

A brief review on strength, deflection and cracking of rectangular, skew and circular reinforced concrete slabs

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Abstract

The paper reviews briefly literature pertaining to the ultimate strength, deflections and cracking of reinforced concrete rectangular, skew and circular slabs and gives a list of 136 publications on the topic.

Key words: Concrete (reinforced), slabs (two-way), membrane action, deflection, crackwidth, reinforcement (layout).

1. Introduction

Reinforced concrete slab is one of the structural elements which is extensively used in civil engineering construction. Two-way structural action, post-cracking non-linear behaviour and development of membrane action complicate the analysis and estimation of the ultimate strength of the slab. Design of the slabs based on limit states also requires a knowledge of their deflections and cracking behaviour. A number of researchers have investigated the flexural strength, deflection and cracking of two-way slabs and these aspects are briefly reviewed here.

2. Methods of slab analysis

2.1 Elastic analysis

The elastic analysis of slabs as thin plate requires the solution of biharmonic equation for specified boundary conditions. Solutions for different shapes of slabs and boundary conditions are found in Timoshenko and Woinowsky-Krieger¹. This analysis is useful to study the elastic behaviour of the slab, but the factor of safety against collapse cannot be assessed. Also it is complicated for slabs of irregular shape, which can however be analysed more easily using yield-line analysis.

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2.2 Yield-line theory

In 1921, the Danish engineer Aage Ingerslev² proposed a method of calculation based upon the constant bending moments acting along the yield lines. Since 1931, K. W. Johansen³⁻⁷ has done pioneering work in this field, in Denmark. His work has been translated into English⁸ in 1962. The yield-line theory assumes that an increase in load causes concentration of strain in steel and concrete along lines of maximum moment. These lines are called yield lines and they spread into a pattern which divides the slab into segments. Near failure, the elastic deformations of each segment are assumed negligible compared to plastic deformations at the yield lines. Consequently all curvatures in the slab at failure are assumed to be concentrated at the yield lines. The collapse load based on assumed mechanism is calculated from the energy equation. As the theory satisfies mechanism and the equilibrium condition, it provides a theoretical upper-bound solution. This theory can be effectively used for the analysis of any irregular type of slab having different type of boundary conditions and loadings. In 1962, European Committee for Concrete⁹ published a report that contains formulae for the analysis of slabs of various shapes subjected to uniformly distributed load, line loads and point loads. Some investigators¹⁰⁻¹² have reported design aids for slabs using yield-line theory. The yield-line theory is suitable for the ultimate load analysis with uniformly distributed (isotropic and orthotropic) reinforcement, but it is difficult to handle if non-uniform or curtailed reinforcement is utilized. Unless the correct mechanism is found the resulting design may be unsafe and the reinforcement volume is uneconomical. This method does not indicate the loadings on edge beams. Moreover it does not provide information regarding the distribution of bending and twisting moments in the interior of the slab away from the yield lines and thus the distribution of reinforcement becomes difficult. However, this theory is finding acceptance due to the fact that the ultimate loads of slabs as determined in tests¹³ are much higher than those predicted by yield-line theory. This difference between yield-line loads and experimental ultimate loads is due to the development of membrane stresses, and this effect is considered in detail separately.

2.3 Lower-bound approach

In the lower-bound approach the solution should be determined to satisfy equilibrium and boundary conditions and provide a statistically admissible moment field without violating the yield conditions anywhere in the slab. This provides a safe solution because the collapse load may be greater than or equal to the calculated value. This solution provides information on the required distribution of positive and negative reinforcement. Some lower-bound solutions¹⁴⁻¹⁹ are available for simply supported, continuous, one short-edge free, rectangular isotropic and orthotropic slabs and circular slabs with isotropic and orthotropic reinforcement. Though most of these solutions do not coincide with upper-bound solutions, they are satisfactory as the differences between the upper- and lower-bound solutions are small. In 1956, Hillerborg²⁰ developed the strip method based on the lower-bound approach. He proposed to design several types of slabs by assuming the load to be carried only by bending moments in two perpendicular

directions. He neglected torsional moments in developing the method. This method is very much suitable for slabs with free, simple or fixed boundary conditions as the slab can be divided into some simple combinations of cantilever and simple beam strips. This is essentially a design method and economy is achieved by varying the reinforcement in different portions of the slab. Wood and Armer²¹ have examined the strip method and concluded that, it provides an exact solution if the reinforcement provided is in accordance with the strip moments with an unlimited number of simultaneous modes. Fernando and Kemp²² attempted to find whether Wood and Armer's conclusion was true for a general case. Their examination shows that strip method does not provide unique solution always, but the exceptions are rare. Harrop²³ studied the application of strip method to simply supported slab-beam panels and found that the minimisation of reinforcement in slabs was counter-balanced by increase in beam reinforcement. Subsequently Harrop²⁴ applied the strip method to the design of skew slabs. He established an affinity relationship between skew and rectangular slabs which enables the design of skew slabs from rectangular slabs. Thakkar and Sridhar Rao²⁵ introduced modifications on Hillerborg's method and presented moment coefficients for rectangular slabs with different boundary conditions. It was claimed that their method could result in a saving of 15 to 20% of reinforcement over IS 456-1964 method. Test results of Armer²⁶, Iyengar *et al*²⁷ and Rajanna and Chandrasekhara²⁸ on rectangular, skew and square slabs (tested under uniformly distributed load) confirm that the slabs designed by strip method behave in a safe and satisfactory way. However, test results of other shapes and boundary conditions are required to support the above finding. Raju²⁹⁻³² gave moment coefficients for rectangular slabs with different boundary conditions. In 1959, Hillerborg³³ extended the simple strip method to complex slabs which include a column support in the interior of the slab. The theoretical treatment is complicated and hence was called the Advanced Strip Method by Crawford³⁴. Alternatively, Kemp³⁵ presented a generalisation of strip method of design suitable for slabs with patch loads and concentrated loads using appropriate distribution of the vertical shear forces instead of applied loads. Mallick and Gupta³⁶ gave tentative proposals for the application of the method. Test results are required to know the behaviour of slabs designed by the above method. Fernando and Kemp³⁷ developed a strip-deflexion method using an elastic theory approach. They introduce compatibility of deflexion criteria of strips. Choosing a torsionless elastic moment field, they demonstrated their method for square and rectangular slabs. This method provides information about the bending moments of lateral beams. They indicated that the method could be extended for point and patch loads. Wilby³⁸ has developed computer programmes using the strip deflexion method which also take into account individual point loads and any support conditions. Subsequently Wilby³⁹ presented design tables for the slabs carrying uniformly distributed load for different boundary conditions including slabs with free edges. Hillerborg⁴⁰ has reported the advanced strip method which includes supports like interior, exterior corner columns and reentrant corners. Gurley⁴¹ has adopted the inversion technique to Hillerborg's simple strip method. Instead of assigning the strip loads he has assigned the moment fields at the critical locations so that the moments are in bimoment equilibrium with the applied loads. He has introduced the stress-resultant bimoment and developed four equilibrium equations. But the extra equation is a material-dependent equilibrium

equation and not as a fourth independent equation. Subsequently Gurley⁴² has presented design methods for slabs considering as torsion-free girders. He has covered the yield lines which are curved in plan also.

2.4 Direct design method

Wood⁴³ proposed this method of design based on elastic stress fields. The steel is determined by using elastic stress distribution at ultimate loads in conjunction with the yield criterion. Recently Hago and Bhatt⁴⁴ reported test results designed by the above method and it was found that the slabs behaved satisfactorily both at working load and at ultimate loads. To design by this method, computer facilities are required and also they have assumed uncracked stiffness of slab throughout the analysis.

3. Studies on rectangular/skew slabs

3.1 Membrane action in restrained slabs

The above mentioned methods have been formulated without considering the effect of change of geometry of the slab. The changes in the geometry of the slab and boundary restraints have considerable influence on the load-carrying capacity of the slab. These effects are called membrane actions and there exist two types of membrane action, *viz.* (1) compressive and (2) tensile membrane actions. This is reviewed briefly in the following sections.

3.2 Compressive membrane action

When a slab is surrounded by and is continuous with either stiff beams or additional slabs, compressive membrane action develops within the depth of the slab. This phenomenon can be explained as follows. After cracking load, there occurs a substantial shift in the position of neutral axis in the cross-section. This effects in the outward movement of the edges. This movement is prevented by the restraints. Hence compressive forces develop in the slab. This compressive force acts above the mid-depth at the centre of the slab and below at the edges of the slab and hence causes arching or dome action. It increases the moment of resistance of the slab cross-section which in turn significantly increases the ultimate flexural resistance of the slab. In 1939, Gvozdev⁴⁵ conducted tests on restrained slabs and showed that the compressive membrane forces were due to the different level of neutral axis in the positive and negative yield lines. Much work has been done on membrane action by Soviet researchers. Their contributions are not available in English. In 1955, Ockleston⁴⁶ conducted load tests on a three-storey reinforced concrete building at Johannesburg and recorded the collapse loads about three or four times of Johansen's yield-line load. Powell⁴⁷ tested nine small-scale restrained, isotropic rectangular slabs with different steel ratio. He noticed that for a slab with 0.25% steel ratio, the ultimate load was 8.2 times higher than

Johansen's load. His test results showed that higher enhancement could be achieved with lower steel ratio. Brotchie⁴⁸, Christiansen⁴⁹, Park⁵⁰, Jacobsen⁵¹, Roberts⁵², Brotchie and Holley⁵³, Hopkins and Park⁵⁴, Desayi and Kulkarni⁵⁵ have presented methods of analysis including compressive membrane action for restrained rectangular slabs based on deformation theory. Also Desayi and Kulkarni have recently critically reviewed the above study¹³. Some investigators have used the flow theory introduced by Sawczuck in 1964. Sawczuck⁵⁶, Janas⁵⁷, Morley⁵⁸, Hung and Nawy⁵⁹ have reported methods of analysis for restrained rectangular slabs using flow theory. Braestrup⁶⁰ has reviewed the theoretical treatments of dome effect of reinforced concrete slabs up to 1978. He made a clear distinction between the flow theory and deformation theory. He developed solutions for a restrained slab strip by flow theory as well as deformation theory and showed that for a given membrane force, the deflection predicted by deformation theory was twice that predicted by flow theory. Also the load-carrying capacity dropped below the flexural collapse load of unrestrained slab, if deformation theory was used. He pointed out that these predictions are due to the assumption made in deformation theory, i.e. concrete is assumed to assert its ultimate compressive stress during unloading. He concluded that deformation theory led to unrealistic predictions. However, Chattopadhyay⁶¹ points out that rigid plastic approach applied to concrete itself is far from reality and hence comparison of the two theories is not appropriate. Also flow theory should not be taken as a suitable method, as it predicts maximum deflection of a slab when total superimposed load ceases to exist. He pointed out that the rigid plastic analysis through either approach is not useful for slabs of practical interest, as the one-way strips may fail before the concrete compressive stress block at the critical section reaches its elastic limit. This leads to the possibility of developing deformation theory whereas flow theory cannot be developed. While the above cited investigations are on restrained rectangular slabs, some work has been done on the strength and behaviour of slab-beam systems. Park⁶², Hayes and Taylor⁶³, Datta and Ramesh^{64, 65}, Desayi and Kulkarni⁶⁶ have proposed methods for estimating ultimate load of rectangular slab-beam system. Tong and Batchelor⁶⁷ reported test results of square two-way bridge slabs subjected to concentrated loads. They have proposed a method of estimating the failure load which includes compressive membrane action. They concluded that if steel of low percentage is used, punching failure could be eliminated and flexural failure mode results in. They recommended a minimum percentage of 0.25% for two-way bridge slabs to avoid instant failure. The study of compressive membrane action has been made on other shapes also. Desayi and Prabhakara⁶⁸ presented a method of analysis for restrained skew slabs including compressive membrane action. The next section deals with tensile membrane action occurring in restrained slabs.

3.3. Tensile membrane action

In restrained slabs, the load-carrying capacity is enhanced by compressive membrane action till the ascending load-deflection curve reaches a peak point. While in the descending portion of the load-deflection curve, the membrane force changes from compression to tension. The load-deflection curve descends to a secondary resistance

point at which the membrane action in the central region of the slab changes to tensile. Beyond this, the tensile membrane action spreads throughout the slab till the fracture of steel. Park⁵⁰, Keenan⁶⁹, Black⁷⁰, Herzog⁷¹ have presented procedures for calculating the incipient collapse deflection capacity. Iqbal and Derecho⁷² have reviewed their contributions in detail.

4. Membrane action in simply supported slabs

In simply supported slabs, membrane forces develop at large deflections. With large deflections at midspan, the central region of the supported edges tend to move inwards but are restrained from doing so by adjacent outer regions. This creates the central area of the tensile membrane stresses within the slab together with a surrounding ring of compression. This effect enhances the load-carrying capacity of the slab. Park⁵⁰, Taylor *et al*⁷³, Hayes⁷⁴, Kemp⁷⁵, Sawczuck and Winnicki⁷⁶, Morley⁵⁸, Desayi and Kulkarni⁷⁷, proposed methods of analysis for rectangular simply supported reinforced concrete slabs including membrane action. Prabhakara⁷⁸ extended the work of Desayi and Kulkarni⁷⁷ to simply supported skew slabs.

The above studies were concerned with the ultimate strength of reinforced concrete rectangular/skew slabs. But these should be complimented by investigation on serviceability limit states namely deflection and crackwidth. In the following sections the literature available on the studies on deflections and crackwidths of reinforced concrete slabs is reviewed.

5. Deflections of two-way reinforced concrete slabs

Deflection is one of the serviceability limit states to be satisfied in the limit-state design of structures. Therefore the estimation of deflection under working loads is often necessary in the design of concrete slabs. But an accurate estimation of deflection is complicated because of the influence of factors like cracking of concrete, creep and shrinkage effects (time-dependent effects) and nonlinear properties of the material. As a convenient way of controlling deflections, some codes of practice specify limiting span to depth ratio. This is conservative because the effect of all factors affecting deflections is included in a single parameter namely depth. Clarke *et al*⁷⁹ have compared the code clauses of 25 countries related to deflections of concrete members. In 1974, ACI Committee 435⁸⁰ reviewed the methods like classical, cross-beam analogy, analogous grid work method, static ratio, wide beam and equivalent frame methods for the determination of deflections of two-way slabs. The ACI manual of concrete practice⁸¹ recommends the procedure developed by Branson based on beam test results for the computation of slab deflections. This requires further examination due to difference in the structural behaviour of the beam and the slab. Scanlon and Murray⁸², Bell and Elms⁸³, Wanchoo and May⁸⁴, Yukio *et al*⁸⁵, Cope and Vasudeva Rao⁸⁶, Schonobrich⁸⁷ have incorporated the influence of cracking in developing finite element analysis to predict the

load-deflection curve of slabs. Bazant *et al*⁸⁸ have provided a summary of the work done on the application of finite element method to reinforced concrete structures up to 1978. This method cannot be readily used in design offices as it requires good computer facilities. Herzog⁸⁹ proposed a trilinear load-deflection plot for two-way slabs from uncracked stage up to failure of the slab. His theoretical load-deflection plot consisted of three zones. Zone 1 is between zero and cracking load, Zone 2 represents the plot from cracking to yield-line load and Zone 3 corresponds to plastic deformation beyond yield-line load. Shukla and Mittal⁹⁰ reported semi-empirical formulae for determining the deflections of rectangular reinforced concrete slabs based on their test results of nine simply supported square slabs tested under uniformly distributed load. Branson⁹¹ gave details of the available methods of computing deflections of two-way slabs. Desayi and Kulkarni^{55,66,77} proposed methods for predicting load-deflection curves for restrained, partially restrained and simply supported rectangular reinforced concrete slabs in the form of piecewise straight lines. They accounted for cracking of concrete and yielding of steel by suitably modifying the flexural rigidity of the slab and changes that occur in the support conditions due to possible yielding after the formation of yield lines. The authors^{92,93} have computed short-time deflections of reinforced concrete rectangular slabs using a deteriorating moment of inertia function. Desayi and Prabhakara⁶⁸ proposed a method for predicting load-deflection curves of restrained skew slabs.

The studies cited above concern only with the estimation of instantaneous deflection of reinforced concrete rectangular/skew slabs. But the serviceability of slabs depends to a large degree on their long-term deflection performance. Hence methods are needed to estimate the long-term deflection of slabs. ACI Code⁸¹ recommends the use of a multiplier constant to short-time deflections to determine long-term deflections. But Taylor's⁹⁴ and Heiman's⁹⁵ investigations showed that ACI Code method did not predict the long-term deflections satisfactorily. Rangan⁹⁶ proposed a method for the computation of instantaneous and long-term deflections of flat plates and slabs. His results were within 24% of the measured values of three existing floors. Rangan and McMullen⁹⁷ presented a rational approach to control both total and incremental long-term deflections of flat slabs and plates by limitation of span-depth ratio. They developed the equations based on a limiting long term total deflections of $L_1/240$ where L_1 is longer centre-to-centre span. They concluded that the use of minimum thickness as per ACI 318-71 and AS 1480-1974⁹⁸ will result in long-term deflections greater than $L_1/240$. IS 456-1978⁹⁹ specified formulae for estimation of short-term, shrinkage and creep deflections. The authors¹⁰⁰ examined these formulae with the use of available 22 rectangular simply supported reinforced concrete slabs (short-term tests only). It was found that the load factors obtained by limiting the total deflection to span/250 were large. In some cases this load factor was found to be as high as 5.93 against the recommended load factor of 1.5 for the limit state of collapse. Hence it was concluded that IS 456-1978⁹⁹ Code clauses probably require further examination in respect of the computation of deflection of slabs including those due to shrinkage and creep. Also more test data on long-term deflections of two-way slabs are required. Gilbert¹⁰¹ extended Rangan's approach for beams to two-way slabs. But this requires finite element simulation. Tam and Scanlan¹⁰² proposed a method for computing immediate deflections

including shrinkage effects. They have compared with the field measurements. Graham and Scanlan¹⁰⁵ have presented a method for estimating the deflection due to shrinkage restraint, construction loads. They proposed multiplier constants for estimating the two-way slab deflections. Limit state of cracking is another serviceability criterion to be satisfied in the limit-state design of structures. The studies pertaining to the prediction of maximum crackwidth in rectangular/skew slabs are presented in the next section.

6. Prediction of crackwidth

In reinforced concrete slabs, the crack appears as soon as the tensile stress due to the bending moment exceeds the tensile strength of concrete. This may impair appearance of the slab or increase the risk of corrosion of the reinforcement. In order to avoid this, the crackwidth should be kept below the acceptable limit. ACI Code, CP 110¹⁰⁴ and CEB-FIP recommendations¹⁰⁵ have specified the maximum allowable crackwidths for structural members under different exposure conditions. A crackwidth study on one-way slabs with welded wire fabric by Srinivasa Rao and Subrahmanyam¹⁰⁶ indicates that a steel stress of 2500 kg/cm² can be allowed without violating the strength and crackwidth limitations for one-way slabs. In developing their mathematical model, they have included the force exerted by transverse bars in longitudinal direction. Nawy and his associates¹⁰⁷⁻¹⁰⁹ have done a major study on cracking of two-way rectangular slabs. The proposed equations were based on the fracture hypothesis on the stress concentration at the intersection of orthogonal bars. The limitations of the proposed equations are in choosing the appropriate multiplying constant for the particular type of loading and specified boundary conditions, and the experimental results being those of rectangular slabs reinforced with welded wire fabric. Desayi and Kulkarni¹¹⁰ extended the method proposed by Desayi¹¹¹ for the determination of maximum crackwidth of partially prestressed concrete beams to two-way rectangular reinforced concrete slabs. Two-way action was included by considering the bearing exerted by transverse bars. A uniform distribution of bond stress over the length of the bar and tensile stress in the stretched concrete area was assumed in their analysis. This was further extended to skew and rectangular slabs by Desayi and Prabhakara¹¹². It was found from their investigation that the assumption of parabolic distribution for bond stress and uniform distribution of bearing stress resulted in a better agreement with the test data of 348 crackwidth measurements on rectangular and skew slabs. In recent times, considerable work has been in progress to study the cracking behaviour of concrete using fracture mechanics concepts. These have yet to find application in the design of slabs.

7. Studies on circular slabs

7.1 Membrane action in restrained/simply supported slabs

Wood¹⁴ analysed the membrane action in isotropic circular slabs using the basic equations of large-deflection plate theory. He assumed the material to be rigid plastic

and computed load-deflection plots for simply supported and restrained isotropic circular slabs. Due to the above assumption, the maximum load-carrying capacity was obtained at zero deflection for restrained circular slabs. Also, the theoretical load-deflection plots start from Johansen's load at zero deflection for simply supported circular slabs. As such, his theoretical curve does not predict the elasto-plastic behaviour of simply supported and restrained circular slabs tested under uniformly distributed load. However his analysis is useful to estimate ultimate strength of slabs, if a suitable value of deflection at ultimate load is assumed. Desayi and Kulkarni¹¹³ extended Wood's approach to orthotropic restrained circular slabs. Two cases of orthotropy, namely, (1) $M_1 > M_2$ and (2) $M_1 < M_2$ were considered in their analysis, where M_1 and M_2 are the plastic moment capacities in circumferential and radial directions respectively. While the above investigations of the section of the slab in circumferential and radial directions were based on deformation theory, some investigators have applied flow theory to circular slabs. Janas⁵⁷ gave rigid plastic flow theory solution not only for restrained circular slabs but also to square slabs and slab strips. The comparison between theoretical and experimental deflection curve was not reported. Morley⁵⁸ used the flow theory for predicting the load-deflection plots for rigid plastic slabs with and without edge restraints. The essential difference in the above two approaches is that, Janas used the dissipation method whereas Morley used equilibrium method. Calladine¹¹⁴ applied flow theory to circular slabs and showed that the upper-bound calculations on the strength of circular plates and slabs turn out to be simpler using a three-dimensional theory instead of two-dimensional slab theory. He illustrated his method on metal plates, sandwich plates and reinforced concrete slabs, simply supported or restrained, under point or uniformly distributed loading. The theoretical load-deflection plots for restrained circular slabs under point loading and simply supported circular slabs under uniformly distributed loading were given in his paper. But theoretical treatment on restrained reinforced concrete slabs under uniform distributed loading was not available in it. Morley¹¹⁵ tested circular slabs with ring beams and reported a method of analysis at the Euromech Conference in 1974. Moondra and Sharma¹¹⁶ reported the behaviour of simply supported circular slabs without and with square, rectangular and triangular openings. They compared the experimental results of seven slabs with the Johansen's load and observed that the ultimate load to be 1.37–1.70 times the yield lines load. They concluded that the ultimate load capacity is independent of the shape of the opening if the area of the opening of different shapes is kept constant. Al-Hassani¹¹⁷ used the deformation theory for the ascending part of the load-deflection curve and flow theory for the descending portion of the curve. He treated elasto-plastic strips using the above procedure. Braestrup and Morley¹¹⁸ proposed a modified rigid plastic theory for circular slabs with ring beams. They assumed that the membrane action starts at an initial elastic deflection. This deflection was assumed empirically as 0.03 times the thickness of slab. An alternative procedure also was suggested by them to obtain the deflection at which membrane forces started developing. From their load-deflection plots, it is observed that the empirical elastic deflection corresponds to that at the Johansen's load of simply supported circular slabs. Hence they assumed a linear load-deflection plot up to Johansen's load of corresponding simply supported circular slab. Their experimental results also showed considerable linear behaviour whereas available literature on other

shapes of slabs showed non-linear behaviour up to Johansen's load. Also the assumption of the same initial deflection irrespective of size of the ring beam is also not clear. However the effect of ring beam on membrane action was considered in their analysis by adding the inplane slab flexibility and ring beam flexibility in the form of a single boundary spring. The agreement between the theoretical and experimental curves is good. But complete details of the experimental work were not presented by them. Chattopadhyay⁶¹ gave some steps to obtain the initial value of deflection according to the theory presented in the paper.

Recently, the authors have developed methods of estimating the ultimate load of simply supported and restrained, isotropic and orthotropic circular slabs subjected to distributed loading taking into account the membrane action, load-deflection behaviour and estimation of widths of cracks that form in these slabs¹¹⁹⁻¹²⁴.

8. Study on slabs with different layouts of reinforcement

Hedley^{14,125} conducted experiments on simply supported square slabs with different arrangements of reinforcement. He kept the total amount of reinforcement to be the same for all slabs. He noticed a reduction of 15% of collapse loads for square slabs with diagonal reinforcement compared with uniform reinforcement parallel to the edges. Taylor *et al*⁷³ tested ten two-way simply supported square slabs to investigate the effect of arrangement of the reinforcement. The slabs were designed to have same flexural resistance under uniformly distributed load. They concluded that slabs with variable reinforcement were stiffer than uniformly reinforced slabs up to Johansen's load. However, the use of variable reinforcement did not enhance the load-carrying capacity. They observed that the use of variable reinforcement reduces crackwidth in the central portion of the slabs, but increases the crack width in the corner regions. Reddy *et al*¹²⁶ tested 12 simply supported square reinforced concrete slabs under central concentrated load to study the effect of direction of reinforcement, effect of banded reinforcement and the effect of circular and spiral reinforcement layout in square slabs. The investigation showed that the best arrangement of reinforcement for strength and stiffness was by having the bars parallel to the sides at a uniform spacing. Ultimate loads of all slabs reported were higher than Johansen's load. Clark¹²⁷ studied the effect of arrangement of reinforcement for reinforced concrete skew bridge slabs. He suggested that saving in steel could be achieved by placing the steel parallel and perpendicular to the abutments in the central region and changing to the arrangement which has steel parallel to the free edge near the edges.

Slabs with different layouts of reinforcement based on optimisation theory have been investigated by Rozvany¹²⁸⁻¹³¹, Melchers¹³², Sharpe and Clyde¹³³, Thakkar and Sridhar Rao¹³⁴. The principal conclusions were: load enhancement in optimised slabs was not much more than conventional slabs; at a particular load, optimised slabs exhibited smaller deflections when compared to conventional slabs; although the reinforcement volume was saved in optimised slabs, the optimal layout was quite complicated. While

much theoretical advancement has been done in optimisation of slabs, reported test results on the same appear to be very few. Muspratt^{135,136} has reported tests on optimally designed slabs. He suggested that Monte Carlo technique could be used for analysis of slabs as the properties of the slab are random variables.

9. Conclusions

1. Many investigations are come across on the determination of ultimate strength of rectangular slabs. Studies on the ultimate strength of skew and circular slabs are comparatively few. While the studies recognize the enhancement in the strength beyond Johansen's load, attempts to include this enhancement in design of slabs are yet to be made.

2. Studies on the determination of deflections and crackwidth in slabs are comparatively less. More experimental work on these aspects including the influence of creep under sustained loading is required in the examination of serviceability limit states of slabs.

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