while moving along a boundary. The penalty function methods, wherein the constrained problem is transformed into a sequence of unconstrained minimization (SUMT) problems, are particularly not precise when the location of the optimum lies in a sharp corner. In such cases direct search methods (non-gradient methods) are preferred. Out of all the direct search methods for constrained optimization, the technique proposed by Box, the Box complex method, is more reliable and efficient[9].

This paper presents a new computational approach using the improved Box complex method for economic dispatch and minimum emission dispatch problems. This method is sufficiently general to be applicable to a wide range of system objectives. It is conceptually very simple, easy to programme and yet does not require large computer storage. It can effectively handle the objectives and the inequality constraints to any degree of nonlinearity. Also, it does not require the derivatives of either the objective or the constraints to find the optimum point and hence is computationally very simple. The method of using random numbers to generate the complex points is a reasonable feature in striving for a global value. The success of any search technique might depend upon the choice of stopping criterion. The algorithm presented in this paper is an improvement over the method proposed by Box regarding the stopping criterion for the search. It is in this sense, the proposed method is termed as improved Box complex method. The feasibility and the efficiency of the proposed method has been demonstrated through a quantitative study on a six-generator system.

2. Problem formulation

2.1 Economic dispatch

The economic dispatch problem is defined as

$$\text{Min } F_t = \sum_{i=1}^{N} (a_i P_i^2 + b_i P_i + c_i) \tag{1}$$

where,

F_t = total input to the system, \$/h
P_i = power output of ith generating unit, MW
a_i, b_i, c_i = cost coefficients of the ith unit
N = number of units

subject to

(i) the operating constraints $P_{i\min} \leqslant P_i \leqslant P_{i\max} \quad i = 1,2,\ldots,N$ (2)

(ii) the demand constraint (neglecting losses)

$$\sum_{i=1}^{N} P_i - P_D = 0 \tag{3}$$

where,

$P_{i\min}$ = the minimum power output of ith unit, MW
$P_{i\max}$ = the maximum power output of ith unit, MW
P_D = total power demand, MW.

2.2 Environmental dispatch

The environmental dispatch problem is defined as to minimize

$$\phi_t = \sum_{i=1}^{N} (d_i P_i^2 + e_i P_i + f_i) \tag{4}$$

where,

ϕ_t = total NO_x emission (In fact, NO_x, SO_x and thermal emissions and particulates can together be treated as a single emission criterion)
d_i, e_i, f_i = emission coefficients of the ith unit.

subject to

(i) the operating constraints

$$P_{i\min} \leqslant P_i \leqslant P_{i\max} \quad i = 1,2,\ldots,N; \tag{5}$$

(ii) the demand constraint (neglecting losses)

$$\sum_{i=1}^{N} P_i - P_D = 0. \tag{6}$$

2.3 *Conversion of equality constraint into inequality constraint*

The Box method is capable of handling only inequality constraints. Normally, an equality constraint can be replaced by two inequality constraints.

$\sum P_i = P_D$ can be replaced by

$$\sum P_i \geqslant P_D - \delta; \tag{i}$$

$$\sum P_i \leqslant P_D + \delta. \tag{ii}$$

The constraints of the nature (*ii*), *i.e.*, $\sum P_i \leqslant P_D + \delta$ are needed only for getting a feasible starting point. These type of constraints are never violated during the process of reaching the optimum point. This is due to the fact that as the complex shrinks the objective function moves towards the constraint of the nature (*i*), *i.e.* $P_i \geqslant P_D - \alpha$. In fact, the cost of generation will be minimum while meeting load requirement when (*i*) is satisfied as equality constraint with $\delta = 0$. Hence, constraints of the nature (*ii*) can be ignored[10].

3. Computational procedure

The following are the steps for the economic/environmental dispatch by the use of improved Box complex algorithm.

Step 1: Set the complex size to K (K = 2N, if the size of X is less than 5, and K = K + 1, otherwise).

Step 2: If ISEQ = 1, the starting feasible point is supplied. Otherwise, the best point of the previous ISEQ is treated as the starting point.

Step 3: Set the iteration count ITR = 1. The remaining K − 1 points are generated by the use of random numbers such that

$$X_j = L_j + R_j(U_j - L_j) \qquad j = 2,3,\ldots,K \tag{7}$$

where, R_j is a set of random numbers uniformly distributed over the interval 0 to 1 and L_j and U_j are the lower and upper bounds for X_j.

This relation will ensure that (K − 1) points so generated will satisfy the lower and upper bounds of the jth decision variables. But, they may not satisfy the inequality constraints. If X_j violates any of the inequality constraints, the trial point is moved halfway towards the centroid of the remaining already accepted points, as

$$X_j = \tfrac{1}{2}\,(X_c + X_j) \tag{8}$$

where, X_c, the centroid for already accepted points, is given by

$$X_c = \frac{1}{j-1} \sum_{l=1}^{j-1} X_l. \tag{9}$$

The process of moving half-way towards the centroid X_c is continued until a feasible point X_j is found.

Step 4: Evaluate the objective function value at each of the K points. Estimate the point X_w at which the function assumes the worst value $f(X_w)$ and the point X_B at which the function value is the best, $f(X_B)$.

Step 5: Check whether $f(X_w) - f(X_B) \leqslant \epsilon_1$ a prespecified tolerance. If satisfied go to next step. Otherwise go to step 9.

Step 6: If ISEQ = 1, go to step 8.

Check ISEQ $\leqslant$ ISMAX (prespecified maximum number of sequential searches and generally three sequential searches are sufficient). If yes, go to step 12. Otherwise go to next step.

Step 7: Check whether the present best function value is the same (or within a prespecified tolerance) as the achieved best function value in the previous sequential search. If satisfied go to step 12. Otherwise go to next step.

Step 8: Set $f(X_{\text{old}}) = f(X_B)$ and increment ISEQ = ISEQ + 1; go to step 2.

Step 9: The worst point X_w is replaced by its reflection X_r such that

$$X_r = (1+\alpha)X_c - \alpha X_w \tag{10}$$

where, α is called the reflection coefficient and its recommended value is 1·3 and X_c is the centroid of all the points except X_w.

Step 10: Check whether the point X_r is feasible and its function value $f(X_r)$ is better than that of (X_w). If satisfied go to step 11. Otherwise reduce the reflection coefficient α and go to step 9. The process is continued till the value of α becomes as small as 10^{-5} and then go to step 2.

Step 11: Check whether ITR has reached the specified maximum number of iterations. If yes go to step 8. Otherwise, increment ITR and go to step 4.

Step 12: Optimum solution is reached and the search is terminated.

4. Application to a sample system

The efficiency of the proposed method has been demonstrated through a quantitative study on a six-generator system[5] for which two cases have been considered. In one case, the generators use the natural gas as fuel whereas in the other oil is used.

Table I
Fuel cost coefficients

Generator no.	Natural gas			Fuel oil		
	a	b	c	a	b	c
1	0·00903	2·28251	44·82131	0·01051	2·65686	52·17248
2	0·00627	2·73377	26·72967	0·00730	3·18216	31·11286
3	0·00168	2·39248	62·18597	0·00196	2·78486	72·38470
4	0·00210	2·26864	73·64796	0·00244	2·64072	85·72651
5	0·00125	2·15151	98·22856	0·00146	2·50439	114·33870
6	0·00106	2·26656	80·34796	0·00124	2·63829	93·52656

Table II
NO_x emission coefficients

Generator no.	Natural gas			Fuel oil		
	d	e	f	d	e	f
1	0·00939	0·73398	31·04487	0·00767	0·80507	363·7048
2	0·00939	0·73398	31·04487	0·00767	0·80507	363·7048
3	0·01530	− 1·22195	90·19784	0·01378	− 1·24885	137·3701
4	0·01530	− 1·22195	90·19784	0·01378	− 1·24885	137·3701
5	0·01033	− 1·14499	96·08599	0·01265	− 1·35522	22·9830
6	0·01033	− 1·14499	96·08599	0·01265	− 1·35522	22·9830

Table III
Operating limits

Generator no.	Natural gas		Fuel oil	
	Lower limit (MW)	Upper limit (MW)	Lower limit (MW)	Upper limit (MW)
1	27	99	27	101
2	27	99	27	101
3	37	236	37	220
4	37	222	37	232
5	144	344	144	344
6	144	344	144	344

The fuel cost and NO_x emission coefficients are given in Tables I and II respectively. The operating limits of the generators are given in Table III. The comparison of the results of economic dispatch on the basis of minimum cost and minimum emission dispatch on the basis of minimum NO_x emission is given in Table IV.

Table IV
Cost and NO_x emission output for economic and minimum emission dispatch

Type of fuel	Operating condition	Cost ($/h)	NO_x emissions (lb/h)
Natural gas	Economic dispatch	3279·80	2633·70
	Minimum emission dispatch	3362·07	2211·88
Fuel oil	Economic dispatch	3819·86	3472·40
	Minimum emission dispatch	3939·74	2855·03

Table V
Optimum power output (in MW) for the six natural gas-fired generating units

Operating condition	P_1	P_2	P_3	P_4	P_5	P_6
Economic dispatch	39·61	28·30	175·56	176·28	335·28	343·98
Minimum emission dispatch	99·00	99·00	185·25	181·06	267·84	267·84

Table VI
Optimum power output (in MW) for the six fuel oil-fired generating units

Operating condition	P_1	P_2	P_3	P_4	P_5	P_6
Economic dispatch	38·54	27·00	195·31	181·18	331·58	326·40
Minimum emission dispatch	101·00	101·00	212·70	212·70	236·30	236·30

Tables V and VI depict the optimum power output of the six generating units. The load demand was selected as 1100 MW.

5. Conclusions

A new computational approach, improved Box complex method, has been proposed and applied for the problems of economic dispatch and emission dispatch. The results reveal that reduced NO_x emissions are possible by proper scheduling. The rescheduling obviously results in deviations from economic dispatch and hence higher operating costs. The increase in cost is more than justified in terms of better public relations by the power industry.

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