SUPERSONIC VELOCITY IN GASES AND VAPOURS.

PART III. VOLUME RESONANCE IN A SUPERSONIC INTERFEROMETER.

By S. K. Kulkarni Jatkar.

INTRODUCTION.

It was shown in Parts I and II (*This Journal,* **21A**, 245, 455) that the complexity of the wave form in a supersonic interferometer, is due to sphericity of the waves, wall reactions, harmonic frequencies and diffraction. The use of a tube, whose diameter is comparable to the wave-length, in conjunction with a crystal oscillator which could be critically controlled, was found to yield satisfactory results.

In this paper the author has presented some of the startling results obtained in narrow tubes, especially when closed with apertures. It was shown previously (Fig. 6, curve *c* and Table VI, Part I, *This Journal,* **21A**, 263) that the quartz crystal was brought into oscillation at certain positions of the reflector *more than a whole wave-length* apart, in a tube which was about 1 cm. in diameter. When the experiments were carried out, in a 8 cm. tube with a 5 cm. reflector, the average spacing between the principal peaks was also *more than a whole wave-length* [Fig. 6, curve *a,* (*loc. cit.*)]. When the size of the reflector in the same tube was decreased to 1 cm., the average distance between the peaks was about 4.5 mm., instead of 3.46 mm. which is the ordinary value of half wave-length in air at the frequency 49.47 Khz. [Fig. 6, curve *b,* (*loc .cit.*)]. In all these experiments the oscillating face of the crystal, which was a rectangle 22 × 3 mm., was covered with a thin plate of microscope cover glass about 16 mm. in diameter. It was, therefore, considered necessary to find out the effect of varying the diameter and also the size of the neck of the tube.

EXPERIMENTAL.

The experimental arrangement was the same as that used in Part I. Two crystal oscillators were used in these experiments, which were 54 × 22 × 3 mm. and 55 × 14 × 4 mm., having frequencies of 49.47 and 48.56 Khz. respectively. The details of the measurements of the frequencies will be published in a future communication. Owing to the diminished reaction of the system when the tube was closed with a narrow aperture, the maximum sensitiveness of the crystal-valve arrangement, described previously, was used. The following tables show the screw readings for the positions of the reflector at which maximum reaction was shown, when the tube was closed with apertures 1–2 mm. in diameter. Some of the typical curves are shown in Fig. 1, which are not all reduced to the same scale. Most of the measurements were carried out at room temperature (23°–26°).

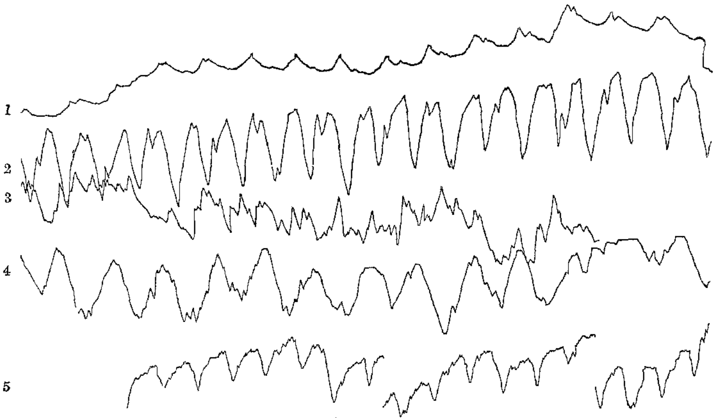

FIG. 1.

TABLE I.

Frequency = 48·56 Khz.
Tube Diameter = 11·0 mm.

Aperture 1·4 mm.						Aperture 2 mm.	
No.	Screw reading mm.	No.	Screw reading mm.	No.	Screw reading mm.	No.	Screw reading mm.
	129·40	8	82·90	18	24·65		50·30
				19	18·90		
2	117·90	10	71·15	20	13·00	2	62·00
3	111·95	11	65·15		16 l		
4	106·40			18–2	93·25	4	73·60
5	100·40	13	53·85	19—3	93·05	5	79·41
6	94·50	14	47·90	20–4	93·40		
7	89·00	15	42·00		93·23		
		16	36·50				
				l = 5·83 mm.		l = 5·82mm	
				$\pi r^2 l = 0 \cdot 536$ c.c.		$\pi r^2 l = 0 \cdot 535$ c.c.	

Table II.

48·56 Khz

Aperture 2·0 mm.

(a) Tube diameter 11·0 mm.				(b) Tube diameter 9·5 mm.			
Subsidiary peak		Main peak		(1)		(2)	
No.	Screw reading mm.	No.	Screw reading mm.	No.	Screw reading mm.	No.	Screw reading mm.
0	18·10				31·30		30·80
		0	20·00				
1	25·10			1	38·60	1	38·40
		1	25·90				
		2	31·40	2	46·00	2	45·90
		3	37·35				
		4	42·80	3	53·40	3	53·45
		5	48·75				
6	53·70			4	61·00	4	61·00
		6	54·65				
		7	60·35	5	68·65	5	68·70
8	64·60						
		8	66·25	6	76·90	6	75·85
3	71·50						
		9	72·20	7	84·50	7	84·20
10	75·30						
		10	77·73	8	92·40	8	89·50
11	82·40						
		11	83·65	9	99·30		
		12	89·10				
		13	95·12	10	106·95		
14	100·70						
		14	101·05	11	114·60		
		15	106·70				
16	110·95			12	121·70	12	121·90
		16	112·65				
		17	117·98	13	129·65		
18	121·75						
		18	124·15	14	136·70		
19	128·85						
		19	130·05	15	144·30		
		20	135·63				
21	140·60			16	152·90		
		21	141·50				
	$10\,l$		$19\,l$		$10\,l$		$10\,l$
18–8	57·15	19–0	110·05	16–6	76·00	12–2	76·00
19–9	57·35	20–1	109·73				
		21–2	110·10				
	57·25		*109·96*				
$l = 5·725$ mm.		$l = 5·79$ mm.		$l = 7·60$ mm.		$l = 7·60$ mm.	
				mean $l = 7·60$ mm.			
$\pi r^2 l = 0·533$ c.c.				$\pi r^2 l = 0·538$ c. c.			

TABLE III.

49·47 Khz.

(a) Tube diameter 12·5 mm. Aperture 1·4 mm.						(b) Tube diameter 9·5 mm. Aperture 9·5 mm.			
25°				97°		Minor peak		Major peak	
No.	Screw reading mm.	No.	Screw reading mm.	No.	Screw reading mm.	No	Screw reading mm.	No.	Screw reading mm.
	137·45	19	56·20		106·00	*1*	128·75	2	125·50
1	133·30	20	51·95	1	100·95	*3*	121·75	4	118·30
2	128·55	21	47·45	2	96·80	*5*	114·75	6	111·00
3	124·35	22	44·10	3	92·35	*7*	107·70	8	103·95
4	120·15	23	40·00	4	86·70	*9*	100·70	10	96·95
5	115·95	24	35·15	5	79·60	*11*	93·20	12	89·70
6	111·60	25	30 65			*13*	86·10	14	82·55
		26	26·30			*15*	79·00	16	75·65
9	99·00	27	22·05	9	59·95	*17*	71·80	18	68·45
10	94·80	28	17·70	10	53·60	*19*	64·95	20	61·35
11	90·30		23 l	11	48·30	*21*	57·80	22	53·90
12	85·95	24–1	98·15	12	44·05	*23*	50·70	24	46·65
13	81·65	25–2	97·90	13	39·35	*25*	43·20	26	39·70
14	77·80	26–3	98·05	14	33·00	*27*	36·20		
15	73·50	27–4	98·10	15	27·50		10 l		10 l
16	69·40	28–5	98·25	16	22·10	1–21	70·95	2–22	71·60
17	65·00			17	16·35	3–32	71·05	4–24	71·65
18	60·45				15 l	5–25	71·55	6–26	71·30
				15–0	78·50	7–27	71·50		
				16–1	78·80				
			98·09		*78·65*		*71·26*		*71·52*
$l = 4·263$ mm.				$l = 5·243$ mm. (97°)		$l = 7·126$ mm.		$l = 7·152$	
						mean $l = 7·14$ mm.			
$\pi r^2 l = 0·522$ c.c.				$\pi r^2 l = 0·642$ c.c.		$\pi r^2 l = 0·527$ c.c.			

Table IV.

49·47 Khz.
Aperture 2·0 mm.

(a) Tube diameter 11 mm.		(b) Tube diameter 9·5 mm.					
		(1)		(2)		(3)	
No.	Screw reading mm.	No.	Screw reading mm.	No.	Screw reading mm.	No.	Screw reading mm.
	59·00		119·20		22·40		46·70
1	64·90	1	111·30	1	30·20	1	53·90
		2	104·50	2	37·20	2	61·35
3	75·90	3	96·30	3	45·00	3	69·20
4	81·60	4	89·25	4	51·70	4	76·50
5	86·90	5	82·30	5	59·65	5	84·00
6	92·50	6	75·35	6	67·00	6	91·60
7	97·70	7	68·30	7	73·70	7	98·50
8	103·55			8	81·40	8	106·70
9	109·20			9	89·10		
10	114·65	10	45·50	10	96·00		
		11	37·50	11	103·00		
		12	30·40	12	109·90	12	135·00
		13	23·40	13	117·90	13	143·00
		14	16·30	14	123·80	14	150·50
	$5\,l$		$10\,l$		$10\,l$		$10\,l$
8–3	27·65	10–0	73·70	10–0	73·60	14–4	74·00
9–4	27·60	11–1	73·80	11–1	73·80		
10–5	27·75	12–2	74·10				
	27·67		*73·87*		*73·70*		
$l = 5{\cdot}534$ mm		$l = 7{\cdot}387$ mm.		$l = 7{\cdot}37$ mm.		$l = 7{\cdot}40$ mm.	
		mean $l = 7{\cdot}38$ mm.					
$\pi r^2 l = 0{\cdot}525$ c.c.		$\pi r^2 l = 0{\cdot}522$ c. c.				$\pi r^2 l = 0{\cdot}523$ c.c.	

Table I (*a*) gives the results of measurements in a tube 11 mm. diameter and having a 1.4 mm. aperture, at 48.56 Khz. The average distance between the peaks is 5.83 mm. The value for the distance between the peaks in a pipe resonator, *i.e.*, for half wave-length, at this frequency, is 3.56 mm. Measurements were recorded after increasing the aperture to 2 mm., some of which are given in Table I (*b*). Although the average distance between the peaks was nearly the same, *viz.,* 5.82 mm. the peaks were inverted (curve 2, Fig. 1), showing that the presence of a resonator increased the intensity of oscillation of the crystal. The volumes corresponding to these lengths [Table I, (*a*) and (*b*)] are 0.536 and 0.535 c.c. respectively.

Table II (*a*) shows the measurements which were carried out under similar conditions on a different occasion. Two sets of peaks were obtained. The readings for the subsidiary and the principal peaks are given side by side in the table. It will be seen that the subsidiary peaks lag behind the main peaks by 1 mm. which is roughly equal to the length of the neck. The average distance of the principal peaks is about 5.79 mm. The volume for this length is 0.533 c.c., which shows that resonance occurs when the distance between the peaks is equal to a fixed volume.

Attempt was then made to see if the values of the apparent lengths varied by changing the diameter of the tube to 9.5 mm. leaving the aperture the same, *viz.,* 2 mms. As the reaction current was considerably decreased, the valve circuit was modified by applying 2 volt grid bias through 5 meg. grid leak and by applying 80 volts to the anode. A resistance of 20,000 ohms was shunted across the tuned anode, and adjusted until the harmonic frequency of the crystal could not be heard on a heterodyne radio receiver and the reaction current at resonance, was increased to 50–70 micro amps. The positions of the major peaks in two experiments are given in Table II (*b* 1, 2). The average value for wave-length in both experiments is 7.60 mm., which again gives 0.538 c.c. as the value for the integral volumes, in agreement with the results obtained previously.

Experiments were next conducted with the quartz oscillator having a frequency 49.47 Khz. Table III (*a*) shows the positions of the peaks observed in a tube 12.5 mm. diameter and having an aperture of 1.4 mm. The average distance between the peaks is 4.26 mm. at 25°, which yields a volume of 0.522 c.c. At 97° the average distance between the peaks increased to 5.24 mm., the corresponding volume being 0.642 c.c.

Table III (*b*) shows the results for the positions of the alternate peaks, which were high and low, in a tube of 9.5 mm. *without an aperture.* The average distance between minor peaks and also between major peaks is 7.14 mm., which gives a volume of 0.527 c.c. The decreased volume is due to increase in frequency.

Table IV (*a*) (curve 5) shows the positions of the principal peaks observed in a tube 9.5 mm. in diameter, and 2 mm. aperture. The average distance between the peaks is 5.53 mms., which gives a volume of 0.525 c.c. in agreement with the average values given in Table III.

Table IV (b, 1 and 2) shows the results of the readings of the peaks observed in a tube 9.5 mm. in diameter, and 2 mm. aperture. The reaction current was considerably reduced and the maximum sensitiveness of the arrangement was employed. The average distances between two peaks of the same intensity are about 7.38 mms. (curve 1). The position of the crystal was 8.00 mm. and the position of the end of the tube was at 9.00 mm. The calculated volume between two peaks is 0.522 c.c.

Table IV (b, 3) shows similar measurements when the position of the crystal was moved 1 mm. away from the tube. The spacing of the principal peaks was about 7.40 mms. giving a volume of 0.523 c.c.

Curve 4 shows the results obtained in a tube 6.4 mm. diameter with 2 mm. aperture. The average distance between the peaks, which are rather flat, is 16.2 mm. The volume is again 0.522 c.c.

The results for the various experiments on volume resonance are summarised in Table V.

TABLE V.

Summary of the Results for Volume Resonance.

Temperature 23°–26° ; * 97°.

Table No.	Tube diameter cm.	Aperture cm.	49·47 Khz.		48·56 Khz.	
			l cm.	$\pi r^2 l$ c.c.	l cm.	$\pi r^2 l$ c.c.
III (a) ..	1·25	0 14	0·426	0·522		
III (a) ..	,,	,,	·524*	·642*		
IV (a) ..	1·1	·2	·553	·525		
I (a) ..	,,	·14	..		0·583	0·536
I (b) ..	,,	·2			·582	·535
II (a) ..	,,	,,			·579	·533
VI (Part I) ..	0·97	·97	·707	·522		
IX (Part I) ..	,,	,,	·708	·522		
IV (b, 1, 2) ..	·95	·2	·738	·522		
IV (b, 3) ..	,,	,,	·740	·523		
II (b) ..	,,	,,			·760	·538
III (b) ..	,,	·95	·714	·527		
Curve 4 ..	·64	·2	1·62	·522		
				0·522		*0·535*

The values of l in an open tube of 0.97 cm. diameter, is the average value of the distance at which the crystal was brought into oscillation as given in Table VI, Part I (*This Journal,* **21A**, 265).

The results show that the average distance, at which principal peaks are observed in a tube which is either open or closed at one end with an aperture, goes on increasing as the diameter of the tube is decreased. Resonance is obtained when the volumes (V_0) have changed by integral numbers of 0.522 c.c. and 0.535 c.c. respectively, which are inversely proportional to the square of the two frequencies within 1%. This observation indicates that we are here dealing with *a new form of a volume resonator*. The conductivity of the *neck* (K) is apparently independent of the diameter of the tube, the value at 49.5 Khz. being 42.3 at 25°. The results which are marked with * were obtained when the tube was heated to 97°, at which temperature the calculated value for the conductivity is about 42. These results show that the value for the conductivity of the neck does not depend on its dimensions, as in the case of the Helmholtz resonator. The frequency of the latter is given by the expression $f_0 = \frac{c}{2\pi}\sqrt{\frac{K}{V_0}}$ where f_0 is the frequency, c is the velocity of sound, K is the conductivity of the neck and V_0 is the volume of the resonator.

As shown in Part I (*This Journal,* 1938, **21A**, 245–71) the apparent internodal distance in air at 50 Khz., was 1.3 times the ordinary half wave-length, when experiments were carried out in a tube of 8 cm. diameter, the size of the reflector being 1 cm., and that of the radiator face, being 1.6 cm. (the approximate diameter of the microscopic cover glass which was stuck on the face of the crystal, was not 1 cm. as erroneously reported in Part I). Similar results have now been obtained in a tube 1.25 cm. in diameter, closed with an aperture of 1.4 mm. The significance of these results will be discussed in a future communication dealing with the theory of the volume resonator.

When the experiments were conducted in a tube of 1 cm. diameter under similar conditions, the average spacing between the peaks was more than double the ordinary $\lambda/2$. Similar results were obtained when the crystal was on the critical side of oscillation, which was strengthened at positions of the reflector *more than a whole wave-length* apart. The crystal was not vibrating at a sub-fundamental or lower frequency in these experiments, as a careful search by heterodyne receiver failed to detect any such oscillation.

The phenomenon might appear to be a case of "sub-harmonics in forced oscillations in coupled dissipative systems", as found in the case of moving coil loud-speakers (Pedersen, 1933). This effect might, indeed, explain the results of Hitchcock (*Proc. Inst. of Radio Eng.,* 1927, **15**, 956) who observed a velocity *double the ordinary* in the electrode gap, where the necessary factors for producing sub-harmonics, such as closed coupling, increased damping due to the electrode gap, and higher power of the oscillator, might have been favourable for relaxation oscillation. In the present investigation, the loose coupling between oscillator and resonating tube and the very small power in the oscillator, precluded the production of any sub-harmonic frequencies. It is obvious that we are here faced with an entirely new version of the Helmholtz resonator working at harmonics of the volume. The fortituous approximate coincidence of the apparent internodal distances with *the wave-length* in the original experiments, is due to the diameter of the tube (0.97 cm.) and the frequency of the crystal (49.47 Khz.) with which the original experiments were conducted.

It was also mentioned in Part I (*loc. cit.*) that *in an open tube* of 0.97 cm. diameter, the effect of minor changes in the oscillator circuit, such as the variation of the grid leak or the tuning condenser, made profound changes in the spacing of the peaks, and that *alternately high and low values* for wave-lengths were obtained, the extent of the peaks being smaller in the lower spacings. The average distances between alternate peaks in the lower spacing is 7.04 mm. while the value for the average spacing of the major peaks is 7.13 mm., yielding a mean average of 7.08 mm. under one set of experimental conditions (*cf.* Table IX A, p. 268, Part I). With increased anode coupling these values were 6.92 and 7.08 mm. respectively (Table IX B, *loc. cit.*). As 6.92 mm. is the normal *wave-length* under these conditions, it appears that an open tube resonates both when the piston displacements correspond to integral number of half wave-lengths corresponding to half the distances between the smaller peaks, and also when its volume is changed by the integral number of 0.522 c.c. corresponding to the average distance of 7.08 mm. between major peaks, in a tube 0.97 cm. in diameter.

It is obvious that the volume resonance occurs at *approximately* the same interval length in a tube of about 1 cm. diameter, as the value of the *wave-length.* This fact not only accounts for the high and low values of the peaks which were observed by the author, throughout his work, but also, for the distortion of the wave form, when the phases of the volume resonance and pipe resonance, are different. The results show that a critically damped crystal oscillator and narrow apertures, favour the occurrence of volume resonance. In an open tube both forms of resonance can occur.

It is therefore important that we should select tube diameters which are not only comparable with wave-length but are such that they will not yield resonating volumes, when the lengths of the tube are changed by integral number of half wave-length. Otherwise 'satellites' due to the superposition of the respective peaks will be observed.

The usual formula for the frequency of the Helmholtz resonator, $f_0 = \frac{c}{2\pi}\sqrt{\frac{K}{V_0}}$ is based on the assumption that the dimensions of the resonator and the neck are small as compared with the wave-length of sound. It is not possible, therefore, to compare the results presented in this paper with the values deduced from the usual formula of the resonator. When the tube is closed at one end by a piston and communicates to an oscillator through a small hole, the usual formula for a closed pipe should hold good and resonance ought to be observed at successive positions of the piston which are $\lambda/2$ apart. Such, however, is not the case. It appears from the results presented in this paper that, tube, whose dimensions are many times the wave-length, resonates when the piston displacements correspond to integral number of equal volumes, which are related to the velocity of sound c and frequency f_0, by the usual formula of the Helmholtz resonator, but which are independent of the conductivity of the neck.

The observations recorded in this paper were made by the author in 1932, and a preliminary note has been published in the *Proceedings of the Indian Science Congress* (1933, p. 117). A simple theory of a resonator working at 'harmonics' of volume will be published in a future communication.

Further work is in progress.

SUMMARY.

Working with tubes of diameters 1.25, 1.1, 0.97, 0.95 and 0.64 cm. and with varying apertures, the distances between consecutive positions at which maximum reaction was shown at 49.47 Khz., was 0.426, 0.553, 0.708, 0.738 and 1.62 cm. respectively at 25°. The corresponding volumes are 0.522 c.c. for each tube, indicating that points of resonance depend on the integral volumes rather than on the wave-length, with a constant value for the conductivity of the neck.

At 48.56 Khz. the resonating volumes are integral number of 0.535 c.c. showing that the volumes are inversely proportional to the squares of the frequencies, as required by the formula of the Helmholtz resonator.

When the neck of the resonator was changed from 0.14 to 0.2 cm. the resonance peaks were inverted, showing that the presence of a tuned resonator increased the intensity of oscillation of the crystal.

The average interval (7.07 mm.), observed between two consecutive positions at which the quartz oscillator was brought into oscillation, and that (7.08 mm.) between two major peaks, observed (*cf.* Part I) in an open tube of 0.97 cm. diameter when the oscillations of the crystal (49.47 Khz.) were critically controlled, are due to volume resonance.

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