HEAT TRANSFER IN A SLOW STEADY FLOW OF AN ELASTICO-VISCOUS FLUID IN A WAVY CYLINDRICAL TUBE

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ABSTRACT

The problem of heat transfer due to the steady slow motion of an elastico-viscous fluid in a wavy cylindrical tube has been considered. Taking the deformation of the boundary to be small, the equations of continuity, momentum and energy have been solved using the perturbation technique. The solution for the velocity field is employed to study the nature of the temperature distribution when the surface of the tube is maintained at a constant temperature.

The velocity field and the vorticity are affected by the stress-relaxation time, while the stress, the skin-friction and the temperature distribution are affected by both the stress-relaxation and the strain-retardation times. The streamlines and the isotherms are similar in nature to that observed in the reference 6.

INTRODUCTION

Using the theory of Fourier Transform Citron[1] has discussed the problem of slow steady motion of a Newtonian fluid in the annular space between two rough coaxial circular cylinders rotating about their common axis. Khamrui[2] has utilised the same procedure to discuss the problem of slow steady flow of Newtonian incompressible fluid through a cylinder assuming the roughness to be small in comparison to the smooth radius of the cylinder. The assumption of vanishing of azimuthal velocity made by Citron is not correct as the roughness of the boundary has been taken in the form of sinusoidal deformation. Belinfante[3] considered the problem of steady flow of Newtonian fluid in a pipe with constrictions using (i) an infinite series for the perturbed velocity field and (ii) a procedure of iteration on the Reynolds number. Following Belinfante, Tyagi[4] extended the same problem to Reiner-Rivlin fluids. Bhatnagar (P. L.) and Mohan Rao[5] have investigated the flow of a Reiner-Rivlin fluid between two wavy cylinders rotating about their common axis using the Fourier series instead of Fourier transforms to avoid the explicit reference to the conditions at infinity. In an earlier investigation[6], we have adopted this method to study the slow steady flow of a Rivlin-Ericksen fluid in a wavy cylindrical tube with heat transfer. The aim of the present investigation is to study the same problem[6] for an elastico-viscous fluid, characterised by the constitutive equation given by Oldroyd[7]. The constitutive equation as given by Oldroyd is

$$E_{ik} = \tfrac{1}{2}\left(U_{k,i} + U_{i,k}\right),$$

$$S_{ik} = P_{ik} - p g_{ik},$$

$$P^{ik} + \lambda_1 \frac{\delta P^{ik}}{\delta T} = 2\mu \left[E^{ik} + \lambda_2 \frac{\delta E^{ik}}{\delta T}\right],$$

$$\frac{\delta B^{ik}}{\delta T} = \frac{\partial B^{ik}}{\partial t} + U^j B^{ik}_{,j} + \Omega^i{}_m B^{mk} + \Omega^k_m B^{im} - E^i_m B^{mk} - E^k_m B^{im},$$

$$\Omega_{ik} = \tfrac{1}{2}\left(U_{k,i} - U_{i,k}\right),$$

where the symbols have the usual meaning. We represent the small deformation in the boundary by a general Fourier series in axial coordinate Z and obtain the solutions to the first power of small deformation and correct to the square of the Reynolds number appropriately defined later in the text.

1. Equations of the Problem

The equations of the problem in additions to the constitutive equation already stated above in tensor notation are:

Continuity equation:

$$U^i_{,i} = 0, \tag{1.1}$$

Momentum equations:

$$\rho U^j U^i_{,j} = S^{ij}_{,j}, \tag{1.2}$$

Energy Equation:

$$\rho C_p U^j T_{,j} = k_1 g^{ik} T_{,jk} + S^{ij} E_{ij}, \tag{1.3}$$

Before proceeding to solve these equations, we render all the physical and dynamical quantities dimensionless by introducing the following dimentionless quantities:

$$r = \frac{r'}{a}, \quad z = \frac{z'}{a}, \quad u = \frac{U_1}{U}, \quad w = \frac{U_3}{U}, \quad p = \frac{P}{\rho U^2}, \quad p_{ik} = \frac{P_{ik}}{\rho U^2}, \quad T = \frac{T'}{T_w}.$$

where a is the average radius of tube, U the velocity on the axis in the absence of deformation in the boundary and T_w the constant temperature of the boundary. In such a scheme the following dimentionless parameters are introduced in the problem:

(i) $R = \rho a U/\mu$ the Reynolds number, ρ being the density of the fluid,

(ii) $\lambda_1' = (\lambda_1 U/a)$, $\lambda_2' = \lambda_2 \mu/\rho a^2$ the dimensionless parameters characterising the stress relaxation time and the strain retardation time respectively,

(iii) $E = U^2/C_p T_w$ the Eckert number, C_p being the specific heat at constant pressure,

(iv) $\sigma = \mu C_p/k_1$ the Prandtl number, k_1 being the thermal conductivity,

(v) $p^{ik} + \lambda_1' (\delta p^{ik}/\delta T) = (2/R) E^{ik} + 2\lambda_2' (\delta E^{ik}/\delta T)$ is the dimentionless deviatoric stress tensor.

Choosing a cylindrical coordinate system (r, θ, z) the boundary conditions of the problem in terms of dimentionless quantities are :

$$\left.\begin{array}{l} u = w = 0, \ T = 1, \ \text{on } r = 1 + \epsilon \sum_{n=1}^{\infty} (a_n \cos \alpha_n z + b_n \sin \alpha_n z) \\ u = 0, \ w_r = 0, \ T_r = 0, \ \text{on } r = 0, \end{array}\right\} \quad [1.4]$$

where $\alpha_n = (n/h)$ and u, w are respectively the radial and axial velocities.

The amplitude of the deformation of the boundary assumed to be small, and the wavelength $2\pi h$ of the periodic deformation can be adjusted by properly choosing ϵ and h respectively.

2. PERTURBATION EQUATIONS AND THEIR SOLUTION

PART A

Velocity Field

We introduce the stream function ψ given by

$$u = -(1/r)\, \psi_z, \quad w = (1/r)\, \psi_r \quad [2.1]$$

so that the equation of continuity [1.1] is identically satisfied and set

$$\left.\begin{array}{l} \psi = \psi^{(0)} + \epsilon \psi^{(1)}, \\ p = p^{(0)} + \epsilon p^{(1)}, \\ p_{ik} = p_{ik}^{(0)} + \epsilon p_{ik}^{(1)}. \end{array}\right\} \quad [2.2]$$

The zeroth order flow is simply the axial flow in a pipe in the absence of deformation in the boundary for which $\psi^{(0)}$ and $p_{ik}^{(0)}$ are functions of the radial distance r alone so that

$$\psi_z^{(0)} = 0,$$

$$\partial (p_{ik}^{(0)})/\partial z = 0.$$

The equations and the boundary conditions determining $\psi^{(0)}$ and $\psi^{(1)}$ are obtained by substituting [2.1] and [2.2] in the dimentionless form of the equations [1.2] and the constitutive equation and equating the various order terms in ϵ.

Thus on integrating the zeroth order equations we obtain

$$\left.\begin{aligned} \psi^{(0)}(r) &= -(R p_z^{(0)}/8)(r^2 - r^4/2), \\ p_z^{(0)} &= \text{constant}, \\ p^{(0)} &= z p_z^{(0)} + \text{constant}; \end{aligned}\right\} \qquad [2.3a]$$

$$\left.\begin{aligned} & p_{rr}^{(0)} = p_{\theta\theta}^{(0)} = p_{r\theta}^{(0)} = p_{\theta z}^{(0)} = 0, \\ & p_{zz}^{(0)} = R p_z^{(0)2} (\lambda_1' - \lambda_2' R)(r^2/2), \ p_{rz}^{(0)} = p_z^{(0)} r/2. \end{aligned}\right\} \qquad [2.3b]$$

Perturbed velocity field:

The boundary conditions satisfied by $\psi^{(1)}$ are:

$$\left.\begin{aligned} & \psi_z^{(1)}(1, z) = \psi_z^{(1)}(0, z) = \psi_r^{(1)}(0, z) = 0, \\ & \psi_r^{(1)}(1, z) = -(R p_z^{(0)}/2) \sum_{n=1}^{\infty} (a_n \cos \alpha_n z + b_n \sin \alpha_n z), \end{aligned}\right\} \qquad [2.4]$$

so that we chose $\psi^{(1)}(r, z)$ in the form

$$\psi^{(1)}(r, z) = -(R p_z^{(0)}/2) \sum_{n=1}^{\infty} [A_n(r) \cos \alpha_n z + B_n(r) \sin \alpha_n z]. \qquad [2.5]$$

The boundary condtions [2.4] then give

$$\left.\begin{aligned} & A_n(0) = A_n'(0) = A_n(1) = 0, \ A_n'(1) = a_n, \\ & B_n(0) = B_n'(0) = B_n(1) = 0, \ B_n'(1) = b_n. \end{aligned}\right\} \qquad [2.6]$$

With the above expression for $\psi^{(1)}(r, z)$ we assume the following expressions for the perturbed stresses:

$$p_{rr}^{(1)} = \sum_{n=1}^{\infty} [C_n(r) \cos \alpha_n z + D_n(r) \sin \alpha_n z], \qquad [2.7a]$$

$$p_{\theta\theta}^{(1)} = \sum_{n=1}^{\infty} [E_n(r) \cos \alpha_n z + F_n(r) \sin \alpha_n z], \qquad [2.7b]$$

$$p_{zz}^{(1)} = \sum_{n=1}^{\infty} [G_n(r) \cos \alpha_n z + H_n(r) \sin \alpha_n z], \qquad [2.7c]$$

$$p_{rz}^{(1)} = \sum_{n=1}^{\infty} [I_n(r) \cos \alpha_n z + J_n(r) \sin \alpha_n z], \qquad [2.7d]$$

and
$$p_{r\theta}^{(1)} = p_{\theta z}^{(1)} = 0. \qquad [2.7e]$$

From [2.?] and [2.7e) we have

$$p_{r\theta} = p_{\theta z} = 0.$$

The motion being slow, we write

$$A_n = A_{0,n} + R A_{1,n} + R^2 A_{2,n}, \quad B_n = B_{0,n} + R B_{1,n} + R^2 B_{2,n}, \qquad [2.8]$$

and similar expressions for C_n, D_n, E_n, F_n, G_n, H_n, I_n and J_n.

From our working, we know that

$$A_{1,n} = B_{1,n} = 0.$$

The equations determining $A_{0,n}$, $B_{0,n}$, $C_{0,n}$, $D_{0,n}$, $E_{0,n}$, $F_{0,n}$, $G_{0,n}$, $H_{0,n}$, $I_{0,n}$ and $J_{0,n}$ are:

$$C_{0,n} = (\alpha_n p_z^{(0)}/r)\,[B'_{0,n} - (1/r)\,B_{0,n}], \qquad [2.9a]$$

$$D_{0,n} = -(\alpha_n p_z^{(0)}/r)\,[A'_{0,n} - (1/r)\,A_{0,n}], \qquad [2.9b]$$

$$E_{0,n} = (\alpha_n p_z^{(0)}/r^2)\,B_{0,n}, \quad F_{0,n} = -(\alpha_n p_z^{(0)}/r^2)\,A_{0,n}, \qquad [2.9c]$$

$$G_{0,n} = -(\alpha_n p_z^{(0)}/r)\,B'_{0,n}, \quad H_{0,n} = (\alpha_n p_z^{(0)}/r)/A_{0,n}, \qquad [2.9d]$$

$$I_{0,n} = -(p_z^{(0)}/2r)\,[A''_{0,n} - (1/r)\,A'_{0,n} + \alpha_n^2 A_{0,n}], \qquad [2.9e]$$

$$J_{0,n} = -(p_z^{(0)}/2r)\,[B''_{0,n} - (1/r)\,B'_{0,n} + \alpha_n^2 B_{0,n}], \qquad [2.9f]$$

$$I''_{0,n} + (1/r)\,I'_{0,n} + (\alpha_n^2 - 1/r^2)\,I_{0,n} - \alpha_n [D'_{0,n} + (1/r)\,D_{0,n}] + \alpha_n H'_{0,n}$$
$$+ (\alpha_n/r)\,F_{0,n} = 0, \qquad [2.9g]$$

$$J''_{0,n} + (1/r), J'_{0,n} + (\alpha_n^2 - 1/r^2)\,J_{0,n} + \alpha_n [C'_{0,n} + (1/r)\,C_{0,n}]$$
$$- \alpha_n G'_{0,n} - (\alpha_n/r)\,E_{0,n} = 0. \qquad [2.9h]$$

On eliminating $C_{0,n}$, $D_{0,n}$, $E_{0,n}$, $F_{0,n}$, $G_{0,n}$, $H_{0,n}$, $I_{0,n}$ and $J_{0,n}$ from the equations [2.9] we get the following equations for the determination of $A_{0,n}$ and $B_{0,n}$:

$$r^3 A^{\text{iv}}_{0,n} - 2r^2 A'''_{0,n} + r(3 - 2\alpha_n^2 r^2)\,A''_{0,n}$$
$$- (3 - 2\alpha_n^2 r^2)\,A'_{0,n} + \alpha_n^4 r^3 A_{0,n} = 0, \qquad [2.10a]$$

$$r^3 B_{0,n}^{iv} - 2r^2 B_{0,n}''' + r\left(3 - 2\alpha_n^2 r^2\right) B_{0,n}''$$
$$-\left(3 - 2\alpha_n^2 r^2\right) B_{0,n}' + \alpha_n^4 r^3 B_{0,n} = 0, \qquad [2.10b]$$

to be solved under the boundary conditions:

$$\left.\begin{aligned} A_{0,n}(1) = A_{0,n}(0) = A'_{0,n}(0) = 0,\ A'_{0,n}(1) = a_n, \\ B_{0,n}(1) = B_{0,n}(0) = B'_{0,n}(0) = 0,\ B'_{0,n}(1) = b_n. \end{aligned}\right\} \qquad [2.10c]$$

The equations determining $C_{1,n}$, $D_{1,n}$, $E_{1,n}$, $F_{1,n}$, $G_{1,n}$, $H_{1,n}$, $I_{1,n}$ and $J_{1,n}$ are:

$$C_{1,n} + (\lambda_1' p_z^{(0)} \alpha_n/4)\left[(r^2 - 1) D_{0,n} + 2\alpha_n p_z^{(0)} A_{0,n}\right] = 0, \qquad [2.11a]$$

$$D_{1,n} + (\lambda_1' p_z^{(0)} \alpha_n/4)\left[(1 - r^2) C_{0,n} + 2\alpha_n p_z^{(0)} B_{0,n}\right] = 0, \qquad [2.11b]$$

$$E_{1,n} - (\lambda_1' p_z^{(0)} \alpha_n/4)(1 - r^2) F_{0,n} = 0, \qquad [2.11c]$$

$$F_{1,n} + (\lambda_1' p_z^{(0)} \alpha_n/4)(1 - r^2) E_{0,n} = 0, \qquad [2.11d]$$

$$G_{1,n} + \lambda_1' p_z^{(0)}\left[(p_z^{(0)}/2)\left\{A_{0,n}'' - (1/r) A_{0,n}'\right\} - r I_{0,n}\right.$$
$$\left.- (\alpha_n/4)(1 - r^2) H_{0,n}\right] = 0, \qquad [2.11e]$$

$$H_{1,n} + \lambda_1' p_z^{(0)}\left[(p_z^{(0)}/2)\left\{B_{0,n}'' - (1/r) B_{0,n}'\right\} +\right.$$
$$\left.(\alpha_n/4)(1 - r^2) G_{0,n} - r J_{0,n}\right] = 0, \qquad [2.11f]$$

$$I_{1,n} + (\lambda_1' p_z^{(0)} \alpha_n/4)\left[(2 p_z^{(0)}/r) B_{0,n} - (1 - r^2) J_{0,n} - (2r/\alpha_n) C_{0,n}\right] = 0, \qquad [2.11g]$$

$$J_{1,n} - (\lambda_1' p_z^{(0)} \alpha_n/4)\left[(2 p_z^{(0)}/r) A_{0,n} - (1 - r^2) I_{0,n} + (2r/\alpha_n) D_{0,n}\right] = 0. \qquad [2.11h]$$

The equations determining $A_{2,n}$, $B_{2,n}$, $C_{2,n}$, $D_{2,n}$, $E_{2,n}$, $F_{2,n}$, $G_{2,n}$ $H_{2,n}$, $I_{2,n}$ and $J_{2,n}$ are:

$$I_{2,n}'' + (1/r) I_{2,n}' + (\alpha_n^2 - 1/r^2) I_{2,n} - \alpha_n\left[D_{2n}' + (1/r) D_{2,n}\right] + \alpha_n H_{2,n}' + (\alpha_n/r) F_{2,n}$$
$$= (\alpha_n p_z^{(0)2}/8.\ \left[(1/r - r) B_{0,n}'' + (1 - 1/r^2) B_{0,n}' + \alpha_n^2 (r - 1/r) B_{0,n}\right], \qquad (2.12a]$$

$$J_{2,n}'' + (1/r) J_{2,n}' + (\alpha_n^2 - 1/r^2) J_{2,n} + \alpha_n\left[C_{2,n}' + (1/r) C_{2,n}\right] - \alpha_n G_{2,n}' - (\alpha_n/r) E_{2,n}$$
$$= -(\alpha_n p_z^{(0)2}/8)\left[(1/r - r) A_{0,n}'' + (1 - 1/r^2) A_{0,n}' + \alpha_n^2 (r - 1/r) A_{0,n}\right], \qquad [2.12b]$$

$$C_{2,n} + (\lambda_1' p_z^{(0)} \alpha_n/4)(r^2 - 1) D_{1,n} = (\alpha_n p_z^{(0)}/r)\left[B_{2,n}' - (1/r) B_{2,n}\right]$$
$$+ (\lambda_2' p_z^{(0)2} \alpha_n^2/4)\left[(1/r - r) A_{0,n}' + (3 - 1/r^2) A_{0,n}\right], \qquad [2.12c]$$

$$D_{2,n}+(\lambda_1' p_z^{(0)}\alpha_n/4)(1-r^2)C_{1,n}=-(\alpha_n p_z^{(0)}/r)[A_{2,n}'-(1/r)A_{2,n}]$$
$$+(\lambda_2' p_z^{(0)2}\alpha_n^2/4)[(1/r-r)B_{0,n}'+(3-1/r^2)B_{0,n}], \qquad [2.12d]$$

$$E_{2,n}-(\lambda_1' p_z^{(0)}\alpha_n/4)(1-r^2)F_{1,n}=(\alpha_n p_z^{(0)}/r^2)B_{2,n}+(\lambda_2' p_z^{(0)2}\alpha_n^2/4)(1/r^2-1)A_{0,n}, \qquad [2.12e]$$

$$F_{2,n}+(\lambda_1' p_z^{(0)}\alpha_n/4)(1-r^2)E_{1,n}$$
$$=-(\alpha_n p_z^{(0)}/r^2)A_{2,n}+(\lambda_2' p_z^{(0)2}\alpha_n^2/4)(1/r^2-1)B_{0,n}. \qquad [2.12f]$$

$$G_{2,n}+\lambda_1' p_z^{(0)}[(\lambda_1' p_z^{(0)2}\alpha_n^2/2)(B_{0,n}+rB_{0,n}')-rI_{1,n}-(\alpha_n/4)(1-r^2)H_{1,n}]$$
$$=-(\alpha_n p_z^{(0)}/r)B_{2,n}'+(\lambda_2' p_z^{(0)2}/4)[4A_{0,n}''-\{4/r$$
$$+\alpha_n^2(1/r-r)\}A_{0,n}'+2\alpha_n^2A_{0,n}], \qquad [2.12g]$$

$$H_{2,n}+\lambda_1' p_z^{(0)}[-(\lambda_1' p_z^{(0)2}\alpha_n^2/2)(A_{0,n}+rA_{0,n}')-rJ_{1,n}+(\alpha_n/4)(1-r^2)G_{1,n}]$$
$$=(\alpha_n p_z^{(0)}/r)A_{2,n}'+(\lambda_2' p_z^{(0)2}/4)\times[4B_{0,n}''$$
$$-\{4/r+\alpha_n^2(1/r-r)\}B_{0,n}'+2\alpha_n^2B_{0,n}], \qquad [2.12h]$$

$$I_{2,n}+(\lambda_1' p_z^{(0)}\alpha_n/4)[(r^2-1)J_{1,n}-(2r/\alpha_n)C_{1,n}+\lambda_1' p_z^{(0)2}\alpha_n rA_{0,n}]$$
$$=-(p_z^{(0)}/2r)[A_{2,n}''-(1/r)A_{2,n}'+\alpha_n^2A_{2,n}]+(\lambda_2' p_z^{(0)2}\alpha_n/8)[(1/r-r)B_{0,n}''$$
$$-(3+1/r^2)B_{0,n}'+\{8/r+\alpha_n^2(1/r-r)\}B_{0,n}], \qquad [2.12i]$$

and

$$J_{2,n}-(\lambda_1' p_z^{(0)}\alpha_n/4)[(r^2-1)I_{1,n}+(2r/\alpha_n)D_{1,n}-\lambda_1' p_z^{(0)2}\alpha_n rB_{0,n}]$$
$$=-(p_z^{(0)}/2r)[B_{2,n}''-(1/r)B_{2,n}'+\alpha_n^2B_{2,n}]-(\lambda_2' p_z^{(0)2}\alpha_n/8)\times$$
$$[(1/r-r)A_{0,n}''-(3+1/r^2)A_{0,n}'+\{8/r+\alpha_n^2(1/r-r)\}A_{0,n}]. \qquad [2.12j]$$

On eliminating $C_{2,n}$, $D_{2,n}$, $E_{2,n}$, $F_{2,n}$, $G_{2,n}$, $H_{2,n}$, $I_{2,n}$ and $J_{2,n}$ from the equations [2.12], we obtain the following equations for the determination of $A_{2,n}$ and $B_{2,n}$:

$$r^3A_{2,n}^{iv}-2r^2A_{2,n}'''+r(3-2\alpha_n^2r^2)A_{2,n}''-(3-2\alpha_n^2r^2)A_{2,n}'+\alpha_n^4r^3A_{2,n}$$
$$=(\alpha_n p_z^{(0)}/4)r^3(r^2-1)[rB_{0,n}''-B_{0,n}'-\alpha_n^2rB_{0,n}]$$
$$+(\lambda_1'^2 p_z^{(0)2}\alpha_n^2/4)r^2[(r^2-1)(r^2A_{0,n}'''+\alpha_n^2A_{0,n}')-4\alpha_n^2r^3A_{0,n}], \qquad [2.13a]$$

$$r^3B_{2,n}^{iv}-2r^2B_{2,n}'''+r(3-2\alpha_n^2r^2)B_{2,n}''$$
$$=-(\alpha_n p_z^{(0)}/4)r^2(r^2-1)[rA_{0,n}''-A_{0,n}'-\alpha_n^2rA_{0,n}]$$
$$+(\lambda_1'^2 p_z^{(0)2}\alpha_n^2/4)r^2[(r^2-1)(r^2B_{0,n}'''+\alpha_n^2B_{0,n}')-4\alpha_n^2r^3B_{0,n}], \qquad [2.13b]$$

to be solved under the boundary conditions :

$$\left.\begin{array}{l} A_{2,n}(0) = A_{2,n}(1) = A'_{2,n}(0) = A'_{2,n}(1) = 0 \\ B_{2,n}(0) = B_{2,n}(1) = B'_{2,n}(0) = B'_{2,n}(1) = 0 \end{array}\right\}, \qquad [2.13c]$$

where a dash denotes differentiation with respect to r.

The stresses, the vorticity and the skin friction are given by

$$\begin{aligned} p_{rr} = \epsilon p_z^{(0)} \sum_{n=1}^{\infty} \alpha_n \Big[& \big\langle (1/r)\,[B'_{0,n} - (1/r)\,B_{0,n}] + R\lambda'_1 p_z^{(0)} \alpha_n/4)\ [(r - 1/r) A'_{0,n} \\ & - (3 - 1/r^2)\,A_{0,n}] + R^2\,[(1/r)\ \{B'_{2,n} - (1/r)\,B_{2,n}\} \\ & - (\lambda_1'^2 p_z^{(0)2}\,\alpha_n^2/16)\ \{(r^3 - 2r + 1/r)\,B'_{0,n} \\ & + (4 - 3r^2 - 1/r^2)\,B_{0,n}\} + (\lambda'_2 p_z^{(0)}\,\alpha_n/4)\ \{(1/r - r)\,A'_{0,n} \\ & + (3 - 1/r^2)\,A_{0,n}\}]\big\rangle \cos\alpha_n z + \big\langle - (1/r)\,[A'_{0,n} - (1/r)\,A_{0,n}] \\ & + (R\,\lambda'_1 p_z^{(0)}\,\alpha_n/4)\,[(r - 1/r)\,B'_{0,n} - (3 - 1/r^2)\,B_{0,n}] \\ & + R^2[-(1/r)\ \{A'_{2,n} - (1/r)\,A_{2,n}\} + (\lambda_1'^2 p_z^{(0)2}\,\alpha_n^2/16)\ \{(r_3 - 2r + 1/r)\,A'_{0,n} \\ & + (4 - 3r - 1/r^2)\,A_{0,n} + (\lambda'_2 p_z^{(0)}\,\alpha_n/4)\ \{(1/r - r)\,B'_{0,n} \\ & + (3 - 1/r^2)\,B_{0,n}\}]\big\rangle \sin\alpha_n z \Big], \end{aligned} \qquad [2.14a]$$

$$\begin{aligned} p_{\theta\theta} = \epsilon p_z^{(0)} \sum_{n=1}^{\infty} \alpha_n \big\langle & \{(1/r^2)\,B_{0,n} + (R\,\lambda'_1 p_z^{(0)}\,\alpha_n/4)\,(1 - 1/r^2)\,A_{0,n} \\ & + R^2\,[(1/r^2)\,B_{2,n} - (\lambda_1'^2 p_z^{(0)2}\,\alpha_n^2/16\,(r - 1/r)^2\,B_{0,n} \\ & + (\lambda'_2 p_z^{(0)}\,\alpha_n/4)\,(1/r^2 - 1)\,A_{0,n}]\}\ \cos\alpha_n z + \{-(1/r^2)\,A_{0,n} \\ & + (R\,\lambda'_1 p_z^{(0)}\,\alpha_n/4)\,(1 - 1/r^2)\,B_{0,n} + R^2\,[-(1/r^2)\,A_{2,n} \\ & + (\lambda_1'^2 p_z^{(0)2}\,\alpha_n^2/16)\,(r - 1/r)^2 A_{0,n} + (\lambda'_2 p_z^{(0)}\,\alpha_n/4)\,(1/r^2 - 1)\,B_{0,n}]\}\ \sin\alpha_n z\big\rangle \end{aligned} \qquad [2.14b]$$

$$\begin{aligned} p_{zz} = (R\,p_z^{(0)2}\,r^2/2)\,(\lambda'_1 - \lambda'_2\,R) + \epsilon p_z^{(0)} \sum_{n=1}^{\infty} \Big[\Big\{ & -(\alpha_n/r)\,B'_{0,n} - R\,\lambda'_1 p_z^{(0)}\,[A''_{0,n} \\ & + \{(\alpha_n^2/4)\,(r - 1/r) - (1/r)\}\ A'_{0,n} + (\alpha_n^2/2)\,A_{0,n}] \\ & + R^2 \big\langle - (\alpha_n/r)\,B'_{2,n} - (\lambda_1'^2 p_z^{(0)2}\,\alpha_n/8)\ \{3\,(1 - r^2)\,B''_{0,n} \end{aligned}$$

$$+ [3\,(r - 1/r) - (\alpha_n^2/2)\,(r^3 - 2r + 1/r)]\,B'_{0,\,n} + 2\,[\alpha_n^2\,(1 - r^2) + 6]\,B_{0,\,n}\}$$

$$+ (\lambda'_2 p_z^{(0)}/4)\,\{4\,A''_{0,\,n} - [\alpha_n^2\,(r - 1/r) + 4/r]\,A'_{0,\,n} + 2\alpha_n^2 A_{0,\,n}\}\rangle\}\,\cos\alpha_n z$$

$$+ \Big\{(\alpha_n/r)\,A_{0,\,n} - R\lambda'_1 p_z^{(0)}\,[B''_{0,\,n} + \{(\alpha_n^2/4)\,(r - 1/r) - 1/r\}\,B'_{0,\,n} + (\alpha_n^2/2) B_{0,\,n}]$$

$$+ R^2 \langle (\alpha_n/r)\,A'_{2,\,n} + (\lambda_1'^2\,p_z^{(0)2}\alpha_n/8)\,\{3\,(1 - r^2) A''_{0,\,n} + [3\,(r - 1/r) - (\alpha_n^2/2) \times$$

$$(r^3 - 2r + 1/r)]\,A'_{0,\,n} + 2\,[\alpha_n^2 (1 - r^2) + 6]\,A_{0,\,n}\} + \lambda'_2 p_z^{(0)}/4) \times$$

$$\{4\,B''_{0,\,n} - [4/r + \alpha_n^2\,(1/r - r)]\,B'_{0,\,n} + 2\alpha_n^2 B_{0,\,n}\}\rangle\Big\} \quad \sin\alpha_n z\,\Big], \qquad [2.14c]$$

$$p_{rz} = p_z^{(0)} r/2 + \epsilon\, p_z^{(0)} \sum_{n=1}^{\infty} \Big[\Big\{ -(1/2r)\,[A''_{0,\,n} - (1/r)\,A'_{0,\,n} + \alpha_n^2 A_{0,\,n}]$$

$$+ (R\lambda'_1 p_z^{(0)} \alpha_n/8)\,[(r - 1/r)\,B''_{0,\,n} + (3 + 1/r^2)\,B_{0,\,n}$$

$$+ \{\alpha_n^2 (r - 1/r) - (8/r)\}\,B_{0,\,n}] + R^2 \langle -(1/2r)\,[A''_{2,\,n} - (1/r)\,A'_{2,\,n} + \alpha_n^2 A_{2,\,n}]$$

$$+ (\lambda_1'^2\,p_z^{(0)2}\alpha_n^2/32)\,\{r^3 - 2r + 1/r) A''_{0,\,n} + (7r^2 - 6 - 1/r^2) A'_{0,\,n}$$

$$+ [\alpha_n^2 (r^3 - 2r + 1/r) - 28r + 12/r]\,A_{0,\,n}\}$$

$$+ (\lambda'_2 p_z^{(0)}\,\alpha_n/8)\,\{(1/r - r)\,B''_{0,\,n} - (3 + 1/r^2)\,B'_{0,\,n}$$

$$+ [\alpha_n^2\,(1/r - r) + 8/r]\,B_{0,\,n}\}\rangle\Big\}\,\cos\alpha_n z + \Big\{-(1/2r)\,[B''_{0,\,n} - (1/r)\,B'_{0,\,n}$$

$$+ \alpha_n^2 B_{0,\,n}] - (R\,\lambda'_1 p_z^{(0)}\,\alpha_n/8)\,[(r - 1/r)\,A''_{0,\,n} + (3 + 1/r^2)\,A'_{0,\,n}$$

$$+ \{\alpha_n^2 (r - 1/r) - 8/r\}\,A_{0,\,n}] + R^2 \langle -(1/2r)\,(B''_{2,\,n} - (1/r)\,B'_{2,\,n} + \alpha_n^2 B_{2,\,n}]$$

$$+ (\lambda_1'^2\,p_z^{(0)2}\alpha_n^2/32)\,\{(r^3 - 2r + 1/r)\,B''_{0,\,n} + (7r^2 - 6 - 1/r^2)\,B'_{0,\,n}$$

$$+ [\alpha_n^2 (r^3 - 2r + 1/r) - 28r + 12/r]\,B_{0,\,n}\} - (\lambda'_2 p_z^{(0)}\alpha_n/8) \times$$

$$\{(1/r - r) A''_{0,\,n} - (3 + 1/r^2)\,A'_{0,\,n} + [\alpha_n^2 (1/r - r) + 8/r]\,A_{0,\,n}\}\rangle\Big\}\,\sin\alpha_n z\Big], \qquad [2.14d]$$

$\vec{\zeta} = \vec{i}_\theta\,\Omega_\theta$, where

$$\Omega_\theta = -(R p_z^{(0)} r/2) + (\epsilon R p_z^{(0)}/2r) \sum_{n=1}^{\infty} \langle \{[A''_{0,\,n} - (1/r)\,A'_{0,\,n} - \alpha_n^2 A_{0,\,n}]$$

$$+ R^2\,[A''_{2,\,n} - (1/r)\,A'_{2,\,n} - \alpha_n^2 A_{2,\,n}]\}\,\cos\alpha_n z + \{(B''_{0,\,n} - (1/r) B'_{0,\,n} - \alpha_n^2 B_{0,n})$$

$$+ R^2\,[B''_{2,\,n} - (1/r)\,B'_{2,\,n} - \alpha_n^2 B_{2,\,n}]\}\,\sin\alpha_n z\rangle, \qquad [2.14e]$$

$$\tau = \int_0^z [2\pi r\, p_{rz}]_r\, dz = 1 + \epsilon \sum_{n=1}^{\infty} (a_n \cos \alpha_n z + b_n \sin \alpha_n z)$$

$$= \pi p_z^{(0)} \Big\langle z + \epsilon \sum_{n=1}^{\infty} \{[(3a_n - A''_{0,n}(1))$$

$$+ R p_z^{(0)} \alpha_n b_n (\lambda'_1 - \lambda'_2 R) - R^2 A''_{2,n}(1)] \sin \alpha_n z - [\{3 b_n - B''_{0,n}(1)\}$$

$$- R p_z^{(0)} \alpha_n a_n (\lambda'_1 - \lambda'_2 R) - R^2 B''_{2,n}(1)] (\cos \alpha_n z - 1)\} \Big\rangle. \qquad [2.14f]$$

Pressure.—The pressure is given by

$$p(r, z) = z\, p_z^{(0)} + \epsilon \sum_{n=1}^{\infty} \Big\langle \int \{ C'_n + (1/r)(C_n - E_n) + \alpha J_n$$

$$+ (R^2 p_z^{(0)2} \alpha_n^2/8)(1/r - r) A_n\}\, dr (\cos \alpha_n z). + \int \{ D'_n + (1/r)(D_n - F_n)$$

$$- \alpha_n I_n + (R^2 p_z^{(0)2} \alpha_n^2/8)(1/r - r) B_n\}\, dr\, (\sin \alpha_n z) \Big\rangle$$

$$+ \epsilon \sum_{n=1}^{\infty} \Big\langle \{(1/\alpha_n) [I'_n + (1/r) I_n] + H_n - (R^2 p_z^{(0)2}/8) [2B_n + (1/r - r) B'_n]\} \times$$

$$\sin \alpha_n z - \{(1/\alpha_n) [J'_n + (1/r) J_n] - G_n + (R^2 p_z^{(0)2}/8) [2 A_n + (1/r - r) A'_n]\} \times$$

$$\cos \alpha_n z \Big\rangle + p'. \qquad [2.15]$$

where p' is some constant pressure and all the quantities in [2.15] are known in terms of A_n and B_n.

PART B

TEMPERATURE FIFLD

As in velocity field, we set

$$T = T^{(0)} + \epsilon\, T^1 (r, z). \qquad [2.16]$$

The equations and the boundary conditions determining $T^{(0)}$ and $T^{(1)}$ are obtained from the non-dimensional form of the energy equation [1.3] and the boundary conditions [1 4] by equating terms independent of ϵ and the coefficient of ϵ respectively.

We have,

$$T^{(0)}(r) = 1 + (E\sigma\, R^2 p_z^{(0)2}/64)(1 - r^4). \qquad [2.17]$$

In view of [2.17] the boundary conditions for $\tau_{r,z}^{(1)}$ reduce to

$$\left.\begin{aligned} T^{(1)}(1, z) &= (E\sigma R^2 p_z^{(0)2}/16) \sum_{n=1}^{\infty} [a_n \cos \alpha_n z + b_n \sin \alpha_n z] \\ T_r^{(1)}(0, z) &= 0, \end{aligned}\right\} \qquad [2.18]$$

which suggest that $T^{(1)}(r, z)$ should be chosen in the form

$$T^{(1)}(r, z) = (E\sigma R^2 p_z^{(0)2}/16) \sum_{n=1}^{\infty} [K_n(r) \cos \alpha_n z + L_n(r) \sin \alpha_n z]. \qquad [2.19]$$

Taking

$$\left.\begin{aligned} K_n &= K_{0,n} + R K_{1,n} + R^2 K_{2,n} \\ L_n &= L_{0,n} + R L_{1,n} + R^2 L_{2,n} \end{aligned}\right\}, \qquad [2.20]$$

the equations determining $K_{0,n}$ $L_{0,n}$; $K_{1,n}$, $L_{1,n}$; $K_{2,n}$ and $L_{2,n}$ are:

$$K_{0,n}'' + (1/r) K_{0,n}' - \alpha_n^2 K_{0,n} = 8 [A_{0,n}'' - (1/r) A_{0,n}' + \alpha_n^2 A_{0,n}], \qquad [2.21a]$$

$$L_{0,n}'' + (1/r) L_{0,n}' - \alpha_n^2 L_{0,n} = 8 [B_{0,n}'' - (1/r) B_{0,n}' + \alpha_n^2 B_{0,n}], \qquad [2.21b]$$

$$\begin{aligned} K_{1,n}'' + (1/r) K_{1,n}' - \alpha_n^2 K_{1,n} &= \lambda_1' p_z^{(0)} \alpha_n [(1 - r^2) B_{0,n}'' + (r - 1/r) B_{0,n}' \\ &\quad + \{\alpha_n^2 (1 - r^2) + 8\} B_{0,n}], \end{aligned} \qquad [2.21c]$$

$$\begin{aligned} L_{1,n}'' + (1/r) L_{1,n}' - \alpha_n^2 L_{1,n} &= - \lambda_1' p_z^{(0)} \alpha_n [(1 - r^2) A_{0,n}'' + (r - 1/r) A_{0,n}' \\ &\quad + \{\alpha_n^2 (1 - r^2) + 8\} A_{0,n}], \end{aligned} \qquad [2.21d]$$

$$\begin{aligned} &K_{2,n}'' + (1/r) K_{2,n}' - \alpha_n^2 K_{2,n} \\ &\quad = 8 [A_{2,n}'' - (1/r) A_{2,n}' + \alpha_n^2 A_{2,n}] \\ &\quad + \alpha_n p_z^{(0)} \sigma [\{(r^2 - 1)/4\} L_{0,n} - (r^2/2) B_{0,n}] - (\lambda_1'^2 p_z^{(0)2} \alpha_n^2/4) [(r^2 - 1) A_{0,n}'' \\ &\quad + (7 r^3 - 6 r - 1/r) A_{0,n}' + \{\alpha_n^2 (r^2 - 1)^2 - 28 r^2 + 12\} A_{0,n}] \\ &\quad - \lambda_2' p_z^{(0)} \alpha_n [(1 - r^2) B_{0,n}'' - (1/r - r) B_{0,n} + \{\alpha_n^2 (1 - r^2) + 8\} B_{0,n}], \end{aligned} \qquad [2.21e]$$

$$\begin{aligned} &L_{2,n}'' + (1/r) L_{2,n}' - \alpha_n^2 L_{2,n} \\ &\quad = 8 [B_{2,n}'' - (1/r) B_{2,n}' + \alpha_n^2 B_{2,n}] \\ &\quad - \alpha_n p_z^{(0)} \sigma [\{(r^2 - 1)/4\} K_{0,n} - (r^2/2) A_{0,n}] - (\lambda_1'^2 p_z^{(0)2} \alpha_n^2/4) [(r^2 - 1) B_{0,n}'' \\ &\quad + (7 r^3 - 6 r - 1/r) B_{0,n}' + \{\alpha_n^2 (r^2 - 1)^2 - 28 r^2 + 12\} B_{0,n}] \\ &\quad + \lambda_2' p_z^{(0)} \alpha_n [(1 - r^2) A_{0,n}'' - (1/r - r) A_{0,n}' + \{\alpha_n^2 (1 - r^2) + 8\} A_{0,n}]. \end{aligned} \qquad [2.21f]$$

The equations [2 21] are to be solved under the boundary conditions:

$$\left.\begin{array}{ll} K_{0,n}(1) = a_n, & K'_{0,n}(0) = 0, \\ L_{0,n}(1) = b_n, & L'_{0,n}(0) = 0 \end{array}\right\} ; \qquad [2.22a]$$

$$\left.\begin{array}{l} K_{1,n}(1) = K'_{1,n}(0) = 0, \\ L_{1,n}(1) = L'_{1,n}(0) = 0 \end{array}\right\} ; \qquad [2.22b]$$

and

$$K_{2,n}(1) = K'_{2,n}(0) = L_{2,n}(1) = L'_{2,n}(0) = 0. \qquad [2.22c]$$

On putting $\lambda'_1 = \lambda'_2 = 0$ in all the equations of Part A and Part B we get the equations for Newtonian fluid and these equations agree with the corresponding equations of reference 6 with $K = S = 0$. The streamline and the vorticity are affected by the stress-relaxation time only while the stresses and the temperature are affected by both the stress-relaxation and the strain-retardation times.

3. Particular Case

The equations obtained in § 2 cannot be solved in close form for any general value of α_n To visualise the flow field and the temperature distribution we take the particular case when the boundary of the tube has sinusodial deformation defined as follows:

(i) $a_n = a'_n = 0$, for all n; (ii) $b_n = b'_n = 0$, for $n > 1$, and $b_1 = b'_1 = 1$.
We choose the following values of the various parameters involved in this problem:

$$\epsilon = 0.05, \ \sigma = p_z^{(0)} = 1 ; \quad \lambda'_1 = 0.2, \ 0.5, \ 0.7 ;$$

$$\lambda'_2 = 0.02, \ 0.06, \ 0.08 ; \quad E = 0.5,$$

$R^2 = 5$, and $h = 1$, so that the wavelength of the periodic deformation is 2π.

I. Velocity Field:

(*a*) *Streamlines*: In this particular case, we have

$$A_{0,1} = 0,$$

for all r in $(0, 1)$ due to homogeneous boundary conditions on $A_{0,1}$. To obtain $B_{0,1}$, $A_{2,1}$ and $B_{2,1}$ we adopt the usual procedure of numerical integration of two point boundary value problem and integrate the equations, [2.10] and [2.13] numerically.

With the known values of $B_{0,1}$, $A_{2,1}$ and $B_{2,1}$ at each point of $(0, 1)$ at a subinterval of 0.1 the stream function is completely determined by the expression

$$\psi = -(R\,p_z^{(0)}/8)\,(r^2 - r^4/2) - (\epsilon\, R\, p_z^{(0)}/2)\,[R^2 A_{2,1} \cos z + (B_{0,1} + R^2 B_{2,1}) \sin z], \qquad [3.1]$$

at every interval of 0.1 for r.

In Fig. 1 we have drawn the streamlines, $\psi' = -8\psi/Rp_z^{(0)} = \text{constant}$, to visualise the flow field. The continuous lines and the dotted lines represent the streamlines for Newtonian and elastico-viscous fluids respectively. The streamlines for elastico-viscous fluids are slightly displaced in the planes $z = \pi/2$ and $z = 3\pi/2$ relative to those for Newtonian fluids.

The streamlines near the boundary of the wavy tube are almost parallel to it. The deformity of the streamlines decreases continuously as we move away from the boundary towards the axis of the tube. On axis of the tube they become just straight due to axial symmetry. This phenomenon is similar to that observed in the case of flow of a Rivlin-Ericksen fluid discussed in reference 6. Table 1 shows the effect of λ_1' on stream function.

(*b*) *Vorticity*: The expression for the vorticity is

$$\vec{\zeta} = \vec{i}_\theta\, \Omega_\theta,$$

where

$$\Omega_\theta = -Rp_z^{(0)}\, r/2 + (\epsilon\, Rp_z^{(0)}/2r)\,\langle R^2 [A_{2,1}'' - (1/r)\, A_{2,1}' - A_{2,1}] \cos z + \{[B_{0,1}'' - (1/r) B_{0,1}' - B_{0,1}] + R^2 [B_{2,1}'' - (1/r) B_{2,1}' - B_{2,1}]\} \sin z\rangle. \qquad [3.2]$$

In the absence of deformation in the boundary, the vortex lines are concentric circles having the same radii in all planes with their centres lying on the axis. Even in the presence of deformation in the boundary, the vortex lines are concentric circles but their radii differ in different planes, the radii being maximum in the plane $z = 3\pi/2$ and minimum in the plane $z = \pi/2$.

Fig. 2 shows the variation of $\Omega_\theta' = -2\Omega_\theta / R\,p_z^{(0)}$ with axial distance z for various value of r. Fig. 3 shows the variation of Ω_θ' with radial distance r in the planes $z = 0$, $\pi/2$, π, $3\pi/2$ and 2π.

From both these figures we note that Ω_θ' increases with radial distance and this increase is maximum in the plane $z = 3\pi/2$ and minimum in the plane $z = \pi/2$.

Table 2 shows the effect of λ_1' on vorticity. The increase in λ_1' results in a decrease in vorticity up to a certain distance from the axis and then the vorticity increases towards the boundary in the plane $z = \pi/2$. The reverse is true in the plane $z = 3\pi/2$.

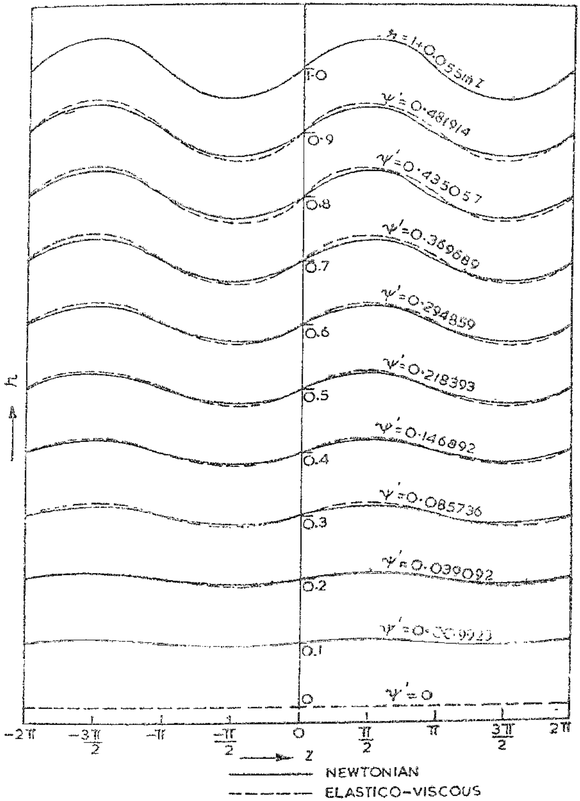

FIG. 1

Streamlines for particular values: $\epsilon=0.05$, $p_z^{(0)}=h=1$, $R_e=8$, $\lambda_1'=0.5$.

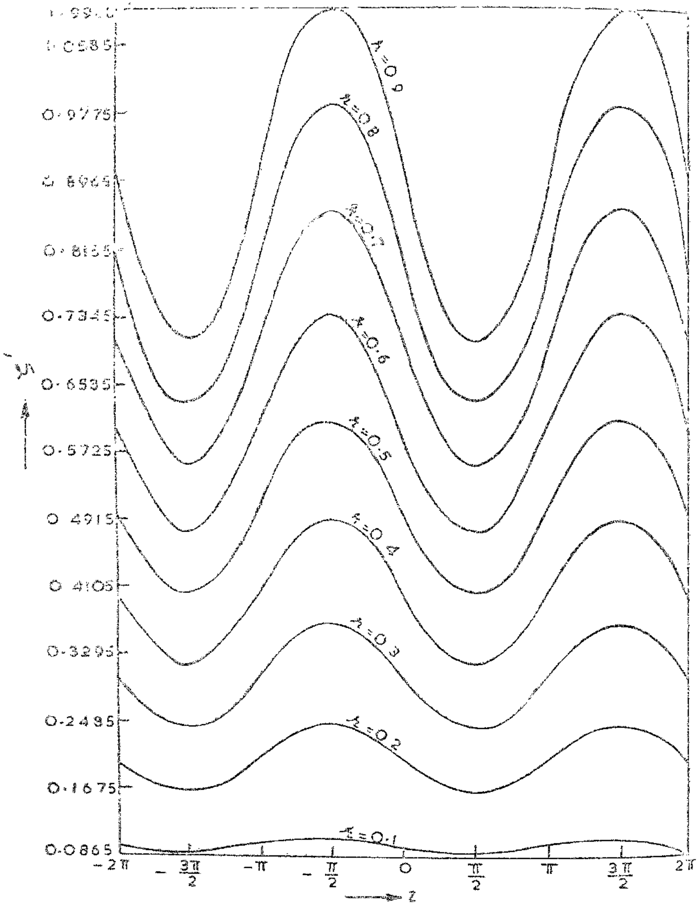

FIG. 2

Variation of vorticity with axial distance for Newtonian Fluids for particular values ; $\epsilon = 0.05$ $p_z^{(0)} = h = 1$, $R^2 = 5$

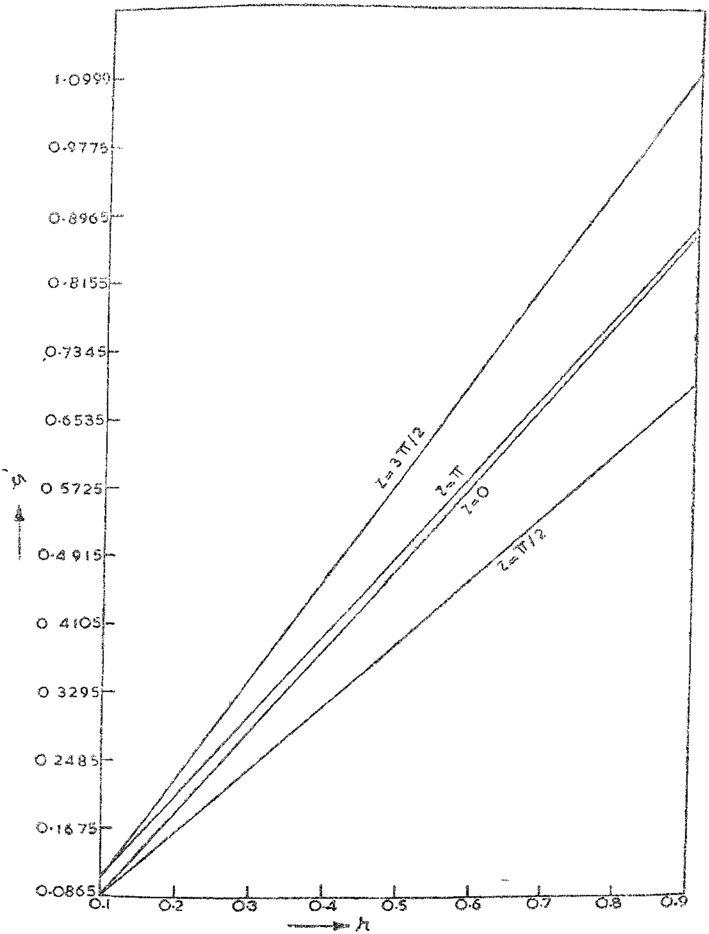

FIG. 3

Variation of vorticity with Radial distance for Newtonian Fluids for particular values: $\epsilon=0.05$, $p_z^{(0)}=h=1$, $R'=5$.

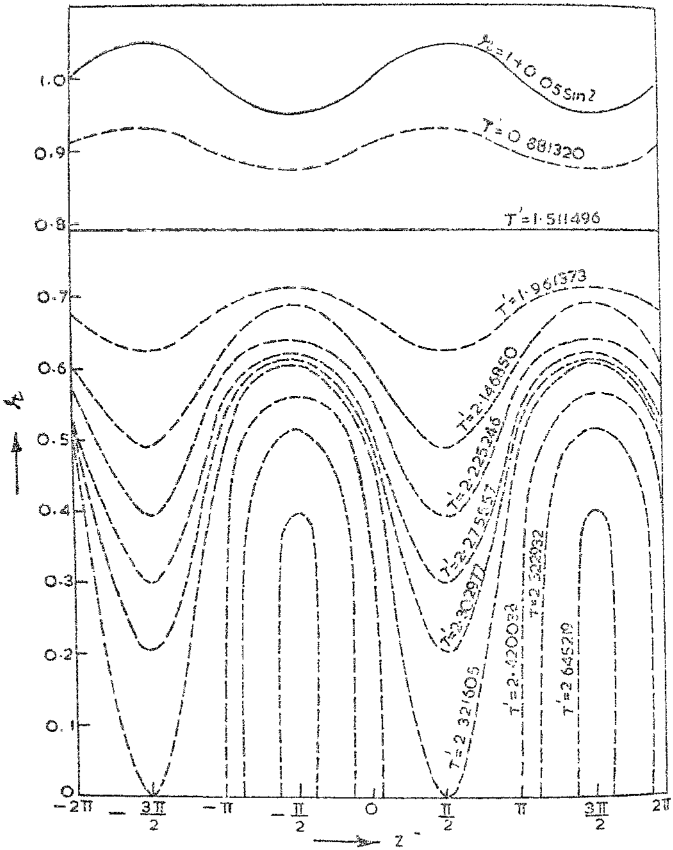

FIG. 4

Isotherms for Newtonian Fluids for $\epsilon = 0.05$, $E = 0.5$, $R^2 = 5$, $\sigma = p_z^{(0)} = 1$.

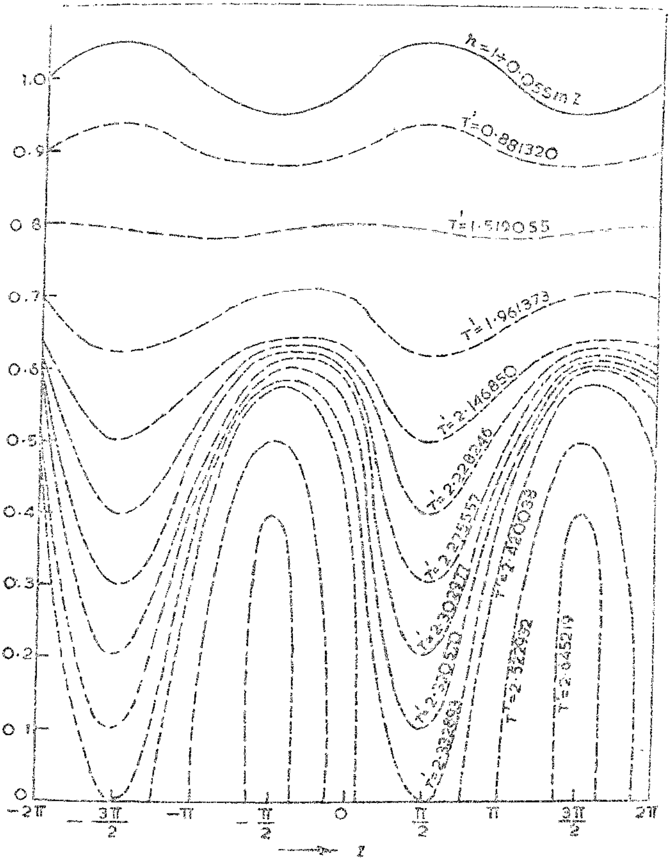

FIG. 5

Isotherms for Elastico-Viscous Fluids for

$\epsilon = 0.05,\ E = 0,\ R^2 = 5,\ \sigma = p_z^{(0)} = 1,\ \lambda_1' = 0.7,\ \lambda_2' = 0.08,\ h = 1$

TABLE 1

r/z	$\psi'(r, z)$					
	0		$\pi/2$	π	$3\pi/2$	2π
0.1	0.009923	(i)	0.009024	0.009977	0.010876	0.009923
		(ii)	0 009021		0.010879	
		(iii)	0.009006		0.010894	
		(iv)	0.008988		0.010912	
0.2	0.039092	(i)	0.035498	0.039308	0.042902	0.039092
		(ii)	0.035485		0.042914	
		(iii)	0.035478		0.420922	
		(iv)	0.035354		0.043046	
0.3	0.085736	(i)	0.077967	0.086164	0.093933	0.085736
		(ii)	0.077942		0.093858	
		(iii)	0.077824		0.094076	
		(iv)	0.077679		0 094221	
0.4	0.146892	(i)	0.134006	0.147508	0.160394	0 146892
		(ii)	0.133968		0.160432	
		(iii)	0.133798		0.160602	
		(iv)	0.133589		0.160811	
0.5	0.218393	(i)	0.200216	0.219107	0.237284	0.218393
		(ii)	0.200170		0 237330	
		(iii)	0.199971		0 237529	
		(iv)	0.199727		0.237773	
0.6	0.294859	(i)	0.272244	0.295541	0.318156	0.294859
		(ii)	0.272198		0 318202	
		(iii)	0.272006		0.318394	
		(iv)	0.271769		0.318631	
0.7	0.369689	(i)	0.344816	0.370211	0.395084	0.369689
		(ii)	0.344779		0 395121	
		(iii)	0.344631		0.395269	
		(iv)	0.344446		0 395454	
0.8	0.435057	(i)	0.411767	0.435343	0.458633	0.435057
		(ii)	0.411746		0.458654	
		(iii)	0.411665		0.458735	
		(iv)	0.411562		0.458838	
0.9	0.481914	(i)	0.466092	0.481986	0 497808	0.481914
		(ii)	0.466087		0 497813	
		(iii)	0.466067		0 497833	
		(iv)	0.466041		0.497859	

N.B. The first entry in each column corresponds to Newtonian fluids, second, third and fourth entries correspond to elastico-viscous fluids for (*a*) $\lambda_1'=0.2$, (*b*) $\lambda_1'=0\ 5$, and (*c*) $\lambda_1'=0.7$.

TABLE 2

r/z	$\vec{\zeta}' = \vec{i}_\theta \Omega'_\theta$				
	0	$\pi/2$	π	$3\pi/2$	2π
0.1	0.098682	(i) 0.092636	0.101318	0.107364	0.098682
		(ii) 0.092628		0.107372	
		(iii) 0.091841		0.108159	
		(iv) 0.091046		0.108954	
0.2	0.196740	(i) 0.159909	0.203260	0.240091	0.196740
		(ii) 0.159707		0.240293	
		(iii) 0.157944		0.242056	
		(iv) 0.155979		0.244021	
0.3	0.296349	(i) 0.237542	0.303651	0.362458	0.296349
		(ii) 0.237299		0.362701	
		(iii) 0.235340		0.364660	
		(iv) 0.233135		0.366865	
0.4	0.396182	(i) 0.313304	0.403818	0.486696	0.396182
		(ii) 0.313046		0.486954	
		(iii) 0.311018		0.488982	
		(iv) 0.308731		0.491269	
0.5	0.496172	(i) 0.396275	0.503828	0.603725	0.496172
		(ii) 0.396012		0.603988	
		(iii) 0.393963		0.606037	
		(iv) 0.391651		0.608349	
0.6	0.596442	(i) 0.469576	0.603558	0.730424	0.596442
		(ii) 0.469279		0.730721	
		(iii) 0.465636		0.734364	
		(iv) 0.462503		0.737497	
0.7	0.698733	(i) 0.549340	0.701267	0.850660	0.698733
		(ii) 0.549474		0.850526	
		(iii) 0.548044		0.851956	
		(iv) 0.547142		0.852858	
0.8	0.800808	(i) 0.625425	0.799192	0.974575	0.800808
		(ii) 0.625701		0.974299	
		(iii) 0.625859		0.974141	
		(iv) 0.626416		0.973584	
0.9	0.902243	(i) 0.700012	0.897757	1.099988	0.902243
		(ii) 0.700455		1.099545	
		(iii) 0.701632		1.098368	
		(iv) 0.703192		1.096808	

N.B. The first entry in each column corresponds to Newtonian fluids, second, third and fourth entries correspond to elastico-viscous fluids for (*a*) $\lambda_1'=0.2$, (*b*) $\lambda_1'=0.5$ and (*c*) $\lambda_1'=0.7$.

TABLE 3

r/s		$T'(r, z)$				
		0	$\pi/2$	π	$3\pi/2$	2π
0.1	(i)	2.503782	2.318000	2.495818	2.681600	2.503782
	(ii)	2.505882	2.318208	2.493718	2.681392	2.505882
	(iii)	2.503520	2.319328	2.496080	2.680272	2.503520
	(iv)	2.501955	2.320570	2.497645	2.679030	2.501955
0.2	(i)	2.488625	2.303985	2.503375	2.688015	2.488625
	(ii)	2.522397	2.303885	2.469603	2.688115	2.522397
	(iii)	2.525391	2.303385	2.466609	2.688615	2.527553
	(iv)	2.527553	2.302977	2.464447	2.689023	2.527553
0.3	(i)	2.465800	2.276247	2.493800	2.683353	2.465800
	(ii)	2.529687	2.276147	2.419913	2.683453	2.529687
	(iii)	2.535404	2.2758[illegible]9	2.414196	2.683761	2.535404
	(iv)	2.539562	2.275557	2.410038	2.684043	2.539562
0.4	(i)	2.417792	2.225376	2.452208	2.646624	2.417792
	(ii)	2.499052	2.225538	2.372948	2.646462	2.499052
	(iii)	2.506199	2.226781	2.365801	2.645219	2.506199
	(iv)	2.511402	2.228246	2.360598	2.643754	2.511402
0.5	(i)	2.326792	2.138132	2.310808	2.549468	2.326792
	(ii)	2.410681	2.138762	2.276919	2.548838	2.410681
	(iii)	2.418182	2.142500	2.269418	2.545100	2.418182
	(iv)	2.423649	2.146850	2.263951	2.540750	2.423649
0.6	(i)	2.161170	1.997502	2.190830	2.354498	2.161170
	(ii)	2.238622	1.998624	2.173378	2.353376	2.238622
	(iii)	2.245553	2.004910	2.106447	2.347090	2.245553
	(iv)	2.250599	2.012177	2.101401	2.339823	2.250599
0.7	(i)	1.888940	1.784361	1.910660	2.015239	1.888940
	(ii)	1.951684	1.785893	1.8479[illegible]6	2.013707	1.951684
	(iii)	1.957295	1.794129	1.842305	2.005471	1.957295
	(iv)	1.961373	1.803656	1.838227	1.995944	1.961373
0.8	(i)	1.469558	1.477649	1.482442	1.474351	1.469558
	(ii)	1.512505	1.479161	1.439495	1.572839	1.512551
	(iii)	1.516312	1.487169	1.435688	1.464831	1.516312
	(iv)	1.519055	1.496407	1.432945	1.455513	1.519055
0.9	(i)	0.857410	1.056322	0.862190	0.663278	0.857410
	(ii)	0.878201	1.057152	0.841399	0.662448	0.878201
	(iii)	0.880014	1.061374	0.839586	0.658226	0.880014
	(iv)	0.881320	1.066284	0.838280	0.653316	0.881320

N.B. The first entry in each column corresponds to Newtonian fluids, second, third and fourth entries correspond to elastico-viscous fluids for (*a*) $\lambda_1'=0.2$, $\lambda_2'=0.2$; (*b*) $\lambda_1'=0.5$, $\lambda_2'=0.06$ and (*c*) $\lambda_1'=0.7$, $\lambda_2'=0.08$ respectively.

(c) Stresses on boundary skinfriction :

We have

$$p_{rr} = \epsilon\, p_z^{(0)} \cos z, \qquad [3.3a]$$

$$p_{\theta\theta} = 0\,, \qquad [3.3b]$$

$$p_{zz} = R\, p_z^{(0)2} (\lambda_1' - \lambda_2' R) \left(\tfrac{1}{2} + \epsilon\right)$$
$$- \epsilon\, p_z^{(0)} \left[\cos z + R\, p_z^{(0)2} (\lambda_1' - \lambda_2' R) \left\{B''_{0,1}(1) - 1\right\} \sin z\right], \qquad [3.3c]$$

and

$$p_{rz} = (p_z^{(0)}) \left[1 + \epsilon \left\{\left[R\, p_z^{(0)} (\lambda_1' - \lambda_2' R) - R^2 A''_{2,1}(1)\right] \cos z \right.\right.$$
$$\left.\left. + \left[2 - B''_{0,1}(1) - R^2 B''_{2,1}(1)\right] \sin z\right\}\right]. \qquad [3.3d]$$

The skin-friction at any point z on the boundary is

$$\tau = \pi\, p_z^{(0)} \left[z + \epsilon \left\{\left[R\, p_z^{(0)} (\lambda_1' - \lambda_2' R) - R^2 A''_{2,1}(1)\right] \sin z \right.\right.$$
$$\left.\left. - \left[(3 - B''_{0,1}(1) - R^2 B''_{2,1}(1)\right] (\cos z - 1)\right\}\right]. \qquad [3.4]$$

The above expressions show that the stresses p_{zz} and p_{rz} and the skin-friction are affected by both the stress relaxation time and strain retardation time. However, the normal stress p_{rr} is same for Newtonian and the class of elastico-viscous fluids dealt herein.

II. Temperature field :

For the particular case of sinusoidal deformation that

$$K_{0,1} = L_{1,1} = 0,$$

for all r, $0.1 \leq r \leq 0.9$, due to homogeneous boundary conditions. For the calculation of $L_{0,1}$, $K_{1,1}$, $K_{2,1}$ and $L_{2,1}$ we have integrated numerically the equations [2.21] and [2.22] for the particular values of the parameter already mentioned earlier in the beginning of this section.

The temperature field is given by

$$T(r, z) = 1 + (E\sigma\, R^2 p_z^{(0)2}/64) \left[(1 - r^4) + 4\epsilon \left\{(R K_{1,1} + R^2 K_{2,1}) \cos z \right.\right.$$
$$\left.\left. + (L_{0,1} + R^2 L_{2,1}) \sin z\right\}\right]. \qquad [3.5]$$

In figures 4 and 5, we have drawn the isotherms in the upper half of the meridian plane for Newtonian ($\lambda_1' = \lambda_2' = 0$) and elastico-viscous fluids respectively for the set of values $\epsilon = 0.05$, $E = 0.5$, $\sigma = p_z^{(0)} = h = 1$, $R^2 = 5$,

$\lambda_1' = 0.7$, $\lambda_2' = 0.08$. The isotherms in the lower half are just the mirror image of those in the upper half. The temperature distribution for Newtonian and elastico-viscous fluids are similar. The main points of difference are the absence of any straight isotherm in Fig. 5 and a slight shift of the isotherms for elastico-viscous fluids in comparison to the corresponding isotherms for Newtonion fluids. The temperature distribution is affected by both the stress relaxation time and the strain retardation time.

We note that the isotherms near the boundary of the wavy cylindrical tube are more or less parallel to the boundary. In the case of Newtonian fluids, depending on the chosen values of the parameters characterising the wavelength of deformation, Reynolds number, Prandtl number and Eckert number, the isotherm becomes parallel to the axis of the tube at a certain height from the axis. As we move towards the axis of the tube from this straight isotherm we find that the deformity of the isotherms increases and becomes more and more pronounced so much so that the isotherms form closed loops between $z = \pi$ and 2π for Newtonian fluids and between $z = \pi/2$ and 2π for elastico-viscous fluids. In Table 3 we have recorded the values of the temperature for Newtonian and elastico viscous fluids for $z = 0, \pi/2, \pi, 3\pi/2$ and 2π and for every subinterval of length 0.1 for r, $0.1 \leq r \leq 0.9$.

The increase in λ_1' and λ_2' results in the decrease of temperature in the planes $z = 0$ and $z = 2\pi$ up to a certain distance from the axis and then the temperature increases towards the boundary of the tube and the reverse is true in the plane $z = \pi/2$. In the plane $z = \pi$ the temperature increases up to a certain distance from the axis and then decreases towards the boundary of the tube. The reverse is true in the plane $z = 3\pi/2$.

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