THE MEASUREMENT OF VOLTAGE GRADIENT ON A STRING OF SUSPENSION INSULATORS.

By G. Yoganandam and R. K. Sen.

SYNOPSIS.

Several methods of measuring the voltage distribution on a string of suspension insulators are described and the distribution under different conditions studied. A theoretical investigation of the voltage distribution under approximately known conditions is given, together with its experimental verification, for a string of new Canadian Porcelain cap and pin type line-insulators.

INTRODUCTION.

Although the adoption of suspension type line-insulators has solved the problem of insulating high tension transmission lines to a large extent, it is well known that difficulties are encountered when the number of insulator units in a string is increased beyond a relatively small figure on account of the inequality of the voltage across the assembled units. This unequal distribution is due to the presence of the stray capacities of the hardware to earth and to line, which capacities have a magnitude comparable to that of each insulator. The equivalent circuit for a string of four insulators is shown in Fig. 1 where C, C_1 and C_2 respectively represent the capacity of the insulator itself, the capacity to earth and the capacity to the line. In addition there exist capacities between the other metal parts at different potentials though these are generally negligible.

The first difficulty encountered in carrying out experimental work of this nature is the choice of one of the many possible methods of making the voltage measurements. It is essential that it should give accurate results and at the same time it be reasonably safe for laboratory work at high voltages. As a good deal of information has been published on high-voltage resistance bridge arrangements, for these purposes, nothing has been included in the following on the use of resistances.

The measurements described were carried out at a frequency of sixty cycles per second and the apparatus used consisted of 10 kVA. testing transformers, built for 115 kV. to ground, supplied by a 30 kVA. alternator.

This machine gives a pure sinusoidal terminal voltage and is of the standard type manufactured by Messrs. The International General Electric Company who also supplied the testing transformers used in the experiments.

PART I. EXPERIMENTAL.

The measurement of voltage gradient on a string of suspension insulators.

The following experimental methods have been tried in the High Tension Laboratory of the Department of Electrical Technology, Indian Institute of Science.

I. AUXILIARY TRANSFORMER METHOD.

This method first described by Fontievelle (*Rev. Gen. d'Elec.*, 1921, **10**, 599) contains so many sources of error that it cannot be regarded as of much value. A known voltage, supplied by an auxiliary transformer, is employed to balance the unknown voltage across any two points on the string of insulators. The diagram of connection is shown in Fig. 2 where T_1 is the main transformer, across which the string of insulators is connected, and T_2 is the auxiliary transformer. The low tension side of T_2 is supplied from the same source as that of T_1 and regulated by a potentiometer. The point A is connected to the metal cap of any one of the insulators, as indicated in Fig. 2, and the voltage of the auxiliary transformer is raised, by means of the potentiometer arrangement, until the galvanometer indicates balance. Thus by successively connecting A to the caps of insulators 1, 2, 3, etc., the voltages of those caps above earth can be measured and the difference of successive readings gives the voltage across individual units. In the experiments described the high tension voltage was indicated by voltmeters connected to 'volt coils' provided in the transformers.

Sources of error.—(1) The capacity and the leakage resistance of the galvanometer terminal, B in Fig. 2, to earth alters the voltage distribution on the string.

(2) The lead A-B connecting the galvanometer and the insulator cap disturbs the existing electrostatic field.

(3) The field of the insulators may be distorted by that of the auxiliary transformer.

(4) The stray displacement currents induced in the conductors A-B and C-D and the direct effect of the resultant field, of the two transformers, on the galvanometer may upset the balance point.

(5) Any time-phase difference between the voltages of the two transformers makes it impossible to obtain a zero deflection of the

galvanometer. If α is the phase angle, the auxiliary transformer voltage, when balance is obtained, is $V\cos\alpha$, where V is the voltage to be measured.

Elimination of errors.—If the auxiliary transformer is sufficiently far from the string of insulators, the effect of the transformer field may become negligible.

The disturbing effect of the connecting lead A-B may be minimised to some extent by using a wire of small diameter placed approximately along an equipotential surface of the insulator field.

If the phase difference between the two transformer voltages is known, a correction can be applied to the actual observations made. In order to measure this phase angle, a low range electrostatic voltmeter, reading up to 1,500 volts, was connected between the high tension terminals of the transformers which were connected in opposition and whose primaries were supplied from the same source. The primary voltage was increased until each of the transformer secondaries gave 60 kV. and it was observed there was practically no deflection on the electrostatic voltmeter, thus no correction for the angle of phase difference was necessary in this instance.

It is not possible to eliminate errors (1) and (4) if the galvanometer is placed in the position shown in Fig. 2, but they may be eliminated as follows:—

A needle point is attached to the exploring terminal A and it is arranged to make and break contact with the metal cap on the insulator while the voltage of the auxiliary transformer is being raised. The point of no sparking between the needle point and the metal cap is a rough indication of balance. It was found, however, that there was no spark over a considerable range of voltage on the low tension side of the transformer T_2, thus this method is not very sensitive. Further the needle point and its attached conductor appreciably disturb the existing field. Another method of balance in which the indicating instrument is connected in series with the string of insulators was found to have advantages and is described below.

Valve circuit balance indicator.—A thermionic valve circuit, consisting of a rectifier and amplifier, was connected in series with the insulator string and at the grounded end as shown in Fig. 3. The deflection of a microammeter in the anode circuit of the second valve was noted for any voltage at which the voltage distribution is required. The lead B-A was then connected to one of the insulator caps and the voltage of the auxiliary transformer raised until the microammeter deflection was the same as before. Then the voltage of the transformer T_2 gives the voltage of the cap above earth. There is a source of error in this method, viz., the induced currents in the lead C-D due to the field of the transformer T_2. This stray effect may be neutralised for all voltages of T_2 by injecting into the valve circuit an electromotive force of the proper phase and magnitude from the primary of the transformer T_2. The lead A-B being disconnected, T_1 is excited and the reading of the microammeter noted. Next T_2 is excited to some voltage and C and R_2 adjusted until the original deflection is repeated. The adjustment is correct when no change in the deflection takes place on varying the voltage of T_2. Then the lead A-B is connected to the insulator string and the voltage of the auxiliary transformer adjusted to give the original reading in the microammeter. The sensitiveness of the balance thus obtained was increased considerably by giving a negative bias to the second valve and increasing the resistance R_1 and it was found possible to get a deflection of ten scale divisions for one volt change in the primary of the transformer T_2.

II. CAPACITY BRIDGE METHOD.

Two variable condensers are connected in series across the string of insulators with a high tension electrostatic voltmeter, as shown in Fig. 4. The valve circuit described above, but without the neutralising arrangement, is inserted in the earth lead of the insulator string. The lead A-B being disconnected, the transformer is excited to any desired voltage and the deflection of the microammeter noted. The lead is then connected to any desired insulator cap: and the condensers C_1 and C_2 adjusted until the microammeter deflection is the same as before. The reading of the voltmeter gives the voltage between the cap and the earth or the cap and the line. The capacity of the voltmeter to earth and its self-capacity greatly reduce the flexibility in the variation of the condensers C_1 and C_2.

In making use of this method, in the laboratory, two fixed metal plates with a movable plate between them constituted the pair of condensers C_1 and C_2. The instrument used was a standard electrostatic attracted disc type voltmeter made by Messrs. Everett, Edgcumbe & Co., Ltd. (see Fig. 11) having a voltage range up to 40 kV. The capacity was altered by adjusting the distance between two plates and the sensitiveness of balance was not the same when the lead A-B was connected to different insulator caps. The chief merit of this method, perhaps, is its simplicity.

III. VOLTMETER METHOD.

The direct measurement of the voltage across any unit is usually too inaccurate for practical purposes on account of the internal capacity of the voltmeter and its external capacity to earth, both of which influence the distribution of voltage on the string of insulators. The method is useful when the earth and the line capacities of the insulators and the earth capacity of the voltmeter are negligible. Error due to the internal capacity of the voltmeter may be eliminated by a method suggested by Fontievelle.

In the experiments described the voltage distribution was measured by means of the high tension electrostatic voltmeter in three ways:—

1. One terminal of the voltmeter was connected to the line and the other to different insulator caps in turn. Differences in the readings gave the voltage across each insulator.

2. This was repeated with one terminal of the voltmeter connected to the earthed end of the string.

3. The voltage on each insulator was measured directly by means of the voltmeter.

COMPARATIVE RESULTS.

The results of tests carried out, by the above methods, on a string of five new standard ten-inch Canadian porcelain suspension insulators of the cap and pin type, a section of which is shown in Fig. 10, are given in Table I. where the voltage across each insulator is stated as a percentage of the line-to-earth voltage. As might be expected from a consideration of the sources of error, the percentage voltage on the line-insulator measured by the auxiliary transformer method with galvanometer balance, or by the direct voltmeter method, is rather too high. The results obtained by the other methods are sufficiently in agreement to be accepted as correct. The capacity bridge method is much simpler than the auxiliary transformer method with the same 'valve circuit balance' because less apparatus is required. On the other hand the latter method, though a little cumbrous, has the advantage of extreme sensitiveness and safety when measurement of the voltage distribution at very high voltages is necessary. The 'No Spark' method of balance yields reasonably accurate results and has been found one of the most expeditious methods available for practical purposes.

VOLTAGE DISTRIBUTION ON A STRING OF SUSPENSION INSULATORS UNDER DIFFERENT CONDITIONS.

1. *Effect of earth and line capacities.*—With a view to studying the influence of the capacity, of the metal parts of a string of insulators, to earth and to line, an insulated metal plate 4 ft. × 2 ft. was placed in a vertical position at a distance of 17 inches from the line. The curve (*a*) Fig. 5 gives the normal voltage distribution *without* the plate and the curves (*b*) and (*c*) give the distributions of voltage with the plate connected to earth and to line respectively. It is interesting to note that there is little departure from the normal (*a*) distribution when the plate is at the earth potential, on the other hand, when the plate is connected to the line the alteration in distribution is considerable. The capacity being inversely proportional to the distance between the string of insulators and the metal plate, it is only at relatively small distances that the capacity effect of the metal plate is considerable. The curve (*d*) of Fig. 5 gives the voltage distribution with the metal plate at *line potential* and only ten inches distant from the line. It is concluded from these observations, that the theory, advanced by F. Olendorff, [*Archiv. f. Elektrot.* 1926, **16**, 261 ; **17**, 79 and 242] that the tower or the pole, from which the insulator string is suspended, has no influence on the voltage distribution, must not be accepted without reservation.

2. *Effect of line-to-earth voltage on the distribution.*—At high voltages the formation of corona, around sharp corners of the metal parts or dust particles on the insulators, reduces the impedance of the insulators and especially that of the line unit. Consequently when the voltage across the string is increased the voltage distribution re-adjusts itself slightly and the gradient tends to become more uniform. Curves showing the voltage across the line insulator at different line-to-earth voltages with various numbers of units in the string are plotted in Fig. 7 and these bear out the above statements.

3. *Influence of the number of insulators in the string on the voltage across the line unit.*—Observations, plotted in Fig. 8, indicate that the voltage across the line insulator falls to a minimum of about 22 per cent. when the number of insulators in the string exceeds six or seven of the type tested.

4. *Effect of the arcing horn.*—The arcing horn exerts a small but appreciable influence on the voltage distribution as shown in Fig. 6. Curve (*c*) gives the distribution when a circular ring 15 inches in diameter made of ½ inch mild steel rod was attached to the tips of the horn.

TABLE I.

Tests made at 60 ~ with 24·6 kV. between line and earth, sine wave form.

Insulator number	Auxiliary Transformer Method			Capacity Bridge Method	Voltmeter Method		
	Galvanometer Balance	Spark Balance	Valve Circuit Balance	—	Voltmeter between earth and Insulator	Voltmeter between line and Insulator	Voltmeter across each Insulator
	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.
1 *line*	51·6	33·2	31·6	30·5	50·0	41·0	41·0
2	13 9	19·5	18·2	19·5	15·9	26·5	18·6
3	10·7	16·6	17·0	17·3	13·0	12·2	14·3
4	10·4	15·2	16·5	16·6	12·1	8·5	10·9
5 *earth*	13·4	15·5	16·7	17·1	9·0	11·8	9·0

Approximate atmospheric conditions during experiments :–

Pressure 26·7 to 30·0 inches of mercury.
Temperature 80° to 90° F.
Relative Humidity 50 per cent. to 60 per cent.

PART II. THEORETICAL.

Predetermination of the voltage gradient on a string of suspension insulators.

Approximate methods of calculating the voltage distribution have been developed by F. W. Peek (*J. Amer. Inst. Elec. Eng.*, 1912, **31**, 717) and A. Salessky (*Arch. f. Elek.*, 1924, **13**, 717). The former assumed equal capacity between each insulator and earth and neglected the capacity to line, while the latter assumed each insulator to have the same capacity to line. A more exact method would be to consider the capacity between each unit and the line to vary inversely as its distance from the line.

It has been suggested by A. Schwaiger, (*Elek. U. Maschin.*, 1919, **37**, 569) that a string of insulators may be treated as a case of distributed capacity for the purpose of calculating the voltage distribution. This method, which is much simpler and more rapidly applied than the numerical methods of Peek and Salessky, may be used for studying the voltage distribution under the following conditions:—

Case (i) Equal capacities to earth and negligible capacities to line.

Case (ii) Equal capacities to earth and equal capacities to line.

Case (iii) Capacities to earth negligible and the capacity, of any element of the string, to line varying inversely as its distance from the line. This represents the conditions when the insulators are suspended from a wooden pole or relatively far from a steel tower.

Case (iv) Equal capacities to earth and capacities to line varying as in case (iii).

The string of insulators with its earth and line capacities are represented by a system of series and parallel capacities as shown in Fig. 1. The following nomenclature will be used in calculating the voltage distribution:—

E_L = Total applied voltage on the string (line-to-earth).
E = Voltage above earth at any point on the string.
x = Distance of any element from the earthed end.
l = Length of the string.
i = Capacity current in the string at a distance x from the earthed end.
C = Self-capacity of the string per unit length.

C_1 = Capacity to earth per unit length of string.
C_2 = Capacity to line per unit length of string.
γ = C_1/C
β = C_2/C
ω = $2\pi f$
f = frequency of supply.
g_l = Voltage gradient at line end of string.

Case (i) Taking the earthed end as the origin the equations for the current and change of current at a distance x from the origin are

$$i = \frac{dE}{dx} w C \quad \ldots\ldots (1)$$

$$\frac{di}{dx} = E w . C_1 \quad \ldots\ldots (2)$$

Differentiating (1) and equating with (2), gives:—

$$\frac{d^2 E}{dx^2} = \gamma E \quad \ldots\ldots (3)$$

The solution of this equation may be written as:—

$$E = A \sinh (x\gamma^{\frac{1}{2}} + B).$$

Since $E = 0$ when $x = 0$ and $E = E_L$ when $x = l$ the voltage distribution is given by:—

$$E = \frac{E_L}{\sinh\ l\gamma^{\frac{1}{2}}} \ \text{Sinh}\ x\gamma^{\frac{1}{2}} \quad \ldots\ldots (4)$$

and the gradient by:—

$$\frac{dE}{dx} = \frac{E_L\ \gamma^{\frac{1}{2}}}{\text{Sinh}\ l\gamma^{\frac{1}{2}}} \cosh x\gamma^{\frac{1}{2}} \quad \ldots\ldots (5)$$

The gradient at the line end is:—

$$g_l = E_L \gamma^{\frac{1}{2}} \coth l\gamma^{\frac{1}{2}}.$$

Equation (5) shows that as the length of the string is increased g_l falls, rapidly, to the practically constant value:—

$$g_{l=\infty} = E_L \gamma^{\frac{1}{2}}$$

Case (ii) The corresponding differential equation is:—

$$\frac{d^2 E}{dx^2} = E\ (\gamma + \beta) - E_L\ \beta \quad \ldots\ldots (6)$$

and the solution may be written as:—

$$E = p + q \sinh\ (\gamma + \beta)^{\frac{1}{2}}\ (x + B) \quad \ldots\ldots (7)$$

Where p, q and B are constants to be determined from the initial and final conditions. It can be shown that:—

$$p = \frac{\beta}{\gamma + \beta} E_L \text{ and}$$

$$q^2 = [4K^2 \{ p^2 + (E_L - p)^2 \} + 4Kp \, (K^2 + 1)(E_L - p)] / (K^2 - 1)^2$$

$$\text{where } K = e^{l(\gamma+\beta)^{\frac{1}{2}}}$$

$$\text{and } B = l \log_e R / \log_e \frac{S}{R}$$

$$\text{where } R = -p/q + \sqrt{p^2/q^2 + 1}$$

$$\text{and } S = \frac{E_L - p}{q} + \sqrt{\frac{(E_L - p)^2}{q^2} + 1}$$

The equation for the potential gradient is given by:—

$$g = q\,(\gamma + \beta)^{\frac{1}{2}} \cosh\,(\gamma + \beta)^{\frac{1}{2}}\,(x + B) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The constant B can be shown to be negative and when $x = B$ numerically the gradient becomes a minimum and equal to $q\ (\gamma + \beta)^{\frac{1}{2}}$. In a string of good insulators, the second or the third unit from the grounded end generally shares the least percentage of the line-to-earth voltage and is referred to as the 'Silent Insulator', in the 'buzzer' tests described by T. F. Johnson (*Elec. World*, 1919, **74**, 568).

Case (iii) In this case C_2 is a varying capacity. Let C_2^0 be the value of C_2 when $x = 0$ and $C_2^0/C = \beta$.

The differential equation in this case is:—

$$(l - x)\frac{d^2E}{dx^2} = -(E_L - E)\ \iota\beta \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

and the solution of equation (9) may be expressed by the series

$$E = E_L + \frac{(l-x)}{\underline{\lvert 1}}\ \frac{dE}{d(l-x)_0} + \frac{(l-x)^2}{\underline{\lvert 2}}\ \frac{d^2E}{d(l-x)_0^2} + \ldots\ldots\ldots\ldots$$

It is more convenient to change the independent variable from x to X where $X = (l - x)$ and put the above equation in the form

$$E = E_L + \frac{X}{\underline{\lvert 1}} \cdot \frac{dE}{dX_0} \div \frac{X^2}{\underline{\lvert 2}} \cdot \frac{d^2E}{dX_0^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

The co-efficients dE/dX_0, d^2E/dX_0^2................etc., can be obtained from the relation $\dfrac{d^nE}{dX_0^n} = \dfrac{\beta^{n-1}}{\underline{\lvert n-1}} \cdot \dfrac{dE}{dX_0} = \dfrac{\beta^{n-1}}{\underline{\lvert n-1}} \cdot B$

where B is a constant determined by the condition $E = 0$ when $X = l$.

Case (iv) The differential equation in this case is :—

$$X.\frac{d^2E}{dX^2} = E\,(X\gamma + \beta l) - E_L.\,\beta l \qquad \text{.......(11)}$$

The solution of equation (11) is similar to that of case (iii). The co-efficients of powers of X in the series are connected by the equation

$$\frac{d^nE}{dX_0^n} = \frac{\beta}{n-1}.\ \frac{d^{n-1}E}{dX_0^{n-1}} + \gamma\frac{d^{n-2}E}{dX_0^{n-2}} \qquad \text{.......(12)}$$

APPLICATION TO PRACTICAL CASES.

Calculated curves of the voltage gradient are shown in Fig. 9 for different values of γ and β and on the assumption that the insulators are perfect capacities which is nearly true when the insulators are dry and clean, the resistance then being very large compared to the capacity impedance. Under wet conditions, or when the surface of the insulator is covered with dust, the leakage resistance is considerably lowered which has the effect of diminishing γ and β. The extent to which the voltage gradient is thereby improved is shown in Fig. 9 which gives curves of gradient for selected values of γ and β.

METHODS OF CONTROLLING THE VOLTAGE DISTRIBUTION.

It is well known and is clearly shown by the curves drawn from the above equations, Fig. 9, that one way to improve the voltage distribution is to use insulators of different capacities in the same string. For example, in case (i), by placing a unit of relatively large capacity near the line end, the voltage on the line unit is reduced. The condition for perfectly uniform distribution of voltage is given by the equation

$$C = C_o + C_1.\,\frac{x^2}{2} \qquad \text{.......(13)}$$

where C is the capacity per unit length of the string at a distance x from the grounded end and C_o is the value of C when $x = 0$.

If there are n insulators in a string, each having a capacity C_1 to earth, then to obtain constant gradient the capacity of the rth insulator from the line end must be made equal to :—

$$C_r = C_0 + \frac{(n-r)\,(n-r-1)}{2}C_1 \qquad \text{.......(14)}$$

where C_o is the capacity of the insulator at the ground end.

In case (ii) the condition is given by :—

$$\frac{C}{C_0}=\frac{x^2}{2}(\gamma+\beta)-x\beta+1 \quad \text{where } \gamma=\frac{C_1}{C_0} \text{ and } \beta=\frac{C_2}{C_0} \text{..........}(15)$$

Another method of equalising the voltages on the insulators is to use on the line a shield of such a shape that at any point of the string, the current flowing through the earth capacity is equal to the current flowing through the line capacity. The above condition can be put in the form of the equation

$$\frac{C_2}{C_1}=\frac{x}{L-x} \text{..}(16)$$

where C_2 is the varying capacity to line.

EXPERIMENTAL AND THEORETICAL RESULTS COMPARED.

It will be seen that the calculated curves of voltage distribution, given in Fig. 9, are qualitatively in agreement with those obtained by experiment. The effect of line and earth capacities predicted by theory is verified by the curves (*a*), (*b*), (*c*) and (*d*) shown in Fig. 5 which represent the measured distribution for a string of five units.

The influence of line capacities due to the presence of the arcing horn, though small, is noticeable in Fig. 6 which shows a rising gradient towards the earthed end of the insulator string. Both theory and experiment indicate that, depending upon the magnitude of the capacities of the insulators to earth, there is a limit to the number of useful insulators in the string. This limiting number, however, increases slightly with high voltages on the line, on account of the formation of corona on the conducting portions of the string of insulators.

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Department of Electrical Technology,
Indian Institute of Science,
Bangalore.

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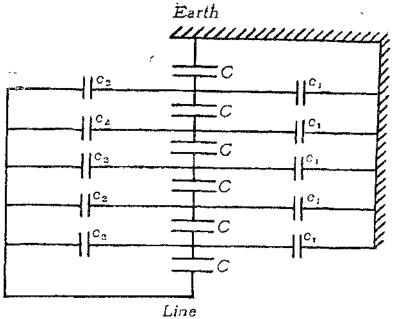

Fig. 1. *Equivalent Circuit*

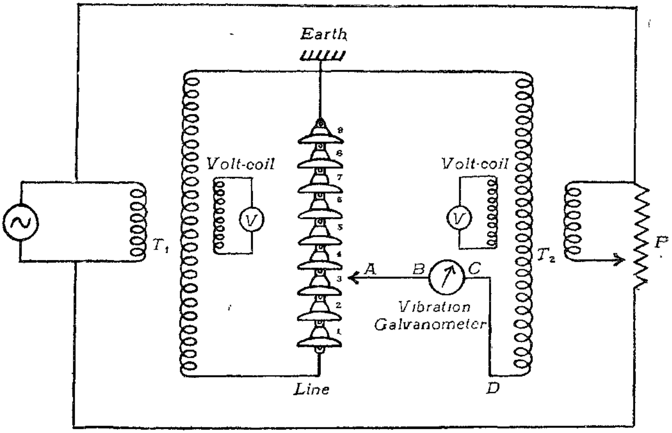

Fig. 2. *Auxiliary Transformer Test Circuit*

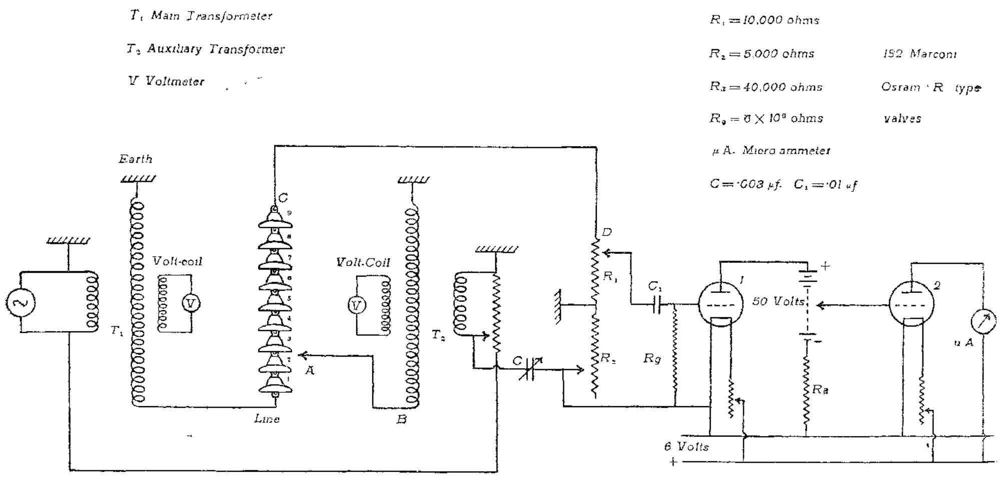

Fig. 3. Thermionic Valve Voltage-balance-detector circuit

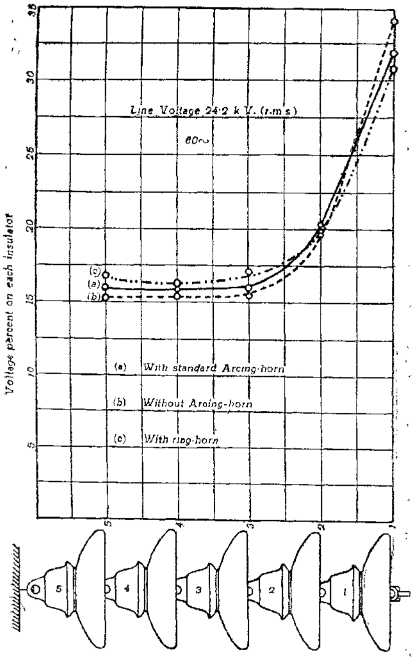

Fig. 6. *Effect of Arcing-horn on the Voltage Gradient*

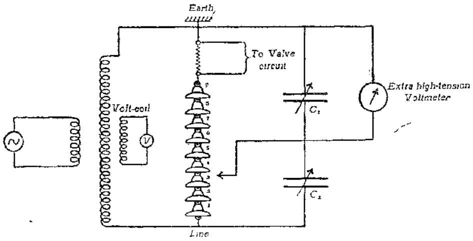

Fig. 4. *Capacity Bridge Circuit*

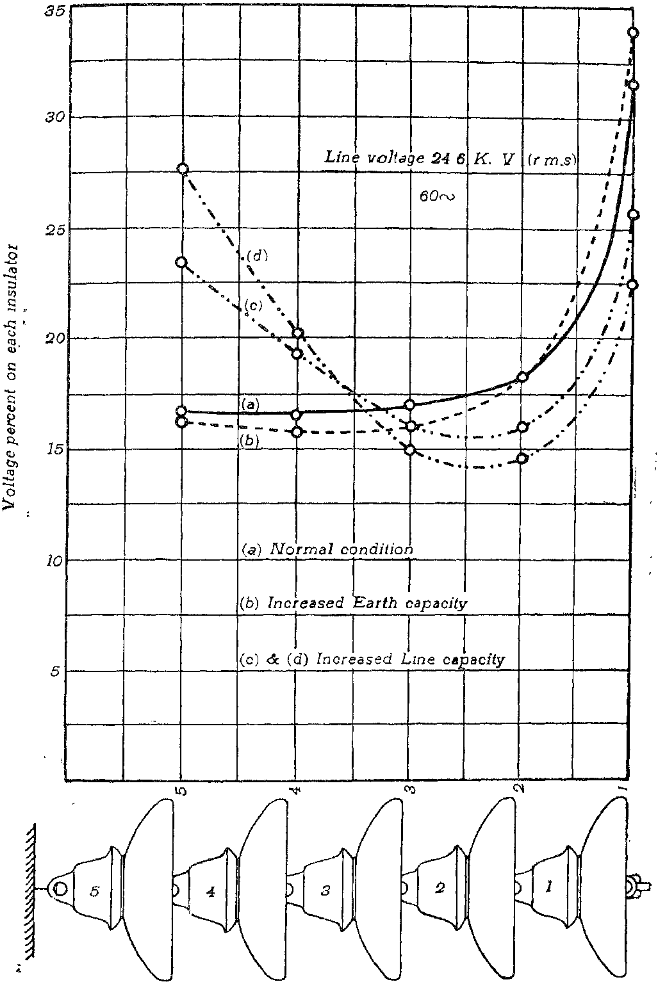

Fig. 5. *Effect of Earth and line capacities on the voltage gradi*

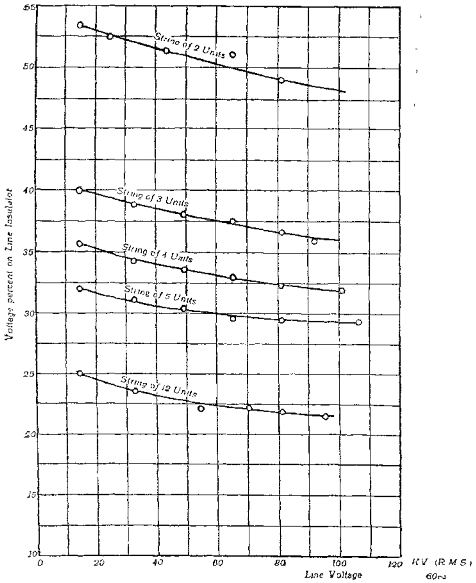

Fig. 7. *Effect of line Voltage*

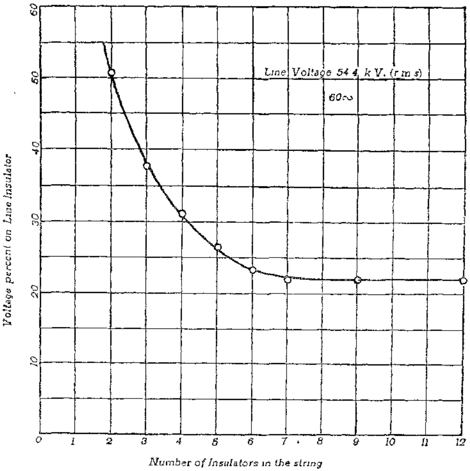

Fig. 8. *Effect of number of units on Voltage across line unit*

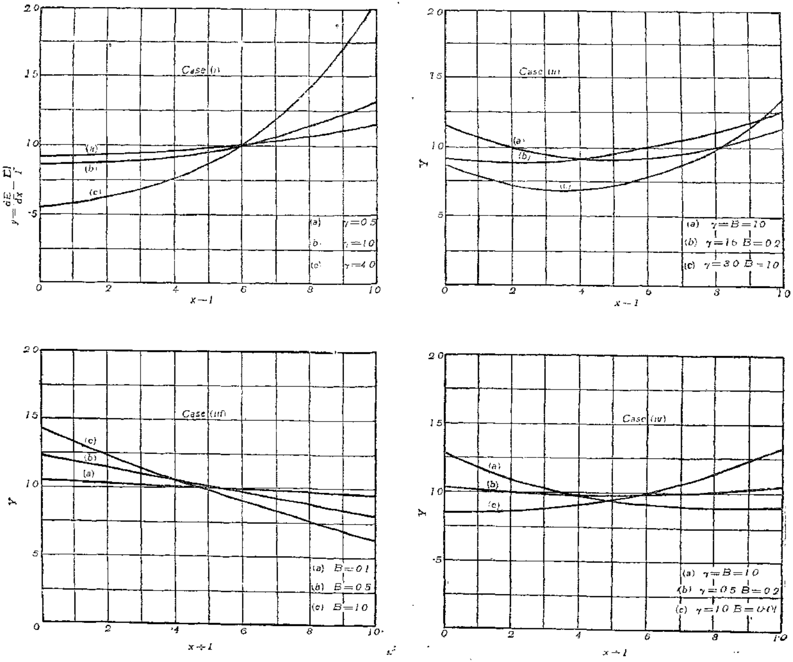

Fig. 9. *Theoretical Voltage Gradient*

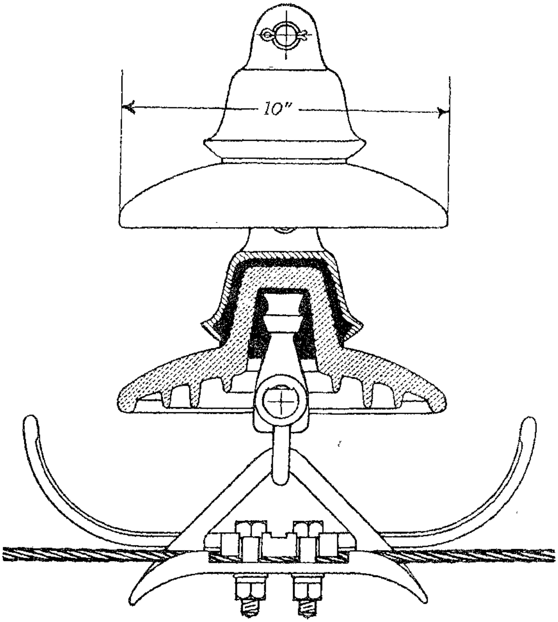

Fig. 10.

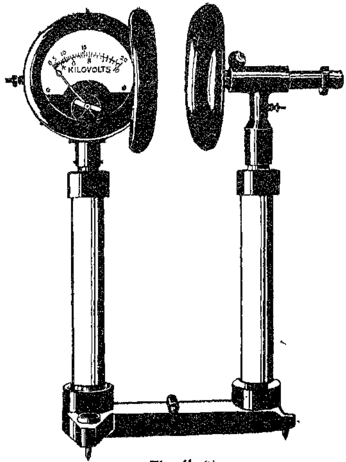

Fig. 11. (a)

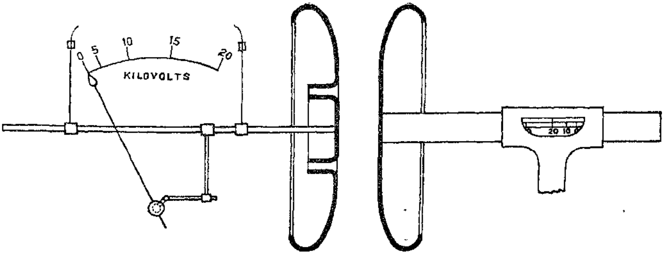

Fig. 11. (*b*)