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BOOK REVIEWS

Conceptions of space and time by Murad Akhundov, translated by Charles Rougle. The MIT Press, 55, Hayward Street, Cambridge, Massachusetts, 02142, USA, 1986, pp. 220, $25. Indian orders to Affiliated East-West Press, 25, Dr. Muniappa Road, Kilpauk, Madras 600010.

"It is only with our mother's milk in early childhood that we acquire our first information about space and time". Does a child display its understanding of time when it extends its arms to its nurse, for "the right temporal concepts must lead to predictions"? To a child, the world appears shallow like the paintings of primitives. The three dimensionality dawns upon later as a necessity for survival though some of us have even learnt to live in eleven dimensions, again for the sake of survival. And why not? Mythology has always charmed scientists who, in gratitude, lend respectability to associations between elementary particles and eight-fold way and bootstrapping and Zenism. In fact, the author suggests mythology and poetry to be good sources of scientific inspiration (as good as a visit abroad?). Perception of time is a function of age, culture and health. "Child seems to talk only of the present and the old only of the past". The increasing immunity to changes is what leads to the conception of homogeneous and continuous time. Are scepticism and intelligence, the inseparable mates? A schizophrenic enjoys the pleasures of living in a manifold of space and time. First, it was water 'chaos' helped by god (or computers) in its wish to be a strange attractor, the cosmos, the beginning of second type of time; a long search from mythology to mythology.

The second chapter is on 'The philosophical evolution of spatial and temporal conception'. What is chaos, a yawning abyss, a darkness, made up of? Chaos is infinite space endowed with matter as well as emptiness. Chaos was first. Then came Gaea. Did Gaea come from within or without? The debate continues. In Newton's world, matter is introduced into a preexisting space and in Einstein and Wheeler's, the space generates the entire matter. "Matter tells space how to curve and space tells matter how to move" thus spake Einstein. Milesians conceived space, time and movement, all eternal, aperion, immortal with infinite divisibility. With motion on their side, necessity and time reigned supreme. For "most powerful of all is necessity for it prevails upon everything and wisest of all is time for it reveals everything". Pythagoreans thought of cosmos to be completely compatible with the limited and the unlimited, represented elements, else and the universe by numbers, the numbers by indivisible points, the mathematical atoms; but in this rationality there was no place for irrational numbers, for the coexistence of a diagonal and a side. The universe of Heraclitus was in a continuous state of evolution with eternal fire its only currency and commodity. Xenophanes extended "all is one" of Heraclitus to "that one" is homogeneous, spherical and free from motion and rest, because motion and rest are subordinate to space and time. Inaccessibility of truth, impossibility of motion, the flying arrow at rest, failure of finite objects in an infinite divisibility, the non-existence of empty space for empty space commands motion and the "the one" did not move, are the cumulus of Xeno's convictions.

The atomists accepted the 'one' of Xeno but divided it into an infinite number of pieces, the atoms filling an infinite empty space. Further divisibility is forbidden. Thus space imbibed duality, boundless like 'Akasa' and with figures and directions like 'Disha'. Since space is democratically empty to big and small, all must move with a constant speed, because the object travels one atom of space in one atom of time as if it disappears and reappears after an atom of space time (germs of quantum mechanics?). Thus Heraclitus with his quivering cosmos and Eleatics with their immobility met the atomists halfway where change took place at an unchangeable rate, setting stage for the appearance of Newton's first law of mechanics. Inspired by the atomists, Plato considered these indivisible particles as the basis of all elements and endowed them with geometrical shapes in the manner of modern thought where the good old and new elementary particles have styles but no substance. Plato's time was younger than eternity whereas Aristotle, had 'no time' for "the past is no longer, the future is not yet and there is only a durationless now" of measure zero, avoided voids for "a place does not perish if the things in it are destroyed".

Philosophic doctrines (finite and filled space, infinite time) distasteful to the Christian sensibilities were brought to medieval Europe through the works of Ibn Rushd. The Christian universe was created out of some of the 'nothing' that god created, the infinite speed implied by emptiness was needed anyway to facilitate 'Farishtas' to frequent the cosmos and to nudge Newton into instantaneous action at a distance in his absolute space.

Time, in mythology runs from past to present to future, in theology, awaiting salvation, the future hoarded at the top of a glass hour is released in controlled quantities through the opening of the present into past. Since humans have this terrible habit of relinquishing the present by regretting the past and reclaiming the future, time keeping was entrusted to soul, and time was left only with psychological existence. The seventeenth-century philosophers thought of time as 'eternity', in relation to god, as duration in relation to material world and as time in relation to human thought. Is there any relationship among these multiplicities? Read the book.

With the shift from geocentric to heliocentric, the recession of the limiting sphere, the rejection of infinite speed for it implied motionlessness, an infinite homogeneous space was set for Kepler, Galileo, Descartes, Newton and others to play the game of classical physics.

Arbitrariness in the definition of time, permitted by classical kinematics was later limited through the introduction of force, the dynamics. Space became a Euclidean three-dimensional differentiable manifold and the continuum of time was represented along the real numerical axis. With the efforts of Euler, d'Alambert and Lagrange mechanics weaned from geometry, became a branch of analysis and moved on to swallow the group-theoretic methods of Lie and topological reconstructions of Wheeler. Conservation laws configured space and time. Geometrics, kinematics and dynamics were woven into a four-dimensional straight space and time. The non-constant gravitational field curved it. Space not only curves but undulates too and when it does so, there is light. No more, it is this or that, 'IT IS' and that is the complete truth. Matter figures only curvature, torsion and a few dimensions of space. Search is on for a more potential space that would account for electromagnetism and particles in classical as well as quantum domains, the swan-song of a scientist.

Well, it's been an uphill task. An enormous inability to say no and the anticipated pleasure of reading in the Saturday afternoon serenity that pours through the eucalyptus and coconut trees, have made it possible for me. You can do it in your own favourite atom of space-time for reading a difficult book.

Indian Institute of Astrophysics
Bangalore 560 034.

VINOD KRISHAN

Iterative aggregation theory by L. M. Dudkin, I. Rabinovich and I. Vakhutinsky. Marcel Dekker, 270, Madison Avenue, New York, NY 10016, 1987, pp. 273, $89.75.

This book, as the title suggests, is about iterative aggregation theory, *i.e.*, iterative algorithms for optimisation which are partly carried out in a decentralised manner by several independent units coordinated by a central coordinator. The subtitle reads 'Mathematical methods of coordinating detailed and aggregate problems in large control systems'. This is a bit misleading. By 'control systems' one usually implies optimisation problems involving dynamical systems evolving in time whereas the present book concerns itself only with 'static' problems devoid of any time evolution. Thus the words 'control systems' in the subtitle should be more appropriately replaced by 'mathematical programming'.

A typical problem falling within the scope of this book has the following general features: Unlike standard iterative schemes that amount to applying iterates of the same map to the data, these algorithms apply two or more maps to the data, alternating with each other. At least one of these 'stages' involves a decentralised computation by several distinct units based on their own data and the putative model of the system (say, the vector of coefficients entering the equations being handled). The results of these computations are transmitted to a central unit which 'aggregates' them and updates the model accordingly. This information is fed back to the decentralised units. This pattern repeats itself.

An important feature of some of these algorithms is that the role of the central unit is only directive and not dictatorial. That is, it is not obligatory for the local units to follow the suggestions of the central unit in toto. Of course, there is a penalty for any deviation from what is proposed, but it is conceivable to have a situation where the local unit may perfer to pay some penalty of this sort for a larger gain elsewhere. Such algorithms often do better than those where the role of the central unit is dictatorial.

This should give a qualitative flavour of the kind of problem the book addresses itself to. A detailed organisation of the book is as follows: It has four parts. The first part is a crash course in optimisation algorithms, inclusive of a survey of traditional methods for decomposing large-scale mathematical programming problems. The second chapter takes up a specific problem for which it proposes iterative aggregation schemes: the interproduct input-output model in economics, which leads to a linear system of equations. More general linear systems are discussed in the thrid chapter while the last chapter is devoted to general constrained and unconstrained optimisation problems.

The entire book is oriented towards proposing specific iterative aggregation algorithms and analysing their convergence. There is little abstract theory. The principal application intended seems to be economic planning, a domain in which the authors are specialists. Thus the language and overall presentation is tuned for an economics-oriented reader, making it at times less than smooth reading for other. All in all, it is a very specialised book. However, one need not undermine its importance on this count. It is a goldmine of ideas and techniques for people actually handling large-scale mathematical programming problems. What is more, it provides a window to the otherwise not easily accessible work of Soviet scientists in this area. Thus it is of great value to the specialist, though perhaps not so for others.

TIFR Centre, IISc Campus,
P.O. Box 1234,
Bangalore 560012.

V. S. BORKAR

Harmonic analysis on homogeneous spaces of $SO_0(1, 2)$ by Yi. A. Vediyev. Hadronic Press, Inc., Nonantum, Massachusetts 02195, USA, 1988, pp. 148.

The study of infinite dimensional representations of Lie groups started with the Gelfand school. The pioneers in this field are Gelfand and his school and Bargmann[1]. Later, this theory took shape in the hands of Harish-Chandra. For the general theory of harmonic analysis on real reductive groups excellent text books are now available (see, *e.g.*, Knapp[2], Wallach[3], Gangolli-Varadarajan[4] in the references below; the last one, even though is not about the general theory it will enable a beginner to get a perspective of the general theory).

The 3-dimensional Lorentz group $SO(1,2)$ or its two-sheeted cover $SL(2,\mathbb{R})$ has played a very important role in the development of the general theory. The book under review deals with harmonic analysis on the first group and the associated homogeneous spaces. The complete classification of the irreducible unitary representation of $SL(2,\mathbb{R})$ was done by Bargmann[1], who also proved the noncommutative version of the Plancheral formula using the eigenfunction expansion corresponding to second-order ordinary differential equations. Later, Harish-Chandra[5], gave a conceptually better proof of this formula which he later tried to extend to higher-dimensional groups. The book starts with various decompositions of the group $SO_0(1,2)$. The group $SO_0(1,2)$ acts transitively on the so-called light cone *i.e.*, the subset

$$\{x := (x_0, x_1, x_2) \in \mathbb{R}^3 : x_0 \geqslant 0, x_0^2 - x_1^2 - x_2^2 = 0\}.$$

The representations are realised on certain spaces of homogeneous functions on the light cone. The reducibility questions are settled *via* the employment of invariant bilinear forms[6]. Then the decompositions of the representations associated with the two-sheeted hyperboloid, the light cone and the one-sheeted hyperboloid are carried out. These are used to decompose the tensor product of unitary representations which is the theme of the last chapter. The author follows the method of Molchanov to do this. It should be mentioned that the first work in this direction is due to Pukansky.

The very explicit calculations in this book are very likely to be of much use to beginners and physicists. The beauty of the subject is marred to certain extent by the hand-filled formulas in the book.

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School of Mathematics
Tata Institute of Fundamental Research
Bombay 400005.

S. KUMARESAN

Oscillation theory of differential equations with deviating arguments by G. S. Ladde, V. Lakshmikantham and B. G. Zhang. Pure and Applied Mathematics Series, Vol. 110, Marcel Dekker, Inc., Madison Avenue, New York, NY 10016, 1987, pp. vi + 308, $89.75.

The existence and location of zeros of solutions of ordinary differential equations are of central importance in the theory of boundary-value problems for such equations. During the last century several results have been published on this topic. The celebrated comparison theorem due to Sturm (*J. Math. Pures Appl.*, 1836, **1**, 106–186) dealing with second-order equations marked the beginning of the study of this problem. The principle involved in comparison theorems is: if a solution of a differential equation has a property P, connected with oscillatory behaviour, then the solution of a second differential equation has property P or some related property under some stated connections between two equations.

Sturm's theorem was generalised to third-order equations by Birkhoff (*Ann. Math.*, 1911, **12**, 103–127) while Reynold extended this theorem to fourth-order equations (*Trans. Am. Math. Soc.*, 1921, **22**, 220–229). Leighton and Nehari's work on the fourth-order equation (*Trans. Am. Math. Soc.*, 1958, **89**, 325–377). Reid Stenberg's work on system of equations (*Am. J. Math.*, 1946, **68**, 237–246, 460 and *Duke Math. J.*, 1952, **19**, 311–322) and Hartman and Wintner's work on partial differential equations (*Proc. Am. Math. Soc.*, 1955, **6**, 862–865) are milestones in the area of oscillation theory. There are several other mathematicians who have enriched this area of research.

The book under review is yet another important development in the theory of differential equations with deviating arguments. The equations of this kind have revealed significant changes from the theory of ordinary differential equations (ODE) without deviating arguments. For example, the theory of oscillations in ODE is centred around second- or higher-order equations because of the fact that first-order ODEs do not possess oscillatory behaviour. However, for equations with delays, even first-order equations possess oscillatory behaviour. For example, it is possible to show that the equations $y'(t) + y(t - (\pi/2) = 0$; $y'(t) + y(t + (\pi/2)) = 0$ possess oscillatory solutions, while $y'(t) + y(t) = 0$ has no oscillatory solutions.

It is to be mentioned that this is the only book now available in English language which deals with oscillation theory of ODEs with deviating arguments. Further, several results in the book establish the importance of deviating arguments since their presence causes or destroys the oscillation phenomena The book incorporates wide variety of results providing an extensive survey of the present-day status of research in the area of theory of oscillations. The bibliography gives a definite indication of the keen interest shown by mathematicians in the oscillation theory for equations with deviating arguments.

The book provides in Chapter 1, the basic elements of the theory of differential equations needed to understand the oscillation theory of equations with delays. A beginner in the area of theory of differential equations can initiate study in this area with the help of this chapter. Chapter 2 includes the oscillation theory of first-order equations. The first-order equations can have single or several delays, or advance arguments. Several theorems proved here reveal the beauty and aesthetics of this area of research. The results are simple in nature and need only a few mathematical tools to prove them. Any graduate student can grasp these results without much effort.

Chapter 3 includes the study of first-order nonlinear equations. Several results concerning nonlinear equations with deviating arguments (single, multiple, advance) are included here. Proving that the solution of $y'(t) - 2y(\lambda t)/(e\ln\lambda)t = 0, 0 < \lambda < 1$ or $y'(t) - 3[y(t + (\pi/2)^{1/3}]\cdot[y(t + 2\pi)]^{2/3} = 0$ is oscillatory based on general theorems is indeed a remarkable result.

Chapter 4 incorporates the results on second-order equations, linear as well as nonlinear. Essentially, it is to be noted that large number of physical phenomena are expressed in terms of second-order equations. The conclusions of this chapter are important from the view point of potential applications to mathematical modelling problems.

Chapter 5 deals with equations of higher order and Chapter 6 includes known results on systems of equations. While the authors have successfully organised these two chapters, it is to be noted that several problems, arising out of these discussions, await solutions.

The book is well written and well presented. It will be useful to researchers since most of the work done so far is now available at one place while it can also be useful to graduate students who wish to advance their knowledge in the theory of differential equations. The analytical tools necessary to understand the theory are not too deep, yet what is revealed through these results is valuable and has potential to take a reader to new frontiers of mathematical discipline.

Department of Mathematics S. G. Deo
Goa University
Bambolim 403 005
Goa.

Differential geometry by Tanjiro Okubo. Marcel Dekker, Inc., 270 Madison Avenue, New York, 10016, 1987, pp. 788, $125.

The field of differential geometry has undergone a thorough change during the past few decades. Theory of differentiable manifolds has become the basic language of modern differential geometry. The geometry of smooth manifolds involves various tensor fields on the manifold. Such fields were defined classically in terms of local transformation properties, but in modern terminology they are interpreted as a cross-section of a suitable tensor bundle associated to the tangent bundle of the manifold. The phenominal growth in the field has created a gap between what could be found in many graduate and undergraduate text books and the most recent research papers in the journals.

To remedy this situation many attempts are being made to construct introductory courses in differential geometry from various view points. For example, the book of Auslander[1] gives an introductory course in differential geometry from the point of view of Lie groups, with the fundamental equations of surface theory arising from the equations of structure of SO(3) and the book of Singer and Thorpe[2] gives an introduction to surface theory in terms of principal bundles and so on.

The book under review is one more addition to the list of existing books, whose purpose according to the author is to provide a text on differential geometry to the advanced undergraduate and graduate students.

The first impression one gets of the book and its contents is that the text is rather bulky and the author covers an immense amount of material at the expense of stating a number of important results without proof. Everything covered in this book is well known and can be found in many of the existing books.

Foundations of differential geometry[3] by S. Kobayashi and K. Nomizu written in early sixties is one of the most well-known books on differential geometry. This is mainly a reference book – a complete treatment of the foundations, and the definitive exposition of the principal bundle point of view. This has helped many aspirants to acquaint themselves with a rapidly growing subject.

Okubo's book appears to have been completely modelled after this book. The content and its organisation is very much influenced by the book of Kobayashi and Nomizu. The influence is so complete that almost everything covered here except for a few minor digressions has a counterpart in the book of Kobayashi and Nomizu, which is often almost identical modulo the difference in style.

The only topics dealt with in this book and not found in the book of Kobayashi and Nomizu are harmonic functions and forms and de Rham cohomology theory. The latter, of course, belongs to the realm of differential topology rather than to differential geometry and this appears to be loosely connected with the rest of the book. The topic on harmonic functions and forms is very well written.

The basic material concerning differentiable manifolds, Lie groups and Lie algebras which runs to about 110 pages is not carefully organised. Nor is it sufficiently self-contained. Such basic results as the connection between Lie groups and Lie algebras are not even discussed. Whereas, some results, which are, of course, pertinent to the subject of this book but which are parts of elaborate theories could have been omitted. For these results the reader could have been referred to specific accounts of these theories. For example, the Weyl theorem that every complex semi-simple Lie algebra has a real form which is a compact type, whose proof requires knowledge of root system.

It is annoying to find in many places references to text books written in Japanese even for standard material available in English texts. For example, for polar decomposition of a non-singular matrix, the reader is referred to T. Yamanouchi and M. Sugiura[4]. There are a few typographical errors which are harmless and a few mathematical errors which are attributable to carelessness For example, on page 6 there is an assertion that topological product inherits topological properties possessed by its factor spaces which is not correct. In many places notation is rather cumbersome.

Despite the publisher's claim that this book is an 'ideal text' for graduate and advanced undergraduate students and 'ideal reference for all mathematicians...', I suspect that there may not be many who would like to consult this book when most of the material covered in this book and much more is covered so neatly in the *Foundations of differential geometry* by Kobayashi and Nomizu.

The book is divided into nine chapters. Chapter 1 gives a brief survey of differentiable manifolds, Lie groups and Lie algebras. Main theorems are stated without proofs.

Chapter 2 deals with the general theory of connections. The chapter begins with a brief resume of definitions and theorems in the theory of fibre bundles that are needed in the text. The definition of a connection on principal bundle is given and the horizontal lift of a curve and parallel translation are established. Curvature form is defined and the structural equation is proved. The holonomy group and the restricted holonomy group are then defined and proved to be Lie groups. Holonomy theorem (Ambrose-Singer) has been established. Existence of connections, connection in associated bundles are discussed. All these results are applied to linear and affine connections.

Chapter 3 is concerned with Riemannian manifold. The definition of Riemannian structure on a manifold is given. The bundle of orthonormal frame is defined and the existence and uniqueness of Riemannian connection is established. Hopf-Rinow theorem on complete Riemannian manifold is proved. Myers and Steenrod theorem on group of isometries on a connected manifold is established. The concept of sectional curvature of a Riemannian manifold is introduced and models of space of constant curvature are constructed. Holonomy groups of Riemannian manifold are discussed.

Chapter 4 gives the fundamental results concerning the geometry of n-dimensional submanifolds immersed in $(n + p)$ dimensional Riemannian manifold. Classical formulas of Weingarten, Gauss and Codazzi are obtained. Fundamental theorem on local isometric embedding of a Riemannian manifold in Euclidean space is given. Chern-Kuiper theorem on immersion is stated and some of its applications are given. Some results on minimal immersion are discussed.

Chapter 5 is concerned with the differential geometric properties of almost complex manifolds and complex manifolds. After some purely algebraic preliminaries the notion of an almost complex structure, its integrability is discussed. Many examples of complex and almost complex manifolds are given. Metric properties of almost Hermitian and Hermitian manifolds are discussed. Notion of Kaehler structure is defined and Schur's theorem for Kaehler manifold is presented.

Chapter 6 is devoted to the study of homogeneous and symmetric spaces. It begins with a brief survey of Lie groups and Lie algebra. The existence and properties of invariant affine connection on reductive homogeneous spaces are discussed. Some basic results in the theory of affine, Riemannian and Hermitian asymmetric spaces are given. A section is devoted to the classification of irreducible orthogonal involutive Lie algebras. The chapter concludes with the classification of simply connected Riemannian symmetric spaces.

Chapter 7 deals with the geometry of G-structures and transformation groups. G-structure on a manifold is defined and geometry of connections of G-structure is studied. It is shown that a group of automorphism of G-structures has the structure of a Lie group if the Lie algebra of G is of finite type. Transformations and infinitesimal transformations which preserve a given linear connection or Riemannian metric are discussed.

Chapter 8 is devoted to the study of variational problems on geodesics. Jacobi fields and conjugate points of a manifold with affine connection are defined and Synge's formulas for the variation of geodesics in Riemannian manifolds are given. Comparison theorems due to Morse, Rauch-Warner and Morse-Shoenberg are given. They are followed by the pinching problem due to Berger, Rauch and Klingenberg. Basic properties of cut loci are proved.

The last chapter covers three topics. They are de Rham cohomology theory, characteristic classes and harmonic functions and forms. In Section 1, axiomatic sheaf cohomology theory is developed. Both de Rham cohomology and the differentiable singular cohomology are shown to be special cases of sheaf cohomology. Alexander-Spanier and Cech cohomology theories are introduced. It is proved that there are canonical isomorphisms of all these cohomology theories on manifolds.

Section 2 deals with differential geometric aspects of characteristic classes. Characteristic classes are defined in terms of curvature form. The basic result of A. Weil is established. Chern classes and Pontrajagic classes are introduced. Section 3 is devoted to the study of harmonic functions and forms. The main object of this section is to establish Hodge decomposition theorem and Hodge theorem on harmonic forms on compact-oriented Riemannian manifolds. The section closes with some interesting results on harmonic forms on Kaehlerian manifolds.

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Department of Mathematics
University of Mysore
Manasagangothri
Mysore 570 006.

B. S. Panduranga Rao

Finite mathematics with applications by Shirley O. Hockett and Martin Sternstein. Robert E. Krieger Publishing Company, Melbourne, Florida, USA, 1984, pp. 393, $31.50.

A strong foundation in basic finite mathematics is a must for the present-day students in business, economics, social and biological sciences. Most of the analytical approaches for decision-making advocated in business or management schools rely heavily on this foundation. Quite often students entering management schools or places of advanced learning in the biological sciences face difficulties in comprehending the analytical methods. This book plays the role of introducing to such students, the basic principles and procedures of finite mathematics. Ample illustrations are provided throughout.

The topics are covered in six chapters: Counting and sets; Probability; Linear functions, Equations and systems; Matrices; Linear programming; Mathematics of finance. Each chapter is divided into several sections, each of which is followed by extensive exercises and tests.

The first chapter gives introduction to the elementary concepts of counting and sets. Venn and tree diagrams, permutations and combinations are discussed with illustrations. A comprehensive coverage of basic concepts of probability is made in the second chapter. The presentation is illustrated through set theoretic concepts. Conditional probability, statistical independence of events and Bayer's law are presented fairly elaborately. Traditional illustrations accompany the sections on binomial probabilities and expected values. The section on finite stochastic processes is somewhat misleading and could have been treated with the inclusion of time-dependent events.

Algebraic expression of functions, equations to lines, system of linear equations and solutions of them are considered in the third chapter. Systems of linear inequalities, which play a prominent role in a later chapter on linear programming, are explained with some very simple but effective examples.

Perhaps the best part of the book is chapter four on Matrices. Apart from the basic concepts related to matrices, matrix operations, products, inverses and their usage in the solution of system of linear equations, the speciality of this book lies in the inclusion of two non-conventional but useful topics under this heading. The first is the introduction of Leontief's input-output analysis in matrix notations thereby taking the reader through one of the best known uses of matrices in economics. The other is the concept of Markov processes which have played a dominant role in present-day analytical approaches in many areas both in behavioural and biological sciences.

In chapter five, the topic of linear programming is presented. In addition to the graphical method for a two variable LP, the simplex method is illustrated for both the minimisation and maximisation objectives. Perhaps the authors could have done a better justification if they had included a section on the formulation of some simple real-world problems as LP models. The final chapter gives the fundamentals of the mathematics of finance. Starting with simple and compound interests, cashflow diagrams, the concept of discounting for time value of money is introduced. The computation of present value of a single future cash flow as well as present value of annuities are covered.

The book ends with two appendices, one on the basics of set theory and the other giving tables of present values of single cash flow and annuities. These are followed by answers to selected exercises and tests and a supplementary errata sheet.

Perhaps the only drawback of the book is its printing style which suffers due to the adaptation of single-spaced typing for a subject full of mathematical symbols and equations. At places, the presentation of the material appears to be too crowded leaving room for ambiguities for a new student who is getting exposed to finite mathematics through this book. Notwithstanding this minor blemish, the book is an excellent introduction to basic finite mathematics for students in social, behavioural and biological sciences.

Department of Management Studies
Indian Institute of Science
Bangalore 560 012.

B. G. RAGHAVENDRA

Introduction to mathematical programming by N. K. Kwak and Marc. J. Schniederjans. Krieger Publishing Co., Inc., P.O. Box 9542, Melbourne, Florida 32902-9542, USA, 1987, pp. 356, $36.50.

This book, as its title suggests, is really an introductory one. It has 14 chapters, appendices of computer programs – one each for linear programming (for which a PC is a must), and another for goal programming on a batch-mode basis (for which a card reader will be necessary). Every chapter has, at its end, a set each of problems and references. The book has an index at the end. Answers to problems, however, are not provided. Every chapter has an overview section to start with and summary as well as a list of key concepts at its end.

There is a chapter on prerequisite mathematics in which relevant material on determinants, matrices, rules of differentiation, partial differentiation and differentiation of implicit functions, and maxima and minima is touched upon. The book first discusses formulation, graphic and simplex methods, duality, sensitivity analysis, transportation, and assignment problems in five chapters devoted to linear programming. There is a chapter on integer programming, two on goal programming, one each on network flows, nonlinear programming, and dynamic programming. The first chapter is a general introduction and the last one talks about the future of the subject.

The author states in the preface that this book is intended for undergraduate courses in management science, operations research, decision science, and applied mathematics. The reviewer feels that it is good for other undergraduates but not to those in applied mathematics, who will appreciate a general treatment but will get bored with separate discussions on simplex method – for maximization, for minimization, and again for problems with mixed constraints (Sections 4.3, 4.4, 4.5.1). Applied mathematics students prefer a unified treatment giving the most general form of an LP problem. Similarly there is no reference to unimodular and totally unimodular matrices through which transportation problem should be treated for students oriented to applied mathematics. There are no theorems that prove the connection between basic feasible solutions and extreme points, convexity of feasible region of an LP, etc. Fundamental theorem of linear programming, weak and strong duality theorems, discussion on complementarity slackness are all important, at least to applied mathematics students, and they do not find a place in this book, which is a drawback.

Undergraduate or even graduate students in management sciences generally do not like mathematical treatment. For them this book is nice, written systematically in slow pace. In one book, they can get introduction to various subtopics in mathematical programming.

There are some printer's devils. For example, on p. 202, 7th line in section 9.2 the word 'their' is wrongly spelt. Similarly on p. 276, 'of' is printed in place of 'if' while giving condition for concavity.

In general, the get up and printing of the book are good. Problems chosen represent down-to-earth situations. This will help enhance the students' interest.

Department of Applied Mathematics
Indian Institute of Science
Bangalore 560 012.

V. G. TIKEKAR

Analysis of volatiles – methods and applications edited by Peter Schreier. Walter de Gruyter, P.O. Box 110240, D-1000 Berlin 30, 1984, pp. 469, $87.

An international workshop was held at the Chemistry Department of the University of Wurzburg, Federal Republic of Germany, from September 28–30, 1983, to bring together analysts working in different fields of analysis of volatiles, *e.g.* in analytical, food, environmental and biological chemistry, but who all essentially use very similar chromatographic and instrumental techniques. This book contains the lectures presented by leading authorities in the field and covering the main topics: sample preparation, analytical techniques and applications. In each lecture the subject is briefly reviewed and an up-to-date appraisal of the developments presented.

A. Sample preparation

Sugisawa (Japan) has discussed in some detail the prerequisites such as the precautions and care to be taken for sample preparation in flavour analysis. The selection of suitable raw material and the steps involved in sample preparation are considered as important parameters in obtaining a meaningful analysis. Leahy and Reineccius (USA) have evaluated the qualitative and quantitative performance of some of the commonly used methods for the isolation of volatiles from foods using the same set of test compounds at ppb concentrations. The methods chosen for study included direct head space, head-space concentration, simultaneous distillation-cum-extraction and solvent extraction using low-boiling organic solvents. Solvent extraction and simultaneous distillation-cum-extraction were found to be efficient methods of wide application and good recoveries. Piringer and Skorier (FRG) have described a selective enrichment of volatiles by gas-water partition in concurrent and countercurrent columns. The example of an application of the above is the identification of off-odour in polyethylene containing food-packaging materials.

B. Analytical techniques

Jennings and Takeoka (USA) have reviewed the state-of-the-art-used silica-capillary gas chromatography and applications in the detection of trace concentrations of sulfur- and nitrogen-containing compounds and sniff evaluation of the head-space gas. Enantio-selective gas chromatography has proved to be a sensitive analytical tool for stereochemical investigations such as the analysis of natural flavours, product control of microbial or enzymatic biotransformations of chiral drugs and pheromoners (Konig, FRG). A comparison of GLC capillaries was studied using the influence of column-wall pretreatments like column material, coating with lacquer, removal of cations by leaching, and inactivation of silanol groups by deactivation. Fused silica offers several advantages like easy deactivation, coating of the capillaries and handling due to flexibility (Gunther *et al*, FRG). The use of passive sampling and thermal desorption has been carried out with fused-silica capillaries in dynamic head-space analysis and the applications include the analysis of solvent vapours in the atmosphere, in printing industry and monitoring nitrous oxide in operation theatres (Kristernsson, Sweden).

Chromatography is essentially a separation technique while spectroscopic techniques are employed for identification and structural elucidation. Multidimensional GC (in coupled columns) and GC-MS have been successfully employed for the analysis of wine aroma (Schomburg *et al*, FRG). The approach of concentration of trace volatiles by packed precolumn-containing polymers like Tenax GC or 2,6-diphenyl-p-phenylene oxide and subsequent thermal desorption has been found useful in the analysis of trace organics in vapour samples like air pollution, flavour volatiles, etc. The application of this technique in combination with multidimensional gas chromatography has been applied for the enrichment and quantitative determination of selected compounds (Nitz and Julich, FRG). The analysis of trace components using the total transfer technique in coupled-column systems (Oreans *et al*, FRG) offers advantages like infection of relatively high concentrations into the capillary column and trapping of the sample. The use of capillary columns for the combined gas chromatograph–Fourier-tranform infrared spectrometer (GC-FTIR) and high-resolution GC-FTIR is described with special reference to applications in the analysis of petroleum products, organic solvents and flavour and fragrances (Herres, FRG). The direct use of carbon-13 NMR spectroscopy is discussed for the analysis of volatiles without prior separation into individual constituents (Kubeczka and Formacek, FRG). This technique is particularly useful for heat-sensitive essential oils obtained by cold-extraction methods.

C. Applications

In this section, there are 14 papers describing the applications of high-resolution capillary columns for flavour analysis (Shibamoto, USA), sensory analysis of GC effluents and isomers (Acrea and Bannard, USA; Drawert and Christoph, FRG), resolution of optically active aroma compounds and synthesis and analysis of some fruity flavour stereoisomers (Tressl and Engel, FRG and Mosandl and Heusinger, FRG). The other papers deal with HRGC-FTIR in tropical fruit flavour analysis (Schreier Idstein and Herres, FRG), high- and low-reduction mass spectrometry in GC-MS coupling for analysing volatiles of plant-tissue culture (Lange and Schultze, FRG), enrichment of flavour volatiles, analysis and correlation of sensory and chemical data (Sugisawa *et al*, Japan; Boland *et al*, FRG; Dirinck *et al*, Belgium; Badings and De Jong, Netherlands; Adam, FRG; Kallio *et al*, Finland; Liardon, *et al*, Switzerland).

On the whole, this volume would be found interesting and of great help to both beginners as well as experts in the area of analysis of volatiles.

Central Food Technological Research Institute Mysore.

M. L. SHANKARANARAYANAN

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