

## BOOK REVIEWS

**General inequalities 5** edited by W. Walter. Birkhauser Verlag, CH-4010, Basel, Switzerland, 1987, pp. 482, S. Fr. 95. Indian orders to Springer Books (India) Pvt. Ltd., 6, Community Centre, Panchasheel Park, New Delhi 110 017.

This volume presents the proceedings of the Fifth Conference on General Inequalities held at Oberwolfach, West Germany during 1986. Though it is issued in the Series of Numerical Mathematics of Birkhauser, the numerical content (be it in the theory or application of inequalities) is really negligible.

Inequalities played and continue to play important role practically in all areas of mathematics. Their occurrence and importance cannot be emphasized much more than what was done in the celebrated book *Inequalities* by Hardy, Littlewood and Polya. In fact, inequalities are part of our life rather than equalities. Equalities like Parseval's Equality are very rare. The presence of right inequalities (for instance, in the case of dynamical systems) means that we have a dissipative mechanism in the system and stability is a consequence. Most of the real systems are dissipative in nature.

Let us cite a few examples of different types of inequalities one usually comes across: inequalities among numbers (e.g.: geometric mean  $\leq$  arithmetic mean), isoperimetric inequalities (e.g.:  $L^2 \geq 4 \pi A$  where  $A$  is the area of a plane domain and  $L$ , its perimeter, inequalities in function spaces (e.g.: Poincare' Inequality), interpolation inequalities (e.g.: Hardy-Littlewood Inequality), differential and integral inequalities (e.g.: Gronwall's Inequality) and so on. The list is endless. These inequalities arise in a variety of contexts including partial differential equations, number theory, geometry, and probability theory. The objects for which equality occurs in the above set of inequalities are rather special. These are important because they describe the best possible constants figuring in the inequalities.

The present book contains research articles which cover a wide range of topics in the field of inequalities. Each type of inequality mentioned above is discussed in some article or the other.

The book begins with the classical Bieberbach Conjecture in the theory of univalent functions. It is shown that how an estimate of the solution to a system of ordinary differential equations can contribute to the resolution of the conjecture. Apart from this, the book contains seven other parts which we describe briefly.

Part I is a collection of papers dealing with inequalities among numbers and integrals. There is one article on Hardy-Littlewood type inequality with weights. The point of the

article is to find the best possible constant in such an inequality. For reasons mentioned earlier, this kind of work gives important information. One finds two articles which are devoted to the study of oscillations in a sequence using inequality conditions.

The second part concerns with inequalities in harmonic and convex analysis. There is a paper which presents some curious properties of zeros of Bessel functions using some inequalities. Wallace studies in his presentation an algebraic problem of finding zeros of a function and shows the problem of finding the most efficient strategy is reduced to a problem of inequality.

The next part presents some inequalities in functional analysis. The second article in this section studies the behaviour of Fourier transformation in some weighted spaces. There is one interesting paper by Clausen who does some small-scale scientific computing with the eigenvalues of Polya operators. This is the only article in this volume where some numerics are involved.

The fourth part is about some inequalities in functional equations. The next section is probably the most important one as far as applications are concerned. It deals with some inequalities arising in PDE and ODE. One can find there articles which study Gronwall-type inequalities, isoperimetric inequalities for eigenvalues, Hardy-Littlewood type inequalities, estimates for the critical value of the eigenvalue parameter in a non-linear problem and inequalities of Sobolev type.

The next section is devoted to certain inequalities appearing in economics and optimization. Included in these is a discussion on various entropies. The last part is full of remarks wherein several open problems connected with the inequalities presented in the earlier sections are thrown.

There is a large part of scientific community including some analysts who come across inequalities but cannot prove. They are mostly interested in applying the inequalities and deriving the consequences. A book like *Analytic inequalities* by Mitrinovic is helpful to them. The usefulness of the present book for such people is not clear. The main drawback of this book is the diverse nature of the areas and topics covered. For a man working in a particular field, the book offers very little. Nevertheless it is found that the survey talk on the Bieberbach Conjecture which presents history, background and proof is very interesting and will save the reader from the tedious search of articles in the literature. The same can be said about the section containing articles on differential operators. This is a set of articles useful to people working in PDE. The book is recommended to researchers in this field.

**Lectures on complex approximation** by Dieter Gaier. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1987, pp. 196, S. Fr. 78. Indian orders to Springer Books (India) Pvt. Ltd., 6, Community Centre, Panchasheel Park, New Delhi 110 017.

This book is a translation of the German edition which appeared in 1980. As the author rightly claims, he has tried to give a synthesis of the concrete, constructive aspects of the complex approximation with the more theoretical results. The book has two parts. The first part deals with the approximation by series expansions of various types and by interpolation. The second part deals with existence theorems.

The first part of the book has two chapters. The first chapter treats the representations of complex functions by orthogonal series and Faber series. The author develops necessary Hilbert space theory. He studies the completeness of polynomials in  $L^2(G)$ . He then introduces domains with PA property (polynomial approximation property). The quality of the approximation has been investigated when the function  $f$  is analytic in  $G$ . The Bergman Kernel is then defined and a series representation of the Bergman Kernel is obtained. As an application of the Bergman Kernel the author constructs certain conformal mappings.

Another type of expansions which is less constructive than the orthonormal expansions is given by Faber expansions. Faber polynomials are introduced and the Faber mapping is studied as a bounded operator. For the approximation inside a curve of bounded rotation the quality of the approximation has been investigated.

The second chapter deals with approximation by interpolation. Hermite's interpolation formula is defined and certain special cases of Hermite's formula are studied. Then the author poses the problem of finding conditions on the interpolation points so that the interpolating polynomials converge to the function. He proves a theorem due to Kalmar and Walsh which says that the interpolating polynomials converge to the function if and only if the interpolation points are uniformly distributed. As examples of uniformly distributed points the author introduces and studies Fejer and Fekete points. Finally, a somewhat detailed study of interpolation in the unit disc has been made and a rational approximation theorem is proved.

As we have already remarked the second part of the book deals with general approximation theorems – existence of approximating rational, entire or meromorphic functions. In chapter III certain approximation theorems on compact sets are proved. First the author proves Runge's approximation theorem which stands at the beginning of complex approximation. The author next proves the celebrated Mergelyan's theorem on polynomial approximation. Next he deals with approximations by rational functions. Alice Roth's construction of a 'Swiss cheese' is given. Bishop's localisation theorem is proved and certain applications are given. Another proof of Bishop's theorem is given using Roth's 'Fusion lemma'.

Finally the last chapter deals with approximations on closed sets. The first section of the last chapter treats uniform approximation by meromorphic functions. Three sufficient criteria for meromorphic approximation are given. The next section deals with

uniform approximation by analytic functions. Arakeljan's approximation theorem and Carleman's theorem on  $\epsilon$ -approximation are proved. Finally some applications of the approximation theorem are given.

The book is well planned and self-contained and as such it is quite readable. I think, it will serve as a nice introduction to the field for graduate students interested in pursuing complex approximation.

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**Banach algebras with symbol and singular integral operators** by Naum Ya. Krupnik, tr. A. Jacob. Birkhauser Verlag, AG, Basel, Switzerland, 1987, pp. 205, S. Fr. 88. Indian orders to Springer Books (India) Pvt. Ltd., 6, Community Centre, Panchasheel Park, New Delhi 110 017.

Singular integral operators have a long history and play a very important role in many problems of analysis. To each singular integral operator, one can associate a function which is called its symbol. This notion was introduced by Mikhlín in studying a regularisation problem for two-dimensional singular integral operators. The idea of studying singular integral operators by means of the symbol turned out to be very useful and elegant as seen from the theory of pseudodifferential operators which came out as the synthesis of differential and singular integral operators.

This book presents a study of certain algebras generated by one-dimensional singular integral operators using Banach algebra techniques. In this book, Gelfand theory of maximal ideals of Banach algebras is used in the construction of symbols of singular integral operators. It is shown that certain algebras don't admit a scalar symbol but only admit what is called a matrix symbol. The Gelfand theory is also used to give a criteria for Fredholmness of singular integral operators.

The purpose of the first chapter is to obtain conditions for invertibility of matrices with entries in a ring in terms of its determinant. Connection between the Fredholmness of a matrix and that of the determinant has also been investigated. The main theme of the second chapter is the explicit calculation of the norms and quotient norms of singular integral operators on certain weighted  $L^p$  spaces. Exact constants in the theorems of Hardy-Littlewood, Babenko and Khvedelidze on boundedness of singular integral operators have been obtained. Certain analogues for spaces of vector-valued functions are given.

The results of the first two chapters are used in the third chapter to study singular integral operators with matrix coefficients. Sufficient conditions for the Fredholmness of singular integral operators with piecewise continuous matrix coefficients is given. A theorem of Simonenko has been extended to matrix analogue of singular integral operators with bounded coefficients. Some necessary conditions for the Fredholmness is also obtained.

The fourth chapter introduces the concept of sufficient families of multiplicative functionals on a Banach algebra and Banach algebras with symbols are defined. Conditions for the existence of scalar symbols in algebras of linear operators have been investigated. On algebras generated by singular integral operators with continuous coefficients on open contours, symbols are constructed.

Chapter five deals with Banach algebras generated by one-dimensional singular integral operators with piecewise continuous coefficients. It is shown that these algebras don't admit a scalar symbol. A matrix analogue of the Gelfand transform is introduced and studied in the next chapter. Finally conditions are given for the existence of a matrix symbol in operator algebras in the last chapter. The book concludes with a number of open problems and conjectures.

The book is more or less self-contained and is written in a simple style. A number of examples are given which help the reader to understand the theory better. But the book looks a little bit monotonous as the author goes on proving theorem after theorem. The author could have given motivations and applications for various theorems proved in the book. But for someone who doesn't bother about applications and takes pleasure in unveiling the hidden beauties of nature, this book will certainly make an interesting reading.

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**Laurent series and their Padé approximations** by Adhemar Bultheel. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1987, pp. 274, S. Fr. 88. Indian orders to Springer Books (India) Pvt. Ltd., 6, Community Centre, Panchasheel Park, New Delhi 110 017.

The present volume in Birkhauser is devoted to the study of Padé approximation of functions. This study can be considered as a topic in various branches of pure and applied mathematics such as approximation theory and complex function theory. The classical Padé approximation problem deals with functions  $f(z)$  which admit power series representation. Principally one is interested in approximating  $f(z)$  by a rational function of the form  $A_m(z)/B_n(z)$  where  $A_m(z)$  and  $B_n(z)$  are polynomials of given degrees  $m$  and  $n$  respectively. The approximation is expected to be global. If  $n = 0$ , this amounts to finding a global polynomial approximation of the function given. There are various classical well-known techniques such as interpolation which do this. However, the procedure in Padé approximation is somewhat different. The idea is to expand  $A_m(z)/B_n(z)$  in powers of  $z$  and match this series with that of  $f(z)$  up to certain order. This provides a linear system for the unknown coefficients of  $A_m$  and  $B_n$ .

The above mentioned process is purely algebraic. This can be viewed as an approximate method of analytic continuation to regions where the power series with which we started may fail to converge. Stated differently, one can consider this as a means to sum a divergent series. This brings out the connection between Padé

approximation and a variety of other fields. For instance, the series got out of a perturbation technique is almost never summable. The question is how to get maximum information out of this divergent series. One can hope to use Padé approximation to answer this.

There are four major issues in the study of Padé approximation. (i) Existence and construction of Padé approximants (ii) study of convergence (iii) connections with other topics and (iv) applications. There is a vast literature on Padé approximation dealing with the aspects mentioned above. All these issues are discussed here in this volume also. The main difference between the existing literature and the present book is the following: instead of the classical assumption that the given function  $f$  admits a power series representation, the author assumes the existence of Laurent series representation for the given function. For certain problems, this later assumption seems to be more natural. The author extends many of the results on classical Padé approximation to this case.

Let us see briefly the contents of this book. It should be mentioned to start with that the numbering of chapters in the contents page and elsewhere do not coincide. Chapters 3 and 7 give various algorithms to construct Padé-Laurent approximation. Their connections with continued fractions and orthogonal polynomials are analysed in chapters 4–6 and 8. Padé tables are introduced and their block structures are explored in the few subsequent chapters.

The next important step is the study of convergence of Padé-Laurent table. This is the object of study in chapters 12–16. The convergence of each column ( $n$  is fixed and  $m \rightarrow \infty$ ) poses no special problem and is analogous to the classical case. On the other hand, the convergence of each row ( $m$  fixed and  $n \rightarrow \infty$ ) is a more serious problem because the simple symmetry argument that is used in the classical Padé approximation case will not be applicable here. It demands some new ideas and is one of the most important results of this monograph. The main task in the convergence theory is to control the location of poles and zeros. Apart from these, there are two chapters reserved for the applications.

The book is well-written and neatly organised. It is essentially self-contained and the required background needed to read it is kept to a minimum. It ought to be useful not only for specialists in Padé approximation, continued fractions and orthogonal polynomials but can also serve as a basic text. Because of the wide range of applications of the subject, this book is also recommended to the people working in several areas of applications such as statistical physics, quantum field theory, turbulence, numerical analysis, linear system theory, stochastic process and so on.

**Seminar on empirical processes** edited by Peter Gaenssler and Winfried Stute. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1987, pp. 110, S. Fr. 32. Indian orders to Springer Books (India) Pvt. Ltd., 6, Community Centre, Panchasheel Park, New Delhi 110 017.

This monograph contains notes based on lectures given in the Seminar on Empirical Processes held at Schloss Mickeln, Düsseldorf, during September 8–13, 1985. There are nine sections in the book, each dealing with some aspect of empirical processes based on independent, identically distributed (i.i.d.) random variables (r.v.s.).

Sections I and II deal with foundations, local and global structure of empirical processes in the i.i.d. set-up. The Markov and Martingale properties of empirical distributions and order statistics are explained. Some useful upper bounds for the tail probabilities of empirical processes are also mentioned in these sections. These properties are exploited in dealing with tests of goodness of fit in Section III. A unified approach is presented in Section III, covering tests of goodness-of-fit based on Smirnov, Pyke, Takacs, Dempster, Birnbaum-Tientz Csörgo, Daniels, Tang, Csaški, Birnbaum-Tingey and Renyi statistics. Using the Poisson representation argument, the authors discuss the finite sample distribution of Kolmogorov-Smirnov statistic. The reviewer feels that this section is *the best* in the book.

Section IV is a brief outline of conditional empirical processes as introduced by Stute. Coupla processes are touched in Section V. The study of empirical processes based on censored data is discussed in some detail in Section VI. Hajek's projection technique is employed in this section. Parameter estimation and boot-strapp are discussed briefly in Sections VII and VIII respectively.

The uniform convergence of  $\mu_n(B)$ , where B is a Borel set is the subject matter of Section IX, the last. This section presents simplified proofs of some theorems on Vapnik-Chernovenkis classes of sets. Some recent results of Gaenssler in this area are also included.

The book serves as a brief introduction to empirical processes highlighting the main approaches in dealing with such processes when the underlying random variables are i.i.d. No new results are presented in the monograph.

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**Solitons and instantons, operator quantisation** edited by V. L. Ginsburg. Nova Science Publishers, Inc., 283, Commack Road, Suite 300, Commack, New York 11725-3401, 1987, pp. 312, \$ 92.

This book consists of four parts, each written by a separate set of authors. The first part, by Saradzhiev and Faynberg deals with the  $A_0=0$  quantisation of non-abelian gauge

theories. The material is in principle standard, but presented with an emphasis and point of view different from that of other books and review articles. Technical delicacies, like the requirement that the gauge function  $S(\vec{x}) \rightarrow 1$  as  $\vec{x}$  are argued with care. The second part, by Fradkin and Bataĭin, deals with the treatment of constraints in the Hamiltonian formulation of quantum gauge theory. The authors are themselves leading researchers on this topic and their intimate familiarity with the subject matter is well reflected in this part of the book. The third part, by Leznoc, Man'ko and Savbel'ev deals with solitons, *i.e.*, localised exact solutions of field equations. Examples include both two-dimensional scalar field systems as well as the monopole and instanton solutions in higher dimensions. There is an extensive discussion of completely integrable non-linear systems, their symmetries and associated integrals of motion. The role of self-dual functions in providing exact solutions of gauge theories is emphasized. The fourth and final part, by Gurevich and Meshcherkin deals with shock wave fronts in hydrodynamics.

The level of presentation in each of the four parts is very sophisticated, the formulation rigorous and general—typical characteristics of the Russian school of pedagogy. There is no attempt to pamper the reader. This is a useful book for any field theorist to have in his library.

However, the book is not without its weaknesses. To start with, the four parts written by four different sets of authors, are not well coordinated with one another. One gets the feeling that four separate review articles have been put together more for publication convenience rather than because of compelling thematic unity. Although the title of the book is solitons and instantons, there is no discussion of solitons in two of the four parts in the book. Instantons are covered to a limited extent in the first part, and do not make an appearance again in the last 75% of the book! Quantisation of solitons is discussed very briefly in one section of the second part. Perhaps the most serious weakness of the book is its preoccupation with exactly integrable systems. Such models are fascinating from the theoretical point of view. But real physical problems are unlikely to be exactly described by such models. It is far more useful to discuss properties of systems which carry solitary waves which are not necessarily strict solitons. Methods for quantising these are well known and well developed. These have been omitted entirely in this book, along with all the insights that these methods yield about the structure of quantum field theory.

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**Thirty years since parity nonconservation – A symposium for T.D. Lee** edited by Robert Novick. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1986, pp. 198, S. Fr. 42. Indian orders to Springer Books (India) Pvt. Ltd., Panchasheel Park, New Delhi 110 017.

The discovery by T. D. Lee and C. N. Yang in 1956 that parity is not conserved in the weak interactions is without doubt one of the most significant developments in



fundamental physics and our understanding of nature. It led to the award of the Nobel Prize to them in 1957. In his talks recounting the development of the concept of the positron and antimatter, Dirac used to stress how hesitant and reluctant he felt to postulate the existence of a fourth new particle to be added to the list of the then known elementary particles, the proton, electron and the photon. Such inhibitions have long since stopped holding anyone back, and it is very common for theorists to postulate the existence of as many yet to be observed particles as seems convenient at the moment. The situation has been somewhat similar with the case of parity violation. Until 1956, it was an implicit assumption – held and expressed so strongly for example by Pauli – that all natural processes had to be left-right symmetric – for how could it be otherwise? (Amazingly enough, in a well-known 1949 paper Dirac had said explicitly that there was no reason to expect this to be so!). After the parity revolution, it was as though a barrier had been broken, and violation of symmetry became an easily accepted concept. The only saving grace today is that one has not yet got to the point where symmetries are broken before they are thought of, but who knows!

On the occasion of Lee's 60th birthday in November 1986, which also coincided with the 30th anniversary of the discovery of parity violation, a special symposium was held at Columbia University to celebrate both the events. This book contains the texts of the talks given at the symposium, accompanied by salty and entertaining introductions to each speaker. Especially for new entrants to the field of elementary particle physics, and also for those with a feeling for history and personalities in physics, this is a very interesting book indeed.

Born in 1926, Lee came as a graduate student from mainland China to the U.S. in 1946, aided by a Chinese Government Fellowship arranged for him by his teacher Ta-You Wu. He studied under Fermi and other masters at Chicago, getting his Ph.D. in 1949. After short stays at Berkeley and the Princeton Institute, he came to Columbia as a Professor in 1953, where he has stayed ever since.

The scene in elementary particle physics in the early and mid 50s, the emergence of the tau-theta puzzle, and the way in which Lee and Yang analysed the problem of parity conservation, are all vividly brought to life in the lectures in this book. It becomes especially clear that Lee has had very close contacts with experimentalists all through his career, constantly suggesting crucial experiments to them, and giving guidance at important moments. While Lee and Yang showed by a masterly analysis that none of the existing data could say anything conclusive one way or the other about parity conservation, it became necessary to suggest key experiments that would clearly answer the question. This they did, and it is no coincidence that the three main experiments were all done in or around Columbia. The  $\text{Co}^{60}$  experiment by Wu *et al* is described by Madame Wu herself. This is a classic of modern physics, and it took close to six months from June 56 to January 57 to get a clear result vindicating the Lee-Yang theory. Any budding experimenter would feel thrilled to read this account. The experiments on the pi-mu-electron chain, however, were done in four days flat – from a Friday to the following Monday in January 1957 – as described by Garwin. The experiments on

nonleptonic hyperon decays took more time, due to poorer statistics, and that is the subject of Schwarz's brief contribution.

Samuel C. C. Ting, another recent Nobel Prize winner in Physics with a Chinese background, speaks of his many encounters with Lee and the ways these helped shape his own ideas and experiments. Drell gives a very humorous and readable account of all the fields of physics in which Lee has made significant contributions; while Lee himself describes his education and growth in the U.S., the way he and Yang came together and pooled their complementary talents to produce physics of the highest class; and the satirical misunderstandings of the later years. A pedagogical talk by Zumino introduces the reader to more recent developments in quantum mechanics related to non-integrable phases, using the machinery of differential geometry, topology and fibre bundles.

Apart from his research, Lee has also been a master teacher of physics. He realised how great an opportunity was made available to him when he was enabled to come to the U.S. as a young student in 1946. To 'make such chances available to others, since 1979-1980 he has organized an extensive program whereby up to now upwards of 700 talented Chinese students of physics have come to the U.S. as graduate students, with the intention of returning to China to pursue research there. Lee had to enlist the support of China's political leaders in setting up this program, which he did. It may be that this activity of the so-called "Lee-scholars" has something to do with recent progress in China in elementary particle and accelerator physics. This book gives an account of this special program, and of some talks presented by its participants as well.

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**Projective relativity, cosmology and gravitation** by Giuseppe Arcidiacono. Hadronic Press, Inc., Nonantum, MA 02195, USA, 1986, pp. 261, price not mentioned.

The book under review presents a 'new theory for the cosmos'. According to the back cover of the book, the author "has developed a new theory of the hyperspherical universe" which "re-unites in a unique mathematical scheme various cosmological and unified theories".

I was, however, disappointed with the book. It is written in a very unclear style; the motivation and the details are often obscure and the reader has to laboriously shuttle back and forth to find the logical connection between the various chapters. The author has also failed to compare his new cosmological theories with the existing ones or with the observations. By and large, the work appears to be an exercise in mathematics presented in the jargon of physics.

The first three chapters of the book deal with standard material - special relativity, thermodynamics and electromagnetism, general relativity and cosmology. In the fourth chapter, the author begins with the group theoretic approach to cosmology and

introduces the 'hyperspherical universe'. This chapter has several new ideas but they are not presented with adequate motivation. The ideas also appear to be disjoint (For example, this chapter has a section (6) describing a unified theory of physical and biological world in just 4 pages!).

Chapter 5 discusses some amount of projective transformation theory and an approach to special relativity based on these transformations. Chapter 6 introduces the projective Fantappie group and works out a cosmological model related to this group. Since this group has richer structure than the usual groups used in cosmology, the author could derive a limited amount of standard cosmological results as special cases of this model. There is, however, no comparison with observations; neither has the author provided detailed motivation as to why he prefers this model to the standard ones. The only place where the reviewer could find some comparison with observation is in Section 6 of Chapter 7. Here the author tries to derive the spiral structure of galaxies from projective mechanics. The result, however, is basically the same as that in Milne's kinematic relativity. The next three chapters discuss the modification in hydrodynamics, electrodynamics, etc., in the new relativity. Chapters 11-13 discuss unified theories of gravity with other forces as well as generalisations of Einstein's theory of relativity based on the larger group structure. Chapter 14 is titled "the deSitter universe and quantum physics", but the discussion which follows is quite different from what one would have expected. The quantum physics which the author discusses is fairly non-standard and appears to be out of date. For example, in page 333 the author claims "As is well known in physics, we have four fundamental forces" - which is not the generally accepted view today.

The book contains some interesting pieces of mathematical information in group theory and projective geometry. Except for that, there is very little to recommend it.

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