PERFORMANCE OF THREE-PHASE INDUCTION MOTORS ON UNBALANCED VOLTAGES.

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INTRODUCTION

An unbalance of voltages may occur on long distance power transmission lines. It occurs not infrequently in the case of generating stations supplying mixed polyphase and single-phase loads, or where the impedances of the three conductors of three-phase railway systems are not of equal magnitude. The predetermination of the performance of three-phase induction motors under such conditions is a matter of importance. It has been recognised by engineers that there is a reduction in the maximum load which an induction motor is capable of carrying safely when supplied with unbalanced voltages, but the extent of this reduction has not received the attention which it merits.

Charters and Hillebrand [1] have carried out experiments on the performance of induction and synchronous motors on unbalanced voltages, but they have assumed that the load limit of the motor is reached when the current in one phase reaches its full load value, and hence their experiments underrate the performance of the motor. In an article in the *Elektrotechnische Zeitschrift*, vol. 39, p. 345 (1918), by Müller, a graphical method is given of resolving an unbalanced voltage system into two balanced voltage systems of opposite phase rotation and the application of the circle diagram to each voltage system.

T. Slepian, in the *Electrical World*,[2] has treated the matter analytically and derived certain simple formulæ to predetermine the output, but his results have not received conclusive experimental verification. Still another analytical treatment has been given by W. V. Lyon,[3] who has in addition described a graphical method of resolving an unsymmetrical voltage system into two symmetrical systems.

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[1] *Am. I.E.E.* Proc. 28, June, 1909, pp. 565-78.
[2] *El. World*, vol. 75, pp. 313-5 (1920).
[3] *Ibid.*, June 5, 1920, pp. 1304-8.

PART I. EXPERIMENTAL

The experiments about to be described had for their object the determination of the behaviour of an induction motor on an unbalanced voltage supply, and of its action as a balancer.

Behaviour of Induction Motor on Unbalanced Supply

The tests were carried out on a 7½ B.H.P., 100-volt, 50-cycle, 1450 r.p.m., three-phase induction motor with Y-connected stator and rotor. Each stator-phase contains 20 conductors and each rotor-phase 21, the full-load current per phase being 45 amperes. The motor was supplied by a three-phase, 100-volt, 50-cycle, 15 K.V.A. alternator, with an accessible neutral.

The unbalancing of the balanced voltages of the alternator was effected in two ways: (1) by injecting into one of the phases of the alternator an opposing e.m.f. derived from the secondary of a transformer whose primary was connected across the alternator phase itself, this condition corresponding to pure unbalance of the *star voltages* with no phase shift; (2) by injecting into one of the phases a quadrature e.m.f. obtained from the secondary of a transformer whose primary was connected across the other two phases, as shown in Fig. 1.

Measurements were made of the star voltages, the line voltages, the line currents, the power in each phase and the output. The method of procedure was to maintain a constant unbalance and load the motor gradually, taking all the readings until the currents far exceeded the normal value. The tests were repeated for various degrees of unbalance.

As is well known, an unbalanced three-phase system may be resolved into two balanced component systems, one of which has the same phase sequence or phase rotation as the unbalanced system, while the other has the opposite phase sequence. For the sake of brevity, we shall in what follows speak of the first balanced component as the *direct phase or direct rotational system*, and of the second as the *reverse phase or counter-rotational system. The unbalance factor of a system is defined to be the ratio of the reverse phase voltage (or current) to the direct phase voltage (or current) and is frequently expressed as a percentage.*

The arrangement of connections shown in Fig. 1 is self-explanatory. The results of the tests are given in Tables I to X. The first seven tables refer to pure unbalanced star voltages with no phase shift, and the next three to unbalanced star voltages combined with phase shift. For purposes of comparison, the performance of the machine as a single phase induction motor is also given in Table XI. Under each table is given the value of the total copper losses corresponding to the output marked by an index.

The impedance of the windings with regard to reverse phase voltages, which is required in predetermining the performance of the motor, was determined by impressing three-phase voltages on the stator and driving the rotor by an auxiliary motor at synchronous speed in a direction opposed to the rotating field. Readings were taken of the p.d. and current. These are given by the curve (*a*) in Fig. 2. Curve (*b*) is the short circuit characteristic, and curve (*c*) the volt-ampere curve obtained for a single-phase current by driving the rotor in the alternating field produced by such a current. The currents corresponding to any reverse phase voltage are seen to be only slightly in excess of those corresponding to the short-circuit test; hence the reverse phase impedance is only slightly (about 5 per cent. in the special case under consideration) smaller than the short-circuit impedance of the motor.

The curves in Fig. 3 exhibit the relation between output and unbalance factor based on two different assumptions. Curve A is based on the assumption that the limit of output corresponding to any degree of unbalance is reached when the total copper losses are equal to the normal full load copper losses; while curve B is based on the assumption that the limit is reached when the current in any one phase reaches the full-load value. Curve A is a straight line up to about 25 per cent. unbalance, which is perhaps the maximum that will ever occur in practice.

It was noticed in the course of the tests that under extreme unbalancing of the supply voltages the motor would not start, and it required to be turned round rapidly by hand before it would begin to accelerate and run up to speed. The supply voltages under such conditions approximate to a single phase supply and the motor has therefore no tendency to start.

It was considered desirable to observe experimentally what the resulting balanced voltage is when the unbalanced supply is corrected by means of a reverse phase series booster, and how far this value agrees with the direct phase component as obtained from the vector analysis of the unbalanced voltages. For this purpose the induction motor under test was connected in series with a three-phase induction regulator arranged to give e.m.f.s of opposite phase rotation. The unbalanced true voltages of supply were 81, 100 and 81. The magnitude and phase of the reverse phase boosting voltages were adjusted until the resulting voltages across the motor were balanced. The resulting balanced voltage was observed to be 85 volts and the currents drawn by the motor were also found to be balanced. Vector analysis of the unbalanced voltages gave 86 volts for the direct phase component and 13 volts for the reverse phase component. The agreement between the two values may, therefore, be considered satisfactory.

Comparison of Temperature Rises obtained with Three-phase and Single-phase Supply.

In Fig. 4 are plotted the results obtained with regard to temperature rise (*a*) when there is the extreme condition of unbalance corresponding to a single-phase supply; and (*b*) when the supply is balanced. The load was in each case 2·6 b.h.p. The much higher temperature rise for a given output when the machine is working on a single-phase supply is amply evident from these curves.

Action of Induction Motor as Phase Balancer

The next test consisted in determining how far the induction motor was capable of correcting the unbalance in the supply voltage. For this purpose one phase of the alternator was loaded with non-inductive as well as inductive loads and the resulting values of unbalanced voltages were taken. Then the induction motor was run from the generator and the voltages were again observed. The results are given in Table XII. The correcting effect of the motor employed was found to be too small to justify further investigation.

PART II. DISCUSSION OF RESULTS

It will be observed from the tables that some of the wattmeter readings are negative. This indicates that the corresponding windings exert a generator action, supplying power to the mains. From this it follows that the total amount of power drawn from the mains by those phases which yield a positive wattmeter reading is in excess of the actual requirements, and that such excess of power is fed back into the remaining generator phase or phases. If the load on the motor is gradually increased, the negative power reduces to zero, and with further increase in load all the phases of the motor draw power from the mains. Any polyphase induction or synchronous motor connected to a distorted supply circuit will behave in such a way as to have a balancing effect on the supply system. This tendency to correct the unbalance is accompanied by a corresponding tendency within the motor to distort the phase relations.

A serious difficulty is encountered when an attempt is made to select a suitable basis for determining the limiting value of the output corresponding to a given degree of unbalance. If there were no interchange of heat between the various parts of the stator, then the limiting value of the output would be obtained when the current in any one phase has reached its full-load value. But in practice it is found that owing to the flow of heat and consequent tendency to equalisation of temperature throughout the stator, the phase carrying the full-load current does not reach the same high temperature as it would do under normal conditions of working on a balanced system. If the interchange of heat were perfect, then the basis for the limiting output would be that $I_1^2 + I_2^2 + I_3^2$ should equal $3I^2$, where I_1, I_2, and I_3 are the phase currents and I is the normal full-load phase current. The true limit of output will lie somewhere between the two extreme conditions considered.

It has been assumed by some writers, for convenience in predetermining the losses, that the actual total copper losses are equal to the sum of the separate losses due to the direct and reverse phase currents, just as if they were of two different frequencies. This is not strictly correct; but when the unbalance is small the values obtained on the above assumption are nearly correct. We give below the losses calculated by the two methods, I_d and I_r standing for the direct and reverse phase components respectively of the unbalanced three-phase currents.

Percentage unbalance	$r\,(I_1^2 + I_2^2 + I_3^2)$	$3r\,(I_d^2 + I_r^2)$
3·5	825	840
10·2	865	853
15·5	824	833
3·1	870	855
9·8	805	855
16·7	798	795

The resolution of the unbalanced voltages or currents into two balanced systems of opposite phase rotation was done graphically. The values of the counter-rotational currents obtained graphically are nearly the same as those obtained experimentally, as will be seen from the following table :—

Reverse phase current (Experimental)	Reverse phase current (Graphical)
6·2	6·1
19·0	18·0
27·5	29·6
7·0	7·5
18·5	16·0
26·5	24·2

In the article by Slepian previously referred to a formula is given for the reverse phase current, which is equivalent to the following :—

$$\text{Reverse phase current} = \frac{\text{Reverse phase voltage}}{\text{Reverse phase impedance}}.$$

The values given by this formula, using the graphically determined reverse phase voltage and the reverse phase impedance given by curve (*a*) of Fig. 2, are in exact agreement with the values determined by graphical analysis of the current system.

The following formula is also given by Slepian for obtaining the values of the direct rotational current and output :—

$$\begin{array}{c}\text{D. R. current}\\ \text{or}\\ \text{output}\end{array} = \sqrt{1 - \left(\frac{\text{Percentage unbalance factor}}{\text{Percentage reactance}}\right)^2} \times \text{full-load current or normal output.}$$

The table which follows gives a comparison of the values of the direct rotational current calculated by means of Slepian's formula with the values obtained by graphical analysis of the current system, and of the output values calculated by means of the formula with those obtained by direct experiment :—

Direct Rotational Current		Output	
From formula	Graphical	From formula	Graphical
44·6	46·0	7·40	6·90
40·0	43·2	6·66	6·00
31·9	35·5	5·31	4·80
44·7	46·2	7·43	7·10
40·4	44·0	6·72	6·00
29·4	38·2	4·90	4·70

The agreement here is by no means close, but since the limiting value of output cannot be decided with any certainty, the lack of close agreement might be partly accounted for by that.

Summing up we may say that single-phase loads on a three-phase system disturb the balance and cause a reduction of output and increased losses in all the polyphase appliances connected to the circuit. Polyphase motors connected to such an unbalanced system tend to restore balance to some extent by taking reverse phase order currents. But the correcting currents taken by polyphase motors are much less than the disturbing currents due to the single phase loads. The correcting effect will be the greater the less the internal impedance of the windings, or, what is the same thing, the larger the motor. If the impedance drop in the windings with regard to the reverse phase order currents is neutralised by injecting a reverse phase e.m.f. by means of a series booster, then complete balance is obtained. The correcting effect of any individual motor will in any case be very small, as its output will always be much smaller than that of the system to which it is connected.

TABLE I

Phase voltages
A B C
53·5 58·0 58·0
Direct phase voltage = 97·2.
Reverse phase voltage = 3·40.
Unbalance factor = 3·5 per cent.

Line voltages
AB BC CA
95·4 100 95·4
Direct phase current = 46·0. } at limiting output.
Reverse phase current = 6·1. }

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
8·6	15·2	16·0	114	300	0	0·4	0·3
9·3	15·5	16·5	228	400	100	0·7	0·8
11·2	19·2	18·0	406	600	400	1·4	1·5
14·8	23·0	20·6	673	900	700	2·3	2·4
19·6	28·0	24·0	940	1100	900	2·9	3·4
25·5	36·0	28·9	1140	1400	1300	3·8	4·4
35·0	45·5	39·0	1905	2100	1900	5·9	5·9
41·0[1]	51·5	45·0	2080	2325	2080	6·5	6·9
46·0	56·0	48·0	2210	2500	2200	6·9	7·4

[1] Copper losses at limiting output = $\cdot 13\,(41^2 + 51{\cdot}5^2 + 45^2) = 825$ watts.

TABLE II

Phase voltages
A B C
43·0 57·2 57·2
Direct phase voltage = 90·8.
Reverse phase voltage = 9·25.
Unbalance factor = 10·2 per cent.

Line voltages
AB BC CA
87·0 100 87·0
Direct phase current = 43·2. } at limiting output.
Reverse phase current = 18·0. }

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
3·7	23·5	24·5	− 102	850	− 320	0·4	0·4
2·8	25·0	24·0	+ 13	1000	− 200	0·8	0·8
5·1	30·5	25·2	190	1800	0	2·0	1·7
9·5	35·0	26·5	380	1500	+ 400	2·3	2·4
10·6	38·5	27·5	457	1700	450	2·6	3·0
19·0	48·0	34·0	813	2100	900	3·8	4·3
31·0[1]	60·5	45·0	1300	2700	1550	5·55	6·0
35·5	67·0	50·0	1520	3000	1900	6·4	6·85.

[1] Copper losses = $\cdot 13\,(31^2 + 60{\cdot}5^2 + 45^2) = 865$ watts.

TABLE III

Phase voltages
A B C
37·0 57·0 57·0
Direct phase voltage = 87·0.
Reverse phase voltage = 13·0.
Unbalance factor = 15·5 per cent.

Line voltages
AB BC CA
81·0 100 81·0
Direct phase current = 35·5. } at limiting
Reverse phase current = 29·6. } output.

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
12·7	33·0	31·5	– 203	1400	– 610	0·6	0·3
12·0	37·0	30·5	– 76	1600	– 400	1·1	1·0
12·5	44·0	31·1	152	1900	– 150	1·9	2·0
16·0	54·0	36·0	456	2550	50	3·5	3·2
21·5[1]	64·0	42·5	775	3100	1000	4·9	4·8
24·5	67·0	45·0	836	3250	1200	5·3	5·1

[1] Copper losses = ·13 $(21{\cdot}5^2 + 64^2 + 42{\cdot}5^2)$ = 824 watts.

TABLE IV

Phase voltages
A B C
30 57·0 57·0
Direct phase voltage = 83·6.
Reverse phase voltage = 17·0.
Unbalance factor = 20·2 per cent.

Line voltages
AB BC CA
78·0 100 78·0

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
19·0	40·0	37·5	– 328	1300	– 750	0.3	0·4
17·2	46·5	36·0	– 50	2100	– 500	1·55	1·4
17·5	55·0	37·5	178	2500	0	2·70	2·4
19·8	62·5	40·5	356	3000	300	3·70	3·4
22·0[1]	65·0	43·0	425	3100	425	4·00	3·8
22·5	70·0	45·0	534	3350	650	4·50	4·65

[1] Copper losses = ·13 $(22^2 + 65^2 + 43^2)$ = 837 watts.

TABLE V

Phase voltages			Line voltages		
A	B	C	AB	BC	CA
24·5	57·0	57·0	73·0	100	73·0

Direct phase voltage = 79·8.
Reverse phase voltage = 21·0.
Unbalance factor = 26·5 per cent.

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
26·7	47·5	44·0	– 254	2150	– 1100	0·8	0·3
25·2	53·5	42·5	– 104	2500	– 800	1·6	1·2
24·0	59·5	42·5	0	2800	– 420	2·4	1·8
24·0[1]	64·0	43·5	100	3025	– 200	2·9	2·3
24·0	68·0	45·0	178	3250	0	3·4	2·7

[1] Copper losses = ·13 $(24^2 + 64^2 + 43{\cdot}5^2)$ = 850 watts.

TABLE VI

Phase voltages			Line voltages		
A	B	C	AB	BC	CA
19·0	57·0	58·0	68·0	100	69·0

Direct phase voltage = 76·8.
Reverse phase voltage = 23·4.
Unbalance factor = 30·5 per cent.

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
35·7[1]	57·5	52·2	– 229	2700	– 1460	1·0	0·3
32·1	69·5	49·5	– 32	3300	– 700	2·6	2·0

[1] Copper losses = ·13 $(35{\cdot}7^2 + 57{\cdot}5^2 + 52{\cdot}2^2)$ = 950 watts.

TABLE VII

Phase voltages			Line voltages		
A	B	C	AB	BC	CA
10·0	57·0	57·0	62·0	100	62·0

Direct phase voltage = 71·0.
Reverse phase voltage = 28·9.
Unbalance factor = 40·7 per cent.

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
47·0[1]	70·1	62·0	– 178	3400	– 1700	1·5	0·3

[1] Copper losses = ·13 $(47^2 + 70{\cdot}1^2 + 62{\cdot}0^2)$ = 1420 watts.

TABLE VIII

Phase voltages
A B C
56·0 56·0 57·0
Direct phase voltage = 98·5.
Reverse phase voltage = 3·0.
Unbalance factor = 3·1 per cent.

Line voltages
AB BC CA
94·0 98·0 100·0
Direct phase current = 46·2 } at limiting output.
Reverse phase current = 7·5 }

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
16·00	6·20	18·60	– 110	229	500	0·6	0·3
16·25	12·20	23·50	200	610	900	1·7	1·9
20·0	20·00	30·50	705	1120	1500	3·3	3·4
27·50	29·50	38·50	1200	1575	1900	4·7	4·9
35·50	42·00	43·00	1500	2035	2550	6·1	5·7
42·00[1]	45·00	54·00	2000	2290	2700	7·0	7·1
59·00	62·00	71·00	2700	3050	3500	9·3	8·9

[1] Copper losses = ·13 $(42^2 + 45^2 + 54^2)$ = 870 watts.

TABLE IX

Phase voltages
A B C
54·3 53·25 52·25
Direct phase voltage = 91·6.
Reverse phase voltage = 9·0.
Unbalance factor = 9·8 per cent.

Line voltages
AB BC CA
85·2 91·0 100
Direct phase current = 44·0. } at limiting output.
Reverse phase current = 16·0. }

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
23·0	6·0	26·0	– 425	229	830	0·6	0·3
21·0	8·5	28·0	– 200	457	1100	1·4	1·1
20·0	12·5	32·0	0	687	1300	2·0	2·0
22·0	22·5	40·5	500	1245	1820	3·6	3·8
26·9	33·0	50·0	900	1500	2050	4·5	5·1
34·0[1]	41·0	58·0					6·1
43·0	50·0	67·0	1450	2490	3200	7·1	7·1

[1] Copper losses = ·13 $(34^2 + 41^2 + 58^2)$ = 805 watts.

TABLE X

Phase voltages
A B C
54·0 51·5 49·5
Direct phase voltage = 87·5.
Reverse phase voltage = 13·3.
Unbalance factor = 16·7 per cent.

Line voltages
AB BC CA
77·0 87·0 100
Direct phase current = 38·2. } at limiting output.
Reverse phase current = 24·2. }

Line currents			Power per phase, watts			Input	Output
A	B	C	A	B	C	K.W.	B.H.P.
30·0	14·2	33·0	– 600	254	1100	0·75	0·3
28·5	15·5	35·5	– 450	432	1300	1·30	0·8
27·0	18·0	40·0	– 200	686	1550	2·0	1·75
25·0	23·0	46·0	100	940	1850	2·9	2·8
26·0	30·0	54·5	1000	1400	2200	3·7	3·75
30·0[1]	36·5	62·0				5·0	4·70
32·0	41·0	68·0	2200	1910	2700	6·8	5·4

[1] Copper losses = ·13 ($30^2 + 36{\cdot}5^2 + 62^2$) = 790 watts.

TABLE XI

Performance as a Single-Phase Induction Motor

Voltage	Current	Input, K.W.	Output, B.H.P.	Efficiency, per cent.	Power factor
100	22·0	0·55	0·30	43·6	0·25
,,	25·0	1·20	1·10	69·0	0·48
,,	31·5	2·20	2·20	75·0	0·70
,,	42·0	3·40	3·30	72·5	0·81
,,	50·0	4·10	4·00	72·3	0·82
,,	65·0	5·20	5·00	71·5	0·80

TABLE XII

Action of Induction Motor as Phase Balancer

—	Non-inductive Load						Inductive Load					
	28 amps.			70 amps.			20 amps.			60 amps.		
	AB	BC	CA	AB	BC	CA	AB	BC	CA	AB	BC	CA
Line voltages of alternator when supplying single-phase load only ...	98·2	100	98·2	97·7	100	97·7	97·8	100	97·8	96·8	100	96·8
Line voltages with single-phase load and induction motor running light	98·2	100	98·2	98·1	100	98·1	98	100	98	97·2	100	97·2
Line voltages with single-phase load and induction motor loaded to about 5 kw.	—	—	—	98·7	100	98·1	—	—	—	—	—	—

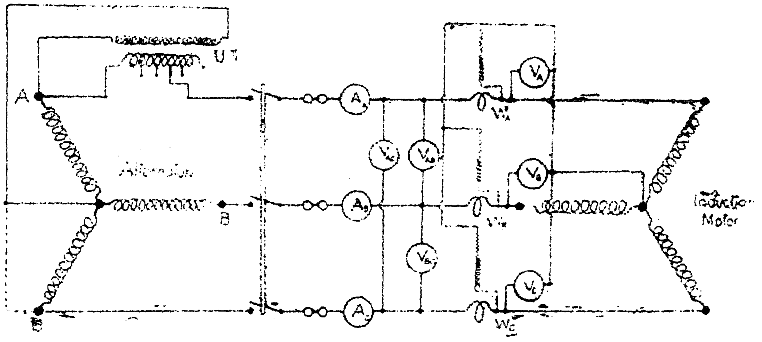

Fig.1

'Diagram of connections.

U Tr, unbalancing transformer; A_A, A_B, A_C, line ammeters; V_{AB}, V_{BC}, V_{CA}, line voltmeters,
W W_B, W_C, wattmeters; V_A, V_B, V_C, star voltmeters

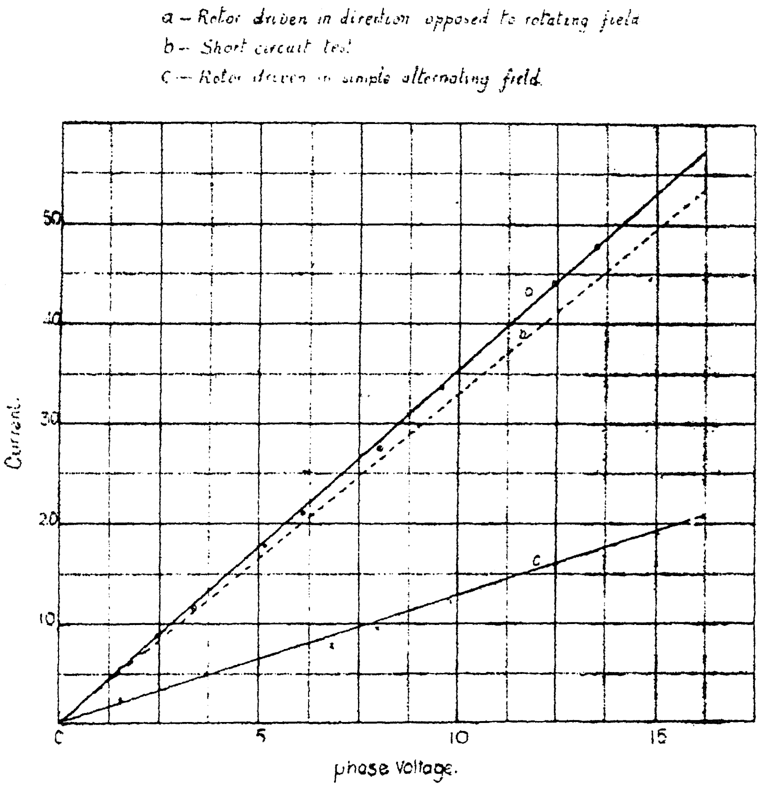

Fig 2 Relation connecting. Stator Voltage and Current.

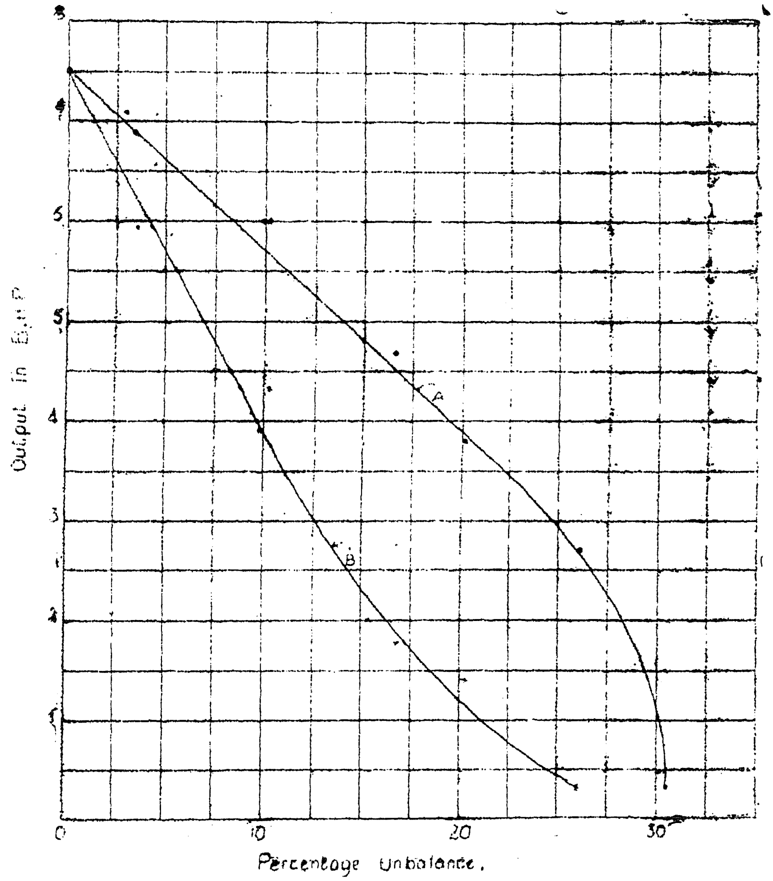

Fig 3; Relation connecting Output and Percentage unbalance

A – when total copper losses equal full load copper losses.
B – when current in any one phase reaches full load value.

CURVES OF TEMPERATURE RISE.

(*a*) Single phase induction motor } Load, 2·6 b.h.p.
(*b*) Three ,, ,, ,,

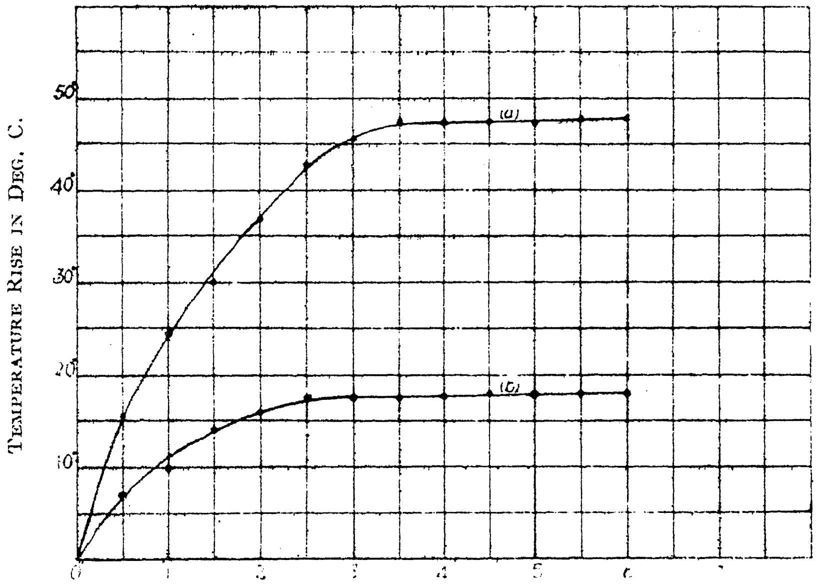

FIG. 4. TIME IN HOURS.