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A method of calculation of a power system forced outage reserve based on universal characteristic of unitary power reserve

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Abstract

A review of the methods of calculation of random outage distribution has been presented. An accurate method of calculation of the universal characteristic curves of relative* power reserve and unitary forced outage* reserve is presented. The input data and the results of calculations of the curves are enclosed. An original procedure for the calculation of forced outage reserve in a power system is described. A critical evaluation of an existing graphical technique of approximate calculation of universal characteristic curves of power reserves is presented.

Key words: Power system, unitary power reserve.

1. Introduction

For reasons of brevity, it is not possible to present in this paper an exhaustive literature review or a more compehensive bibliography. Instead only a review of the known methods of calculation of probability distribution and probability distribution function of random outages in power systems is presented. Knowledge of outage distribution allows one to determine the power reserve for outages.

In section 1.2 an existing method for calculation of the random outage power reserves connected with the individual generating units is presented. This forms a basis for the determination of the reserve for the whole power system. This technique may be treated as a starting point for the elaboration of the new method which is presented in sections 2 and 3.

1.1 Review of the methods of calculation of random outage distribution

In applying the most frequently used methods, the real system (series, and parallel-series combinations) of boilers, turbines and generators, as well as hydrounits are represented

The equivalence of the following terminology may be noted: unitary outage reserve–unit outage reserve relative per unit. Ed.

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in the form of a system of the n generating units, of capacities equal to the nominal capacities M_j of individual generators, with equivalent forced outage rates q_j, and equivalent availability rates $p_j = 1 - q_j$, for $j = 1,2, \ldots, n$.

Let $y_j (j = 1,2, \ldots, n)$ represent the outage capacity of the jth generating unit. This capacity, y_j, may take the values 0 and M_j with probabilities p_j and q_j, respectively. The variable of Y_j may be treated as a random variable which is determined in the following way:

$$P(Y_j = 0) = p_j; \qquad (1.1)$$
$$P(Y_j = M_j) = q_j.$$

Several methods exist for the calculation of forced outage distribution; the most representative ones are discussed here.

1.1.1. The accurate method

The probabilities of occurrence of all possible states of generating units corresponding to the specified capacities of random (forced) outages are obtained by the products of the following binominals:

$$(p_1 + q_1)(p_2 + q_2)\ldots(p_n + q_n) = p_1 p_2 \ldots p_n + q_1 p_2 p_3 \ldots p_n + \ldots + q_1 q_2 \ldots q_n = 1. \qquad (1.2)$$

These probabilities determine explicitly the distribution and distribution function of forced outages. Calculations for real power systems, with a large number of generating units, by the accurate method are time consuming since they involve 2^n combinations.

1.1.2. Methods using the Poisson's or Bernoulli's distribution

The real system of n generating units is represented by an equivalent system of the n_z identical generating units each of capacity, M_z and forced outage rate, q_z. Next the distribution of forced outages is calculated from the Poisson's or Bernoulli's expressions.

1.1.3. Method of a power system division on groups of generating units

A higher calcualtion accuracy may be obtained by the division of generating units into several groups, each group having units of equal capacity and forced outage rate. Each group is represented by the equivalent system of identical generating units, and for each such sub-system the distribution of forced outages is calculated from the Bernoulli's equation. With the convolution of distributions of random outages of consecutive groups, the distribution of total random outages in a power system is obtained.

1.1.4. Methods of utilising the higher order parameters of the distribution of forced outages

a) Method of calculation of distribution parameters

Parameters of the kth order of probability distribution as the moments α_k, central moments μ_k, and cumulants $\mathcal{X}_k$ $(k = 1,2,\ldots)$ may be easily calculated from the definition, when the probability distribution is known. It may be shown that any order parameters of forced outage distribution can be calculated without knowing this distribution, but only distributions (equation 1.1) of forced outage of generating units. The cumulant method[1] was used[2,3] for calculation of the parmeters of forced outage distribution in a power system

$$Y = \sum_{j=1}^{n} Y_j . \tag{1.3}$$

A more general approach has been presented[4,5] for the determination of the generating unit and its forced outage distribution (the additional states of partial failure have been distinguished).

b) Methods utilising asymptotic expansions derived from the normal distribution (Type A Gram-Charier's series, and Edgeworth series)

Kozik presented the method of calculation of probability density and distribution functions of forced outages using asymptotic expansions derived from the normal distribution[4]. It has been shown that the expansion of orthogonal polynomials (Type A Gram-Charier's series) is less accurate than the Edgeworth series. The results of calculations show the high accuracy of approximation of distribution and distribution function by the Edgeworth series[5].

The probability density function $f(s)$ and probability distribution function $F(s)$ of standardized forced outages

$$s = \frac{Y - \alpha_1}{\sqrt{\mu_2}} \tag{1.4}$$

may be approximated by the Edgeworth series

$$f(s) = \varphi(s) + \sum_{k=1}^{\infty} B_k(s) \tag{1.5}$$

$$F(s) = \phi(s) + \sum_{k=1}^{\infty} L_k(s) \tag{1.6}$$

where $\varphi(s)$, $\phi(s)$ are the normal density and distribution functions, respectively, and $B_k(s)$, $L_k(s)$ are the kth terms of expansion. The terms $B_k(s)$ are determined from:

$$\left.\begin{aligned} B_1(s) &= \frac{1}{6}\frac{\mathcal{X}_3}{\mathcal{X}_2^{3/2}} H_3(s)\varphi(s) \\ B_2(s) &= \left[\frac{1}{24}\frac{\mathcal{X}_4}{\mathcal{X}_2^2} H_4(s) + \frac{1}{72}\frac{\mathcal{X}_3^2}{\mathcal{X}_2^3} H_6(s) \right] \varphi(s) \\ &\dots \end{aligned}\right\} \tag{1.7}$$

The terms $L_k(s)$ are determined from:

$$\left.\begin{aligned} L_1(s) &= -\frac{1}{6}\frac{\mathscr{X}_3}{\mathscr{X}_2^{3/2}}\, H_2(s)\,\varphi(s) \\ L_2(s) &= -\left[\frac{1}{24}\frac{\mathscr{X}_4}{\mathscr{X}_2^2}\, H_3(s) + \frac{1}{72}\frac{\mathscr{X}_3^2}{\mathscr{X}_2^3} H_5(s)\right]\varphi(s) \end{aligned}\right\} \quad (1.8)$$

where $\mathscr{X}_k$ is the kth order cumulants of forced outage distribution, and $H_k(s)$, the Hermite's polynomial.

Detailed expressions for parameters calculation (α_k, μ_k, $\mathscr{X}_k$) of forced outage distribution for $k = 1,2,\ldots,7$ as well as of the terms $B_k(s)$ and $L_k(s)$ for $k = 1,2,\ldots,5$ are presented elsewhere[4,5]. The results of calculations show that the accuracy of the method depends on the number of cumulants (2 to 7) taken into account.

The results of the more recent works of other authors concerned with the use of cumulant method and asymptotic expansions derived from the normal distribution, were presented in the IEEE PES Summer Meetings in 1979[6–8], 1980[9], 1981[10], 1982[11,12], 1984[13,14], as well as in the IEEE PES Winter Meeting in 1983[15,16].

1.2 The Russian method of calculation of forced outage reserve

The approximate method of determination of forced outage reserve based on universal characteristic curves of unitary power reserve was presented by Lialik and Urvancev[17]. The term unitary power reserve means the relative value of forced outage to be provided for each individual generating unit. In this case, the amount of forced outage reserve R in a power system is the sum of the unitary reserves:

$$R = \sum_{j=1}^{k} r_j' n_j M_j \quad (1.9)$$

where k is the number of groups of identical generating units,
n_j, the number of generating units in the jth group,
M_j, the generating unit capacity in the jth group,
r_j', the unitary relative forced outage reserve of the jth generating unit.

The unitary power reserve is determined from a fictitious power system which consists of n identical generating units. In this case, equation (1.9) will take the form:

$$R = r'nM = r'P_i \quad (1.10)$$

where P_i is the total capacity of all generating units.

The unitary power reserve may be expressed in terms of the generating unit capacity or the total power system capacity:

$$r' = \frac{R/n}{M} = \frac{R}{P_i} \tag{1.11}$$

Both the forced outage reserve and the unitary reserve depend upon the following factors: number of generating units (n), generating unit capacity (M), forced outage rate (q), peak load (P_s), cumulative distribution function ($Z(x^*)$) of relative ($x^* = x/P_s$) power demand x, unitary cost of annual reserve (k_R), unitary cost of unsupplied energy ($k_{\Delta A}$):

$$r' = f\left[n, M, q, P_s, Z(x^*), k_R, k_{\Delta A} \right] \tag{1.12}$$

The concept of relative generating unit capacity is now introduced. This quantity is expressed as

$$u = \frac{M}{P_s} \tag{1.13}$$

It is now possible to express the unitary power reserve in terms of u, instead of using the capacity of individual generating unit; at the same time the total power system capacity is assumed to be equal to the peak load ($P_i = P_s$). In that way the characteristic curves of demand become independent of the total power system capacity and of the number of generating units, as these values are related to each other by the equation:

$$n = \frac{P_i}{M} = \frac{P_s}{M} = \frac{1}{u}. \tag{1.14}$$

The criterion to justify the installation of the next stand-by generating unit is expressed as:

$$t_d^*(h) > \frac{k_R}{Tk_{\Delta A}} \geqslant t_d^*(h + 1) \tag{1.15}$$

where $t_d^*(h)$, $t_d^*(h + 1)$ are relative loss of load-time duration after installation of hth and $(h+1)$th generating unit, respectively, and T is the planning period (8760 h).

Expression $k_R/(Tk_{\Delta A})$ determines the time duration of the relative loss of load which is economically justified, and which in Russian conditions fluctuates in a range from 0·001 to 0·01[17]; usually the value of 0·001 is postulated. Such a value has been assumed as critical $t_{d,kr}^* = 0{\cdot}001$.

The portion of the CDF of the load that has a decisive influence on the value of unitary forced outage reserve corresponds to the load range from 85 to 100% of the peak load. The CDF $Z(x^*)$ in that range may be considered a constant [17]. Finally the unitary power reserve is a function of relative generating unit capacity and forced outage rate:

$$r' = f(u, q). \tag{1.16}$$

The characteristic curves $r' = f(u)$ with q = const. have been presented[17] for q in a range from 0·005 to 0·15.

2. Accurate method of calculation of universal characteristic curves of relative forced outage reserve and unitary forced outage reserve

2.1. Calculation of universal characteristic curves of ralative forced outage reserve

Determined from condition (1.15) the forced outage reserve is burdened with an error

$$\Delta R \leqslant M. \tag{2.1}$$

Inaccurate determination of forced outage reserve causes an error in the calcualtion of the relative forced outage reserve (in section 1.2 named as the unitary relative forced outage reserve)

$$\Delta r_a \leqslant u. \tag{2.2}$$

It may be shown, however, that the characteristic curves of relative forced outage reserve may be obtained with any required accuracy.

Let us assume that the power system consists of N identical generating units of capacities M and forced outage rate q. The inverse CDF $\bar{Z}(x) = 1 - Z(x)$ of power demand x, and the discrete distribution $f(p_d)$ of available capacity p_d are shown in fig. 1*a*, whereas the inverse CDF $\bar{Z}(x^*)$ of relative power demand $x^* = x/P_s$ and the discrete distribution $f(p_d^*)$ of relative available capacity $p_d^* = p_d/P_s$ are demonstrated in fig. 1*b*.

Between the installed capacity $P_z = NM$, peak load P_s, forced outage reserve R, the number of generating units N, and the capacity of generating unit M, there exists the following relation:

$$P_z = NM = P_s + R \tag{2.3}$$

and in p. u. values

$$\frac{P_z}{P_s} = \frac{NM}{P_s} = 1 + \frac{R}{P_s}. \tag{2.4}$$

From equation (2.4) results the dependence of relative forced outage reserve $r_a = R/P_s$ on the relative generating unit capacity $u = M/P_s$ and the number of generating units N

$$r_a = Nu - 1. \tag{2.5}$$

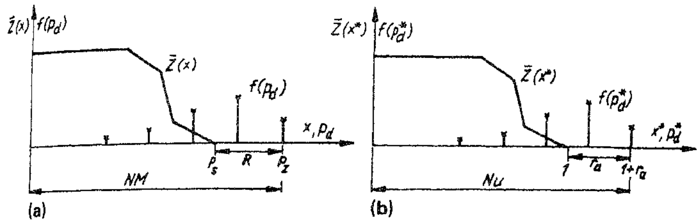

FIG. 1. Inverse CDF of power demand and discrete distribution of available capacity.

The relative loss of load-time duration t_d^* may be determined from discrete distribution of relative available capacity $f(p_{d,i}^*)$ and inverse CDF of relative power demand $Z(x^*)$ on the basis of the equation

$$t_d^* = \sum_{i=0}^{N} f(p_{d,i}^*).[\bar{Z}(x^* = p_{d,i}^*) - P(x^* = p_{d,i}^*)] \tag{2.6}$$

where $P(x^* = p_{d,i}^*)$ is the probability of occurrence of relative load $x^* = x/P_s$ equal to relative available capacity $p_{d,i}^* = p_{d,i}/P_s$.

In the case under consideration the probability distribution of outage is a binomial distribution, and hence the discrete distribution of relative available capacity is obtained from

$$f(p_{d,i}^*) = \binom{N}{i} q^i (1-q)^{N-i};\ i = 0, 1, \dots, N \tag{2.7}$$

where

$$p_{d,i}^* = 1 + r_a - i\frac{r_a + 1}{N}. \tag{2.8}$$

Bringing formula (2.7) into expression (2.6) the final equation is given as

$$t_d^* = \sum_{i=0}^{N} \binom{N}{i} q^i (1-q)^{N-i} \left[\bar{Z}\left(1 + r_a - i\frac{r_a+1}{N}\right) - P\left(x^* = 1 + r_a - i\frac{r_a+1}{N}\right)\right]. \tag{2.9}$$

With the known number of generating units N, constant forced outage rate q, and constant relative power demand $CDF\ Z(x^*)$, the relative loss of load-time duration t_d^* is dependent upon the relative forced outage reserve r_a.

Calculating the relative loss of load-time duration for ascending values of forced outage reserve (starting from zero) the functional characteristic $t_d^* = f(r_a)$ of relative loss of load-time duration *vs* relative forced outage reserve may be obtained. For the critical value of loss of load-time duration $t_{d,kr}^*$, the value of relative forced outage reserve r_a required may be read directly from this curve.

In computer calculations it is convenient to use iterative technique which allows one to determine automatically the value of relative forced outage reserve when the critical value of loss of load-time duration is given. In the first iteration it must be assumed that $r_a = r_{a,1} = 0$; and in the second one that $r_a = r_{a,2} = q$; in the later iterations the values of $r_a = r_{a,k}$ are calculated from the equation

$$r_{a,k} = r_{a,k-1} + \frac{t_{d,kr}^* - t_d^*(r_{a,k-1})}{t_d^*(r_{a,k-1}) - t_d^*(r_{a,k-2})} (r_{a,k-1} - r_{a,k-2}) \, . \tag{2.10}$$

In each iteration the following inequality is checked

$$(t_{d,kr}^* - \epsilon) < t_d^*(r_{a,k}) \leqslant (t_{d,kr}^* + \epsilon) \, . \tag{2.11}$$

Calculations are finished when condition (2.11) is met.

After the relative forced outage reserve has been calculated, the relative generating unit capacity is determined from the expression resulting from (2.5)

$$u = \frac{r_a + 1}{N} \tag{2.12}$$

The family of universal characteristics of relative forced outage reserve may be obtained by performing calculations for adequately selected values of N (considering expression 2.12), with N value ranging from N_o to ∞ for different forced outage rates. Number N_o is evaluated from the following relation

$$(\frac{\log t_{d,kr}^*}{\log q} + 1) \geqslant N_o > \frac{\log t_{d,kr}^*}{\log q} \tag{2.13}$$

The following assumptions have been made in the calculations:
- representative[5] relative power demand *CDF* of national power system for December standard working days (Table I),
- critical value of loss of load-time duration $t_{d,kr}^* = 0{\cdot}001$,
- calculational accuracy $\epsilon = 10^{-5}$.

The results of calculation of one universal characteristic curve of relative forced outage reserve for generating unit forced outage rate $q = 0{\cdot}05$ are shown in Table II.

Table I
Inverse CDF $\bar{Z}(x^*)$ for December

No.	x^*	$\bar{Z}(x^*)$	No.	x^*	$\bar{Z}(x^*)$
1	0·6641	1·00	12	0·9581	0·10
2	0·7088	0·95	13	0·9629	0·09
3	0·7230	0·90	14	0·9649	0·08
4	0·7386	0·80	15	0·9666	0·07
5	0·7725	0·70	16	0·9744	0·06
6	0·8179	0·60	17	0·9751	0·05
7	0·8619	0·50	18	0·9789	0·04
8	0·8836	0·40	19	0·9805	0·03
9	0·9066	0·30	20	0·9859	0·02
10	0·9263	0·20	21	0·9906	0·01
11	0·9452	0·15	22	1·0000	0·00

The universal characteristic curves of relative forced outage reserve obtained from the present method are equivalent to the characteteristic curves of unitary power reserve[17], because of the relation (1.14):

$$n = \frac{1}{u} \tag{2.14}$$

where n is the number of generating units resulting from the expression $P_i = nM = P_s$ assumed by Lialik and Urvancev[17]. From equation (2.14) a non-integer number is usually obtained; however, it does not matter. The characteristic curves differ in calculation accuracy; with the constant number of generating units n (without stand-by ones) the increment of forced outage reserve is equal to the generating unit capacity[17], with the result that with the critical condition (1.15) an error $\Delta r_a \leqslant u$ is related. In the

Table II
Characteristic curves $r_a = f(u)$ and $r = f(u)$ for $q = 0{\cdot}05$

N	u	r_a	r
1000	0·0011	0·0590	0·0557
500	0·0021	0·0646	0·0606
200	0·0054	0·0785	0·0728
145	0·0075	0·0864	0·0796
105	0·0104	0·0958	0·0875
80	0·0138	0·1070	0·0967
50	0·0227	0·1335	0·1178
25	0·0479	0·1966	0·1643
17	0·0743	0·2637	0·2087
13	0·1010	0·3126	0·2381

proposed method with constant number of generating units N (together with stand-by ones) the increment of forced outage reserve is obtained by the reduction of peak load. The peak load variations may be as low as required and hence the relative forced outage reserve defined by the expression (2.11) may be determined with any accuracy ϵ that is chosen.

2.2 *Calculation of universal characteristic curves of unitary relative forced outage reserve*

The capacity of each of N identical generating units may be divided into two components:
- capacity M_x taking part in covering the peak load,
- capacity M_R utilised as a forced outage reserve.

$$M = M_x + M_R \,. \tag{2.15}$$

The unitary relative forced outage reserve r represents a part of forced outage reserve which falls on one generating unit with relation to its capacity:

$$r = \frac{R/N}{M} = \frac{r_a}{Nu} \,. \tag{2.16}$$

Taking into consideration expression (2.5) in equation (2.16) the final relation between unitary relative forced outage reserve and the relative forced outage reserve is obtained as

$$r = \frac{r_a}{r_a + 1} \,. \tag{2.17}$$

The relative generating unit contribution w in covering the peak load, part of it falling on one generating unit with relation to its capacity is given by:

$$w = \frac{P_s/N}{M} = \frac{1}{1+r_a} = 1 - r \,. \tag{2.18}$$

The above components of generating unit capacity are determined from expressions:

$$\left.\begin{aligned} M_x &= wM = (1 - r)M \\ M_R &= rM. \end{aligned}\right\} \tag{2.19}$$

The universal characteristic curves of unitary relative forced outage reserve may be obtained by the following analogue techniques, similar to those used when calculating the universal characteristic curves of relative forced outage reserve r_a, with the only difference that after calculating r_a one has to calculate r from formula (2.17).

The universal characteristic curves of unitary relative forced outage reserve are presented in fig. 2, whereas the results of one characteristic curve calculations are shown in Table II.

3. Calculation of forced outage reserve in a power system

The forced outage reserve related to the capacity of n generating units equal to the peak load is assumed by Lialik and Urvancev[17] as the unitary forced outage reserve. In a final phase of calculations in a power system one must consider that not only n generating units but also h stand-by units exist. From the definition of unitary reserve it follows that:

a) reserve R determined from formula (1.9) should be obtained with the condition that:

$$\sum_{j=1}^{k} n_j M_j = P_s \,. \tag{3.1}$$

b) reserve R is a result of participation in a power system of additional generating units. Components of power reserve (consecutive groups of generating units) $r_j' n_j M_j$ have to consist of generating units of capacity M_j and their number has to be equal to $r_j' n_j$ for $j = 1,2,\ldots,k$. Expression $r_j' n_j$ is usually a non-integer number. It brings in difficulties that are impossible to overcome. From equation (1.9) a forced outage reserve is obtained which can not be realised, because of the non-integer number of stand-by generating units and the discrete values of installed capacities of some generating units.

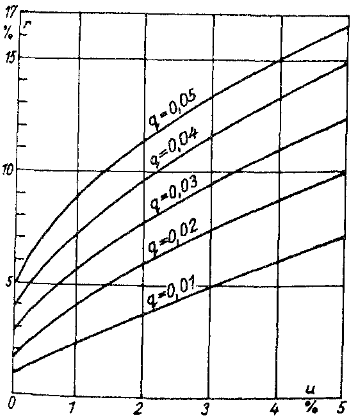

Fig. 2. Universal characteristic curves of unitary relative forced outage reserve.

To avoid these difficulties, in section 2.2. different definitions of unitary relative forced outage reserve have been introduced. The unitary relative forced outage reserve is defined as the forced outage reserve related to N generating unit capacity equal to $NM = P_s + R$. In the number N the stand-by units are also included.

The input data for calculations are:

a) forecast peak load of a power system P_s,
b) capacities M_j, forced outage rates q_j and number n_j of identical generating units for k groups of units.

The following procedure for forced outage reserve calculation is proposed:

a) for each generating unit of jth group, calculation of its relative capacity:

$$u_j = \frac{M_j}{P_s}; \tag{3.2}$$

b) determination from the unitary characteristic curves (for u_j and forced outage rate q_j) of unitary forced outage reserve r_j and next calculation of forced outage reserve for the jth group:

$$R_j = r_j n_j M_j \ ; \tag{3.3}$$

c) calculation of contribution of jth generating unit group in the covering of the peak load:

$$P_{u,j} = (1-r_j)n_j M_j \ ; \tag{3.4}$$

d) summation of forced outage reserves for consecutive generating unit groups:

$$R = \sum_{j=1}^{k} R_j \ ; \tag{3.5}$$

e) calculation of total contribution in a covering of peak load:

$$P_u = \sum_{j=1}^{k} P_{u,j} \ ; \tag{3.6}$$

f) comparison of the contribution p_u with the peak load P_s. Three cases may occur:

1) $P_u = P_s$ a forced outage reserve calculated from equation (3.5), is considered as the forced outage reserve in demand.

2) $P_u < P_s$, the system of k generating unit groups under consideration does not ensure the covering of the peak load; the additional generating units should be installed (units scheduled in a power system development plan) and the calculations supplemented with

the procedure described in the points from a to f. If, after the introduction of a subsequent generating unit, the expression $P_u > P_s$ is satisfied the calculations are stopped.

3) $P_u > P_s$, the system of k generating unit groups covers the peak load in surplus; some units should be withdrawn from calculations starting from the last one envisaged in a power system extension schedule. If, after the elimination of 1th generating unit, the condition $P_u < P_s$ is satisfied, the calculations are stopped. The forced outage reserve in demand is the reserve obtained after the elimination of (1–1) units.

In the calculations performed in points 2 and 3 usually some power surplus equal to $P_u - P_s$ is obtained. This difference may be used when the final structure of generating units is decided.

4. Conclusions

1. The proposed accurate method of calculation of universal characteristic curves of relative forced outage reserve and unitary relative forced outage reserve is simple as compared to other techniques.

2. Universal characteristic curves of unitary relative forced outage reserve make it possible to calculate the participation of generating unit in covering the peak load. This participation is an additional factor (apart from the forced outage rate and normal maintenance duration) characterizing the usefulness of a generating unit.

3. The proposed method of forced outage reserve calculations makes it possible to calculate without difficulty the maintenance envelope based on the procedure described by Kozik *et al*[18].

4. The method of forced outage reserve calculation, being an approximate technique, is very sensitive to the change of generating unit parameters which may be utilized in maintenance scheduling optimization.

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