SOME PERTURBATION EFFECTS IN MICROWAVE CAVITIES OPERATING IN DEGENERATE MODES

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Abstract

It has been shown that the perturbation caused by the introduction of a metallic rod into a cylindrical cavity operating in the companion modes $TE_{01n} - TM_{11n}$ produces different amounts of changes in the Q and the resonant frequencies for the two modes. The effect of increasing the radius and the depth of insertion of the rod on the Q and the changes in the resonant frequencies has been discussed.

Introduction

The paper presents the results of a theoretical investigation into some of the effects of perturbation on the characteristics of cylindrical cavities that are under construction in this laboratory. If a micro-wave cavity operates in a non-degenerate mode, and if the frequency of this mode is well separated from those of the other modes, then a slight deformation of the boundary wall may not alter appreciably the electromagnetic field configurations inside the cavity. In such a case, the Q of the cavity will remain practically unaltered. But if the cavity operates in a degenerate mode, then any deformation of the boundary wall in the form of a coupling hole or unsymmetrical tuning mechanism, will affect the electromagnetic field configurations inside the cavity. This will result in coupling between the degenerate modes, and the Q of the cavity will be lowered.

In cylindrical cavity resonators, the resonant frequencies f_{lmn} for the TE_{lmn} and TM_{lmn} modes are given by the following expression.

$$f_{lmn} = \frac{c}{2} \left[(2x_{lm}/\pi D)^2 + (n/L)^2\right]^{\frac{1}{2}},$$

where x_{lm} represents the m-th root of $J_l'(x) = 0$ and of $J_l(x) = 0$ for the TE and TM modes respectively, where

D = Diameter of the cavity,

L = Length of the cavity,

c = Velocity of light.

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The resonant frequencies f_{01n} and f_{11n} for the TE_{01n} and TM_{11n} modes are identical as $J_0'(x) = -J_1(x)$. This is an important case of degeneracy. In the case of high Q cavities, such as the echo box, n may vary from 1 at about 1 kmc to 50 at about 50 kmc.

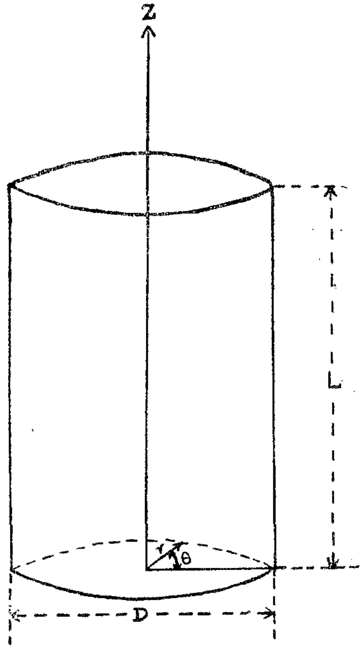
In the design of micro-wave cavities, it is necessary to select a suitable operating area on the mode chart so as to have the largest possible tuning range and the minimum number of interfering and cross-over modes. Even, if the operating area is properly selected for a cavity operating in the TE_{01n} mode at about 9 kmc there are about twelve interfering modes and nine cross-over modes including the TM_{11n} and the self interfering $TE_{0,1,(n+1)}$ modes. For a cavity operating at about 25 kmc, the crossing modes are about forty in number with hundreds of interfering modes. In order that a micro-wave cavity may be useful for practical purposes, the unwanted modes should be suppressed. This requires a thorough knowledge of their field configurations, and suitable technique such as the placing of polyiron disc, water ring, etc., at the back of the tuning disc may be employed to damp out the undesired modes. For a cavity operating in the TE_{01n} mode, it is rather difficult to suppress completely the companion TM_{11n} mode. So, in practice, the companion mode is filtered out at the output of the cavity by suitably locating the coupling hole at the wall or end-plates of the cavity.

The above discussion shows that in designing a micro-wave cavity it is necessary to know about the changes in Q and resonant frequencies of the cavity operating in $TE_{01n} - TM_{11n}$ mode when a tuning rod is introduced into the cavity. The case of TM_{11n} mode will be considered in detail and the case of TE_{01n} will be mentioned briefly for comparison. The case of TE_{01n} mode has been treated elsewhere[1] in connection with the study of the effects of perturbation caused by the introduction of a dielectric rod into a cylindrical cavity. The properties of micro-wave cavities of certain simple geometry have been theoretically investigated by several authors.[2, 3, 4] The effects of perturbation caused by the deformation of the boundary wall of a cavity have also been studied by some authors.[5, 6, 7, 8, 9]

FIELD COMPONENTS IN A CYLINDRICAL CAVITY (TM_{11n} MODE)

Consider the case of a right circular cylinder type of cavity and assume that the tangential components E_{tan} of the electric field and the normal component H_{nor} of the magnetic field vanish at the boundary surfaces. Then the normal mode fields in a completely lossless cavity excited in TM_{lmn} mode are given as follows.[10]

$$\left.\begin{aligned}
E_r &= -\frac{k_3}{k} J_l'(k_1 r) \cos l\theta \sin k_3 z \\
E_\theta &= l \frac{k_3}{k} \frac{J_l(k_1 r)}{k_1 r} \sin l\theta \sin k_3 z \\
E_z &= \frac{k_1}{k} J_l(k_1 r) \cos l\theta \cos k_3 z \\
H_r &= -l \frac{J_l(k_1 r)}{k_1 r} \sin l\theta \cos k_3 z \\
H_\theta &= -J_l'(k_1 r) \cos l\theta \cos k_3 z \\
H_z &= 0
\end{aligned}\right\} m > 0 \qquad (1)$$



where $k_3 = n\pi/L$, $k = 2\pi/\lambda$, also $k^2 = k_1{}^2 + k_3{}^2$.

In the case of TM_{11n} mode $k_1 = x_{11}/R$ and the expressions in (1) reduce to the following:

$$\left.\begin{aligned}
E_r &= -\frac{k_3}{k} J_1'(k_1 r)\cos\theta\sin k_3 z\\
E_\theta &= \frac{k_3}{k}\frac{J_1(k_1 r)}{k_1 r}\sin\theta\sin k_3 z\\
E_z &= \frac{k_1}{k} J_1(k_1 r)\cos\theta\cos k_3 z\\
H_r &= -\frac{J_1(k_1 r)}{k_1 r}\sin\theta\cos k_3 z\\
H_\theta &= -J_1'(k_1 r)\cos\theta\cos k_3 z\\
H_z &= 0
\end{aligned}\right\} \tag{2}$$

FREQUENCY SHIFT OF THE CAVITY

If an ideal cavity of volume V is designated by 1, and by 2 when it is slightly deformed by the introduction of a cylindrical metallic rod of volume v, then the change Δf in the resonance frequency f of the ideal cavity due to this deformation is given by the following expression.[5]

$$\frac{\Delta f}{f} = \frac{\int_{(1)-(2)} (E_1^2 - H_1^2)\, dv}{2\int_{(1)} E_1^2\, dV} \tag{3}$$

where,

$\int_{(1)-(2)}$ = integral over that part of the volume of the cavity (1) which is not present in (2)

$\int_{(1)}$ = integral over the volume of the cavity (1). $\mathbf{E}_1$ and $\mathbf{H}_1$ represent the amplitudes of the electric and magnetic fields respectively.

Equation (3) has been deduced on the assumption that the field configurations inside the perturbed cavity remains unaltered from that of the ideal cavity. The error introduced by this assumption can be shown to be very small.

If the cylindrical rod of radius a is introduced to a depth d into the cavity of radius R, then we get from equation (2) the following expressions:

$$\int_v E_r^2 dv = \int_{z=0}^{d}\int_{r=0}^{a}\int_{\theta=0}^{2\pi} \frac{k_3^2}{k^2} r\,[J_1'(k_1 r)]^2 \cos^2\theta \sin^2 k_3 z\, dr\, d\theta\, dz$$

$$= \frac{\pi k_3^2}{k^2}\left(\frac{d}{2} - \frac{\sin 2k_3 d}{4k_3}\right)\left[\left(\frac{a^2}{2} - \frac{1}{2k_1^2} + \frac{1}{k_1}\right) J_0^2(k_1 a) + \left(\frac{a^2}{2} - \frac{1}{2k_1^2}\right) J_1^2(k_1 a)\right] \tag{4}$$

$$\int_v E_\theta^2 dv = \frac{k_3^2}{k^2}\int_0^d\int_0^a\int_0^{2\pi} r\,\frac{J_1^2(k_1 r)}{k_1^2 r^2}\sin^2\theta \sin^2 k_3 z\, dr\, d\theta\, dz$$

$$= -\frac{\pi k_3^2}{2k^2 k_1^2}\left[\frac{d}{2} - \frac{\sin 2k_3 d}{4k_3}\right][J_0^2(k_1 a) + J_1^2(k_1 a)] \tag{5}$$

$$\int_v E_z^2 dv = \frac{k_1^2}{k^2}\int_0^d\int_0^a\int_0^{2\pi} r\, J_1^2(k_1 r)\cos^2\theta \cos^2 k_3 z\, dr\, d\theta\, dz$$

$$= \frac{\pi a^2}{2}\frac{k_1^2}{k^2}\left(\frac{d}{2} + \frac{\sin 2k_3 d}{4k_3}\right)\left[J_0^2(k_1 a) + J_1^2(k_1 a) - \frac{2J_0(k_1 a)\, J_1(k_1 a)}{k_1 a}\right]. \tag{6}$$

Similarly,

$$\int_v H_r^2 dv = -\frac{\pi}{2k_1^2}\left[\frac{d}{2} + \frac{\sin 2k_3 d}{4k_3}\right][J_0^2(k_1 a) + J_1^2(k_1 a)] \tag{7}$$

$$\int_v H_\theta^2 dv = \pi\left[\frac{d}{2} + \frac{\sin 2k_3 d}{4k_3}\right]\left[\left(\frac{a^2}{2} - \frac{1}{2k_1^2} + \frac{1}{k_1}\right) J_0^2(k_1 a) + \left(\frac{a^2}{2} - \frac{1}{2k_1^2}\right) J_1^2(k_1 a)\right] \tag{8}$$

and

$$\int_v E_1^2\, dv = \frac{L}{2}\left[\frac{\pi R^2}{2} - \frac{\pi k_3^2}{k^2 k_1^2} + \frac{\pi k_3^2}{k^2 k_1}\right] J_0^2(x_{11}) \tag{9}$$

as $\qquad J_1(x_{11}) = 0$ and $\sin 2k_3 L = 0$.

For small values of d, and neglecting powers of sine higher than the third, we get the following expression for the frequency shifts Δf from equations (3) to (9)

$$\left(\frac{\Delta f}{f}\right)_{TM_{11n}} = \frac{(A-B)\, J_0^2(k_1 a) + (A-C)\, J_1^2(k_1 a) - (D/k_1 a)\, J_0(k_1 a)\, J_1(k_1 a)}{LE\, J_0^2(x_{11})} \tag{10}$$

where,

$$A = \left(\frac{d}{2} - \frac{1}{3} k_3^2 d^3\right) \frac{\pi (a^2 - 1)}{2k^2} + \frac{\pi d}{2}\left(\frac{a^2 k_1^2}{2k^2} - \frac{k_3^2}{k^2 k_1^2} - \frac{1}{2k_1^2}\right)$$

$$B = \pi \left(d - \frac{1}{3} k_3^2 d^3\right)\left(\frac{a^2}{2} - \frac{1}{k_1^2} + \frac{1}{k_1}\right)$$

$$C = \pi \left(d - \tfrac{1}{3} k_3^2 d^3\right)\left(\frac{a^2}{2} - \frac{1}{k_1^2}\right)$$

$$D = \frac{\pi a^2 k_1^2}{k^2}\left(d - \tfrac{1}{3} k_3^2 d^3\right)$$

$$E = \left[\frac{\pi R^2}{2} - \frac{\pi k_3^2}{k^2 k_1^2} + \frac{\pi k_3^2}{k^2 k_1}\right]$$

If $a \to R$, equation (10) reduces to

$$\left(\frac{\Delta f}{f}\right)_{a \to R} = \frac{(A - B)\, J_0^2(x_{11})}{LE\, J_0^2(x_{11})} = \frac{A - B}{LE} \tag{11}$$

as $J_1(x_{11}) = 0$.

If $d \to L$, we get the following expression for Δf from equations (3) to (9)

$$\left(\frac{\Delta f}{f}\right)_{d \to L} = \frac{(A' - B')\, J_0^2(k_1 a) + (A' - C') J_1^2(k_1 a) - (D'/k_1 a)\, J_0(k_1 a)\, J_1(k_1 a)}{LE\, J_0^2(x_{11})} \tag{12}$$

as $\sin 2k_3 d = 0$ when $d \to L$

where,

$$A' = \frac{\pi d}{2}\left(\frac{a^2 k_1^2}{2k^2} - \frac{k_3^2}{k^2 k_1^2} - \frac{1}{2k_1^2}\right)$$

$$B' = \frac{\pi d}{2}\left(\frac{a^2}{2} - \frac{1}{k_1^2} + \frac{1}{k_1}\right)$$

$$C' = \frac{\pi d}{2}\left(\frac{a^2}{2} - \frac{1}{k_1^2}\right)$$

$$D' = \frac{\pi a^2 d k_1^2}{2k^2}.$$

The frequency shift for the cavity operating in the TE_{01n} mode is found as follows by utilising the field components,[10] equation (3) and adopting the same procedure as above, and neglecting powers of sine higher than the third

$$[VJ_0^2(x_{01})]\cdot\frac{\Delta f}{f}=\Big\{-v\Big(1-\frac{2}{3}\frac{\pi^2n^2d^2}{L^2}\Big)\Big[J_0^2\Big(\frac{x_{01}a}{R}\Big)+J_1^2\Big(\frac{x_{01}a}{R}\Big)\Big]$$
$$+\Big[\frac{\lambda^2 v}{8\pi^2}\Big(1-\frac{2}{3}\frac{\pi^2n^2d^2}{L^2}\Big)\Big(\frac{4\pi^2}{\lambda^2}+\frac{\pi^2n^2}{L^2}\Big)$$
$$-\frac{v}{2}\Big(\frac{x_{01}}{R}\frac{\lambda}{2\pi}\Big)^2\Big]\cdot\Big[\frac{2J_0(x_{01}a/R)\,J_1(x_{01}a/R)}{(x_{01}a/R)}\Big]\Big\} \tag{13}$$

If $a\rightarrow R$ equation (13) reduces to

$$\Big(\frac{\Delta f}{f}\Big)_{a\rightarrow R}=-\frac{v}{V}\Big(1-\frac{2}{3}\frac{\pi^2n^2d^2}{L^2}\Big) \tag{14}$$

as $$-J_0'(x_{01})=J_1(x_{01})=0.$$

If $d\rightarrow L$, then by utilising the expressions for the field components[10] for the TE_{01n} mode and equation (3) it can be shown that

$$\Big(\frac{\Delta f}{f}\Big)_{d\rightarrow L}=-\frac{0\cdot958\lambda^2 v}{\pi^2 aRV}\cdot\frac{J_0(x_{01}a/R)\,J_1(x_{01}a/R)}{J_0^2(x_{01})} \tag{15}$$

The negative sign for Δf in equations (14) and (15) indicates a decrease in the resonant frequency.

Maximum Energy Stored in the Electric Field of the Cavity (TM_{11n} Mode)

The energy stored in the electric field per unit volume is $\frac{1}{2}\,\epsilon E^2$, where ϵ may be considered as unity when the dielectric is air. Let it be assumed that the boundary wall of the cavity is perfectly conducting and that the field components (equation 2) remain unaltered when there is slight deformation of the cavity caused by the introduction of the rod. Then the maximum energy W_E stored in the electric field of the loaded cavity is obtained as follows:

$$W_E=\frac{1}{2}\Big[\frac{k_3^2}{k^2}\int_{r=0}^{R}\int_{\theta=0}^{2\pi}\int_{z=0}^{L} r\,[J_1'(k_1r)]^2\cos^2\theta\sin^2 k_3z\,dr\,d\theta\,dz$$
$$+\frac{k_3^2}{k^2}\int_0^R\int_0^{2\pi}\int_0^L r\,\frac{J_1^2(k_1r)}{k_1^2r^2}\sin^2\theta\sin^2 k_3z\,dr\,d\theta\,dz$$
$$+\frac{k_1^2}{k^2}\int_0^R\int_0^{2\pi}\int_0^L r\;J_1^2(k_1r)\cos^2\theta\cos^2 k_3z\,dr\,d\theta\,dz\Big]$$
$$=\frac{L}{4}\Big[\frac{\pi R^2}{2}-\frac{\pi k_3^2}{k^2k_1^2}+\frac{\pi k_3^2}{k^2k_1}\Big]J_0^2(x_{11}) \tag{16}$$

POWER LOST IN THE LOADED CAVITY (TM_{11n} MODE)

The average power P_r absorbed by the cylindrical wall per unit length is found as follows by setting $\mathbf{H}_{tan} = H_\theta$ and using the Poynting vector.

$$P_r = \tfrac{1}{2}\sqrt{\frac{\pi f\mu}{\sigma}} \int_{z=0}^{L}\int_{\theta=0}^{2\pi} R\,[J_1'(k_1 r)]^2 \cos^2\theta \cos^2 k_3 z\, dz\, d\theta$$

$$= \frac{\pi RL}{4}\sqrt{\frac{\pi f\mu}{\sigma}}\, J_0^2(x_{11}) \tag{17}$$

as $J_1(x_{11}) = 0$.

Power lost P_z in both the end plates is obtained as follows by setting

$$\mathbf{H}_{tan} = \sqrt{H_r^2 + H_\theta^2}$$

$$P_z = 2\Big[\frac{1}{2}\sqrt{\frac{\pi f\mu}{\sigma}} \int_{r=0}^{R}\int_{\theta=0}^{2\pi} R\,[J_1'(k_1 r)]^2 \cos^2\theta \cos^2 k_3 z\, dr\, d\theta$$

$$+ \tfrac{1}{2}\sqrt{\frac{\pi f\mu}{\sigma}} \int_{r=0}^{R}\int_{\theta=0}^{2\pi} R\,\frac{J_1^2(k_1 r)}{k_1^2 r^2} \sin^2\theta \cos^2 k_3 z\, dr\, d\theta\Big]$$

$$= \pi\sqrt{\frac{\pi f\mu}{\sigma}}\left\{\frac{R^2}{2} - \frac{1}{k_1^2} + \frac{1}{k_1}\right\} J_0^2(x_{11}) \tag{18}$$

as $\cos k_3 z = 1$ at $z = 0$ or $z = L$ and $J_1(x_{11}) = 0$.

Similarly, the power P_r' absorbed by the cylindrical rod per unit length is given by the following expression:

$$P_r' = \tfrac{1}{2}\sqrt{\frac{\pi f\mu}{\sigma}} \int_{z=0}^{d}\int_{\theta=0}^{2\pi} a\,[J_1'(k_1 r)]^2 \cos^2\theta \cos^2 k_3 z\, dz\, d\theta$$

$$= \frac{\pi a}{2}\sqrt{\frac{\pi f\mu}{\sigma}}\left[\frac{d}{2} + \frac{\sin 2k_3 d}{4k_3}\right][J_1'(k_1 a)]^2 \tag{19}$$

Q OF THE CAVITY

The loaded Q (Q_2) of the cavity is defined as follows:

$$Q_2 = \omega\frac{W_E}{P_r + P_r' + P_z}. \tag{20}$$

So the loaded Q for the TM_{11n} mode is given from equations (16) to (20) as follows;

$$(Q_2)_{TM_{11n}} = \frac{L\sqrt{\pi f\sigma}\left[\frac{R^2}{2} - \frac{k_3^2}{k^2}\left(\frac{1}{k_1^2} - \frac{1}{k_1}\right)\right] J_0^2(x_{11})}{\sqrt{\mu}\left[\left\{R^2 + \frac{RL}{2} - \frac{2}{k_1}\left(\frac{1}{k_1} - 1\right)\right\} J_0^2(x_{11}) + \frac{a}{2}\left(\frac{d}{2} + \frac{\sin 2k_3 d}{4k_3}\right)[J_1'(k_1 a)]^2\right]} \tag{21}$$

The unloaded $Q(Q_1)$ for the TM_{11n} mode can be similarly found to be as follows:

$$(Q_1)_{TM_{11n}} = \frac{L\sqrt{\pi f\sigma}\left[\frac{R^2}{2} - \frac{k_3^2}{k^2}\left(\frac{1}{k_1^2} - \frac{1}{k_1}\right)\right]}{\sqrt{\mu}\left[\left\{R^2 + \frac{RL}{2} - \frac{2}{k_1}\left(\frac{1}{k_1} - 1\right)\right\}\right]}. \tag{22}$$

The loaded Q of the cavity operating in the TE_{01n} mode is found as follows by adopting the same procedure as outlined elsewhere[1] when the perturbation is caused by the insertion of a dielectric rod.

$$(Q_2)_{TE_{01n}} = \frac{V\sqrt{\pi f\sigma}\, J_0^2(x_{01})}{\lambda^2\sqrt{\mu}\left[\frac{3\cdot 672L}{\pi R} J_0^2(x_{01}) + \frac{\pi n^2 R^2}{2L^2} J_0^2(x_{01}) + \frac{7\cdot 342a}{\pi R^2}\left(\frac{d}{2} - \frac{\sin 2k_3 d}{4k_3}\right) J_0^2\left(\frac{x_{01}a}{R}\right.\right.}$$

The unloaded $Q(Q_1)$ is similarly given as follows:

$$(Q_1)_{TE_{01n}} = \frac{V\sqrt{\pi f\sigma}}{\lambda^2\sqrt{\mu}\left[\frac{3\cdot 672L}{\pi R} + \frac{\pi n^2 R^2}{2L^2}\right]} \tag{24}$$

When $d \to L$, equation (21) reduces to

$$(Q_2)_{TM_{11n}} \to \frac{L\sqrt{\pi f\sigma}\left[\frac{R^2}{2} - \frac{k_3^2}{k^2}\left(\frac{1}{k_1^2} - \frac{1}{k_1}\right)\right] J_0^2(x_{11})}{\sqrt{\mu}\left[\left\{R^2 + \frac{RL}{2} - \frac{2}{k_1}\left(\frac{1}{k_1} - 1\right)\right\} J_0^2(x_{11}) + \frac{aL}{4}[J_1'(k_1 a)]^2\right]} \tag{25}$$

and equation (23) reduces to

$$(Q_2)_{TE_{01n}} \to \frac{V\sqrt{\pi f\sigma}\, J_0^2(x_{01})}{\lambda^2\sqrt{\mu}\left[\frac{3\cdot 672L}{\pi R} J_0^2(x_{01}) + \frac{\pi n^2 R^2}{2L^2} J_0^2(x_{01}) + \frac{7\cdot 342aL}{2\pi R^2} J_0^2\left(\frac{x_{01}a}{R}\right)\right]}$$

If $a \to R$ equation (21) reduces to

$$(Q_2)_{TM_{11n}} \to \frac{L\sqrt{\pi f\sigma}\left[\frac{R^2}{2} - \frac{k_3^2}{k^2}\left(\frac{1}{k_1^2} - \frac{1}{k_1}\right)\right]}{\sqrt{\mu}\left[\left\{R^2 + \frac{RL}{2} - \frac{2}{k_1}\left(\frac{1}{k_1} - 1\right)\right\} + \frac{R}{2}\left(\frac{d}{2} + \frac{\sin 2k_3 d}{4k_3}\right)\right]} \tag{27}$$

and equation (23) reduces to

$$(Q_2)_{TE_{01n}} \to \frac{V\sqrt{\pi f \sigma}}{\lambda^2 \sqrt{\mu}\left[\frac{3 \cdot 672 L}{\pi R} + \frac{\pi n^2 R^2}{2L^2} + \frac{7 \cdot 342}{\pi R}\left(\frac{d}{2} - \frac{\sin 2k_3 d}{4k_3}\right)\right]} \tag{28}$$

DISCUSSION

The accuracy of the relations given in (10) and (13) depends on the degree of the validity of the assumption that the field configurations inside the perturbed cavity is the same as that in the ideal cavity. The error introduced due to this assumption can be shown[5] to be of the order of $[(1/Q_2) + \Delta f/(f \pm \Delta f)]$ and this is extremely small. It has been further assumed in deducing the relations that the cavity is oscillating only in a singular mode. The accuracy of the relations, therefore, will depend on to what extent the secondary modes have been suppressed. It is expected that the relations will hold good in practice, so long as, $a \ll R$ and $d \ll L$ and the cavity is designed to operate in the desired mode free from other extraneous modes.

When $d \to L$, the simple cavity reduces to the case of a coaxial cavity, and the field components involve z functions, instead of the simple Bessel functions of the first kind. The results given in equations (12) and (15) are therefore approximate and may not agree well with experimental results.

It is evident from the expressions (10) and (13) that the percentage changes in resonant frequencies for the two modes for the same a and d will be different. The following case will illustrate this. If $n = 3$, $d = 0 \cdot 1$ cm., $a = 0 \cdot 1$ cm., $\lambda = 3$ cm., $R = 2 \cdot 616$ cm., $L = 7 \cdot 366$ cm., and $f = 10$ kmc., then the percentage change in the resonant frequencies as calculated from (10) and (13) are approximately $0 \cdot 18$ and $0 \cdot 01$ in the TM_{11n} and TE_{01n} modes respectively.

In deriving the relations (21) and (23) it has been assumed that the boundary wall of the cavity is perfectly conducting and that the field configurations inside the cavity remain unaltered even when the cavity is loaded. The validity of this assumption depends on to what degree the condition $\sigma \gg \omega\epsilon$ is satisfied. In the case of microwave cavities, as the inside surface is silvered, this assumption will hold good for all practical purposes so long as $a \ll R$ and $d \ll L$. For a cavity of the dimensions given above and operating at 3 cm., and for a and d equal to $0 \cdot 1$ cm., the percentage changes in Q with respect to the unloaded Q for the two modes are inappreciable. But if a and d are increased to 1 cm. for the same cavity, the percentage changes in Q are approximately $0 \cdot 23$ and $4 \cdot 4$ for TM_{11n} and TE_{01n} modes respectively as calculated from expressions (21) to (24).

From expressions (22) and (24), it may be shown that for the cavity having dimensions given above, the unloaded Q for the TM_{11n} mode is only 44% of the unloaded Q for the TE_{01n} mode. The Q for the TM_{11n} mode may be further decreased for a cavity desired to operate in the TE_{01n} mode by placing a dissipative material at a suitable position of the cavity.

References

1. Chatterjee, S. K. .. *Jour. Ind. Inst. Sci.*, 1952, under publication.
2. Hansen, W. W. .. *Jour. Appl. Phys.*, 1938, **9**, 654.
3. Bourgnis, F. .. *Ann. Phys.*, Lpz., 1939, **35**, 359.
4. Condon, E. U. .. *Jour. Appl. Phys.*, 1941, **12**, 129.
5. Bethe, H. A. and Schwinger, J. *N.D.R.C. Rep.*, D 1–117, March 1942.
6. Banos, A. .. *M.I.T. Rep.*, RL–630, November, 1944 ; RL–631, November 1944.
7. Kahan, T. .. *C.R. Akad. Sci.*, 1945, **221**, 536 and 694.
8. Bernier, J. .. *L'onde Electrique*, 1946, **26**, 305.
9. Cunliffe, A. and Mathias, L. E. S. *J. I. E. E.*, 1950, **97**, Part III, 367.
10. Kinzer, J. P. .. *B. T. L. Case*, 23458–5, January 8, 1943.