PHASE TRANSFORMATION AND PHASE BALANCING.

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INTRODUCTION.

The general term "*Phase Transformation*" may be regarded as including in its widest sense two distinct classes of problems, *viz.*, (1) the passage from a system of polyphase currents either to another polyphase system with a different number of phases, or to a single-phase system (and *vice versa*); (2) the change of phase of either a polyphase or a single-phase alternating current.

Both these classes of problems have since the earliest days of polyphase currents engaged the attention of numerous inventors, and many ingenious solutions have been proposed from time to time.

Closely connected with phase transformation is another problem, *viz.*, that of Phase Balancing. This latter problem has recently attracted a great deal of attention, owing mainly to the increasing use of single-phase electric furnaces and single-phase railways

TRANSFORMATION OF ONE SYSTEM OF POLYPHASE CURRENTS INTO ANOTHER CONTAINING A DIFFERENT NUMBER OF PHASES.

The numerous possible solutions of this problem may be grouped under two heads, *viz.*, those utilising stationary apparatus and those in which running machinery is employed.

A. *Methods depending on the use of stationary apparatus:—*

(*a*) S. P. Thompson's method. The first suggestion of this, one of the most general methods of passing from any number of phases to any other number, is due to the late Professor S P. Thompson*. As originally described by the inventor, the core of the phase transformer consisted of two cylindrical portions separated from each other by an air-gap, like the stator and rotor

*S. P. Thompson Polyphase Electric Currents, 2nd Edition (1900) Page 332.

of an induction machine. If the outer core is provided with two independent Gramme ring windings, and if one of these is supplied with a polyphase system of currents by connecting the polyphase supply mains to suitable points in the winding, then another polyphase system of any desired number of phases may be obtained from the second winding. It is obvious that instead of ring windings, drum windings or polyphase windings of any type may be used. Further improvements are also immediately obvious: there being no need to provide an air-gap, as there is no relative motion of the central core and the outer ring, this gap may be entirely suppressed, which will greatly reduce the magnetising current and improve the power-factor of the phase transformer. The windings would then be contained in tunnels in the core. This latter arrangement is described in a recent French patent.* It is evident that in certain cases instead of two distinct windings a single winding might be employed, to which connections are made at suitable points. The arrangement then becomes a phase auto-transformer. The voltage ratio of the two systems is easily determined from the ratio of the corresponding chord lengths in the circular topographic diagram of the winding, in a manner similar to that employed in connection with rotary converters.

In Fig. 2 are shown the results obtained with a small induction motor (whose rotor was locked) used for three to two phase transformation for both non-inductive and inductive loads on the secondary. The motor had a wound three-phase rotor and a two-phase stator winding. Details of the connections used are shown in Fig. 1. As might have been expected from the presence of the air-gap, both the voltage regulation and the power-factor of the primary winding are very poor, even with a non-inductive load. When the load is highly inductive, the voltage regulation is somewhat worse than for a non-inductive load, and the power-factor is, of course, uniformly low.

Fig. 3 gives the results obtained with the same motor for a non-inductive load when the transformation was from two to three-phase currents.

A further example of the use of this method is given in Fig. 5, which shows the results of tests on a small (5 kw.) converter used as a stationary auto-transformer for converting three-phase to two-phase currents. The corresponding arrangement of connections is shown in Fig. 4. This machine had laminated

* Brevet Francais No. 483. 749 of 1917 See Revue Générale de l'Électricite', Vol. 5, p. 130 (1919).

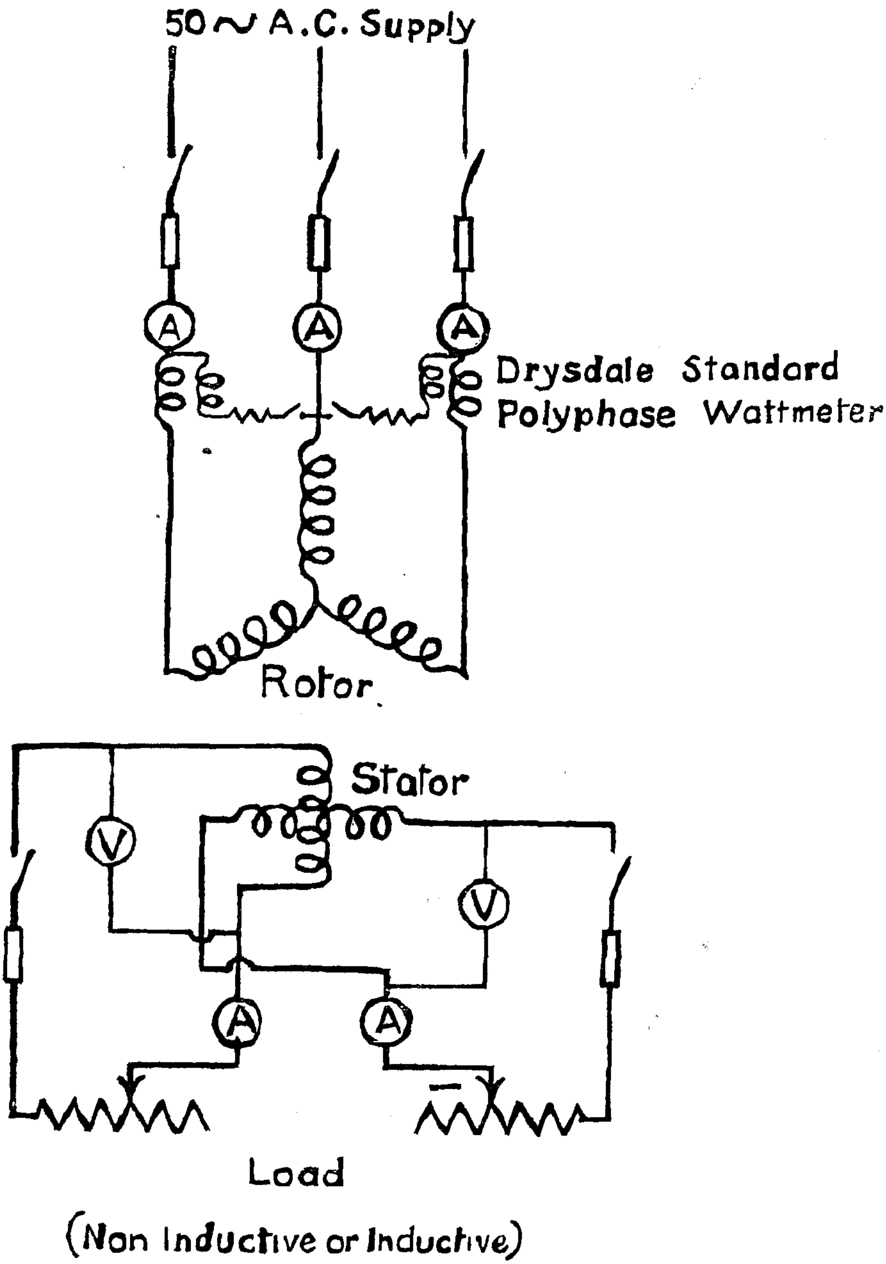

Fig. 1. Connection of Induction Motor used as three-phase to two-phase Stationary Transformer.

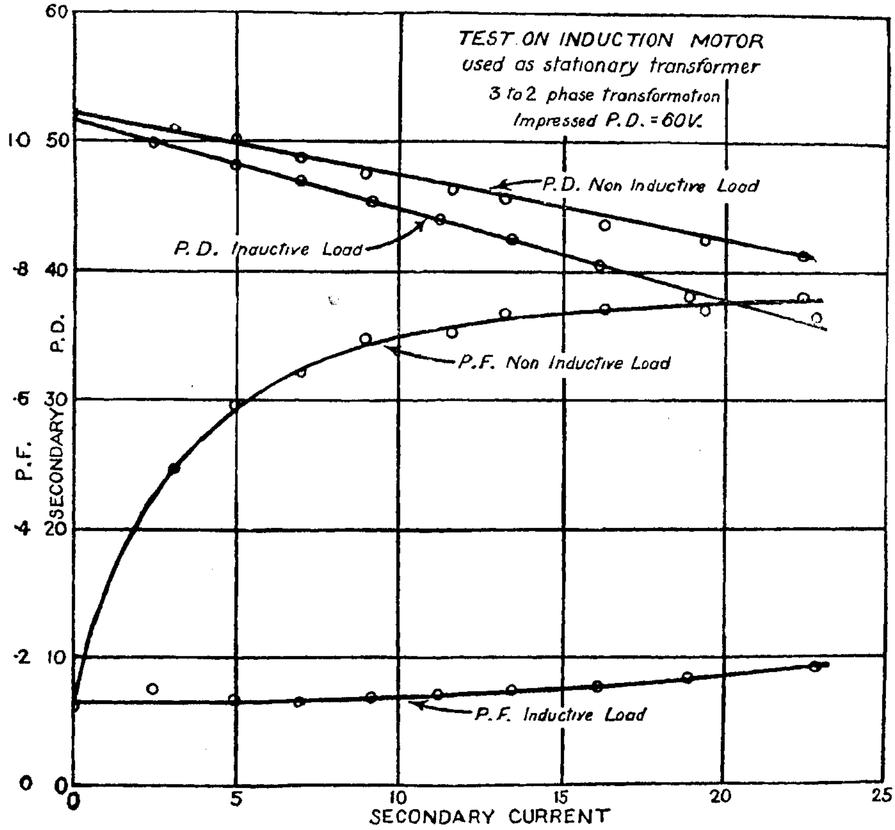

Fig. 2. Results of Tests on Induction Motor used as Stationary Transformer.

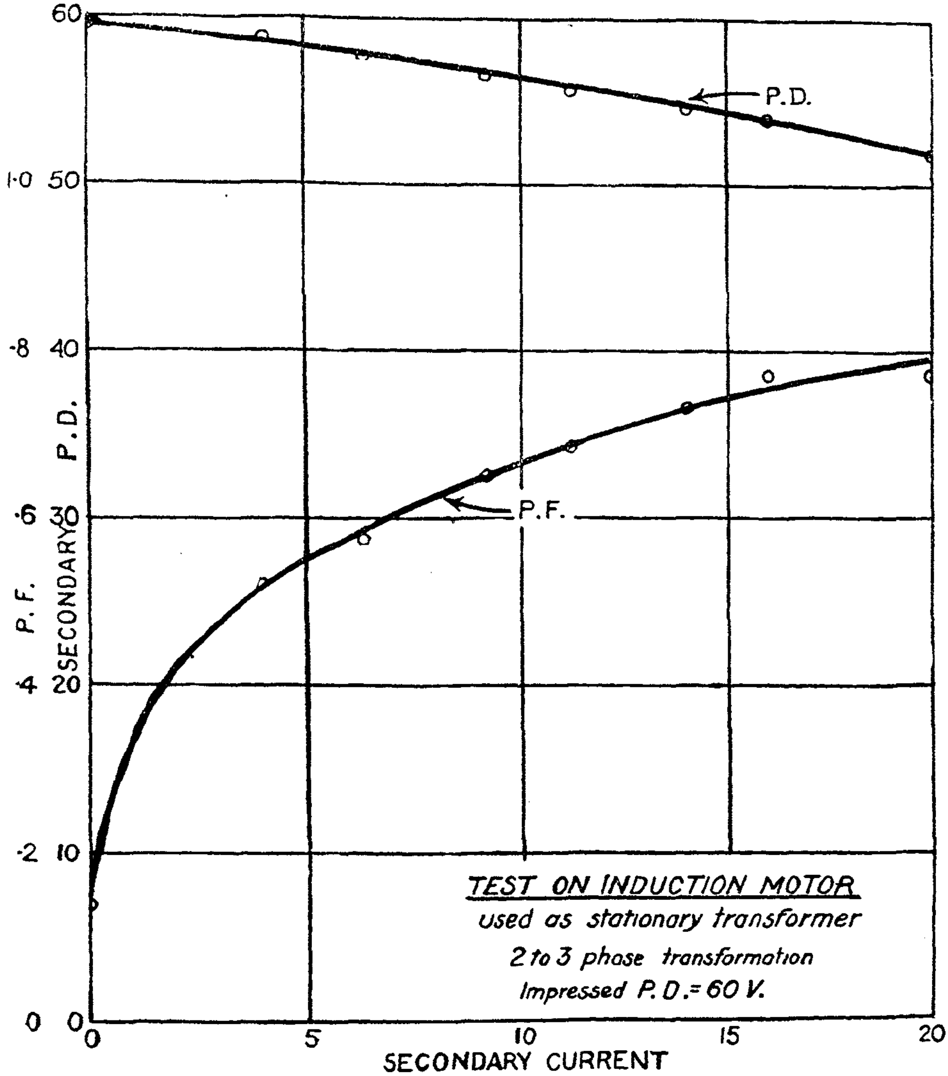

Fig. 3. Results of Tests on Induction Motor used as Stationary Transformer.

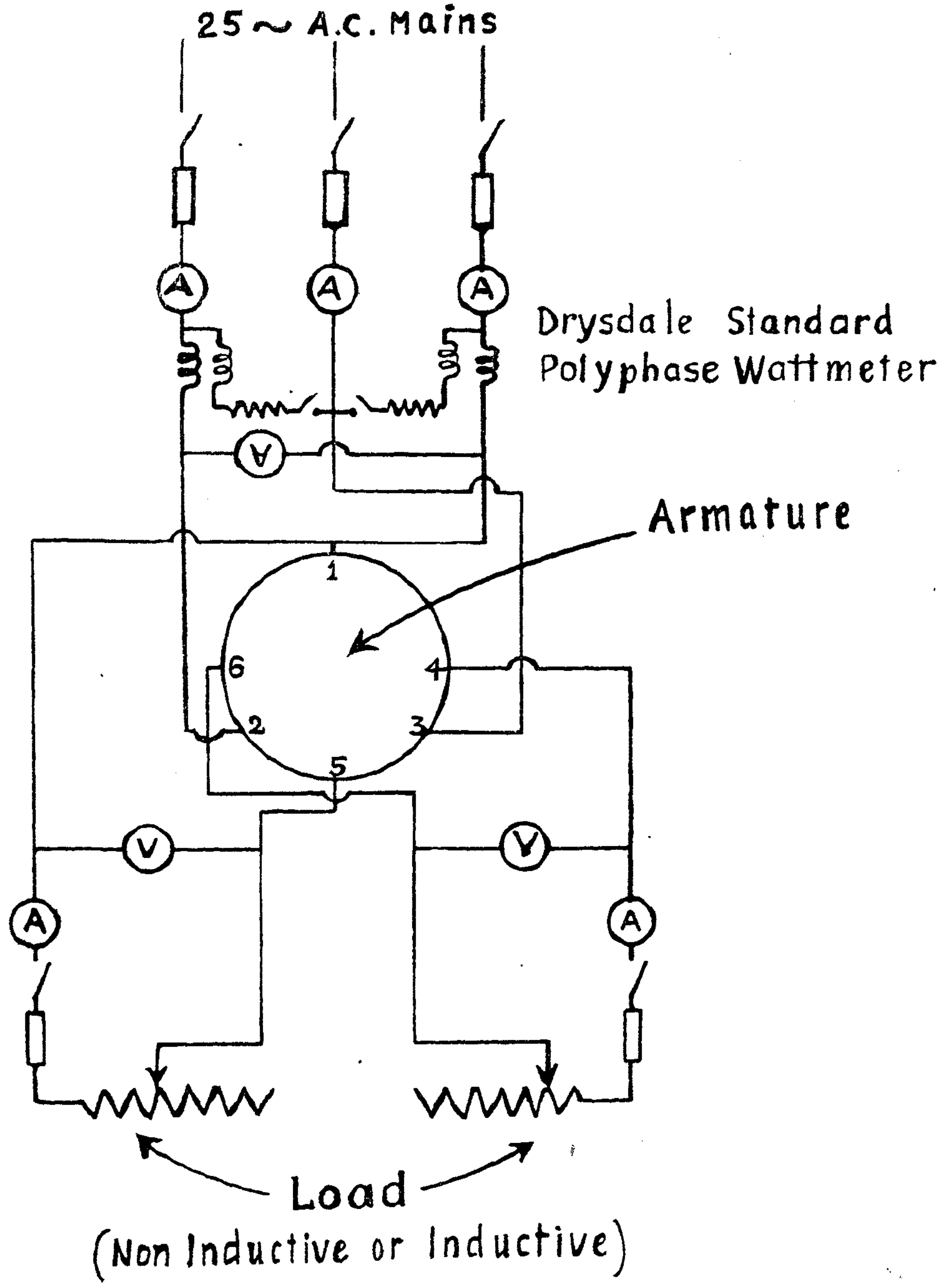

Fig. 4. Connections of Armature of Rotary Converter when used as Stationary Auto-transformer.

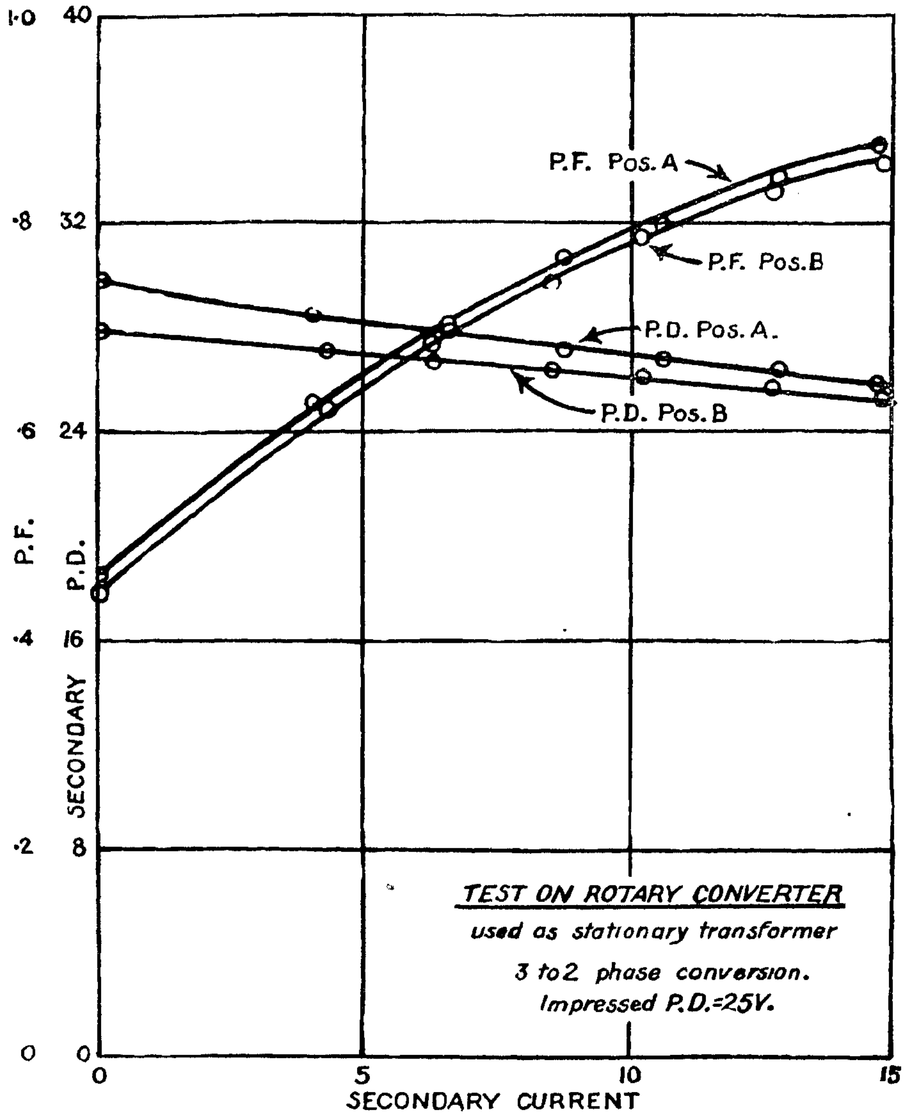

Fig. 5. Results of Tests on Rotary Converter used as Stationary Auto-Transformer.

poles, but a solid yoke and solid reversing-pole cores. The armature was fitted with slip rings for both two and three-phase currents. During the experiments the brushes were lifted off the commutator. The results were found to depend to some extent on the position of the armature relatively to the field, and the curves of Fig. 5 refer to the two extreme positions. As will be seen, considering the small size of the machine, the voltage regulation is fairly good, while the power-factor, though necessarily low at light loads, rises as high as .87 at the higher loads. The load was a non-inductive one.

The method just described has not hitherto been generally used in practice, and the reason is not far to seek : it requires the use of transformers of special construction, the cost of which is considerably higher than that of ordinary transformers. In most practical cases we have to deal with the passage from three to two phases or *vice versa,* and in such cases the transformation may be effected more simply by the Scott method, in which power transformers of ordinary construction are employed.

(*b*) Scott's Method. This method is applicable to the special case of three-phase to two-phase transformation, or *vice versa.* The method is sufficiently well known not to require any description. Attention may, however, be drawn to one point connected with this method, *viz.*, that the utilisation of the materials in one of the two transformers required is not as advantageous as in transformers used under ordinary conditions. For the sake of brevity, we shall denote the transformer which has no tapping (or in which an 86·6 per cent. tapping is used) by T_o, and the transformer with a mid-point tapping by T_t. As regards T_o, this works under conditions precisely similar to those in any ordinary transformer. The transformer T_t, however, works under less favourable conditions, as will be evident from the following considerations. The number of turns in the winding of T_t is $\frac{2}{\sqrt{3}}$ times that in T_o (or the working part of the winding of T_o) and since the current in every part of the winding of T_t has the same r. m. s. value (assuming a balanced load) as the current in T_o, it follows that if we were to use conductors of the same cross-section in the windings of T_t and T_o the loss in the tapped winding of T_t would be $\frac{2}{\sqrt{3}}$ times that in the corresponding winding of T_o. In order to equalise the copper losses in the two transformers the cross-section of the conductor in the tapped winding would have to be increased $\frac{2}{\sqrt{3}}$ times, so that for equal copper losses the amount of copper in the tapped winding of T_t would have to be $\left(\frac{2}{\sqrt{3}}\right)^2$ or $\frac{4}{3}$ that of the corresponding winding of T_o. This increase is due to the fact that whereas in T_o the ampere-turns of the various sections of the winding are in phase with each other, in T_t the ampere-turns of one half of the winding are out of phase with those of the other, and hence a larger arithmetical number of ampere-turns must be provided.

The Scott system therefore involves a larger outlay on transformers than would be necessary in the case of transformers for ordinary purposes.*

(c) This method is of limited application, enabling us to effect one particular kind of transformation, *viz.*, a doubling of the number of phases of a system originally containing an odd number.

In order to arrive at a clear understanding of the principle of the method, let us suppose that we have a system of $(2n+1)$ phases, and that the primaries of $(2n+1)$ single-phase transformers are connected to the system. The $(2n+1)$ secondaries may be connected in any suitable way. A simple mesh or star connection would give us a $(2n+1)$ phase system on the secondary side. Suppose, however, that we connect the middle points of all the secondary windings to a common or neutral point Consider any secondary winding and assume that directions away from the middle point are reckoned positive, as shown by the full line arrows in the circuit diagram of Fig. 6 (*a*). Let us draw the corresponding vector diagram, taking the instant at which the e. m. f. in the winding is at its maximum value, and has the direction from B to A as shown by the dotted arrow in Fig. 6 (*a*). Then in accordance with the assumptions made, the e. m. f. in the half OA of the winding is at its maximum positive value, and the vector representing it will occupy a vertical position with the arrow head pointing upwards, as shown by OA in Fig. 6 (*b*). But the e. m. f. in the half OB is opposed to the direction assumed as positive for this half of the winding, and will therefore be represented by the vector OB in Fig. 6 (*b*). We may thus regard the entire winding AB as formed of two phases, OA and OB, with the e. m. f. s in phase opposition (directions away from O being taken as positive). Supposing all the secondary windings to be connected in this manner, and taking

*The Woodbridge system (*General Electric Review*, Vol. 22, p. 1040) is characterised by greater economy of material, but is not generally used on account of a number of counterbalancing disadvantages.

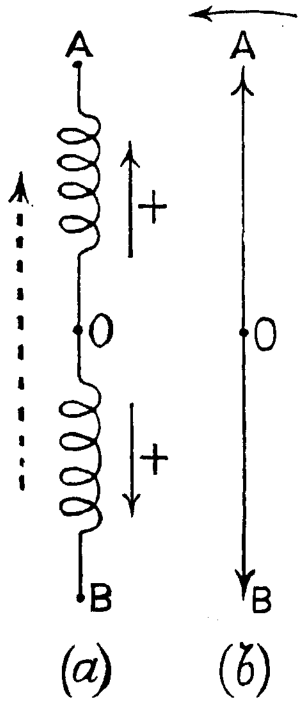

Fig. 6. To illustrate method of doubling the number of phases.

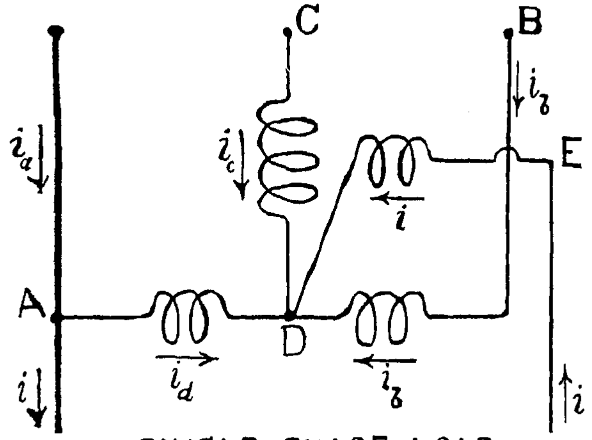

Fig. 7. Quadrature Phase Converter.

directions away from O as positive, it is obvious that in the vector diagram we shall have two opposite vectors for each winding. Now if in the vector diagram of the original system we had an odd number $(2n+1)$ of equally spaced vectors, then the result of reversing each vector will be to double the total number of vectors, the new or reversed vectors falling exactly half way between the original vectors. This shows that the new system is a star connected system of $2\,(2n+1)$ phases.

If, however, we start with an even number of phases, then the reversal of each vector for such a system does not give rise to a new vector, but yields a vector which is coincident with one of the existing vectors. The method is therefore inapplicable to systems containing an even number of phases.

The best known and widely used application of this method is to be found in the transformation of a three-phase into a six-phase system, as exemplified by the diametral method of connecting six-phase converters to three-phase mains. It is unnecessary in this case actually to connect the middle points of the transformer secondaries to a common point, but virtually the six terminals of the secondary windings represent the terminals of a star connected six-phase system.

We could not, it may be noted, use the same method for passing from a six-phase to a twelve-phase system in order to supply a twelve-phase converter, since, as already explained, the method is limited to systems with an odd number of phases.

(*d*) This method, although not used to any appreciable extent in practice, is interesting as being even more general than method (*a*); for it enables us, by the use of ordinary transformers, to build up a polyphase system of any desired number of phases from a source of two alternating voltages which differ in phase. Let us suppose that we have two transformers, the primaries of which are supplied at the given voltages. In order to obtain a secondary voltage of any given magnitude and phase, we have merely to use suitable components of the two secondary voltages. The phase difference between the secondary voltages being fixed, the directions of the two components are also determined, and by varying the magnitudes of the components we can obtain a resultant of any desired magnitude and phase. It might in some cases be necessary to reverse the phase of one of the components in order to obtain the desired result. Suppose now that we wish to obtain a polyphase system of secondary voltages. Then it will in general be necessary to provide as many secondaries on each transformer as there are phases in the required system, and the winding of each phase will consist of a pair of secondaries connected in series, one belonging to each transformer. The turns of the two secondaries are so adjusted as to give the required magnitude and phase of the secondary voltage corresponding to the given phase of the required polyphase system.

Scott's method may be regarded as a special application of this more general method of phase transformation.

B. *Methods making use of Rotating Machinery.*

(*a*) *Motor-Generator Method.* Although a possible method. this is so obviously inferior, as regards first cost, maintenance and efficiency, to the methods considered under A that it would ordinarily never be used.

(*b*) *Converter Method.* In this the power winding of the machine would consist of two entirely independent polyphase windings, or of a single winding of the continuous current type, to suitable points of which connections would be made. It is evident that any continuous current machine could be adapted for use as a converter of this type by providing it with a suitable double set of slip-rings.

Although superior to the Motor-Generator Method as regards first cost and efficiency, this method would never be normally used, and for the same reasons.

It is quite possible, however, that in cases of emergency one or other of these two methods might be found convenient on occasion.

Transformation of a Polyphase System of Currents into a Single-Phase System.

This problem is one of great practical interest, and admits of a large variety of solutions.

(*a*) One of the most obvious and simplest methods of effecting this transformation is to utilise two of the mains of the polyphase system. No apparatus is required for this purpose and the method is, of course, in everyday use. Owing to the extreme simplicity of the method no other methods would ever be employed, were it not for the fact that when the single-phase load to be supplied is large in comparison with the output of the station, a serious unbalancing of the polyphase voltages takes place. In such cases one or other of two remedial measures must be resorted to: either a different method of transformation must be adopted, or else the unbalancing of the phases must be corrected by the use of suitable apparatus.

(*b*) Motor-Generator Method. This method has the great advantage of taking a balanced load from the polyphase system and of enabling the power-factor of this load to be adjusted to a value approaching unity. The main objection to the method is the high cost of the Motor-Generator.

(*c*) The polyphase and single-phase windings, instead of being wound on two independent stators as in method (*b*), may be concentrated on the same stator. This would result in a reduction of weight and first cost as compared with (*b*). To render such an arrangement effective, special means must be provided to suppress the effects of the single-phase armature reaction, so as to enable the machine to draw a balanced load from the three-phase mains. This method has recently been developed by Prof. Miles Walker, and a detailed description of the special type of converter designed by him for use with a single-phase furnace will be found in his paper on *The Supply of Single-Phase Power from Three-Phase Systems**

(*d*) A *quadrature phase converter* may be used for obtaining a single-phase load from a three-phase system without any unbalancing of this latter. The action of a quadrature phase converter is dealt with in a later section of this paper. Here it will be sufficient to state that such a phase converter enables us to shift the phase of a voltage (or current) by 90°. The arrangement about to be described, which is in actual use in the United States, is shown diagrammatically in Fig. 7. Across two of the three-phase mains is connected an auto-transformer ADB, and between the midpoint D of its winding and the remaining three-phase main C is introduced the primary of the quadrature phase converter. The secondary DE of the converter is connected in series with AD (or BD), and the single-phase load is supplied across AE (or across BE).

When there is no load, the auto-transformer windings AD, BD and the primary CD of the converter draw certain magnetising currents from the mains. Since when the load is switched on there is no very great change in the supply voltages, we may approximately assume that the *resultant ampere-turns on the auto-transformer and converter remain constant under all conditions of load.* From this it follows that the load currents which become superposed on the magnetising currents must be

*Journal of the Institution of Electrical Engineers, vol 57, p. 109.

such that their magnetic effects neutralise each other, both in the auto-transformer and the converter.

It will conduce to simplicity of treatment if in what follows we confine our attention to the *load* currents, and leave out of consideration the magnetising current system. Let the instantaneous load currents be represented as in Fig. 7. Then we must have

$$i_a = i_d + i \quad \dots \quad \dots \quad (1)$$

$$i + i_c + i_d + i_b = 0 \quad \dots \quad (2)$$

Again, D being the mid-point of the auto-transformer winding, the number of turns in AD equals that in DE, and since the magnetic effects of the load currents in AD and DE must neutralise each other, we must have

$$i_b = i_d \quad \dots \quad \dots \quad (3)$$

Similarly, if we denote by S_1 the number of turns in the primary DC of the converter, and S_2 that in the secondary, we must have

$$I_c = \frac{S_2}{S_1} I \quad \dots \quad \dots \quad (4),$$

where I_c and I denote the r. m. s. values of the currents concerned.

We also know that, as a result of the action of the converter, i is in phase quadrature with i_c.

It is well known that in a balanced three-phase system the total instantaneous power is constant throughout a cycle. Now the power supplied to the single-phase load comes partly from AD, and partly from CD. From the topographic voltage diagram of Fig. 8 it is evident that the voltage in DC is in phase quadrature with that in AD; and since the voltage in DE is, by the action of the converter, in quadrature with that in CD, it follows that the voltage vectors of AD and DE are coincident in direction, so that each of them is displaced by the same angle from the current vector of i. Hence the amounts of power contributed by AD and DE are proportional to their voltages, and the instantaneous power contributed by AD to the load is in phase with the instantaneous power contributed by DE. But by the converter action the instantaneous power in DC is in quadrature with that in DE, and therefore also with that contributed to the load by AD. Hence the total power passing into the load is

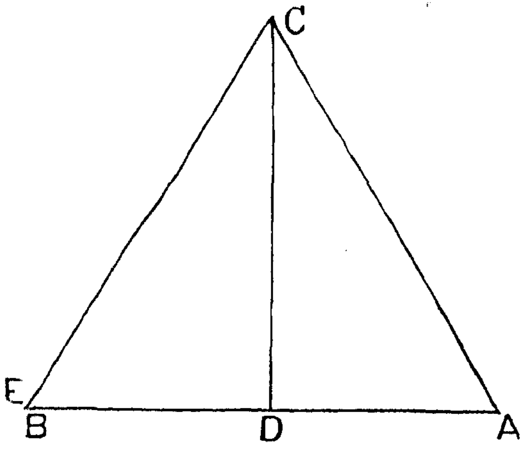

Fig. 8. Topographic diagram corresponding to Fig. 7.

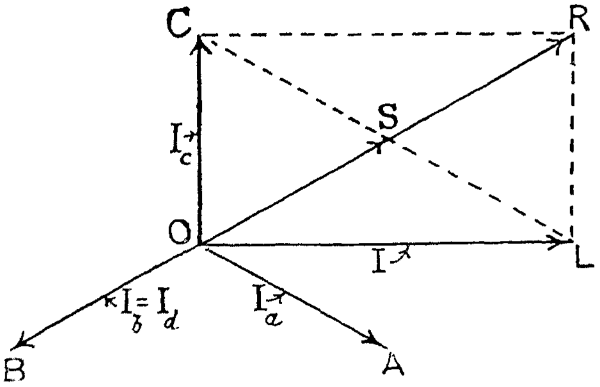

Fig. 9. Diagram of load currents.

made up of the two instalments of power contributed by AD and DC, which are in phase quadrature; and if the total power from the three-phase mains is to remain constant throughout the cycle, the two quadrature components of it in AD and DC must be equal. We have already shown that the contributions of power corresponding to AD and DE are proportional to their voltages, so that equality of the power components involves equality of voltages in AD and DE. But since (see topographic diagram, Fig. 8) the voltage of DC is $\sqrt{3}$ times that of AD—and therefore of DE—the number of turns in DC must be $\sqrt{3}$ times that in DE, *i. e.*, $S_1 = \sqrt{3}\ S_2$ or $\frac{S_2}{S_1} = \frac{1}{\sqrt{3}}$. Substituting this value for $\frac{S_2}{S_1}$ in (4), we find

$$I_c = \frac{1}{\sqrt{3}} I \qquad \dots \qquad \dots \qquad (5)$$

We can now construct the vector diagram of *load currents*. Let a vector OL (Fig. 9) be drawn to represent the current I taken by the load. At right angles to this lay off a vector OC to represent I_c. By equations (2) and (3) above, $i_b = i_a = -\frac{1}{2}(i + i_c)$. Hence, to obtain I_b or I_a, find the resultant of I & I_c, bisect it at S, and then reverse OS, obtaining OB, which represents both I_b and I_a. Lastly, by equation (1), in order to obtain I_a, add OL vectorially to OB, thus obtaining $OA = I_a$. It is easily seen from the geometry of the figure that OA = OB = OC, *i. e*, the three-phase system of load currents is a balanced one.

It will be noticed that the working voltage across AE is equal to the line voltage of the three-phase supply.

Since the secondary of the quadrature phase converter is connected *in series* with the load, the converter when so used has been termed a *series phase balancer*.

Phase Shifting.

In numerous practical applications we have to deal with either a polyphase or a single-phase system, the phases of whose voltages it is necessary to alter. Such a change of phase may be brought about by what are generally known as phase shifting devices.

The problem of phase shifting may present itself in practice in a variety of forms, of which the following are the most important.

(1) The derivation, from a polyphase system, of another polyphase system the phases of whose valtages may be continuously varied with respect to the phases of the voltages of the original system.

(2) The derivation, from a polyphase system, of a single-phase system with continuously variable phase of its voltage.

(3) The derivation, from a single-phase voltage, of another voltage in phase quadrature with it.

We shall consider these three problems in the above order and point out the practical applications in connection with which they arise.

(1) The problem of continuously shifting the phases of the voltages of a polyphase system has received a wellknown practical solution in the induction regulator, which is extensively used for the voltage regulation of rotary converters. No description of this well-known device is necessary, but it may be pointed out that an induction regulator is merely a transformer of the type considered under method A (*a*) above, but with the windings arranged to be movable relatively to each other.

Another (but much more costly) solution of the same problem is presented by a synchronous motor-generator set, the stator of one of the machines being arranged to be movable.

(2) If in an induction regulator we substitute a single-phase winding for one of the polyphase windings, or if we simply use one phase only, or two phases in series, of the polyphase winding, we obtain a solution of the problem in question. The practical application of such phase-shifting transformers to the testing of alternating current meters is well known to all engineers interested in such tests.

(3) The derivation, from a single-phase voltage, of another voltage in quadrature with it, is a problem of the highest practical importance, as its satisfactory solution enables us to combine the simplicity of single-phase power distribution with the advantages attending the use of polyphase motors. A very large number of solutions of this problem have been proposed. The various methods available may be grouped under two heads, *viz.*, methods which make use of rotating machinery and those which do not. The latter class of methods involves the use of various combinations of resistances, inductances and capacities, and is, generally speaking, only of use in connection with certain classes

of electrical measurements, where questions of efficiency and power-factor are unimportant. For handling large amounts of power, the use of rotating machines is indispensable. It is only this latter class of methods that we propose to consider in the present paper.

A synchronous single-phase motor-generator set, with suitable displacements of the stator windings of the two single-phase machines, offers an obvious solution, but suffers from the disadvantages of large bulk and weight, and high first cost. An improvement is brought about by mounting both stator windings on the same core and using a single rotor, the arrangement forming a quadrature phase converter. This may be provided with either a polarised rotor resembling that of an ordinary a. c. generator, in which case the converter will run at synchronous speed, or else with a laminated rotor having a squirrel-cage winding, in which case the machine will run slightly below the speed of synchronism. The first arrangement corresponds to a two-phase synchronous machine in which one winding is a motor winding and the other a generator winding, while the second arrangement corresponds to a two-phase induction machine, in which one winding is a motor winding and the other a generator winding. It is evident from this that two-phase synchronous and induction machines of ordinary construction could be used as quadrature converters, and that no special type of machine is required for this purpose. In either type of converter there is a periodic storage of mechanical energy in, and discharge of mechanical energy from, the rotor of the machine. That such must be the case is evident from the fact that while the flow of energy into the motor winding and the supply of energy from the generator winding are both fluctuating, the fluctuations are in quadrature with each other, so that the instant of maximum supply of power to the motor winding corresponds to zero power in the generator winding and *vice versa.*

It may be interesting to consider in detail the function of the squirrel-cage winding in a quadrature converter of the induction type. The action of the converter depends on the fact that a rotating squirrel-cage transforms an alternating magnetic field into a rotating one. This action may be explained as follows. Consider first a squirrel-cage which is rotating very slowly in a steady magnetic field occupying a fixed position in space. Owing to the low frequency of the e. m. f. s. induced in the rotor conductors, the currents in these conductors will be practically in phase with their e. m. f. s, and hence the magnetic axis of the rotor currents will be in space quadrature with the field; in other words, the rotor currents will give rise to a cross-field, but will exert no direct magnetising or demagnetising action on the impressed field.

Consider now the other extreme—that in which the rotor is running at a very high speed in a steady field fixed in space. Owing to the high frequency of the e. m. f. s in the rotor conductors, the currents will now practically be in time quadrature with their e. m. f. s, and the magnetic axis of the rotor currents will be coincident in position with that of the impressed field and in direct opposition to it. We now have no appreciable cross-field but only a direct demagnetising effect.

To sum up, then, we may say that slow rotation of a squirrel-cage in a steady magnetic field produces no appreciable demagnetising, but only a distorting, effect on the field; while rapid rotation results in practical wiping out of the impressed field.

Since these effects depend on the relative motion of the rotor and the field, they will obviously continue to exist if, the relative velocity remaining unaltered, a velocity of the same magnitude and direction be impressed on both rotor and field.

Hence if a squirrel-cage rotor is made to rotate in a *rotating* magnetic field at a speed not differing greatly from that of the field itself, the slow relative motion will give rise to a cross-field; but the magnitude of the original rotating field will not be appreciably affected. On the other hand, if the rotor speed be made to differ greatly from that of the field—corresponding to a high relative speed between the two—the original field will be largely wiped out.

We may now pass to the case of a squirrel-cage rotating in an *alternating* impressed field. As is well known, such a field may be replaced by two equal and oppositely rotating fields. If now the rotor is made to run in the direction of one of the component fields at a speed not differing greatly from that of this component, the value of the component will remain practically unaltered; while the other component, whose direction of rotation is opposed to that of the rotor and which consequently has a high speed of rotation relatively to it, will be practically wiped out. The final result, then, is a conversion of the original impressed alternating field into a more or less pure rotating field which has the same direction of rotation in space as the rotor. It may be noted that not only will the rotor effect the conversion of an alternating into a rotating field, but that the space distribution of the flux corresponding to the rotating field will be a close approach to a sine distribution. As a result, even

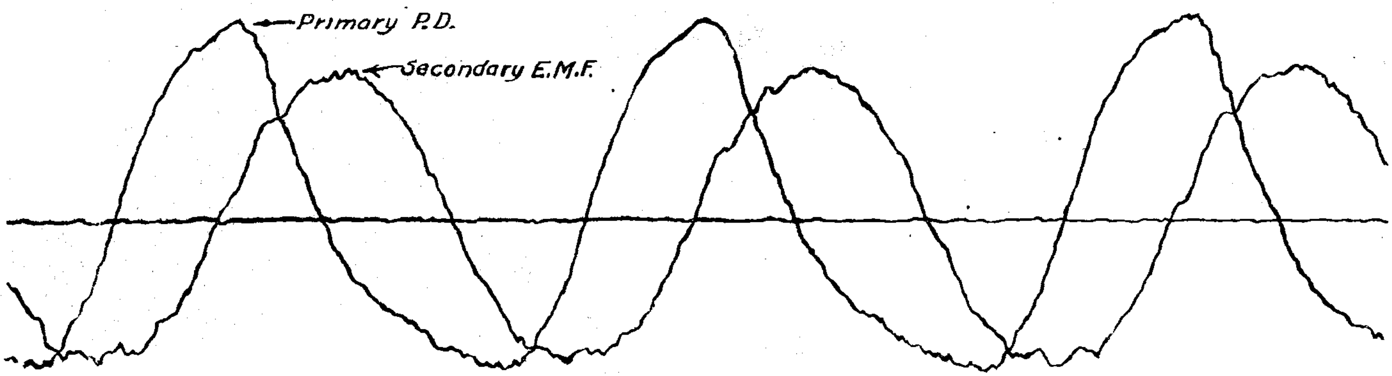

Fig. 10. Showing smoothing effect of Rotor Current on E. M. F. Wave-shape.

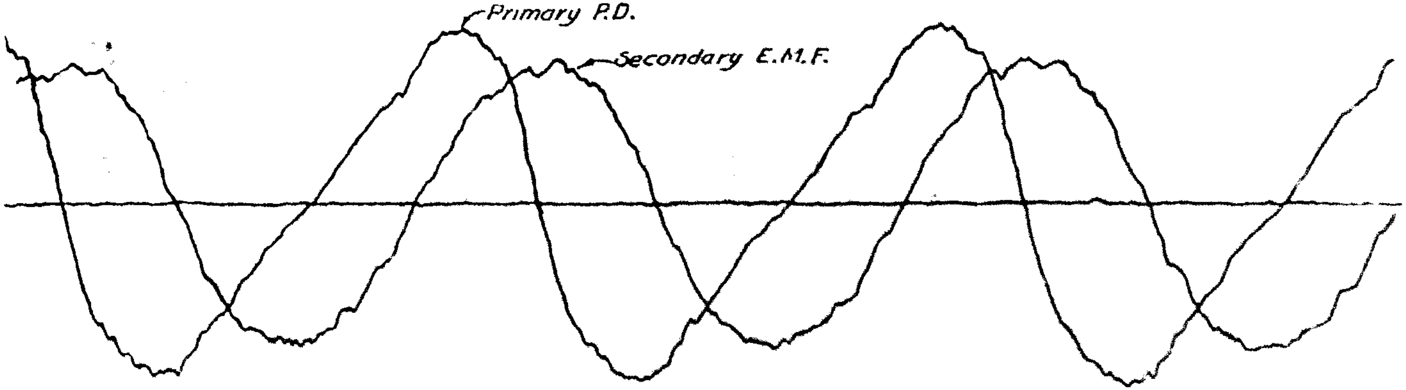

Fig. 11. Showing smoothing effect of Rotor Current on E. M. F. Wave-shape

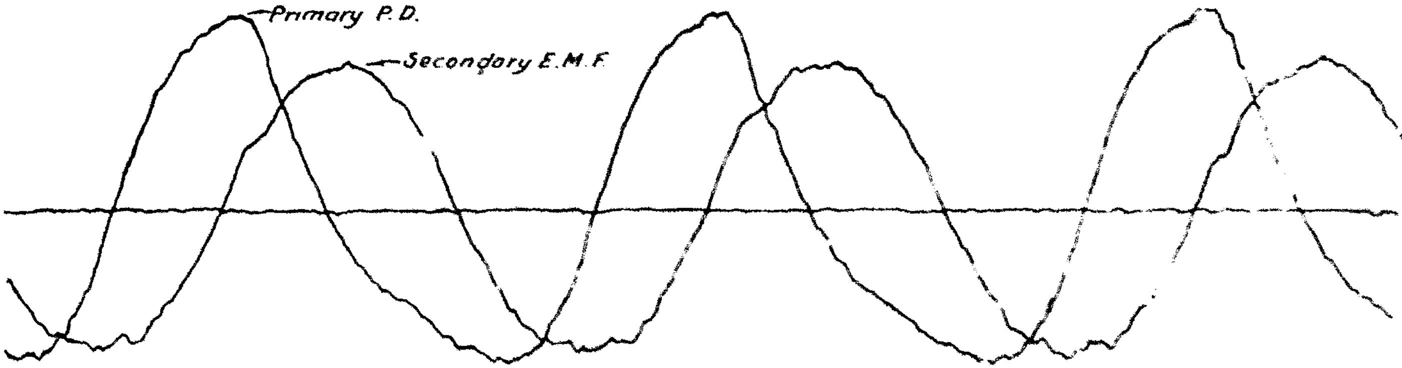

Fig. 12. Showing smoothing effect of Rotor Current on E. M. F. Wave-Shape

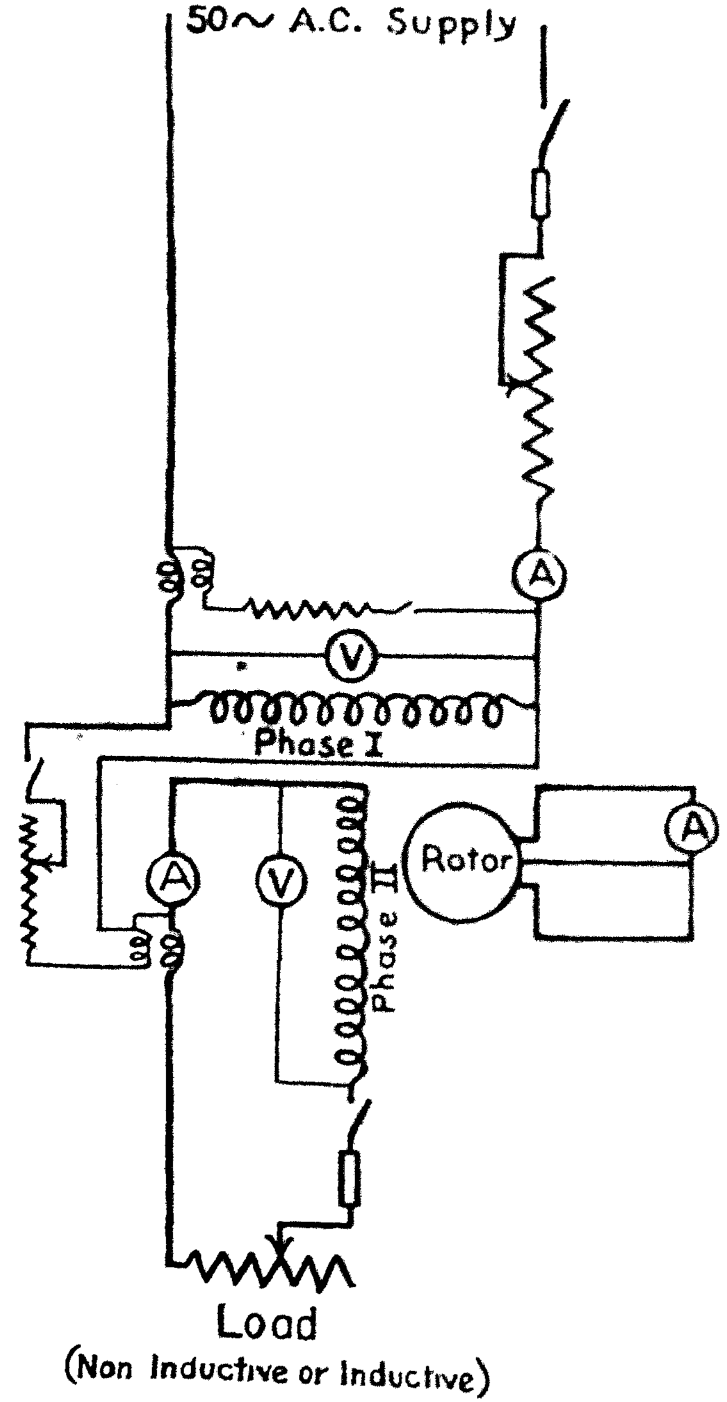

Fig. 13. Connections of Induction Motor used as Induction Quadrature Converter

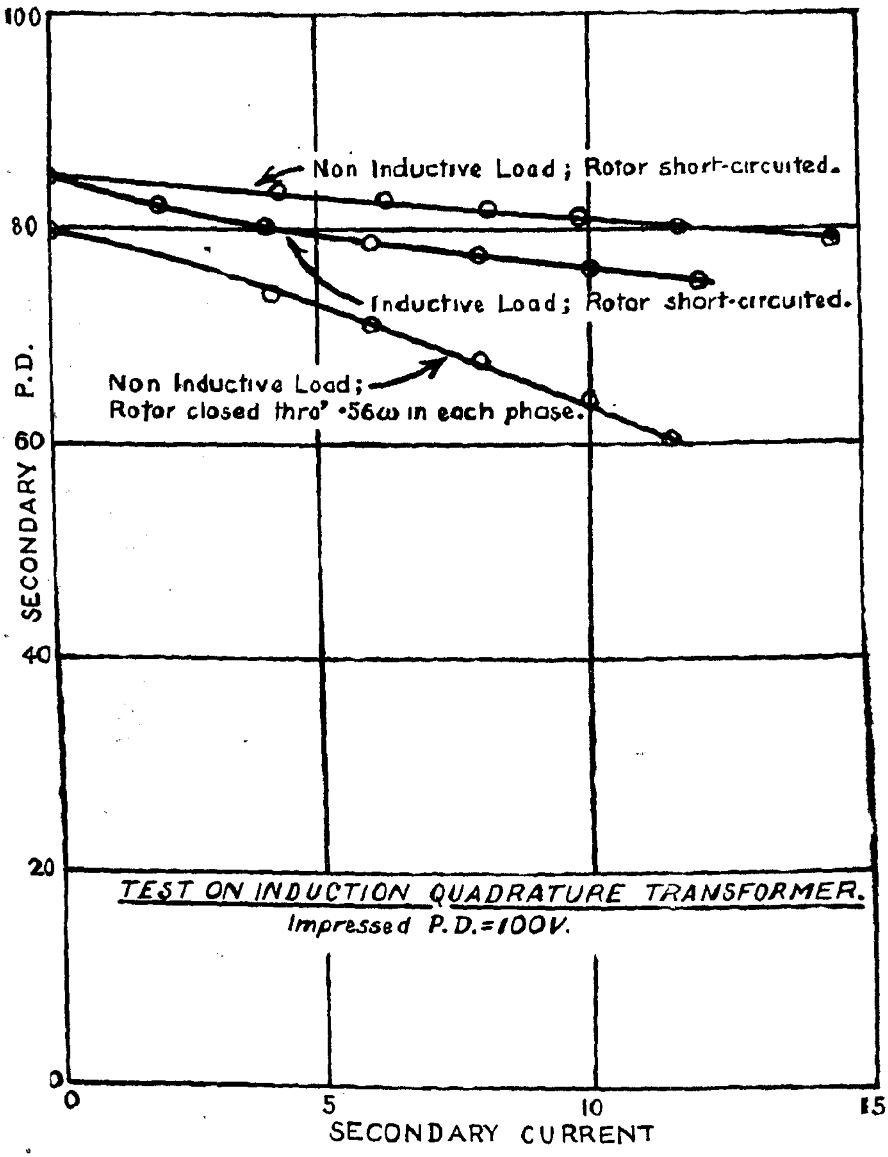

Fig. 14. Voltage Regulation curves of Induction Quadrature Converter.

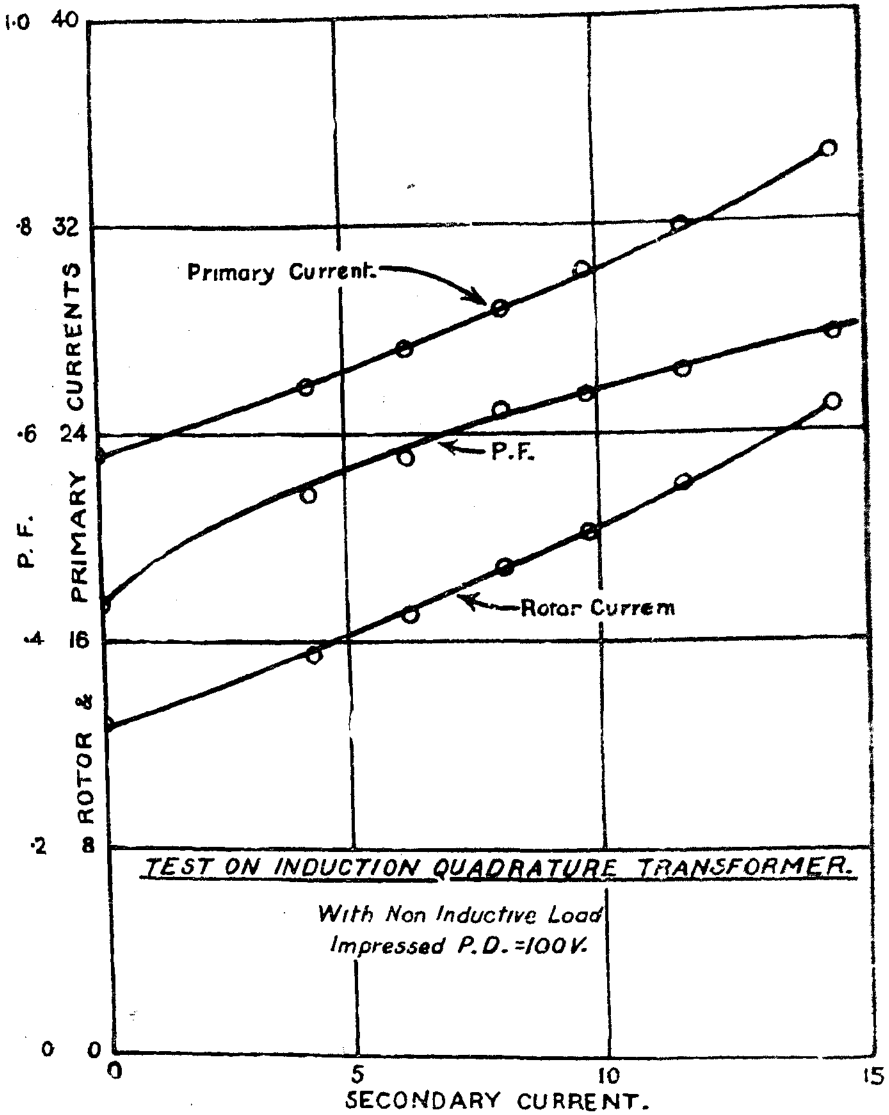

Fig. 15. Relation connecting Primary Current and Power Factor of Induction Quadrature Converter with Secondary Current.

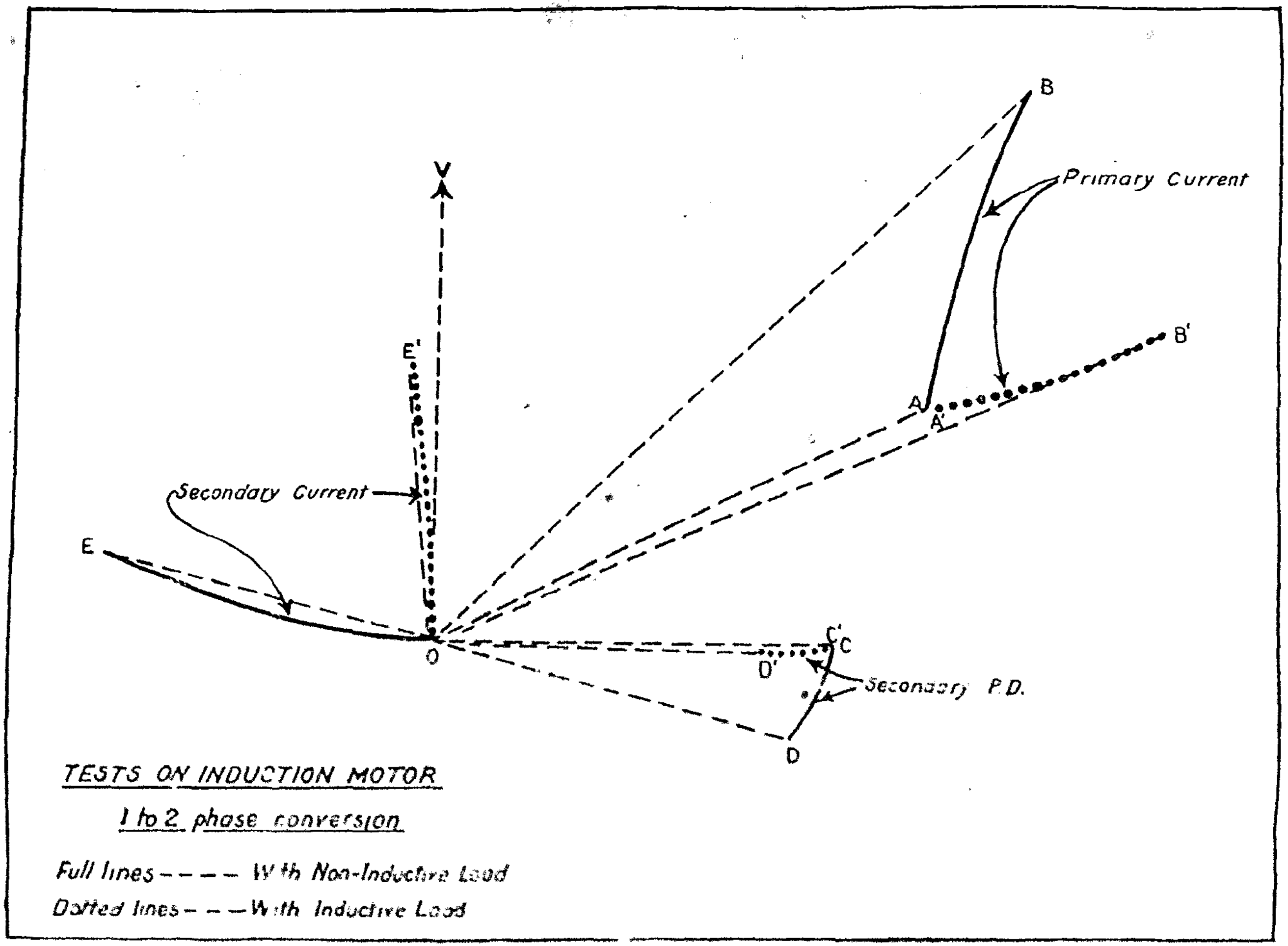

Fig. 16. P. D. and Current Loci of Induction Quadrature Converter

if we apply a highly distorted wave of p. d. to one phase of a two-phase induction quadrature converter, the e. m. f. wave obtained on the other phase will be a close approach to a sine wave.

The above deductions are fully borne out by the Ondograph records shown in Figs. 10—12. These records show the results of applying three different highly distorted wave-shapes to one phase of a $7\frac{1}{2}$ b. h. p. two-phase induction motor used as a quadrature converter, the shape of the e. m. f. wave in the second phase being taken on open circuit. The wave distortion was obtained by the use of two alternators connected in series and coupled to a motor, the number of poles in one of the alternators being twice that in the other. The peculiar kind of distortion obtained is due to the use of an even harmonic. The Ondograph records show in a striking manner both the quadrature displacement of, and the practical elimination of the even harmonic from, the e. m. f. wave in the second phase. It may be remarked that the ripples in the waves are mainly due to irregularities in the working of the Ondograph, and do not represent real effects. This will be noticed from the fact that the zero line itself is thrown into slight ripples.

Some experimental results obtained with the $7\frac{1}{2}$ b. h. p. induction motor to which the Ondograph records refer are shown in Figs. 14 and 15. The arrangement of connections is shown in Fig. 13. The voltage regulation curves for both a non-inductive and a highly inductive load are given in Fig. 14. An additional curve shows the effect on the regulation of adding resistance to the rotor circuit. It will be seen that with the rotor short-circuited the regulation is fairly close, whether the load be non-inductive or inductive. The curves of Fig. 15 show the relation connecting the primary current, primary power-factor and rotor current, with the secondary current when the the load is non-inductive. It will be noticed that there is almost exact proportionality between the primary and rotor currents. and that the rotor currents are relatively large even when the secondary is open-circuited. This is accounted for by the ampere-turns which the rotor provides for the suppression of the oppositely rotating component of the impressed alternating field.

Fig. 16 is a polar diagram which shows the variations in the magnitude and phase of the various quantities involved as the load is increased. The full line curves OE, CD and AB refer to a non-inductive load. The vector OV, which represents the primary p. d., is taken as the fixed reference line of the diagram. On open secondary circuit the secondary voltage OC is practically in exact phase quadrature with the primary p. d., and the primary current OA lags behind the primary p. d. by the large angle VOA. For the sake of clearness, the secondary current locus is shown reversed in phase. With a non-inductive load, as the extremity of the secondary current vector (reversed) traces out the locus O E, the secondary p. d. vector traces out the locus C D, and the primary current vector the locus AB. The corresponding loci for a highly inductive load are shown by the dotted curves CE′, C′D′ and A′B′. The diagram shows clearly that the quadrature relation of the primary and secondary voltages is maintained quite satisfactorily. The main weakness of the arrangement is the low value of the power-factor.

In order to determine the effect of exciting the rotor, thereby changing the converter from an induction into a synchronous machine, a series of readings was obtained with the connections arranged as shown in Fig. 17. Two of the windings of the three-phase rotor were connected in parallel and joined in series with the third winding, and a current was supplied to the rotor from a battery through a regulating resistance. Fig. 18 gives the voltage regulation curves for two values of the exciting current, in the case of a non-inductive load, and for one value of the exciting current in the case of an inductive load. A great improvement took place in the power-factor due to the substitution of a synchronous for an induction machine.

Phase Balancing.

The direct supply of a single-phase load from two mains of a three-phase system is, as already pointed out, accompanied by an unbalancing of the three-phase voltages. The extent of this unbalancing depends on the total drop occasioned by the single-phase load, i. e.., on the product of the single-phase current into the total effective impedance of the circuit external to the single-phase load—including the two mains and the generator windings.

If we suppose that the windings of the generator may be regarded as possessing a definite fixed impedance, then the unbalancing effect of a single-phase load of any given power-factor may be determined as follows.

Let Fig. 19(*a*) represent the circuit diagram of the generator and the single-phase load supplied by it, the curved and straight arrows indicating the directions assumed as positive for the delta e. m. f. s* and star currents respectively. Then in

*The positive directions of the p. d. s. are taken as being opposed to the positive directions of the e. m. f. s.

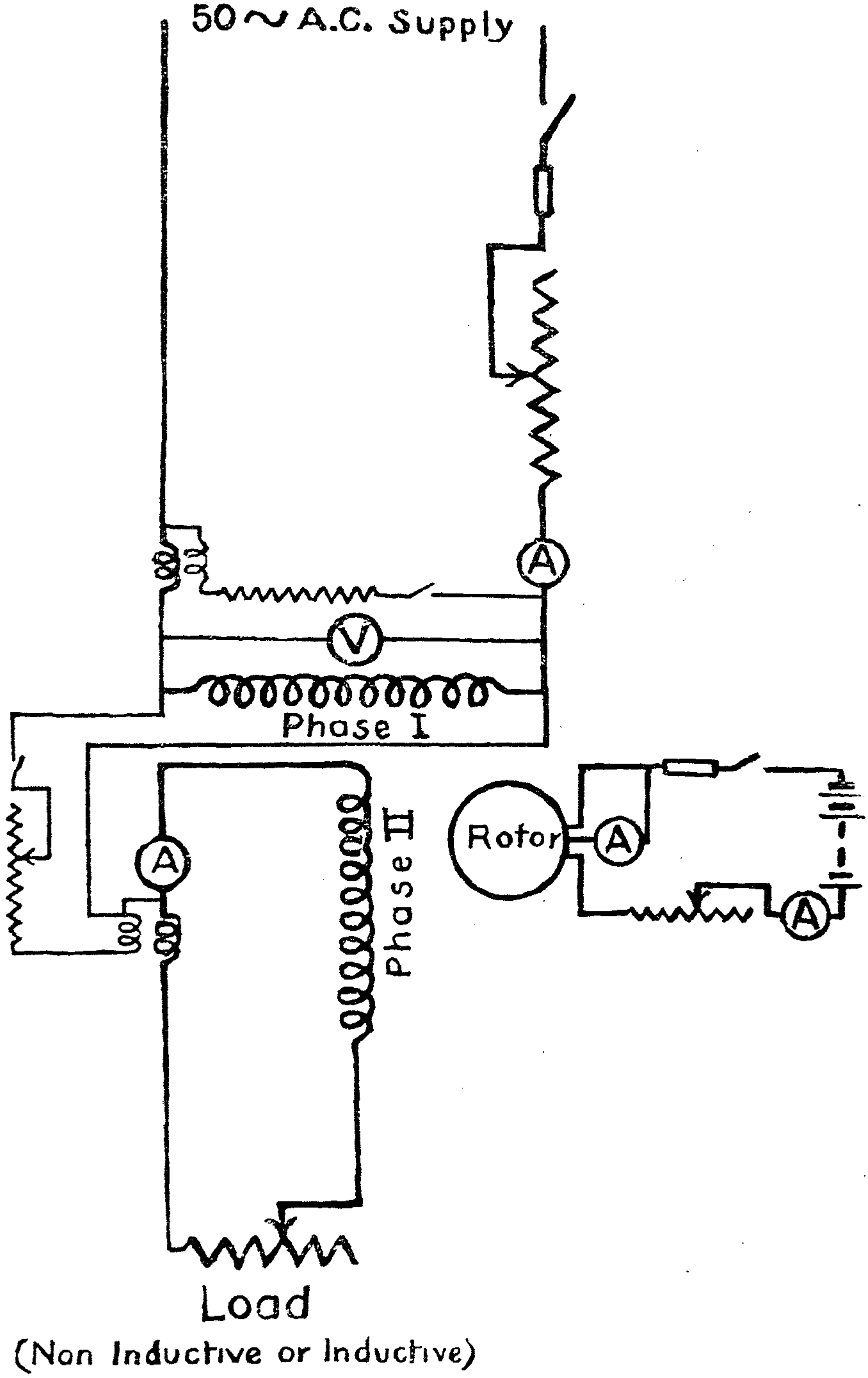

Fig. 17. Connections of Induction Motor used as Synchronous Quadrature Converter.

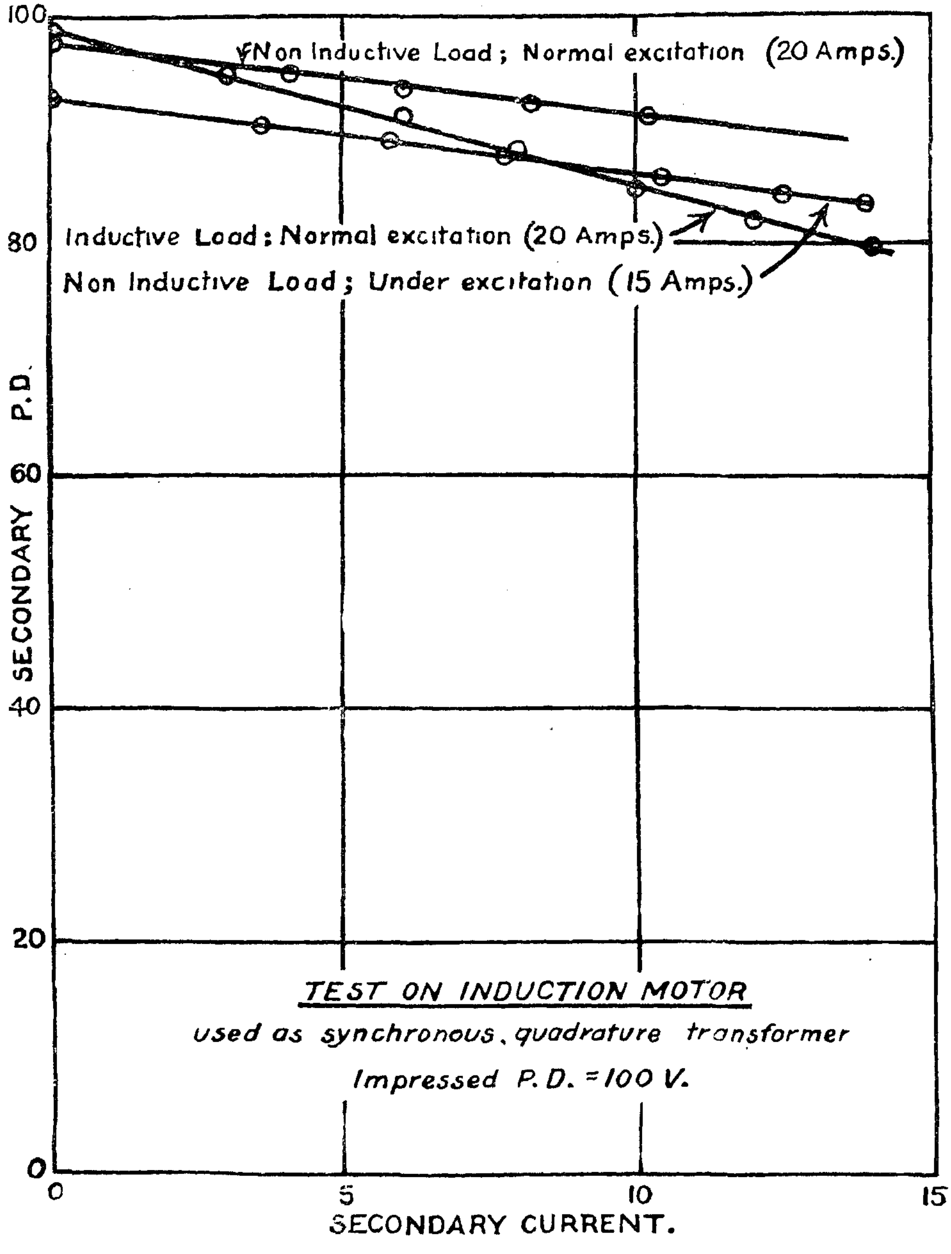

Fig. 18. Voltage Regulation Curves of Synchronous Quadrature Converter.

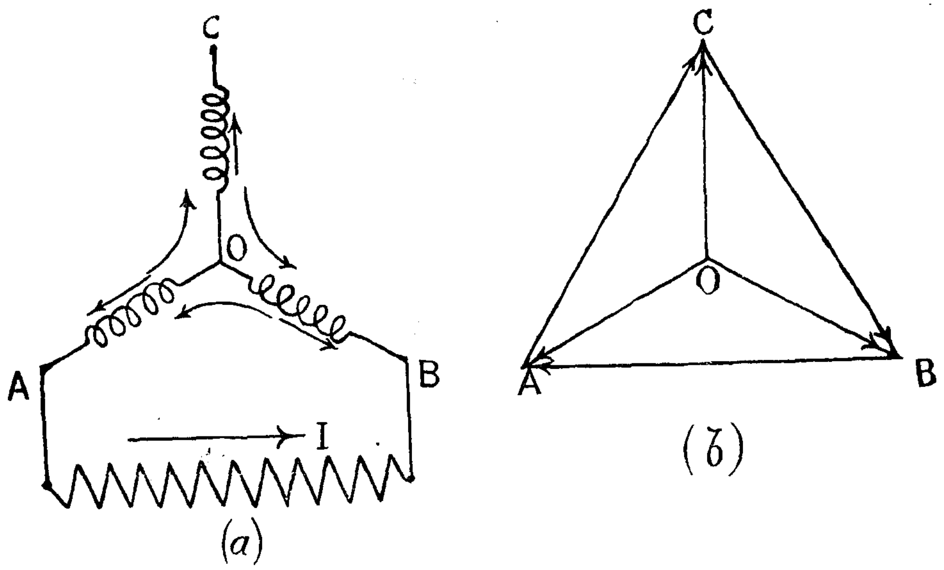

Fig. 19. Circuit and Topographic diagrams.

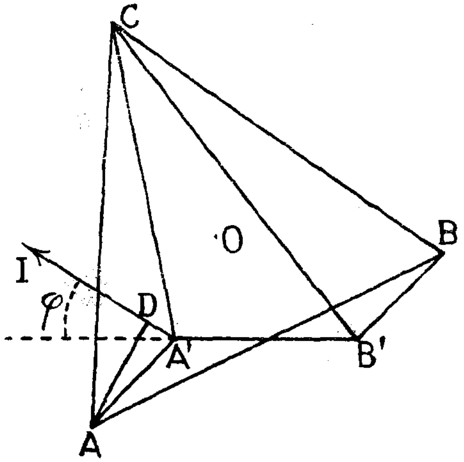

Fig. 20. Construction for finding unbalancing effect of Single-phase load.

Fig. 19 (*b*) the points A, B, C, O represent the topographic diagram of the terminals and neutral point of the unloaded machine; and the vectors represent the vector diagram of the various delta and star voltages. Let cos ϕ be the power-factor of the load, and let in Fig. 20 B′ A′ represent the p. d. across the terminals AB in Fig. 19 (*a*) when the machine is loaded. Let the current be represented by I, making an angle ϕ with B′ A′. The impedance drop in each phase is given by A′ A; A′ D being the resistance drop and DA the reactance drop. Now consider the phase OA in Fig. 19 (*a*). Since the direction assumed for the current as positive is from O to A, and since this is the same as the positive direction for the e. m. f. generated by OA, it follows that the e. m. f. will be obtained from the p. d. by adding to the latter (vectorially) the impedance drop. Now since A′ is one end of the vector representing the star p. d. of the phase OA (the position of the other end of this vector being as yet unknown) it follows that by drawing a vector A′ A from A′ equal to the impedance drop, we find the position A of the extremity of the star e. m. f. vector of the phase OA. Considering next the phase OB we notice that whereas the positive direction for the e m. f. is from O to B, the direction of the current which has been taken as positive is from B to O. Hence the e. m. f. will be obtained from the p. d. by *subtracting* vectorially from this latter the impedance drop A′A. If then we draw from B′ the vector B′ B, making it equal and opposite to A′ A, we obtain the position B of the extremity of the star e. m. f. vector of the phase OB. Thus AB in Fig. 20 is the open-circuit p. d. across the terminals AB of Fig. 19 (*a*). We may now complete the equilateral triangle ABC of the open-circuit terminal p. d. s, thus obtaining the point C. If O is the centre of this triangle, then OA′ and OB′ give us the star p. d. s of the phases OA and OB in Fig. 19 (a), while OC gives the p. d. across the remaining phase OC in Fig. 19 (*a*). Since this latter phase carries no current, the vector OC remains unaltered when the machine is loaded with a single-phase load. The terminal p. d. s are given by A′ B′, B′ C and CA′.

Since the resistance drop A′ D is generally small in comparison with the synchronous reactance drop DA, the total drop may be taken to be approximately represented by DA. If we make this assumption, then in the case of a non-inductive load, DA becomes perpendicular to A′ B′, and we get the vector diagram of Fig. 21 (*a*), in which the triangle of terminal voltages is represented by A′ B′ C, whose sides are all unequal. If, on the other hand, the load is purely inductive, the vector diagram assumes the form shown in Fig. 21 (*b*), the triangle of voltages being in this case isosceles.

If the impedance drop in the generator winding could be made sufficiently small, approximate balance of voltages would be maintained without the use of any special devices, and it is evident that this condition is approximately satisfied by a generator carrying a single-phase load which is only a small fraction of the full-load output of the machine. With heavy single-phase loads the unbalancing must be corrected by the use of suitable auxiliary apparatus.

One method of greatly improving the performance of a three-phase generator carrying a single-phase load is to provide its field magnet with a sufficiently heavy squirrel-cage. As already explained, such a squirrel-cage will largely suppress all pulsations or angular oscillations of the magnetic flux, so that the windings of each phase will be swept by approximately the same resultant flux, and so have equal e. m f. s. induced in them. The unbalancing in this case would arise mainly from the resistance and the leakage reactance drop, the inequalities in the armature ampere-turns of the different phases being corrected by the action of the squirrel-cage.

From a purely geometrical point of view, the problem of converting an unbalanced into a balanced three-phase system is that of converting a triangle with unequal sides into an equilateral triangle. This may be done in an infinite variety of ways, as will be obvious from the fact that if a triangle such as ABC (Fig. 22 of unbalanced voltages be given we may select any equilateral triangle A′ B′ C′ at random and join its vertices to those of the given triangle. If OA, OB and OC are the three original star voltages, then by adding to them vectorically the voltages AA′, BB′ and CC′ respectively we obtain the balanced system A′ B′ C′ of delta voltages.

To restore balance, therefore, we have to provide the three boosting voltages AA′, BB′ and CC′. It would obviously be convenient if these boosting voltages could be provided by a single machine of standard construction, *i. e.*, by an ordinary three-phase generator. Since in such a machine the three voltages AA′, BB′ and CC′ would be equal and would have phase differences of 120°, the problem arises as to whether it is possible to select the triangle A′ B′ C′ so as to fulfil these requirements. Such a choice is, as shown in the Appendix, possible, and the necessary geometrical construction was first given in a paper by R. E. Gilman and C. le G. Fortescue, on "Single-Phase Power Service from Central Stations", read in 1916 before the American Institute of Electrical Engineers.*

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* Transactions of the American Institute of Electrical Engineers Vol 35, p. 1329.

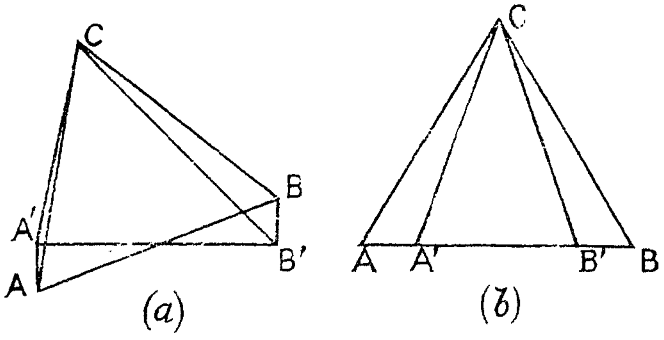

Fig. 21. Unbalancing due to non-inductive and inductive loads.

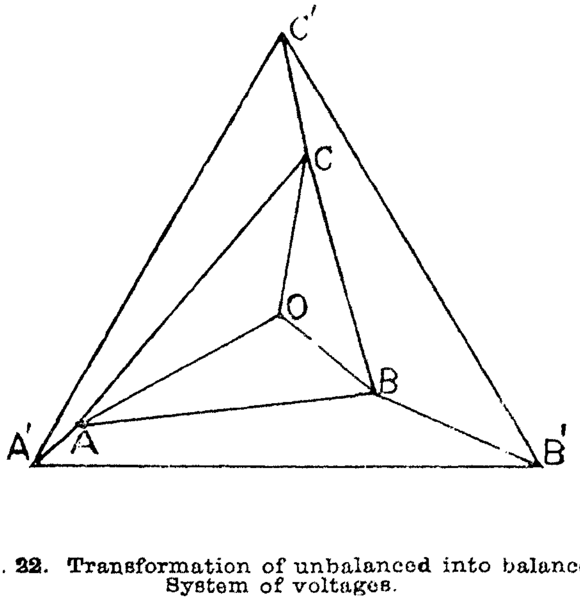

Fig. 22. Transformation of unbalanced into balanced System of voltages.

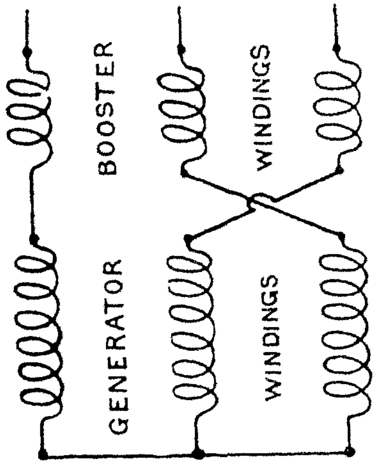

Fig. 23. Diagram of booster connections.

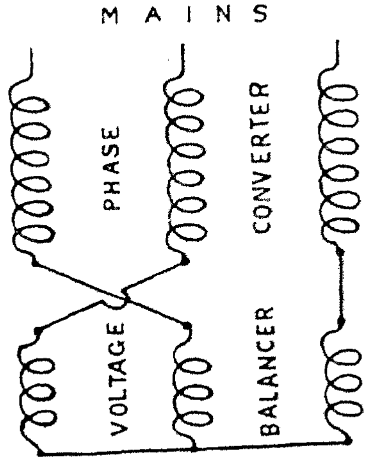

Fig. 24. Connections of Shunt phase balancer

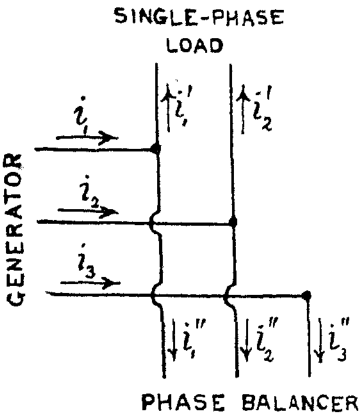

Fig. 25. Circuit diagram.

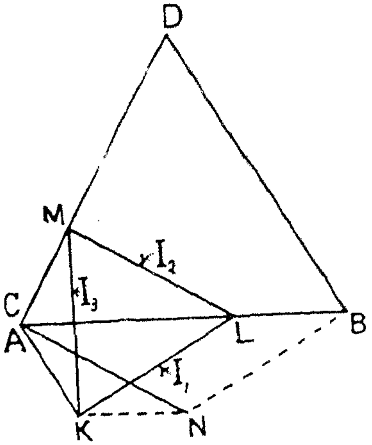

Fig. 26. Construction for transforming single-phase into two balanced three-phase systems.

The final result of the investigation given in the Appendix is to show that an unbalanced three-phase system may always be restored to balance by means of a balanced system of three-phase boosting voltages whose phase sequence is the reverse of the phase sequence of the given unbalanced system.

One method, therefore, of maintaining a balance of voltages across the mains of a three-phase system which has to carry a heavy single-phase load is to connect between the generator and the mains a three-phase booster whose voltages have the required magnitude and phase, and a phase sequence opposed to that of the generator. This arrangement is represented diagrammatically in Fig. 23. It is not used in practice, as a more economical arrangement—to be described presently—is available. The disadvantage of the method shown in Fig. 23 is that, although balance of voltages is obtained across the mains, the generator currents are unbalanced, and in consequence the generator cannot be loaded up to its full capacity.

The principle of the method actually adopted in practice is as follows. If across three-phase mains supplying an unbalanced load there be connected an ideal three-phase machine —whether of the synchronous or the induction type—whose internal impedance drops are negligible, then the terminal p. d. s of such a machine must for every condition of load be equal to its e. m. f. s, and since the e. m. f. s, are balanced, the p. d. s must likewise be balanced. Such an ideal machine will therefore maintain exact balance of voltages under all conditions of load. Further, owing to the balance of voltages, the generator currents will balance* and the generator may be loaded up to its full capacity.

Although such an ideal machine with negligible drops is unattainable, its equivalent may be practically realised by injecting into the windings of the machine e. m. f. s equal to the impedance drops. Now we have already seen that the unbalanced p. d. s of the machine may be rendered balanced by superposing on them a three-phase system of voltages of suitable phase and magnitude and of opposite phase rotation. Such voltages are provided by means of the voltage balancer, the connections being arranged as shown in Fig. 24. The first machine, which is directly across the mains, is known as a phase converter, because it draws currents from the mains such that when these are added vectorially to the unbalanced load currents, the resultant system —which is the generator system of currents—is a balanced one ;

*Since the drops in the generator windings are not negligible, a balance of p. d. s could not be obtained without a balance of currents.

he action of the phase converter in the case of a single-phase load is to transform such a load into a balanced three-phase load, before it is dealt with by the generator.

The combination of phase converter and voltage balancer forms a *phase balancer,* and since this is connected across the mains in parallel with the load, it is termed a *shunt phase balancer,* in order to distinguish it from the *series phase balancer* described under (*d*) above.

We shall now consider the relations of the currents in the generator, phase balancer and load. To fix ideas, we shall take the case of a simple single-phase load connected across two of the three-phase mains. The circuit diagram is given by Fig. 25, and the directions assumed as positive are indicated by arrows. The triangle of the load currents in this case degenerates into a straight line as shown in Fig. 26, where AB represents I'_1 and BC (which is identical with BA) I'_2. The vector CA, which is of zero length, is the non-existent load current in the third main. We now apply the geometrical construction explained in the appendix to the degenerate triangle ABC. On AB as base we construct the equilateral triangle ABD. We trisect CD, thus obtaining the vector CM, which defines a system of three equal and equally spaced vectors, by the application of which to the angular points of the triangle ABC we transform this latter into the equilateral triangle MLK. In doing so, we have to remember that if we proceed around the triangle ABC in a counter-clockwise direction, the transforming vectors of which CM is the first have to be taken in clockwise order (thus to C, we apply the vector CM; to A, the vector AK; and to B, the vector BL). The triangle KLM so obtained represents the balanced generator currents, KL, LM and MK representing I_1, I_2 and I_3 respectively. Referring now to the circuit diagram of Fig. 25, we see that

$$i''_1 = i_1 - i'_1$$
$$i''_2 = i_2 - i'_2$$
$$i''_3 = i_3$$

Interpreting the above equations vectorially, we see that I''_1 is obtained by subtracting from I_1 (KL in Fig. 26) the vector AB (or adding the vector BA). The result is the vector NA, and this is seen to be the resultant of NK and KA, or BL and KA. Similarly, I''_2 may be shown to be the resultant of LB and AM; and I''_3, the resultant of MA and AK. The simplest way of obtaining these resultants is to draw the star of currents corresponding to AM, BL and AK, as in Fig. 27—where OM',

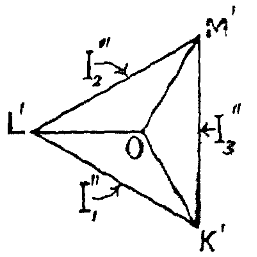

Fig. 27. Phase Converter currents.

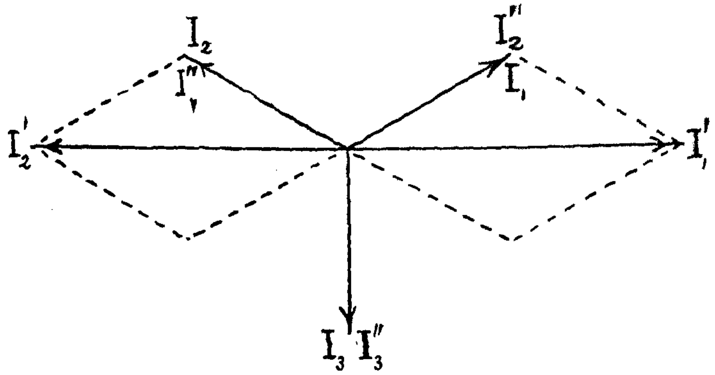

Fig. 28. Complete current diagram.

OL′ and OK′ are equal to CM, BL and AK of Fig. 26 respectively —and to form the corresponding delta K′ L′ M′. It will then be seen that K′ L′, L′ M′ and M′ K′ represent I''_1, I''_2 and I''_3 respectively. From the geometry of Fig. 26 it is not difficult to show that the lengths of the vectors representing the generator currents I_1, I_2 and I_3 are equal to those of the vectors I''_1, I_2'', I''_3, which represent the phase balancer system of currents. The complete system of currents is shown in Fig. 28, and we see that a single-phase load current is obtainable by superposing two equal and balanced three-phase current systems of opposite phase rotation.

A similar construction is applicable to the general case of any kind of unbalanced load.

The voltage balancer must provide voltages sufficient to maintain the necessary balanced system of currents of opposite phase rotation through the combined impedance of its own windings and those of the phase converter. With change of load the magnitude and phase of the voltage balancer e. m. f. s. must vary. In practice the voltage balancer is provided with two field windings arranged in electrical space quadrature, and each winding has its own exciter. The necessary variation in the magnitude and phase of the balancer voltage is obtained automatically by means of two Tirrill regulators, which act on the field windings of the voltage balancer exciters.*

A few words may be said in conclusion regarding the conditions which determine whether the *series* or the *shunt* type of converter is the more suitable. This point has been dealt with by Alexanderson†. Of the two types of machine, the series phase converter is the simpler and cheaper. Hence if it is merely desired to suppress the unbalancing effect which would arise from a heavy single-phase load at a given point of the system, the *series* phase converter should be used. On the other hand, if there were a number of large single-phase loads distributed throughout the supply system, and if these were connected across the various phases in such a manner as to yield an approximately balanced three-phase load, it is obvious that the use of a series phase balancer with each load would be wasteful and unnecessary, a very much smaller single machine of the shunt type being sufficient for restoring balance.

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* Further details of the arrangement as used by the Philadelphia Electric Company in the United States will be found in the *General Electric Review*, vol 21, p 19c.

† *General Electric Review*, vol. 19, p. 1101 (1916).

APPENDIX.

The balancing of an unbalanced three-phase system by the use of balanced three-phase boosting voltages.

In what follows, we shall find the following lemma useful. If from the vertices of an equilateral triangle F G C (Fig. I) we draw three straight lines FK, CL and CM of equal length and making angles of 120° with each other, then on joining K, L and M by straight lines we obtain a new equilateral triangle KLM.

In order to prove this, join KG, LC and MF. Then in the triangles FKG, GLC we have FK=GL, FG=GC, and $\angle$KFG = $\angle$LGC (since GC is FG rotated through 120°, and GL is FK rotated through 120°). Hence the triangles are equal, so that KG=LC, $\angle$FKG= $\angle$GLC, and hence KG and LC make and angle of 120° with each other.

Considering next the triangles KGL and LCM, we have KG=LC (as just shown), GL=CM by construction, and $\angle$KGL=$\angle$LCM (since LC is KG rotated through 120°, and CM is GL rotated through 120°). Thus the triangles are equal, and from this it immediately follows that KL=LM, and that KLM is 120°. Hence KLM is an equilateral triangle.

If, therefore, from the vertices of any equilateral triangle we draw three straight lines making angles of 120° with each other, and if, starting from the vertices of the original triangle, three points be made to travel with the same constant velocity along the three straight lines, then the triangle formed at any instant by the three moving points is equilateral.

Considering now an unbalanced system of three-phase voltages, and starting with the unbalanced triangle ABC (Fig. II) let us select one of its sides, say AB, and on this side construct an equilateral triangle ABD. Bisect AB at H, and join HD, HC. Draw FG at right angles to HC, and from C draw CF, CG making angles of 30° with CH. Then obviously CFG is an equilateral triangle. Join AF, BG. Then triangle AFH is equal to triangle BGH, and BG is equal and parallel to AF.

Join DC, and produce AF to meet CD at E.

Then the triangles HFA and HCD are similar; for the angle AHD is equal to FHC, being a right angle; and if we

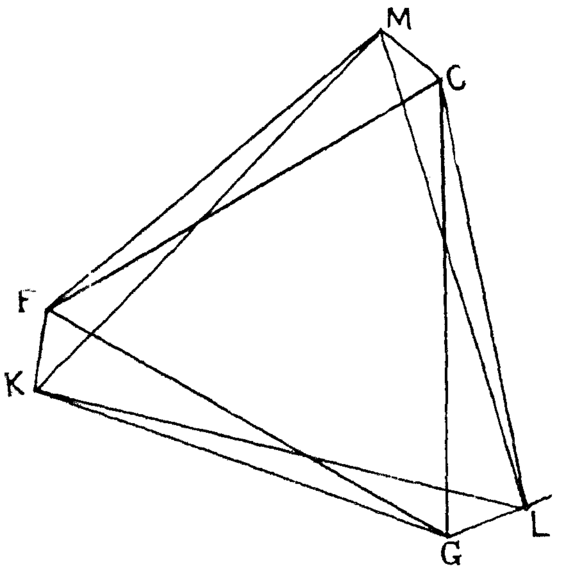

Fig. I.

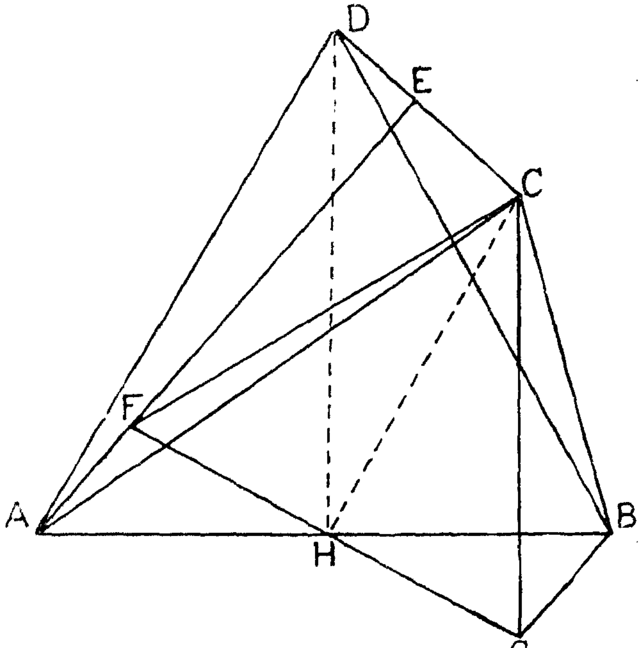

Fig. II.

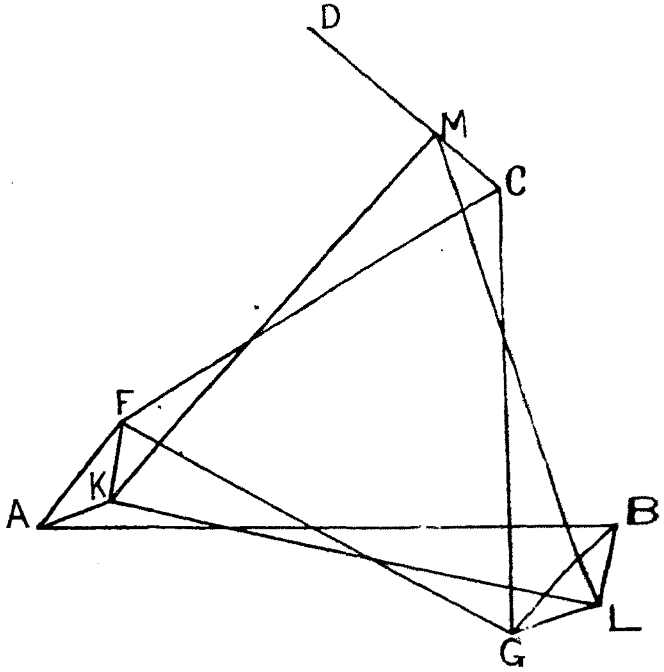

Fig. III.

subtract from each the same angle FHD, we get $\underline{|AHF} = \underline{|DHC}$; also, $AH = \frac{1}{\sqrt{3}}$ HD, and $FH = \frac{1}{\sqrt{3}}$ HC, hence $\frac{AH}{HD} = \frac{FH}{HC}$, and thus the triangles HFA and HCD are similar.

Hence $\underline{|HAF} = \underline{|HDC}$, and since AH is perpendicular to HD, we must have AE perpendicular to DC.

Further, since $\frac{AF}{CD} = \frac{AH}{HD} = \frac{1}{\sqrt{3}}$, we have $AF = \frac{1}{\sqrt{3}}$ CD.

On AF and GB as bases construct the isosceles triangles AKF and GLB (Fig. III) having base angles of 30° and from C lay off CM = FK = GL. Since FK makes an angle of 30° with FA, it makes an angle of 60° with DC (which is perpendicular to FA) or an angle of 120° with CM. Again GL which is parallel to AK makes an angle of 120° with FK. Thus the three vectors CM, FK and GL form a system of three equal vectors spaced 120° apart. Similarly CM, BL and AK form another such system. Join the points K, L and M by straight lines. Then since CM, FK and GL are three equal lines drawn from the vertices of the equilateral triangle CFG and making angles of 120° with each other, it follows by the lemma proved above that KLM is an equilateral triangle.

We thus see that the equilateral triangle KLM is derivable from the original triangle ABC by drawing from the vertices of this latter the three vectors AK, BL and CM, which are equal and spaced 120° apart.

We have therefore shown that any unbalanced three-phase system of voltages may be restored to balance by the use of a balanced three-phase system of boosting voltages. It will be further noticed that if we proceed around the triangle ABC in a counter-clockwise direction, the vectors AK, BL and CM have to be taken in a clockwise direction. In other words, the phase sequence of the balanced three-phase boosting voltages is the reverse of that of the original unbalanced system.

The magnitude of the required boosting voltages is easily determined. For since $CM = AK = \frac{1}{\sqrt{3}}$ AF, and since AF is, as we have shown, equal to $\frac{1}{\sqrt{3}}$ CD, we find that $CM = \frac{1}{3}$ of CD.

Hence the following construction for finding the magnitude and phase of the boosting voltages.

On any side AB of the unbalanced triangle of voltages ABC construct an equilateral triangle ABD. Join the vertices C and D of the triangles and trisect CD. If M is the point of tri-section nearest C, then CM gives one of the required boosting voltages, the remaining two voltages being equal to it in magnitude and displaced from it by 120° and 240° in phase.

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