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BOOK REVIEWS

The physical effects in the gravitational field of black holes edited by M. A. Markov. Nova Science Publishers, Inc., 283, Commack Road, Suite 300, Commack, New York 11725, 1987, pp. 262, $ 83.

Probably no word in modern times transports one instantaneously to the realms of the strange and bizarre as the word black hole. Yet black holes are not only the most pristine macroscopic objects fabricated solely from space-time concepts but the simplest as well since general relativity leads to their uniqueness. As far back as 1783 John Michell speculated about objects that would be invisible because their gravitational potential would exceed the specific kinetic energy of a projectile moving with the speed of light. This argument later appeared in the 1796 edition of Laplace's famous *Exposition du syseme du Monde* without acknowledgement adds B. Carter! The notion of a black hole was revived neither with the coming of Einstein's general relativity (1916) nor with Schwarzschild's (1916) discovery in the relativistic context of the Michell radius, mainly due to the coordinate singularity at the horizon. The issue of gravitational trapping of light became more serious only after Chandrasekhar's (1931) discovery of the mass limit for white dwarfs and Oppenheimer and Volkoff's (1939) limit on the mass of neutron stars. The modern impetus that led to the notion of a black hole was Oppenheimer and Snyder's work (1939) on spherically symmetric gravitational collapse. In the late 1950s and early 1960s Wheeler argued forcibly that gravitational collapse to a singularity was one of the most serious problems for fundamental physics. Subsequently when quasars were discovered astrophysicists looked for a ready explanation in terms of gravitational collapse and began investigating other observational manifestations of collapsed objects. In 1968 the collapsed object was picturesquely christened as black hole by Wheeler and the time was ripe for a new phase in the history of black holes. Investigations proceeded along two directions; one was the mathematical foundation of black-hole configurations and the other, black-hole astrophysics. The former led to the remarkable results on black-hole thermodynamics by Bardeen, Bekenstein, Carter and Hawking that certain black-hole parameters are analogous to macroscopic thermodynamic variables. Meanwhile, it was speculated that in addition to black holes formed by gravitational collapse, primordial black holes could form by density fluctuations in the very early universe. In the process of studying such mini black holes Hawking discovered that black holes are not completely black after all but emitters of a thermal radiation. This effect is quantum mechanical and thus of significance only for mini black holes; the black holes of gravitational collapse are at such low temperatures that they are virtually black. The surprising result of the black-hole radiance was based on quantum field theory in curved space time and this is the broad subject area the volume under review is concerned with.

The first five chapters of the book are based on the doctoral dissertation of V. P. Frolov and deal with the following: black holes and the problem of self energy, quantum particle creation in the gravitational field of a black hole, vacuum polarization in the presence of a black hole and the problem of spherically symmetric collapse in quantum gravity. The next three articles are by A. I. Zel'nikov, P. A. Bolashenkov, and V. Ye. Kur'yan respectively each jointly with V. P. Frolov. These deal with further details of vacuum polarization by and nonsingular models of black holes, as well as a related problem of radiation from accelerated mirrors. The last article on one-loop calculations of photon splitting in a relativistic quantum plasma is totally out of place and has no relation whatsoever with the rest of the book. It seems an editorial gaffe that could easily have been fixed. Luckily the article is brief.

Quantum field theory in curved space time is a particular case of the general problem of physical systems in external fields. In the late sixties research in this discipline was in connection with particle production in cosmological backgrounds. I discovered that Frolov and Markov were the first to discuss quantum effects in black hole backgrounds in 1970. This was followed by the discovery of superradiance and Zeldovich-Starobinsky-Unruh nonthermal emission for rotating black holes but Hawking effect was the glamour event that led to the explosion of activity in quantum field theory in curved space time. This led to the publication of a very comprehensive monograph by N. D. Birrell and P. C. W. Davies as early as 1979. The present book covers material with a different emphasis: black holes. It implements quantization by Schwinger's dynamical principle and illustrates how functional methods significantly simplify computation. The problem of self energy of charges is handled differently from the usual ADM approach and it is proved that point charges do not exist and the smallest charges called friedmons are stable. A unified treatment of creation of particles of different spins by rotating black holes is given. The particle creation will eventually start reacting back on the gravitational field and the first step in this computation is to study the expectation value of the energy momentum tensor. V. P. Frolov has been involved in one of the most detailed studies of the expectation value of the energy momentum tensor around black holes and this volume provides at one place his contribution to the subject. Finally these quantum effects could modify the formation of singularities so rigorously proved by the theorems of Penrose and Hawking. A model calculation presented here shows that higher derivative terms dominate the evolution near the singularities and may be could eliminate it. The treatment of various topics is rigorous and the book is heavy on formalism. What I missed was an adequate physical appreciation of results obtained; *e.g* on page 87 when the thermal nature of the radiation is related to the equivalence principle the discussion is left dangling. In view of the fact that the book does not address itelf to classical and astrophysical effects of black holes a more precise title would have been 'quantum effects in the gravitational field of black holes'. Further, the value of the book would have been enhanced if it contained related work by other Soviet groups as well. May be the editors will be more generous next time!

Barring the above minor reservations the book is a welcome addition to the existing literature on the subject. It makes more accessible the work done in the USSR on quantum field theory in black hole space times and it is interesting to note that many

results available in Western (English) literature have their counterparts in Soviet journals. To speed up their availability would it not be better if such pedagogical and technical material appeared as review articles rather than as books? May be, then, such work would be less dated, and instead of parallel science movements in the Soviet and Western blocks, one would have better feedback that enriches and goes a long way in improving the general quality of research. The book is reasonably well produced and free from obvious typographical errors. A couple of minor ones are: on the contents page where friedmon states has become Friedman states, on page 87 Gitman is reference number 185 rather than 176. I also found the translation of some technical terms unusual *e.g.* early black holes for primordial black holes, perpetual black holes for eternal black holes, selections of vacuum states for choice of vacuum states, powerful gravitational field for strong gravitational field, achieve infinity for reach infinity and law of preservation for law of conservation. Lastly, like other reviewers these days, I cannot but point out the exorbitant cost of volumes in this series. Will not this defeat the very purpose of bringing out such volumes and propagate the selection effect in priorities in favour of western science?

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Expanding protons: Scattering at high energies by Hung Cheng and Tai Tsun Wu. The MIT Press, 55, Hayward Street, Cambridge, Mass. 02142, USA, 1987, pp. 285, 84 figures, $ 40. Indian orders to Affiliated East-West Press Pvt. Ltd., 6, Roselyn Gardens Apartment, 20/A Barnaby Road, Madras 600 010.

The book under review covers a specialised area in particle physics: the asymptotic behaviour, in gauge field theories, of high-energy hadronic collisions in the kinematic region where transverse momenta transferred to particles in the final state are small. These events constitute the dominant component of hadronic collisions at high energies. Notwithstanding the sustained interest in this area and the importance of this topic in high-energy physics, available materials are scattered over numerous papers and review articles. The book thus fulfils a long-standing need of the interested readers and researchers.

Starting with a brief introduction to relativistic gauge field theory the authors quickly proceed to develop the machinery for the exposition of the model of expanding protons discovered by them. This model, it should be stressed, is only one among several models popular in the theory and phenomenology of high-energy hadronic collisions. What distinguishes it from the rest is the rare distinction that experiments confirmed the predictions of this model years after they were first made. In the year of its birth, all significant predictions of the model of expanding protons seemed to be wrong experimentally. Total cross sections in all hadronic collisions seemed to approach from *above* to asymptotically energy-independent constant. The elastic fraction of the total cross section seemed to decrease with increasing collision energy suggesting a Regge

behaviour, and phenomenologists were struggling with mixed success to verify a shrinking elastic diffraction peak, the *sine qua non* of Regge behaviour. Things took a dramatic turn with the announcement of the first results on proton–proton scattering at the Intersecting Storage Ring at CERN. The total cross section, instead of decreasing to a constant, seemed to have taken an upturn as predicted by the authors three years earlier. Whatever lingering doubt one had, melted away with the observation in 1979 in the CERN proton-antiproton collider of an increase in the ratio of the integrated elastic cross section and the total cross section, nine years after the theoretical predictions were made.

The key to this remarkable success of the model seems, in retrospect, to the present reviewer, to be the choice by Cheng and Wu of a realistic field theory, the gauge field theory, as the testing ground for their asymptotic studies. But for this choice the asymptotic growth of the tower diagrams would not have summed to a power of the energy that violates the Froissart or the unitarity bound. This is crucial for their final predictions of an asymptotic growth for the total cross section saturating unitarity and of the increase of the ratio of the integrated elastic cross section and the total cross section to an asymptotic value half. These predictions are, in fact, the results of implementing unitarity in the direct channel through the eikonal representation, with the energy dependence of the eikonal determined by the tower diagrams.

The picture of the expanding proton would be incomplete without an exposition of the monumental work and hard calculations that led to it. In unfolding the exciting story of the expanding protons the authors have two types of readers in mind. The first eight chapters of the book are intended primarily for those who are interested only in an intelligent understanding of the main features of high-energy hadronic collisions and the theoretical basis of the model of expanding protons, without being bogged down in the details of complicated calculations. In these eight chapters calculational procedures for vector-meson exchange diagrams for fermion–fermion elastic scattering are first developed, the droplet and the Regge pole models for potential scattering at high energy are reviewed, the kinematical origin and relative importance of the fragmentation and the pionisation components of multi-particle production are explained, and finally, the model of expanding proton is developed and its predictions compared with the experimental data. The remaining four chapters (chapters 9 to 13) together with Appendices B and C elaborate on the computational details and techniques. These are addressed mainly to researchers interested in high-energy behaviour in perturbative gauge field theories.

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Group theory, gravitation and elementary particle physics (Proceedings of the Lebedev Physics Institute, Volume 167) edited by A. A. Komar. Nova Science Publishers, Inc., 283, Commack Road, Suite 300, Commack, New York 11725, 1987, pp. 365, $ 98 (US & Canada), $ 117 (elsewhere).

This volume, a translation from Russian, is a collection of scholarly articles mainly dealing with coupled quasi-classical interacting quantum systems described under various situations including gravitational fields and applications of geometric and group theoretic methods to integrable non-linear equations with an attempted understanding of the associated dynamical symmetries. There is also an article on theories with general type of constraints and their canonical formalism, one on the initial state of the universe and another on the Aharonov-Bohm Effect. Altogether a rather wide sophisticated specialised selection of subjects! At this stage, however, one must caution prospective readers of the book! Not only are the articles written in a rather terse style with little introduction, going straight into the troubled depths of the respective subjects making it hard work even for the well-initiated but what makes it even harder work also is the rather poor printing, which manifests itself in various parts of the book.

For instance, in the very opening article dealing with the 'Initial state of the universe' by Markov and Mukhanov, right in the abstract itself one sees whole expressions and formulae omitted! Probably one has to fill in the blanks ofter reading the whole article. This article deals with the modified Einstein field equations in which the gravitation constant is a function of the energy density E in the early universe, decreasing with increase of density (in spirit with asymptotic freedom of gravitational interactions) vanishing as $E \to \infty$. At the same time it is claimed that what vanishes is the product of the energy-density-dependent gravitational constant and the energy-momentum tensors, each particle becoming increasingly of lower mass as E increases. The cosmological constant Λ is also made a function of E. It is claimed that the singularity can then be avoided. The authors have not mentioned similar works of various other authors who have pointed out several difficulties with these kinds of models. Moreover, it is not clear how the vanishing of the energy momentum tensor, as the density becomes infinite, smoothly gives rise to a de Sitter state as the initial state of the universe.

The second article by Dodonov and Man'ko deals with a whole range of quasi-classical (mainly dissipative systems with linear classical equations) systems and their quantum description by means of Wigner functions and the corresponding Fokker-Planck equation with the conditions of normalisation, hermiticity and positive definiteness of the density matrix. Some of the problems dealt with by this approach include the damped quantum oscillator, eigenfunctions of quadratic Hamiltonians, charged particles in nonuniform electromagnetic fields, singular oscillators, quantum oscillator relaxation in a magnetic field, etc. Also the quantum corrections to the classical equilibrium distribution functions and thermodynamics potentials are calculated. The generalisations and quantum analogs of classical integral Poincaré-Cartan invariants for these systems are elucidated thus giving a geometric interpretation to the above problems. This article is the longest (84 pages) in the book and makes difficult reading with many equations. The reader really has to work through. Again as is the case with all the articles, the poor

printing has rendered many equations incomplete and invisible! Particularly faint is eqn (5.25) and the equations on pages 47, 51, 52 and 56 though on many of the other pages it is no better! There are many minor points like the Casimir operator being spelt with a 'k' as on p. 53. Of course, many of the topics are rigorously treated like the relaxation of the quantum oscillator in a magnetic field with the various limiting cases clearly identified, along with the thermodynamics involved. The third article by Zel'nikov and Frolov deals with vacuum polarisation effects of massive fields in gravitational fields such as near-rotating black holes. The differences from the massless cases are clearly brought out.

The vacuum-polarised energy-momentum tensor for massive scalar, vector and spinor fields is found for the case of algebraically special spaces. An interesting gravitational analog of the dimagnetism effect is found for the scalar field in the Kerr metric background with the possibility of shielding or antishielding depending on the interaction of the spin of the field with the angular momentum of the black hole. Very detailed expressions are given for all the components of the energy momentum tensor and general expressions for effective actions in arbitrary gravitational fields.

The fourth article by Glauber and Man'ko deals with damping and fluctuations in systems of coupled quantum oscillators, *i.e.* properties of equilibrium states to which a system of N-coupled oscillators interacting with a heat bath is driven. The heat bath is simulated by an infinite number of oscillators, each of the N oscillators, interacting with its own bath characterised by a temperature. Forced oscillators and their correlation functions are also considered. Also the study is extended to N-dimensional forced oscillators. Although some of these topics may have some interesting applications, for example to gravitational wave detectors, as is the case with all the other articles, bad printing again takes its toll! For example on p. 166, in eqn (5.36) the exponentials are missing in the first row of the matrix and in eqn (5.39) the 'dagger' symbol is not visible.

Such examples of indifferent 'filling up' of equations abound throughout the book requiring more 'slog' from the reader.

The fifth article by Skarzhinskiy deals with different aspects of the Aharanov-Bohm Effect. Solenoid-charged particle scattering is considered in some detail. The effect is investigated for bound states including Landau levels. Unlike many of the other articles this has a clear introduction. The next article by Dodonov *et al* surveys select results in parametric excitation of quantum systems having dynamical symmetry, *i.e.* systems whose Hamiltonian is a function of time with values in a certain Lie algebra; the evolution operator belonging to a representation of the Lie group which may be explicitly found. Examples are non-stationary harmonic oscillators and spin motion in magnetic fields. Several new mathematical sum rules and generating functions are derived. The article by Fradkina deals with canonical formalism and S-matrix of theories with constraints of the general type, *i.e.* first and second class constraints of arbitrary rank. The author claims to have derived correct expressions for S matrix of theories that are momentum-quadratic with canonical gauges and including ghost fields. String theories are generalised to relativistic membranes (higher dimensional objects) and it is demonstrated that the theory of membranes in (n+1) dimensional space is a system with

rank-n constraints. Generalized [illegible] is done for these theories. One would have hoped the author had derived interesting analogue such as the critical dimension for membranes (like 26-dimensional space strings) but these questions have not been dealt with. The treatment here is very sketchy and many recent references are not included.

The last two articles deal with completely integrable non-linear equations. The first by Dokborov, gives a technique for ensuring complete integrability and finding the Lax pair for a non-linear equation. A new class of Backlund transformations are derived, for the Ernst equations (for axially symmetric gravitational fields) and applied to various Einstein-Rosen's. Algebraic formulation of monopole configurations are examined for the gauge group (SO(N)). The second by Leznov *et al* gives a purely algebraic method of finding soliton solutions of non-linear equations not requiring use of inverse-scattering. The symmetry basis of the method is related to infinite dimensional Lie algebras. A useful list compiling some thirty-two completely integrable non-linear equations is a main feature of this article. Transition to quantum models and multidimensional generalisations is not discussed.

Despite many of the shortcomings discussed above, this book is on the whole a useful collection of articles for serious students specialising in quantum mechanics of dissipative open systems and group theoretical methods applied to non-linear systems.

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Topics on nonlinear theory wave-plasma interactions by Klaus Baumgartel and Konrad Sauer. Birkhauser Verlag, CH-4010, Basel, Switzerland, 1987, pp 222, S. Fr. 52. Indian orders to Springer Book (India) Private Ltd., 6, Community Centre, Panchsheel Park, New Delhi 110 017.

This monograph is the latest in the market on the subject of nonlinear interactions in the plasmas. It has been translated from the original German version. Naturally there are some deficiencies in this department whenever books are translated. However, lots of serious efforts have gone into planning the monograph. Even though it is a monograph it can be comprehended even by beginners. There is a chapter on basic plasma equations followed by linear wave theory. Three chapters have been devoted to weakly nonlinear interactions. Strong nonlinear waves have been treated in the last two chapters.

Basic equations have been given for simulated Raman scattering, two plasmon decay, and their threshold derived. The treatment is based on fluid equations. Subjects like nonlinear Landan Damping have been excluded. There is chapter devoted to Brillouin scattering in homogeneous and inhomogeneous plasmas in the convective and absolute instability domains. Some effort has also been made to understand saturations of such instabilities. Resonance absorption has been treated very well. Solitons, shock-like structures, solitary waves through the use of Zakharov equations occupy lots of space. If one wants a quick grasp for wave steepening and wave-breaking, this book is the right place. Odd-integer half harmonic have been treated reasonably well. There is an attempt

to compare the theoretical results with experiments, but the monograph is weak in this department. There are a large number of references which is positive point, but there is very little critical review of these. The reader, inadvertently, is made to believe as if all the quoted papers are correct. Certain references are conspicuous by their absence, particularly those appearing in the *Advances in plasma physics* by A. Simon and W. B. Thampson (eds), John Wiley. References to the English translation of the Russian Journals have not been given.

The monograph will certainly be welcome by the plasma community. Any one trying to understand the subject will find it a very friendly book to read.

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Special functions of mathematical physics by Arnold F. Nikiforov and Vasilii B. Uvarov (translated by R. P. Boas) Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1988, pp.427, S. Fr. 168. Indian orders to Springer Book (India) Private Ltd., 6, Community Centre, Panchsheel Park, New Delhi 110 017.

Mathematical description of many physical phenomena leads to partial differential equations of rather complex form. The usual way of simplifying the equations is to use the method of separation of variables which reduces the partial differential equations into ordinary differential equations. These simplified equations are of the generalised hypergeometric type. A knowledge of the explicit solutions of these equations is necessary to get a better insight into the qualitative features of the original problem. So a study of the hypergeometric functions is inevitable and these functions include, as special cases, the so-called special functions of mathematical physics, namely, the classical orthogonal polynomials, spherical harmonics and Bessel functions.

There are many books available on the topic of special functions. But the present book presents a unified and elementary treatment of the special functions based on the generalisation of the Rodrigues formula satisfied by the classical orthogonal polynomials. This approach makes it possible to give explicit integral representations for the special functions. From these representations using only elementary complex analysis, the authors derive most of the important properties of the special functions. Another novel feature of this book is the treatment of classical orthogonal polynomials of a discrete variable. These polynomials are of interest in the theory of difference methods.

Let us now move to the description of the contents of various chapters. The first chapter introduces the differential equations of hypergeometric type. For the polynomial solutions of such equations Rodrigues formula is proved and integral representations are obtained. From the Rodrigues formula many basic properties are derived. The second chapter deals with classical orthogonal polynomials in details. For the Jacobi, Laguerre and Hermite polynomials basic formulas and expansion results are proved. As far as the

qualitative behaviour of these polynomials are concerned certain inequalities and asymptotic expansions are established. Certain convergence theorems are proved and some applications to quantum mechanics are given.

Another important class of special functions is the spherical harmonics. These are very useful in atomic spectra and scattering theory. They are obtained by considering bounded solutions of the Laplace equation in spherical polar coordinates. The role of group representations has been made clear in the study of generalised spherical harmonics which are used in the theory of angular momentum. Classical orthogonal polynomials of a discrete variable are studied as solutions of difference equations of hypergeometric type. Useful information about Hahn, Chebyshev, Kravchuk, Meixner and Charlier polynomials are gathered. Another useful topic treated in this chapter is the classical orthogonal polynomials on non-uniform lattices.

The third chapter deals with Bessel functions. Typical problems that lead to Bessel functions arise in solving the Helmholtz equation in cylindrical polar coordinates. Bessel functions and Hankel functions are introduced and various representations of them are obtained. All the basic properties like recursion relation, differentiation formulas, asymptotic expansions are proved. Graf and Gegenbauer's addition theorems for Bessel functions which are useful in mathematical physics are obtained. This chapter concludes with a brief description of the semiclassical approximation for the solutions of equations of second order getting asymptotic formulas for classical orthogonal polynomials and Bessel functions of large order.

The classical orthogonal polynomials and Bessel functions are special cases of the more general hypergeometric functions. In the fourth chapter the authors study basic properties of the hypergeometric functions. Various special functions are expressed in terms of hypergeometric functions. The last chapter gives applications of special functions to certain concrete problems of mathematical physics, quantum mechanics and numerical analysis. The authors give easy solutions of some basic problems in quantum mechanics like the problem of the motion of the particles in a central field and solutions of the Schrödinger, Dirac and Klein-Gardon equations for the Coulomb potential. The connection between Clebsch-Gordon coefficients and Hahn polynomials is presented.

Since familiarity with the properties of gamma function is necessary for the study of special functions, the authors have included a brief account of the theory of gamma function in the appendix. In the appendix they have also collected all the basic formulas and main properties of the most important special functions. The book contains a large amount of material and the exposition is concise and lucid. Many basic formulas are given in the form of tables for quick reference. This book will certainly make a nice reading and will serve as a good reference book for physicists and applied mathematicians.

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Set theory—An introduction by R. L. Vaught, Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1985, pp. 141, S. Fr. 76. Indian orders to Springer Book (India) Private Ltd., 6, Community Centre, Panchsheel Park, New Delhi 110 017.

A fine new introductory book in set theory. The book confines itself to the most basic ideas and results in the subject. Unlike most of the books written on the subject in the last two decades, it first presents the naive set theory, a la Cantor, then the formal set theory is developed. At the end a minimum amount of logic is presented to prepare those who wish to plunge into the deeper parts of set theory. The book is written in a very pleasant style and is quite easy to read. However much of the proofs are left as an exercise for the reader. Therefore a student, or a person new in the area, must work out the problems, till he is able to do the exercises in his head.

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Finite difference methods on irregular networks by Bernd Heinrich, Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1987, pp. 206, SFr. 68. Indian orders to Springer Book (India) Private Ltd., 6, Community Centre, Panchsheel Park, New Delhi 110 017.

Finite element method (FEM) and finite difference method (FDM) offer possibilities to discretize a given set of equations including boundary conditions starting from a suitable 'triangulation' of the domain. There is a vast literature on these topics. Needless to add that FDM is much older compared to FEM. It is well known that FEM has the following advantages:

(i) domain is better approximated;

(ii) symmetry properties of the original equation are preserved;

(iii) Natural boundary conditions are treated in a better way;

(iv) yields to the analysis of convergence using Hilbert space methods;

(v) can handle weak solutions with very little smoothness.

All these are possible because FEM is based on the so-called averaged formulation of the problem. On the contrary, the classical FDM is based on the so-called point-wise formulation of the problem. Usually, FDM requires solutions to be smooth of certain order. In fact, all the advantages listed above may be lost in FDM. The main point in which FDM scores over FEM is in the simplicity of the construction of the finite difference schemes (FDS). In spite of its drawbacks, FDM is more commonly used by engineers just because of its conceptual simplicity.

Thus the following question arises naturally. Can one modify the classical FDM in such a way that it enjoys all the advantages of FEM and it retains its simplicity to some extent? The answer to this question is provided not long ago. It is mainly available in many research articles in engineering literature and the method is known under various names such as : box integration method, balance method, finite control volume method, etc. So far, there was no comprehensive study of the mathematical analysis of this method and the present volume attempts to fill in this gap.

It presents the description of the method, its convergence analysis and error estimates in some model problems which are second-order elliptic boundary value problems (BVP) in the plane

Let us briefly take a look at the contents of the book. After introducing the BVP, the author takes a 'triangulation' of the domain by triangles and rectangles. The first step in the modified method is to introduce a secondary triangulation by means of the boxes (PB and MD) associated to each node of the first triangulation. The idea of the balance method in constructing FDS is to integrate the equation by parts in these boxes and discretize this equation *via* a suitable FDS. This is done in chapter 3.

In chapter 4, various properties of the difference operators A_h are discussed. Symmetry, monotonicity and positive definiteness are among them. Much of the work in this analysis resembles that of FEM. For instance, discrete versions of L^2 and H^1 norms and Poincaré Inequality are needed to prove positive definiteness. Chapter 5 is devoted to proving error estimates in terms of the parameter of discretization. In chapter 6, the author provides an extension of the method to the case where the first-order terms are present in the operator. This corresponds to the classical upwind schemes. Generalization of the method to oblique derivative problems and fourth-order problems are presented in chapter 7.

It is unfortunate that not much space is devoted to the discussion of the implementation of the method. Thus people who are primarily interested in the practical side of the method may not derive much benefit from this book. On the other hand, the book is useful to the readers interested in the proofs of convergence of the method.

The most important part of the book is the construction of the FDS which is done in chapters 2 and 3. Here a lot of notations are introduced. It may be difficult for the reader to digest all these in a quick way. For people who are not familiar with the subject of FEM, the reading may be heavy and rough. The printing of the book does not offer any help to the reader in this regard. Thus the book is definitely not suitable for beginners in the subject. To appreciate the value of the book in a better way and see the analogy with FEM, the reader is advised to go through first some of the standard and simple texts on FEM (e.g. Ciarlet: *The finite element method*; TIFR Lecture Notes, 1975)

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Contemporary mathematics, selected papers on algebra and topology by G. Birkhoff, edited by G. C. Rota and J. S. Oliveira. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1987, pp. 628, SFr. 138. Indian orders to Springer Book (India) Private Ltd., 6, Community Centre, Panchsheel Park, New Delhi 110 017.

The volume is a welcome addition to the mathematical literature. It has been brought out by Birkhauser under the series 'Contemporary mathematicians', and is edited by Gian-Carlo Rota and Joseph S. Oliveira. The volume is divided into six sections, *viz.*, Lattices; Universal algebra; Topology; Lie groups and Lie algebras; Lattice-ordered algebraic structures; and History of algebra. It contains 39 publications in these areas by Garrett Birkhoff, who has written not only the preface to the volume but also his own comments for each section. His own beginning words in the preface characterize the attempt of selection and importance: "The present volume of reprints are what I consider to be my most important and influential papers on algebra and topology. To tie them together and to place them in context, I have supplemented them by a series of brief essays sketching their historical background (as I see it). In addition to these I have listed some subsequent papers by others which have further developed some of my key ideas".

There is an autobiographical element in the entire volume. While the author describes the development of his papers, he is also dwelling on his circumstances, personal likes and dislikes, opinions on others, father's influence on his academic thinking, compulsions by events in history, etc. The volume presents only about one-fifth of the total works of Garrett Birkhoff, the remaining four-fifths belonging to applied mathematics areas. The same is apparent from the list of his Ph.D. students and their topics in a span of 40 years. In spite of this greater time given to research papers in applied mathematics, his book-writing has been more or less evenly divided between algebra and applied mathematics.

The four papers listed under the section heading 'History of algebra' are worthy of special mention. The first two of these are well known and have been well read. They are: 'Current trends in algebra' (*Am. Math. Mon.*, 1973, **80**, 760–782) and 'The role of modern algebra in computing' (*Computers in algebra and number theory*, G. Birkhoff and M. Hall (eds), Am. Math. Soc., 1971, 1–47). The next two of the papers under this section give the rise of modern algebra up to 1950, in two parts, — the first one up to 1936 and the second one, the rest — and give a racy account of the rise and growth of modern algebra from Emmy Noethers times, through algebraic geometry, covering various British, American and German traditions, the influence of high-speed digital computers, problems of optimization and computational complexity, etc. These four papers carry the autobiographical undercurrent rather explicitly. They scintillate the mathematical dynamism of Garrett Birkhoff. One cannot also escape noticing his slight bias in favour of stressing the Anglo-French influence in the early development of algebra and related processes of thinking.

For both the present and the future generations of mathematicians, this is a captivating collection.

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Cadmium in the environment: Experimentia Supplement Vol. 50 edited by H. Mislin and O. Ravera. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1986, pp. 7–144, SFr 64. Indian orders to Springer Book (India) Private Ltd., 6, Community Centre, Panchsheel Park, New Delhi 110 017.

This multi-author volume is based on parts of two reviews published in the Journal, *Experimentia*. The text has been divided into three parts: (1) Cadmium in the environment, (2) Bioaccumation of Cadmium, (3) Cadmium and human health.

Environmental science is a fast developing field of current research. A wealth of data has already been accumulated in this field on diverse aspects. Yet the field is wide open for further exploration, as there are many inconsistencies in the available results due to a number of controlling factors unattempted or ununderstood.

Cadmium is one of the environmental metal pollutants which has attracted the attention of many environmental scientists in the recent years. In fact, it has become a subject of serious concern to even multidisciplinary researchers primarily due to its detrimental and dangerous consequences on the 'living world'. As one of the authors (S. Ray) has aptly put in his article; in spite of tremendous progress made over the years, the role of Cd in the environment and consequential outburst to living systems is still very nebulous since mostly the data has come from laboratory studies. Hence the basic misunderstanding of the entire process, in a broader perspective, is likely to remain so until some of the rudimentary processes like uptake, storage and elimination, etc., are fully investigated and understood. In this context, this volume offers a rich panorama of wide range of results and contents to stimulate further investigations, although data largely pertinent to developed countries (UK, USA, Japan, etc.) only has been taken into account.

Some of the salient features, and shortcomings of this volume are:

The editor has strived to give an exhaustive overview of the subject by selecting articles of specialists in the filed.

The book not only serves as an excellent source book information (appended with a list of numerous references to specialised scientists working in the field, but also in creating a general awareness about Cd toxicity, environmental role and effects on human health as far as any non-specialist (individual) is concerned.

An apparent drawback of the book seems to be that the editor has overlooked, in selecting and including quite a few articles in each part which overlap to quite a significant extent in their objectives, data contents and presentation. The division of the

articles into the three different parts appears to be very arbitrary. This is quite evident from the titles of the articles which are self-explanatory to their contents and more so as one glimpses at the content sub-titles.

The first part of the book comprises seven articles which deal with aspects ranging from the geochemistry of Cd to its place and role in atmosphere and waters of all kind. A noteworthy article of this part is the 'Evaluation of methods for the speciation of cadmium' by M. Astruce. This is a well-presented article, helpful to not only an analytical and environmental chemist but equally to a synthetic inorganic and bio-inorganic chemist.

The second part comprises six articles which could broadly be divided into two streams. One presents facts about aquatic chemistry of Cd and its influence on aquatic living species. While the other ravels the agricultural and phytochemical facets of Cd pollution with a brief remark on zoo technology of Cd in one of the articles.

The last part of the volume spans biochemistry of Cd relevant to human system (largely with reference to food and health hazards). Two articles in this part are quite significant. One deals with the analytical aspect of Cd exposure and toxicity to human organs (Biological indicators of Cd exposure and toxicity by Z. A. Shaikh and R. M. Smith) which could be regarded as a complementary article to that by M. Astruce (Part I). While the other discusses the uncalities of environmental Cd pollution problems to human health (Cd–the environment and human health—An overview by K. J. Yost), this article thrashes out the very objective and data contents of the rest of te articles and is unique as it lays emphasis to show that Cd, as an environmental toxic pollutant, does not appear to pose a major health problem of serious concern. Indeed, this article is in total contradiction to the very spirit of this edited volume. However, the editor deserves compliments for inclusion of this unique article.

It is rather not an easy task to bring forth in a comprehensive manner, the review/overview highlights of a multi-authored compilation of this kind, spanning on varied aspects from article to article, yet sharing with the overlapping contents.

The present venture, however, does deserve a place in research libraries.

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Basic organometallic chemistry by Ionel Haiduc and J. J. Zuckerman. Walter de Gruyter and Co., P.O. Box 11020, D-1000, Berlin 30, 1985, pp. 404, DM 169/$ 29.90.

This book is based on another book by Professor Haiduc in the Rumanian language, published in 1974.

The book is divided into three parts. In part I, apart from introductory material, are to be found a general and elementary discussion on bonding in organometallics, a section

on laboratory techniques, and a section on the literature of organometallics. Part II deals with non-transition metal organometallics, and part III with transition metal organometallics.

In part II discussion is group-wise. In part III, the major classification followed is based on the number of electrons contributed by the ligands. Thus we have chapters on two-electron ligands, three-electron ligands, etc. Within each chapter, however, discussion is group-wise. There is also a short introductory chapter on 'The electronic structure and classification of transition-metal organometallic compounds', at the beginning of part III. The last two chapters in part III deal with 'Organometallic compounds derived from acetylenes' and with 'Organometallic compounds with π-metal-carbon bonds'. There are a total of eighteen chapters.

The first question that arises about this book is about the intended audience, which is not clearly defined. It is hard to see towards whom the book is directed. If it is meant for a first course, it appears to contain too much detail and too many reactions. There are neither worked-examples nor problems to aid the student. If the book is for an advanced course, it probably lacks depth and more importantly, references. The book is obviously not meant as a reference text.

Similarly, it is not clear whether the book is directed at organic or inorganic chemists. No attempt has been made to give even an idea of the amazing applications of organometallic chemistry to organic synthesis. There may be a slight bias towards inorganic chemists—indeed, Professor Haduc has written text-books on inorganic chemistry. Be that as it may, the style of the book, being descriptive, is out of tune with the times. An approach where unifying themes were stressed would have been more appropriate. At times, to many organic chemists, the book may bring back terrifying memories of descriptive inorganic chemistry.

Another important shortcoming concerns language and printing. As was mentioned above, this book is based on a Rumanian text-book. In this reviewer's opinion, the translation has been less than professional and may leave many in this country smiling with satisfaction. An illustrative example, from p. 45 is: 'The great reactivity of the alkali metal compounds and the small differences between the behaviour of the sodium, potassium and the heavier alkali metal derivatives has limited interest to those of sodium, the most readily available of them'. Another example, which may be particularly pleasing to Indian ears is (from p. 147): 'Few germanols, R_3GeOH, are known; usually $R = C_6H_5$ or C_6F_5.'

Similarly, printing and other errors abound in the book, especially in the first half. Indeed, errors start from the word go. The note on the cover illustration says: 'Decaphenylstannocene, $[\eta^5\text{-}(C_6H_5)_5]_2Sn^{II}$: The first symmetrical main-group sandwich compound' (Get it?!)

My estimate is that the book averages nearly one printing or other error per page. However, I suppose the printer is hardly responsible for unbalanced equations and groups which mysteriously transform themselves from 'R' on one side to 'Me' on the other. Here is a sampling:

p.29 : 'receipies'
p.65 : Structure of RmgBr $(OEt)_2$ is wrong
p.75 : second equation, right-hand side: $Hg(C \equiv CH)_2$ should be $Hg(C \equiv CR)_2$
p.91 : 'a unusual boron-nitrogen compound'
p.129: second equation not balanced, also n = ?
p.167:'$R_3Sn\text{-}NMe_2 + EtNH_2 \rightarrow Me_3Sn\text{-}NEt\text{-}SnMe_3 + Me_2NH$'
p.168: 'In solid $Ph_3Sn\text{-}S\text{-}SnPh_3$, the bond angle is nearly tetrahedral....' (which bond angle?)
p.213: $[RN_3]^+$ should be $[RN_2]^+$
p.243: last equation - 'diglime'
p.281: 'Thus triphenylcyclopropenium cation form a salt.....'
p.293: Structure of α-pyrone is wrong, lacking a double bond.
p.296: first equation – (2 LiR) should be (−2 LiBr)
p.341: 'tropyllium'
p.357: second equation – 'R' groups missing on right-hand side.
p.375: last equation – ferrocene has a stray Hg attached.

All in all, therefore, recommending purchase is out of the question. Which is not to say the book is useless. Instructors (*i.e.* not students) may find it useful if they can find the time for intelligent supplementation. A pity, for the book is otherwise beautifully got up. And, a surprise, for it is from a major publisher.

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Dinosaurs and their living relatives by R. H. Hedley. Cambridge University Press, Cambridge B2 2RU, UK, 1985, pp. 72,£ 4.95. Indian orders to: Affiliated East-West Press, 6, Roselyn Gardens Apartments, 20/1A, Barnaby Road, Madras 600 010.

It is a completely new approach to the study in trying to discover the relationship between the dinosaurs and other related animals – both living and extinct.

The present edition has re-designed chapters adding on new information throughout the text.

A great deal of imagination and effort have gone into the book to simplify the subject by an exploratory approach with simple questions and answers, in step by step stages, using a series of full colour photographs of the many collection of the National History Museum's famous dinosaur collection.

This edition has further simplified the subject and made it more absorbing by creating a better understanding in the relationship of the characteristics of the dinosaurs from the knowledge of fossils and the animals seen today which are physically related to them like the birds and crocodiles. It has successfully used drawings, illustrations, photographs and colours to show similar basic patterns or to distinguish the misleading similarities in anatomy, embryo development and other structures like horn skin and feathers.

This book is an excellent publication in the hands of an evolutionary biologist and a layman wanting to know more about the dinosaurs and its linkages with the present living animals.

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