COMBINATION OF X-RAY FOCUSSING MIRRORS IN AN X-RAY MICROSCOPE

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Summary

The paper contains a theoretical study of combinations of spherical crystal reflectors which can give X-ray images similar to those in an optical microscope. The only possible combination is found to be that in which both the mirrors are concave and both have the same radius of curvature, so that the two mirrors form parts of the same spherical surface. Under these conditions, the Bragg relation is satisfied at both mirrors for any number of multiple reflections. By making use of N reflections, the optical path of the "microscope" for given magnification can be reduced to 1/N of that required with only one reflection.

1. Introduction

The reflection of a beam of mono- or poly-chromatic X-rays by a lattice plane in a crystal (plane or deformed) is governed by the same optical conditions as ordinary light but for the restriction that, for each direction of incidence, only selected wavelengths are reflected which satisfy the Bragg relation. Applying this principle (Ramachandran, 1951), focussed images of a wire mesh magnified upto twenty times were obtained using reflections from a cleavage flake of mica elastically deformed into a concave spherical surface (Ramachandran & Thathachari, 1951).

Since the publication of the above articles, the authors have come across reports of similar work undertaken by Y. Cauchois (1946, 1950). Cauchois has also suggested the same principles as mentioned above and has indicated possible applications of the method. Although some photographs have been published in both the articles, no details are given of the experimental technique. In the second paper (1950) Cauchois has suggested the possibility of combining crystal reflectors to form optical systems for X-rays, whereby a microscope can be constructed having a large magnification, at the same time having a much smaller optical path than what would be necessary in a single stage. No definite results, however, seem to have been worked out by her.

Similar possibilities have also been theoretically considered by the authors and some of the results were referred to in the previous communications. Although the reflection of X-rays from a single mirror is perfectly analogous to that of ordinary light, in the case of a combination of two such mirrors a further restrictive condition occurs, *viz*., that a particular ray must satisfy the Bragg condition on both the mirrors. The purpose of this paper is to consider the conditions under which two crystal reflectors can be combined to produce an X-ray image.

2. THE CONDITIONS FOR THE FORMATION OF AN X-RAY IMAGE BY A SYSTEM OF TWO SPHERICAL CRYSTALLINE REFLECTORS

Consider a system of two spherical crystalline reflectors (A & B) concave or convex and suppose that X-ray reflections are obtained from the lattice planes parallel to the spherical surfaces. Considering first a ray proceeding along the common axis of the two mirrors (P_1 P_2 of Fig. 1), the

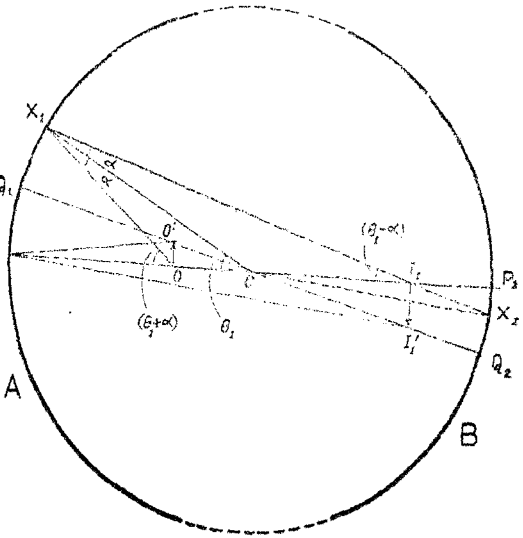

FIG. 1. Figure showing angular relations in the formation of images

Bragg condition will be satisfied at both the mirrors for such an axial ray if and only if the lattice spacing is the same for both of them. In other words, both A & B should be of the same crystalline material and the planes employed for selective reflection should have the same Miller indices in either case. Again, considering another ray (*e.g.*, CQ_1) incident normally on mirror A, and therefore reflected back normally, it should again be incident normally on the surface of mirror B, if the Bragg relation is to be satisfied at both A and B. Thus $Q_1 CQ_2$ must be normal to B. A similar condition holds for every ray incident normally on A, so that we have the general condition that every normal to A should be a normal to B as well. In other words, *A and B should be portions of the same spherical surface.* When this is so it is clear that the Bragg relation is automatically satisfied at both the mirrors for every possible ray, whatever be its angle of incidence (*e.g.*, X_1X_2); for the lines joining points of incidence of the ray on A and B after successive reflection from the two mirrors will always be a chord of the sphere which will be equally inclined to the tangents or the normals at the points of incidence. Thus a ray can successively be reflected by the two mirrors any number of times, and the angle of incidence would be the same at every reflection.

3. Expression for the Position of the Object and Image in the Case of N Reflections successively from A and B

In the case of multiple reflections, the image (real or virtual) formed by each reflection serves as the source for the next reflection. Let suffixes 1, 2,....N attached to different quantities denote the ordinal numbers of the reflection with which they are associated. We shall adopt the convention that the various distances pertaining to each reflection are measured from the corresponding pole, the direction pole to centre being always reckoned positive. Let R denote the common radius of curvature of A and B and u, v the distances of the object and image respectively from the pole. Let

$$\frac{u_s}{R} = \xi_s \text{ and } \frac{v_s}{R} = \eta_s. \tag{1}$$

For concave mirrors (with which we are concerned) the image formed after s reflections will be real only if

$$\eta_s \geqslant 0. \tag{2}$$

Fig. 1 represents a diametral section of the sphere. Let P_1, P_2 be the poles of the two mirrors and C the common centre of curvature. Let O be the position of a point object along the common axis of A and B. Consider a ray OX_1 incident at an angle α to the normal CX_1 at X_1 and let CX_1 be inclined at an angle θ_1 to P_1P_2. Let I_1 be the image of O due to the first reflection from A. We shall assume θ_1 to be so small that its square and higher powers can be neglected. In other words, we shall confine ourselves to paraxial rays which alone can be focussed without appreciable spherical aberration. Under these conditions, we have

$$P_1X_1 = u_1 (\theta_1 + \alpha) = R_1\theta_1 = v_1 (\theta_1 - \alpha).$$

Hence eliminating α,

$$\eta_1 = \xi_1/(2\xi_1 - 1).$$

In general, for the sth reflection

$$\eta_s = \xi_s/(2\xi_s - 1). \tag{3}$$

Now

$$\xi_2 = 2 - \eta_1 = (3\xi_1 - 2)/(2\xi_1 - 1).$$

Hence

$$\xi_3 = (3\xi_2 - 2)/(2\xi_2 - 1) = (5\xi_1 - 4)/(4\xi_1 - 3)$$

and in general,

$$\xi_s = \frac{(2s-1)\,\xi_1 - (2s-2)}{(2s-2)\,\xi_1 - (2s-3)}, \tag{4}$$

$$\eta_s = \frac{(2s-1)\,\xi_1 - (2s-2)}{2s\xi_1 - (2s-1)}. \tag{5}$$

Considering an extended object OO′, the first image is I_1I_1'. It is clear from the similar triangles P_1OO', $P_1I_1I_1'$ of Fig. 1 that the magnification due to the first reflection is

$$M_1 = \frac{I_1I_1'}{OO'} = \frac{v_1}{u_1} = \frac{\eta_1}{\xi_1}.$$

In general, the magnification produced by the sth reflection is

$$M_s = \frac{\eta_s}{\xi_s} = \frac{(2s-2)\,\xi_1 - (2s-3)}{2s\xi_1 - (2s-1)}. \tag{6}$$

M_s may be positive or negative; if positive it means that both the object and the image are real or that both are virtual and if negative, one will be real, while the other will be virtual. The total magnification M due to N reflections is given by the product of the magnifications due to each. Thus

$$\begin{aligned} M &= M_1M_2 \ldots M_s \ldots M_N \\ &= 1/[2N\xi_1 - (2N-1)]. \end{aligned} \tag{7}$$

M may be positive or negative, but the important quantity is the magnitude. Its sign does not give one an idea as to whether the final image is real or virtual, for the sign depends upon the number of times the transition virtual to real has taken place in between. However, the condition for the final image to be real is given by

$$\eta_N \geqslant 0.$$

This leads to two possibilities:

either

$$\xi_1 \geqslant (1 - 1/2N) \tag{8 a}$$

or

$$0 \leqslant \xi_1 \leqslant (1 - 1/\overline{2N-1}) \tag{8 b}$$

(8*a*) corresponds to positive values of M and all values of the magnification between 1 and ∞ are possible within the range of ξ_1. (8 *b*) on the other hand, corresponds to negative values of M and the magnification can only assume values between $1/(2N-1)$ and $(2N-1)$. In either case, if the total magnification M is given, then the values of ξ_1 and η_N giving the positions of the object and the final image are

$$\xi_1 = 1 - \frac{1}{2N} + \frac{1}{2NM} \tag{9}$$

and

$$\eta_N = 1 + \frac{M-1}{2N}. \tag{10}$$

If we denote by η' the value of v/R corresponding to the same magnification (M) obtained in a single stage, then it can be shown that

$$\frac{\eta_N}{\eta'} = \frac{1}{N}\left[1 + \frac{2(N-1)}{(M+1)}\right]. \tag{11}$$

Thus, for large magnifications, the optical path (v) employing N reflections is only about 1/Nth of that when only one reflection is used. This is a great advantage particularly for large magnifications. Further, it also appears that, in such cases, the spherical aberration is much less when N reflections are employed than with a single reflection using the same aperture and magnification. Exact expressions for the spherical aberration and other defects of the image are being worked out and will be reported later.

4. Practical Considerations

In order to be able to observe the final image on a screen or to photograph it, the image should be formed in the region outside the sphere in Fig. 1. This means that $v_N \geqslant 2R$, *i.e.*, $\eta_N \geqslant 2$. From Eq. (10), this means that

$$M \geqslant (2N+1)$$

and

$$1/2 \leqslant \xi_1 \leqslant 1. \tag{13}$$

Thus, the object should be situated in between the two mirrors. This appears to be an essential condition for obtaining large magnifications. Therefore, special arrangements have to be made regarding the illuminating system so as to practically realise such conditions. Suppose that the object is irradiated so that the first reflection occurs at mirror A. The source of X-rays must then be to the right of B and it is convenient to have the image to the left of A (Fig. 2).

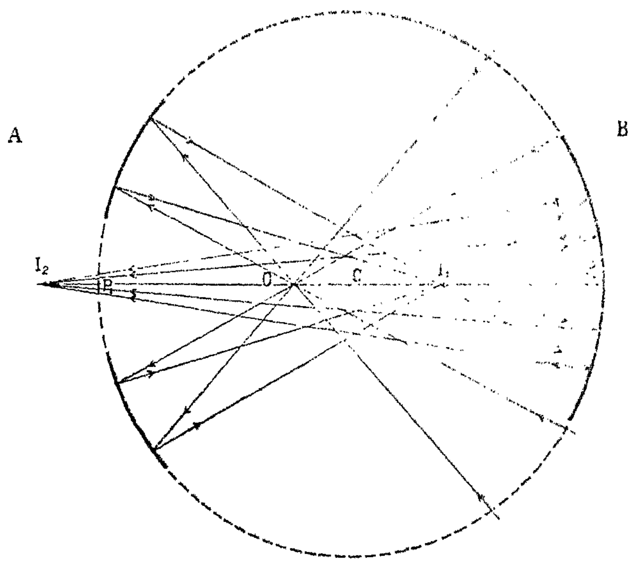

FIG. 2. Ray paths showing formation of an image after two successive reflections (Total magnification 6)

It can be readily shown that it is not possible to illuminate the object through a central aperture in B. This is so because all the rays incident on the object through the aperture and reflected by A would again be incident on B *within the* area of the aperture (if $\frac{1}{2} \leqslant \xi \leqslant 1$). Consequently an arrangement similar to that shown in Fig. 2 must be used, *i.e.*, the cone of rays incident on the object O must come from *outside* the mirror B, so that the rays first reflected by A can again be reflected by B and so on. This means that the source of X-rays must be ring-like, which may be produced by electrostatically rotating the focal spot or mechanically rotating the X-ray tube about an eccentric axis. In a way, this is an advantage, since with the former arrangement, large tube currents can be employed as with a rotating anode. Obviously mirror A must have a central aperture to let out the rays forming the image. Fig. 2 has been drawn to show the ray paths in the case of two reflections ($N = 2$) and a total magnification of 6 ($M = 6$).

Attempts are being made to realise these ideas in practice. It is obvious that many practical difficulties will have to be surmounted before the apparatus can be made to work. However, the theoretical results are being published because the combination described above seems to be the only possible arrangement of spherical crystal reflectors which can give an image of a finite extended object.

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