MICROWAVE RESONATOR

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Abstract

The characteristics of a microwave resonator consisting of a cylindrical conductor terminated at both ends by large circular metallic plates and excited in pure E or H modes and also when the modes are coupled are studied theoretically.

Introduction

The resonance phenomena in microwave cavity resonators of simple and some complicated shapes have been studied by several authors[1-29]. The present paper is concerned with a theoretical study of the resonant properties of an open type of microwave cavity resonator consisting of a metallic wire of radius d, terminated at both ends by large circular metallic plates, each of radius $a\,(>>d)$. The resonator shown in Fig. 1 is open on all sides, except at the two ends. In the case of a resonator closed on all sides by metallic walls, there is no loss of energy by radiation and E or H modes can exist independently. Whereas, in the case of an open type resonator, due to the discontinuity which is invariably present at the edge $(\rho = a)$ of the end plates, some energy may be lost by radiation. As the radiated wave is a T-wave, E_ρ^e, H_ρ^e and E_ρ^h, H_ρ^h of the E and H modes respectively, must vanish inside the resonator or approach zero value at $\rho = a$. But the radial components of E and H modes of the non-radiating standing wave part of the total field within the resonator cannot independently become zero. So, it may be said that $E_\rho^e + E_\rho^h = 0$ or $H_\rho^e + H_\rho^h = 0$ at $\rho = a$, which means that the E and H modes are coupled. The primary object of the paper is to derive expression for Q of the resonator when it is excited in a pure E or H modes and also when the E and H modes are coupled.

Field components:

The field components for the E and H modes in cylindrical coordinate (ρ, θ, z) system are as follows (Appendix A1)

E Wave:

$$E_z = -B\cos\theta\,[xH_1^{(1)}(\gamma_e\rho) - H_1^{(2)}(\gamma_e\rho)]\exp(-jhz)$$

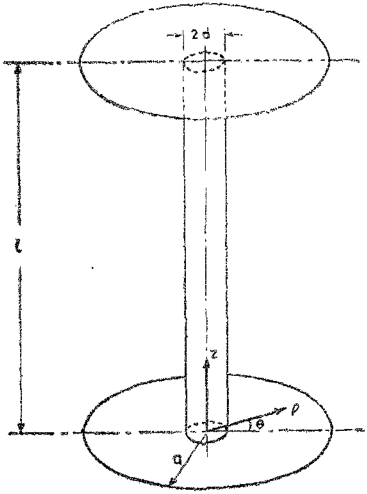

FIG. 1
Coordinate system used for the resonator

$$E_\rho = B\,(h\gamma_e/\omega^2\mu_0\epsilon_0)\cos\theta\,[x\,H_1^{(1)\prime}(\gamma_e\rho) - H_1^{(2)\prime}(\gamma_e\rho)]\exp(-jhz)$$
$$E_\theta = -B(h/\omega^2\mu_0\epsilon_0)\,(1/\rho)\sin\theta\,[x\,H_1^{(1)}(\gamma_e\rho) - H_1^{(2)}(\gamma_e\rho)]\exp(-jhz) \quad (1)$$

$$H_z = 0$$
$$H_\rho = B\,(j/\omega\mu_0)\,(1/\rho)\sin\theta\,[x\,H_1^{(1)}(\gamma_e\rho) - H_1^{(2)}(\gamma_e\rho)]\exp(-jhz)$$
$$H_\theta = B\,(j\gamma_e/\omega\mu_0)\cos\theta\,[x\,H_1^{(1)\prime}(\gamma_e\rho) - H_1^{(2)\prime}(\gamma_e\rho)]\exp(-jhz)$$

H Wave :

$$E_z = 0$$
$$E_\rho = D\,(j/\omega\epsilon_0)\,(1/\rho)\cos\theta\,[y\,H_1^{(1)}(\gamma_h\rho) - H_1^{(2)}(\gamma_h\rho)]\exp(-jhz)$$
$$E_\theta = -D\,(j\gamma_h/\omega\epsilon_0)\sin\theta\,[y\,H_1^{(1)\prime}(\gamma_h\rho) - H_1^{(2)\prime}(\gamma_h\rho)]\exp(-jhz)$$
$$H_z = -D\sin\theta\,[y\,H_1^{(1)}(\gamma_h\rho) - H_1^{(2)}(\gamma_h\rho)]\exp(-jhz)$$
$$H_\rho = -D(h\gamma_h/\omega^2\mu_0\epsilon_0)\sin\theta\,[y\,H_1^{(1)\prime}(\gamma_h\rho) - H_1^{(2)\prime}(\gamma_h\rho)]\exp(-jhz)$$
$$H_\theta = -D\,(h/\omega^2\mu_0\epsilon_0)\,(1/\rho)\cos\theta\,[y\,H_1^{(1)}(\gamma_h\rho) - H_1^{(2)}(\gamma_h\rho)]\exp(-jhz) \quad [2]$$

tanding Waves :

The standing waves are represented on a vector basis as follows :

$$E_{zs} = E_{z+} + E_{z-} \qquad H_{zs} = H_{z+}H_{z-}$$

$$E_{\rho s} = E_{\rho +} + E_{\rho -} \qquad H_{\rho s} = H_{\rho +}H_{\rho -}$$

$$E_{\theta s} = E_{\theta +} + E_{\theta -} \qquad H_{\theta s} = H_{\theta +}H_{\theta -} \qquad [3]$$

with $H_{zs} = 0$ for the E wave and $E_{zs} = 0$ for the H-wave. The subscripts $+$ and $-$ indicate components of the wave travelling in the $+z$ and $-z$ directions respectively. Due to the reflections taking place at $z = 0$ and $z = l$, the following relations hold good.

E Wave

$$E_{z+} = + E_{z-}$$

$$E_{\rho +} = - E_{\rho -}$$

$$E_{\theta +} = - E_{\theta -}$$

$$H_{\rho +} = + H_{\rho -}$$

$$H_{\theta +} = + H_{\theta -} \qquad [4]$$

H Wave :

$$H_{z+} = - H_{z-}$$

$$H_{\rho +} = + H_{\rho -}$$

$$H_{\theta +} = + H_{\theta -}$$

$$E_{\rho +} = - E_{\rho -}$$

$$E_{\theta +} = - E_{\theta -} \qquad [5]$$

Resonant Waves :

The boundary conditions at $z = 0$ and $z = l$ are $E_{\rho s} = 0$ and $E_{\theta s} = 0$ which lead to the condition $\sin hz = 0$ *i.e.*, $h = m_z \pi/l$ where m_z is a positive integer and indicates number of half-cycle variations in E_z and H_z in the case of E and H modes respectively. The field components are $f(\rho)$ and the constants A, B, C, D and hence x and y are complex quantities. The field components of the resonant waves are (Appendix A 2).

E mode :

$$E_{zr}(\rho) = 2\left[\chi J_1(\gamma_e \rho) + \chi' H_1^{(2)}(\gamma_e \rho)\right] \cos\theta \cos(m_z \pi/l) z$$

$$E_{\rho r}(\rho) = 2j(h\gamma_e/\omega^2\mu_0\epsilon_0)\left[\chi J_1'(\gamma_\rho \rho) + \chi' H_1^{(2)'}(\gamma_e \rho)\right] \cos\theta \sin(m_z \pi/l) z$$

$$E_{\theta r}(\rho) = -2j(h/\omega^2\mu_0\epsilon_0)(1/\rho)\left[\chi J_1(\gamma_e \rho) + \chi' H_1^{(2)}(\gamma_e \rho)\right] \sin\theta \sin(m_z \pi/l) z$$

$$H_{zr}(\rho) = 0$$

$$H_{\rho r}(\rho) = -2j(1/\omega\mu_0)(1/\rho)\left[\chi J_1(\gamma_e \rho) + \chi' H_1^{(2)}(\gamma_e \rho)\right] \sin\cos(m_z \pi/l) z$$

$$H_{\theta r}(\rho) = -2j(\gamma_e/\omega\mu_0)\left[\chi J_1'(\gamma_e \rho) + \chi' H_1^{(2)'}(\gamma_e \rho)\right] \cos\theta \cos(m_z \pi/l) z \qquad [6]$$

H mode :

$$E_{zr}(\rho) = 0$$

$$E_{\rho r}(\rho) = -2\,(1/\omega\epsilon_0)\,(1/\rho)\,[\psi\, J_1(\gamma_h\,\rho) + \psi'\, H_1^{(2)}(\gamma_h\,\rho)] \cos\theta \sin(m_z\,\pi/l)\,z$$

$$E_{\theta r}(\rho) = 2\,(\gamma_h/\omega\epsilon_0)\,[\psi\, J_1'(\gamma_h\,\rho) + \psi'\, H_1^{(2)}(\gamma_h\,\rho)] \sin\theta \sin(m_z\,\pi/l)\,z$$

$$H_{zr}(\rho) = -2\,j\,[\psi\, J_1(\gamma_h\,\rho) + \psi'\, H_1^{(2)}(\gamma_h\,\rho)] \sin\theta \sin(m_z\,\pi/l)\,z$$

$$H_{\rho r}(\rho) = 2\,(h\,\gamma_h/\omega^2\mu_0\epsilon_0)\,[\psi\, J_1'(\gamma_h\,\rho) + \psi'\, H_1^{(2)}(\gamma_b\,\rho)] \sin\theta \cos(m_z\,\pi/l)\,z$$

$$H_{\theta r}(\rho) = 2\,(h/\omega^2\mu_0\epsilon_0)\,(1/\rho) + \psi\, J_1(\gamma_h'\,\rho) + \psi'\, H_1^{(2)}(\gamma_h'\,\rho)] \times \cos\theta \cos(m_z\,\pi/l) \qquad [7]$$

$J_1(\gamma\,\rho)$ is an oscillatory function, so the term containing this function represents a standing wave. The Hankel function $H_1^{(2)}(\gamma\,\rho)$ represents a standing wave. The Hankel function $H_1^{(2)}(\gamma\,\rho)$ represents an outward travelling wave. Hence the term involving $H_1^{(2)}(\gamma\,\rho)$ accounts for the radiation of energy in the radial direction. Consequently, the field inside the resonator consists of a non-radiating standing wave and an outward travelling wave, a part of which is reflected back into the resonator due to the discontinuityat $\rho = a$.

Condition of Resonance :

(i) *E mode* :

If the energy is to be confined within the resonator without any loss by radiation, the term containing $H_1^{(2)}(\gamma\,\rho)$ must be zero which requires $\chi' = 0$. So, the field components inside the cavity oscillating in pure E mode are

$$E_{zr}(\rho) = 2\,\chi \cos\theta \cos[(m_z\,\pi/l)\,z]\; J_1(\gamma_e\,\rho)$$

$$E_{\rho r}(\rho) = 2\,j\,(h\,\gamma_e/\omega^2\mu_0\epsilon_0)\,x \cos\theta \sin[(m_2\,\pi/l)\,z]\; J_1'(\gamma_e\,\rho)$$

$$E_{\theta r}(\rho) = -2\,j\,\chi\,(h/\omega^2\,\mu_0\,\epsilon_0)\,(1/\rho) \sin\theta \sin[(m_2\,\pi/l)\,z]\, J_1\,\gamma_e\,\rho)$$

$$H_{\rho r}(\rho) = -2\,j\,\chi\,(1/\omega\mu_0)\,(1/\rho) \sin\theta \cos[(m_z\,\pi/l)\,z]\; J_1(\gamma_e\,\rho)$$

$$H_{\theta r}(\rho) = -2\,j\,\chi\,(\gamma_e/\omega\,\mu_0) \cos\theta \cos[(m_z\,\pi/l)\,z]\, J_1'(\gamma_e\,\rho)$$

$$H_{zr}(\rho) = 0 \qquad [8]$$

The condition $\chi' = 0$ is satisfied if $b = a$ and $b' = a'$ *i.e.* $A = B$ which leads (Appendix A.3) to $x = -1$ *i.e.*

$$H_1^{(2)}(\gamma_e\,d) = -H_1^{(1)}(\gamma_e\,d) \qquad [9]$$

which yields $J_1(\gamma_e\,d) = 0$ [10]

as the condition of resonance

(ii) H *mode* :

The condition of no loss of energy requires that $\psi' = 0$. Consequently, the field components inside the cavity oscillating in pure H mode are

$$H_{zr}(\rho) = -2j\psi \sin\theta \sin[(m_z \pi/l) z] J_1(\gamma_h \rho)$$
$$H_{\rho r}(\rho) = 2\psi (h\gamma_h/\omega^2\mu_0\epsilon_0) \sin\theta \cos[(m_z \pi/l) z] J_1'(\gamma_h \rho)$$
$$H_{\theta r}(\rho) = 2\psi (h/\omega^2\mu_0\epsilon_0)(1/\rho) \cos\theta \cos[(m_z \pi/l) z] J_1(\gamma_h \rho)$$
$$E_{\rho r}(\rho) = -2\psi (1/\omega\epsilon_0)(1/\rho) \cos\theta \sin[(m_z \pi/l) z] J_1(\gamma_h \rho)$$
$$E_{\theta r}(\rho) = 2\psi (\gamma_h/\omega\epsilon_0) \sin\theta \sin[(m_z \pi/l) z] J_1'(\gamma_h \rho)$$
$$E_{zr}(\rho) = 0 \qquad [11]$$

The condition $\psi' = 0$ is satisfied if $c = d$ and $c' = d'$ *i.e.* $C = D$ which leads to (Appendix A.3) $y = -1$, *i.e.*

$$H_1^{(2)'}(\gamma_h d) = -H_1^{(1)'}(\gamma_h d) \qquad [12]$$

which yields

$$J_1'(\gamma_h d) = 0 \qquad [13]$$

as the condition of resonance. The eigenvalues $\gamma_h d$ which satisfy the above equation is obtained when $J_1(\gamma_h d)$ is maximum, *i.e.*, $\gamma_h d = 1.84$, 8 54, 14.85 etc.

(iii) *Coupled E and H modes* :

The conditions [10] and [13] hold good when the cavity is oscillating in pure mode. But due to the finite size of the end plates, higher order modes may be generated resulting in the coupling of E and H modes. The non-existence of radial components in the vicinity of $\rho = a$ is satisfied by the condition

$$[E^e_{\rho r} + E^h_{\rho r}] = 0 \qquad [14]$$

or,

$$[H^e_{\rho r} + H^h_{\rho r}] = 0 \qquad [15]$$

when the modes are coupled. The condition [14] yields

$$\frac{B}{D} = \frac{-j\omega\mu_0}{h\gamma_{eh}a} \frac{y H_1^{(1)}(\gamma_{eh} a) - H_1^{(2)}(\gamma_{eh} a)}{x H_1^{(1)'}(\gamma_{eh} a) - H_1^{(1)}(\gamma_{eh} a)} \qquad [16]$$

The boundary condition $(E^e_{\theta r} + E^h_{\theta r})|_{\rho = d} = 0$ leads to

$$\frac{B}{D} = -j\frac{\gamma d\omega\mu}{h} \frac{y H_1^{(1)'}(\gamma_{eh} d) - H_1^{(2)}(\gamma_{eh} d)}{x H_1^{(1)}(\gamma_{eh} d) - H_1^{(2)}(\gamma_{eh} d)} \qquad [17]$$

Using equations [16], [17], appropriate recurrence relations, and $x = -1$, $y = -1$, the following characteristic equation is obtained.

$$\frac{-H_1^{(1)}(\gamma a) - H_1^{(2)}(\gamma a)}{-H_0^{(1)}(\gamma a) + \frac{H_1^{(1)}(\gamma a)}{\gamma a} - H_0^{(2)}(\gamma a) + \frac{H_1^{(2)}(\gamma a)}{\gamma a}}$$

$$= \gamma^2 a d \frac{-H_0^{(1)}(\gamma d) + \frac{H_1^{(1)}(\gamma d)}{\gamma d} - H_0^{(2)}(\gamma d) + \frac{H_1^{(2)}(\gamma d)}{\gamma d}}{-H_1^{(1)}(\gamma d) - H_1^{(2)}(\gamma d)} \quad [18]$$

where γ is used for γ_{eh}. The above equation reduces to

$$\frac{J_1(\gamma a)}{\gamma a\, J_0(\gamma a) - J_1(\gamma a)} = \frac{\gamma d\, J_0(\gamma d) - J_1(\gamma)}{J_1(\gamma d)} \quad [19]$$

But
$$\frac{J_1(\gamma a)}{\gamma a\, J_0(\gamma a) - J_1(\gamma a)} = \text{constant.}$$

Differentiating the above equation with respect to $\gamma\alpha$, yields

$$\gamma a\, J_0^2(\gamma a) - 2 J_1(\gamma a)\, J_0(\gamma a) + \gamma a\, J_1^2(\gamma a) = 0 \quad [20]$$

Let the roots of eq [20] be δ_n $(n = 1, 2, 3 \cdots)$.

For the mth mode, the eigen values γ_{eh} is given by $\gamma_{eh} = \delta_m/a$ and the condition of resonance is

$$\frac{m_z \pi}{l} = \left[\frac{4\pi}{\lambda_0^2} - \frac{\delta_m^2}{a^2}\right]^{1/2} \quad [21]$$

Since $m_z \pi/l$ is positive and real $(\delta_m/a) < (2\pi/\lambda_0)$ and $(\delta_m^2/a^2) << (4\pi^2/\lambda_0^2)$. Hence [19]

$$l = m_z \lambda_0/2 \quad [22]$$

i.e., the resonance condition is established when the distance between the two end plates is an integral multiple of half wavelength.

In practice $d << a$ and if $\gamma d << 1$, then by making small argument approximations of $J_0(\gamma d) \cong 1$ and $J_1(\gamma d) \cong \gamma d/2$

eq. [19] reduces to

$$J_0(\gamma_{eh} a) = 0 \quad [23]$$

which gives the successive eigen values γ_{eh} when the resonator is oscillating under condition that the modes are coupled, *Electric and Magnetic Energies :*

The energies stored in the electric field (W_E) and in the magnetic field (W_M) of the resonator are

$$W_E = \frac{\epsilon_0}{2} \int_{\theta=0}^{2\pi} \int_{\rho=d}^{a} \int_{z=0}^{l} |E|^2 \rho \, d\rho \, d\theta \, dz \qquad [24]$$

$$W_M = \frac{\mu_0}{2} \int_{\theta=0}^{2\pi} \int_{\rho=d}^{a} \int_{z=0}^{l} |H|^2 \rho \, d\rho \, d\theta \, dz \qquad [25]$$

where,

$$|E|^2 = |E_{zr}|^2 + |E_{\theta r}|^2 + |E_{\rho r}|^2$$

$$|H|^2 = |H_{\rho r}|^2 + |H_{\theta r}|^2 \qquad [26]$$

for Ewave and

$$|E|^2 = |E_{\theta r}|^2 + |E_{\rho r}|^2$$

$$|H|^2 = |H_{zr}|^2 + |H_{\rho r}|^2 + |H_{\theta r}|^2 \qquad [27]$$

for H wave

The electric and magnetic energies W_E^E, W_M^E and W_E^H W_M^H for the E and H modes respectively are obtained by using equations [8], [11] and [24-27] respectively.

$$W_E^E = \pi \, l \, \epsilon_0 \, \chi^2 \Big[J_1^2 (\gamma_e a) \left\{ \tfrac{1}{2} a^2 - \tfrac{1}{2} \frac{h^2 \gamma_e^2 a^2}{\omega^4 \mu_0^2 \epsilon_0^2} + \frac{h^2}{\omega^4 \mu_0^2 \epsilon_0^2} \right\}$$

$$- J_1^2 (\gamma_e d) \left\{ \tfrac{1}{2} d^2 - \tfrac{1}{2} \frac{h^2 \gamma_e^2 d^2}{\omega^4 \mu_0^2 \epsilon_0^2} + \frac{h^2}{\omega^4 \mu_0^2 \epsilon_0^2} \right\}$$

$$- J_0^2 (\gamma_e a) \left\{ \tfrac{1}{2} \frac{h^2 \gamma_e^2 a^2}{\omega^4 \mu_0^2 \epsilon_0^2} - \frac{h^2 \gamma_e}{\omega^4 \mu_0^2 \epsilon_0^2} - \frac{h^2}{\omega^4 \mu_0^2 \epsilon_0^2} \right\}$$

$$+ J_0^2 (\gamma_e d) \left\{ \tfrac{1}{2} \frac{h^2 \gamma_e^2 d^2}{\omega^4 \mu_0^2 \epsilon_0^2} + \frac{h^2 \gamma_e}{\omega^4 \mu_0^2 \epsilon_0^2} - \frac{h^2}{\omega^4 \mu_0^2 \epsilon^2} \right\}$$

$$- \tfrac{1}{2} a^2 J_0 (\gamma_e a) J_2 (\gamma_e a) + \tfrac{1}{2} d^2 J_0 (\gamma_e d) J_2 (\gamma_e d) \Big] \qquad [28]$$

$$W_M^E = - (\pi \, l \, \chi^2 / \omega^2 \mu_0) \left[(1 - \tfrac{1}{2} \gamma_e^2 d^2) J_1^2 (\gamma_e d) - \tfrac{1}{2} \gamma_e^2 d^2 J_0^2 (\gamma_e d) \right.$$

$$\left. + \tfrac{1}{2} \gamma_e^2 a^2 J_0^2 (\gamma_e a) - [1 - (\gamma_e^2 a^2 / 2)] J_1^2 (\gamma_e a) \right] \qquad [29]$$

$$W_E^H = (\pi l \psi^2/\omega^2 \epsilon_0) \left[\left(\tfrac{1}{2}\gamma_h^2 a^2 - 1\right) J_1^2(\gamma_h a) + \tfrac{1}{2}\gamma_h^2 a^2 J_0^2(\gamma_h a) - \left(\tfrac{1}{2}\gamma_h^2 d^2 - 1\right) J_1^2(\gamma_h d) - \tfrac{1}{2}\gamma_h^2 d^2 J_0^2(\gamma_h d) \right] \quad [30]$$

$$\begin{aligned} W_M^H = \pi l \mu_0 \psi^2 \Big[& J_1^2(\gamma_h d) \left\{ \tfrac{1}{2} d^2 - \tfrac{1}{2}\frac{h^2 \gamma_h^2 d^2}{\omega^4 \mu_0^2 \epsilon_0^2} + \tfrac{1}{2}\gamma_h^2 + \frac{h^2}{\omega^4 \mu_0^2 \epsilon_0^2} \right\} \\ & - J_1^2(\gamma_h a) \left\{ \tfrac{1}{2} a^2 - \tfrac{1}{2}\frac{h^2 \gamma_h^2 a^2}{\omega^4 \mu_0^2 \epsilon_0^2} + \tfrac{1}{2}\gamma_h^2 + \frac{h^2}{\omega^4 \mu_0^2 \epsilon_0^2} \right\} \\ & + J_0^2(\gamma_h a) \left\{ \tfrac{1}{2}\frac{h^2 \gamma_h^2 a^2}{\omega^4 \mu_0^2 \epsilon_0^2} + \tfrac{1}{2}\gamma_h^2 + \frac{1}{\gamma_h^2} - \frac{h^2}{\omega^4 \mu_0^2 \epsilon_0^2} \right\} \\ & - J_0^2(\gamma_h d) \left\{ \tfrac{1}{2} d^2 - \tfrac{1}{2}\gamma_h^2 + (1/\gamma_h^2) - (h^2/\omega^4 \mu_0^2 \epsilon_0^2) \right\} \\ & - \tfrac{1}{2} d^2 J_0(\gamma_h d) J_2(\gamma_h d) + \tfrac{1}{2} a^2 J_0(\gamma_h a) J_2(\gamma_h a) \Big] \end{aligned} \quad [31]$$

Total Power Lost:

The total power (P) lost in the resonator is equal to the sum of the power lost (P_e) in the end plates, power lost (P_w) in the wire and power lost by radiation. If it is assumed that the total energy is contained within the resonator and there is no loss of power by radiation, then

$$P = P_e + P_w \quad [32]$$

where

$$P_e = 2 \times \tfrac{1}{2} \sqrt{\frac{\pi f \mu_0}{\sigma_e}} \int_{\theta=0}^{2\pi} \int_{\rho=d}^{a} |H_{\tan}|^2 \, \rho \, d\theta \, d\rho \quad [33]$$

for both the end plates, and

$$P_w = \sqrt{\frac{\pi f \mu_0}{\sigma_w}} \int_{\theta=0}^{2\pi} \int_{z=0}^{l} |H_{\tan}|^2 d \, d\theta \, dz \quad [34]$$

where, σ_e and σ_w represent the conductivity of the end plates and the wire respectively. The elements of area on the surface of the end plates and wire are $\rho \, d\theta \, d\rho$ and $d \, d\theta \, d\rho$ respectively. For E mode

$$|H_{\tan}|^2_{\text{end plates}} = |H_{\rho r}|^2 + |H_{\theta r}|^2$$

$$|H_{\tan}|^2_{\text{wire}} = |H_{\theta r}|^2 \quad [35]$$

and for H mode

$$|H_{\tan}|^2_{\text{end plates}} = |H_{\rho r}|^2 + |H_{\theta r}|^2$$

$$|H_{\tan}|^2_{\text{wire}} = |H_{zr}|^2 + |H_{\theta r}|^2 \qquad [36]$$

The total power lost P_E and P_H for the E and H modes respectively are obtained by using equations [8], [11] and [32-36] appropriately.

$$P_E = -\frac{4\pi\chi^2}{\omega^2\mu_0^2}\left(\frac{\pi f \mu_0}{\sigma_e}\right)^{1/2}\Big[-\tfrac{1}{2}\gamma_e^2 d^2 J_0^2(\gamma_e d) - \tfrac{1}{2}\gamma_e^2 a^2 J_0^2(\gamma_e a)$$

$$-(1-\tfrac{1}{2}\gamma_e^2 a^2) J_1^2(\gamma_e a)\Big] - \frac{\pi d l \chi^2 \gamma_e^2}{\omega^2\mu_0^2}\left(\frac{\pi f \mu_0}{\sigma_w}\right)^{1/2} J_0^2(\gamma_e d) \qquad [37]$$

$$P_H = 4\pi\psi^2\frac{h^2}{\omega^4\mu_0^2\epsilon_0^2}\left(\frac{\pi f\mu_0}{\sigma_e}\right)^{1/2}\Big[(\tfrac{1}{2}\gamma_h^2 a^2 - 1) J_1^2(\gamma_h a) + \tfrac{1}{2}\gamma_h^2 a^2 J_0^2(\gamma_h a)$$

$$-(\tfrac{1}{2}\gamma_h^2 d^2 - 1) J_1^2(\gamma_h d) - \tfrac{1}{2}\gamma_h^2 d^2 J_0^2(\gamma_h d)\Big]$$

$$+\pi l d \psi^2\left(\frac{\pi f\mu_0}{\sigma_w}\right)^{1/2}\left[\frac{h^2}{\omega^4\mu_0^2\epsilon_0^2 d} - 1\right] J_1^2(\gamma_h d) \qquad [38]$$

Determination of h:

At resonance, $W_E = W_M$. Using the relations $W_E^E = W_M^E$ and $W_E^H = W_M^H$ in the case of E and H modes respectively and $\epsilon_0 = 8.85\times10^{-12}$ F/m, $\mu_0 = 4\pi\times10^{-7}$ H/m, the following equations from which h can be evaluated are obtained

(i) *E mode :*

$$26\times10^{-9} f^{-4} h^2\Big[-\gamma_e^2 a^2\{J_1^2(\gamma_e a) + J_0^2(\gamma_e a)\} + \gamma_e^2 d^2 J_0^2(\gamma_e d) + 2J_1^2(\gamma_e a)$$

$$+2\gamma_e\{J_0^2(\gamma_e a) + J_0^2(\gamma_e d)\} + 2\{J_0^2(\gamma_e a) - J_0^2(\gamma_e d)\}\Big]$$

$$= 22\times10^{14} f^{-2}\Big[J_1^2(\gamma_e a) + \tfrac{1}{2}\gamma_e^2 d^2 J_0^2(\gamma_e d) - (\gamma_e^2 a^2/2)\{J_0^2[(\gamma_e a) + J_1^2(\gamma_e a)\}\Big]$$

$$-\tfrac{1}{2}a^2\{J_1^2(\gamma_e a) - J_0(\gamma_e a) J_2(\gamma_e a)\} - \tfrac{1}{2}d^2 J_0^2(\gamma_e d)$$

(ii) *H mode:*

$$26\times10^{29} f^{-4} h^2\Big[-\gamma_h^2 d^2 J_1^2(\gamma_h d) + 2\{J_1^2(\gamma_h d) + J_0^2(\gamma_h d)\}$$

$$+\gamma_h^2 a^2\{J_1^2(\gamma_h a) + J_0^2(\gamma_h a)\} - 2\{J_1^2(\gamma_h a) + J_0^2(\gamma_h a)\}\Big]$$

$$= 22\times10^{14} f^{-2}\Big[(\gamma_h^2 a^2/2)\{J_1^2(\gamma_h a) + J_0^2(\gamma_h a)\}$$

$$-(\gamma_h^2 d^2/2)\{J_1^2(\gamma_h d) + J_0^2(\gamma_h d)\} - \{J_1^2(\gamma_h a) - J_1^2(\gamma_h d)\}\Big]$$

$$-\tfrac{1}{2}d^2\{J_1^2(\gamma_h d)-J_0^2(\gamma_h d)-J_0(\gamma_h d)J_2(\gamma_h d)\}$$
$$+\tfrac{1}{2}a^2\{J_1^2(\gamma_h a)-J_0(\gamma_h a)J_2(\gamma_h a)\}$$
$$-\gamma_h^2/2\{J_1^2(\gamma_h d)-J_1^2(\gamma_h a)-J_0^2(\gamma_h a)+J_0^2(\gamma_h d)\}$$
$$-(1/\gamma_h^2)\{J_0^2(\gamma_h a)-J_0^2(\gamma_h d)\} \qquad [40]$$

(iii) *Coupled Modes*:

To determine h for EH mode γ_e in eq. [39] is replaced by γ_{eh} and for HE mode γ_h in eq. [40] is replaced by $\gamma_{he}=\gamma_{eh}$. The expressions for h obtained by using the relation $J_0(\gamma_{eh}a)=0$ are as follows:

$$26\times10^{29}f^{-4}h^2[-\gamma^2a^2J_1^2(\gamma a)+\gamma^2d^2\{J_1^2(\gamma d)+J_0^2(\gamma d)\}+2\{J_1^2(\gamma a)$$
$$-J_1^2(\gamma d)\}+2\gamma J_0^2(\gamma d)-2J_0^2(\gamma d)]$$
$$=-22\times10^{11}f^{-2}[\{J_1^2(\gamma d)-J_1^2(\gamma a)\}-\gamma^2d^2\{J_1^2(\gamma d)+\tfrac{1}{2}J_0^2(\gamma d)\}$$
$$+(\gamma^2a^2/2)J_1^2(\gamma a)]-\tfrac{1}{2}a^2J_1^2(\gamma a)+\tfrac{1}{2}d^2\{J_1^2(\gamma d)-J_0(\gamma d)J_2(\gamma d)\} \qquad [41]$$

for EH mode, and

$$26\times10^{29}f^{-4}h^2[-\gamma^2d^2J_1^2(\gamma d)+2\{J_1^2(\gamma d)+J_0^2(\gamma d)\}+\gamma^2a^2J_1^2(\gamma a)$$
$$-2J_1^2(\gamma a)]$$
$$=22\times10^{14}f^{-2}[(\gamma^2a^2/2)J_1^2(\gamma a)-(\gamma^2d^2/2)\{J_1^2(\gamma d)+J_0^2(\gamma d)\}$$
$$-\{J_1^2(\gamma a)-J_1^2(\gamma d)\}]-\tfrac{1}{2}d^2\{J_1^2(\gamma d)-J_0^2(\gamma d)-J_0(\gamma d)J_2(\gamma d)\}$$
$$+\tfrac{1}{2}a^2J_1^2(\gamma a)-(\gamma^2/2)\{J_1^2(\gamma d)-J_1^2(\gamma a)+J_0^2(\gamma d)\}+(1/\gamma^2)J_0^2(\gamma d) \qquad [42]$$

for HE mode. In equations [41] and [42] $\gamma=\gamma_{eh}$. The guide wave length λ_g inside the resonator is determined from h. It is evident that λ_g is a function of f_0, d and a.

Resonator Q:

The quality factor of the resonator is defined as

$$Q=\omega(W_E/P) \qquad [43]$$

By using proper values of W_E and P in eq. [43], expressions for Q when the resonator is excited in pure and coupled modes are obtained as follows:

$$Q^E=\langle[\{2-\gamma_e^2a^2\}J_1^2(\gamma_e a)-\{\gamma_e^2a^2-2\gamma_e-2\}J_0^2(\gamma_e a)$$
$$+\{\gamma_e^2d^2+2\gamma_e-2\}J_0^2(\gamma_e d)]f^{-3/2}h^2l\times81\cdot64\times10^{29}$$
$$+[a^2J_1^2(\gamma_e a)-a^2J_0(\gamma_e a)J_2(\gamma_e a)-d^2J_0^2(\gamma_e d)]f^{5/2}l\times1\cdot57\rangle$$
$$\div\{-120\cdot31\times10^{-39}[-\tfrac{1}{2}\gamma_e^2d^2J_0^2(\gamma_e d)-\tfrac{1}{2}\gamma_e^2a^2J_0^2(\gamma_e a)$$
$$-(1-\tfrac{1}{2}\gamma_e^2a^2)J_1^2(\gamma_e a)]-52\cdot2\times10^{-39}\gamma_e^2dlJ_0^2(\gamma_e d)\} \qquad [44]$$

$$Q^{H} = 179 \times 10^{8} f^{-1} l \left[\left(\tfrac{1}{2}\gamma_h^2 a^2 - 1\right) J_1^2(\gamma_h a) + \tfrac{1}{2}\gamma_h^2 a^2 J_0^2(\gamma_h a) - \left(\tfrac{1}{2}\gamma_h^2 d^2 - 1\right) J_1^2(\gamma_h d) - \tfrac{1}{2}\gamma_h^2 d^2 J_0^2(\gamma_h d)\right]$$
$$\div \left\{694 \cdot 22 \times 10^{22} h^2 f^{-7/2} \left[\left(\tfrac{1}{2}\gamma_h^2 a^2 - 1\right) J_1^2(\gamma_h a) + \tfrac{1}{2}\gamma_h^2 a^2 J_0^2(\gamma_h a) - \left(\tfrac{1}{2}\gamma_h^2 d^2 - 1\right) J_1^2(\gamma_h d) - \tfrac{1}{2}\gamma_h^2 d^2 J_0^2(\gamma_h d)\right] + 2 \cdot 61 \times 10^{-7} d\, l \sqrt{(f)}\left[52 \times 10^{29}(h^2/d) f^{-4} - 1\right] J_1^2(\gamma_h d)\right\} \quad [45]$$

$$Q^{EH} = 81 \cdot 64 \times 10^{29} f^{-3/2} h^2 l \left[\{2 - \gamma^2 a^2\} J_1^2(\gamma a) - \{2 - \gamma^2 d^2\} J_1^2(\gamma d) + \{\gamma^2 d^2 + 2\gamma - 2\} J_0^2(\gamma d)\right] + 1 \cdot 57 f^{5/2} l \left[a^2 J_1^2(\gamma a) - d^2 J_1^2(\gamma d) + d^2 J_0(\gamma d) J_2(\gamma d)\right]$$
$$\div \left\{-120 \cdot 31 \times 10^{-39}\left[-\tfrac{1}{2}\gamma^2 d^2 J_0^2(\gamma d) + \left(1 - \langle\gamma^2 d^2/2\rangle\right) J_1^2(\gamma d) - \left(1 - \tfrac{1}{2}\gamma^2 a^2\right) J_1^2(\gamma a)\right] - 52 \cdot 2\, \gamma^2 d\, l \times 10^{-39}\left[J_0^2(\gamma d) + \langle J_1^2(\gamma d)/\gamma^2 d^2\rangle - \langle 2 J_0(\gamma d) J_1(\gamma d)/\gamma d\rangle\right]\right\} \quad [46]$$

where, $\gamma = \gamma_{eh}$

$$Q^{HE} = 179 \times 10^{8} l f^{-1} \left[\left(\tfrac{1}{2}\gamma^2 a^2 - 1\right) J_1^2(\gamma a) - \left(\tfrac{1}{2}\gamma^2 d^2 - 1\right) J_1^2(\gamma d) - \tfrac{1}{2}\gamma^2 d^2 J_0^2(\gamma d)\right]$$
$$\div \left\{694 \cdot 22 \times 10^{22} f^{-7/2} h^2 \left[\left(\tfrac{1}{2}\gamma^2 a^2 - 1\right) J_1^2(\gamma a) - \left(\tfrac{1}{2}\gamma^2 d^2 - 1\right) J_1^2(\gamma d) - \tfrac{1}{2}\gamma^2 d^2 J_0^2(\gamma d)\right] + 2 \cdot 61 \times 10^{-7} l\, d f^{1/2}\left[52 \times 10^{29} f^{-4} h^2 d^{-1} - 1\right] J_1^2(\gamma d)\right\} \quad [47]$$

where, $\gamma = \gamma_{he}$

Large Argument Approximations :

When the argument of the Bessel function and γ are large equations [39—47] reduce to the

$$h^2 \left[\gamma_e a \left\{\cos^2(\gamma_e a - \langle 3\pi/4\rangle) + \cos^2(\gamma_e a - \langle \pi/4\rangle)\right]\right. \times \left[\langle 1 \cdot 92 \times 10^{-31} f^4/\gamma_e a\rangle + 4 \cdot 23 \times 10^{-16} f^2 \gamma_e a\right] + 3d^2 f^4 \times 10^{-31}\left\{\langle 2 J_1(\gamma_e d) J_0(\gamma_e d)/\gamma_e d\rangle - J_0^2(\gamma_e d)\right\} \quad [48]$$

$$h^2 \langle 2 \{J_1^2(\gamma_h d) + J_0^2(\gamma_h d)\} - \gamma_h^2 d^2 J_1^2(\gamma_h d) + (2\gamma_h a/\pi)\{\cos^2[\gamma_h a - (3\pi/4)] + \cos^2(\gamma_h a - \pi/4)\}\rangle$$
$$= \cdot 846 \times 10^{-15} f^2 \langle (\gamma_h a/\pi)\{\cos^2[\gamma_h a - (3\pi/4)] + \cos^2(\gamma_h a - \pi/4)\} - (\gamma_h^2 d^2/2)\{J_1^2(\gamma_h d) + J_0^2(\gamma_h d)\} + (2/\pi\gamma_h a) J_1^2(\gamma_h d)\rangle$$
$$- 3\cdot 84 \times 10^{-31} f^4 \langle \tfrac{1}{2} d^2 \{J_1^2(\gamma_h d) - [2 J_1(\gamma_h d) J_0(\gamma_h d)/(\gamma_h d)\} - (\gamma_h/\pi a)\{\cos^2(\gamma_h a - 3\pi/4) + \cos^2(\gamma_h a - \pi/4)\} + (\gamma_h^2/2)\{J_1^2(\gamma_h d) + J_0^2(\gamma_h d)\}\rangle \quad [49]$$

$$h^2[(4/\pi\gamma a) - (2\gamma a/\pi)\cos^2(\gamma a - 3\pi/4) - 2 J_1^2(\gamma d) + (2\gamma - 2) J_0^2(\gamma d)]$$
$$= 4 \times 10^{-31} f^4 \langle [-(a/\pi\gamma)\cos^2(\gamma a - 3\pi/4) + \tfrac{1}{2} d^2 \{J_1^2(\gamma d) - [2 J_1(\gamma d) J_0(\gamma d)/(\gamma d)] - J_0^2(\gamma d)\}\rangle$$
$$- 0\cdot 846 \times 10^{-14} f^2 \langle (1 - \gamma^2 d^2) J_1^2(\gamma d) - (2/\pi\gamma a) \times [1 - (\gamma^2 a^2/2)]\cos^2(\gamma a - 3\pi/4) - (\gamma^2 d^2/2) J_0^2(\gamma d)\rangle \quad [50]$$

where, $\gamma = \gamma_{eh}$

$$h^2[(2 - \gamma^2 d^2) J_1^2(\gamma d) + 2 J_0^2(\gamma d) + (2/\pi\gamma a)(\gamma^2 a^2 - 2)\cos^2(\gamma a - 3\pi/4)]$$
$$= \cdot 846 \times 10^{-15} f^2 \{(2/\pi\gamma a)[(\gamma^2 a^2/2) - 1]\cos^2(\gamma a - 3\pi/4) + [1 - (\gamma^2 d^2/2)] J_1^2(\gamma d) - (\gamma^2 d^2/2) J_0^2(\gamma d)\} + 4 \times 10^{-31} f^4 \times$$
$$\{[d J_1(\gamma d) J_0(\gamma d)/\gamma] - [(d^2/2) + (\gamma^2/2)] J_1^2(\gamma d) + [(a/\pi\gamma) + (\gamma/\pi a)\cos^2(\gamma a - 3\pi/4) - [(\gamma^2/2) - (1/\gamma^2)] J_0^2(\gamma d)\} \quad [51]$$

where, $\gamma = \gamma_{he}$

$$Q^E = \{[-(2\gamma a/\pi)\{\cos^2(\gamma a - 3\pi/4) + \cos^2(\gamma a - \pi/4)\} + 2\gamma J_0^2(\gamma d)] f^{-3/2} h^2 l \times 81\cdot 64 \times 10^{29} + [(2a/\pi\gamma)\{\cos^2(\gamma a - 3\pi/4) + \cos^2(\gamma a - \pi/4)\} - (4/\pi\gamma^2)\cos(\gamma a - 3\pi/4)\cos(\gamma a - \pi/4) - d^2 J_0^2(\gamma d)] f^{5/2} l \times 1\cdot 57\} \div \{120\cdot 31 \times 10^{-39}[\tfrac{1}{2}\gamma^2 d^2 J_0^2(\gamma d) + (\gamma a/\pi)\cos 2\gamma a] - 52\cdot 2 \times 10^{-39}\gamma^2 d l J_0^2(\gamma d)\} \quad [52]$$

where, $\gamma = \gamma_e$

$$Q^H = 179 \times 10^8 f^{-1} l [(\gamma a/\pi) \{\cos^2 (\gamma a - 3\pi/4) + \cos^2 (\gamma a - \pi/4)]
- (\tfrac{1}{2}\gamma^2 d^2 - 1) J_1^2 (\gamma d) - \tfrac{1}{2}\gamma^2 d^2 J_0^2 (\gamma d)]
+ \{694{\cdot}22 \times 10^{22} h^2 f^{-7/2} [(\gamma a/\pi) \{\cos^2 (\gamma a - 3\pi/4)
+ \cos^2 (\gamma a - \pi/4)\} - (\tfrac{1}{2}\gamma^2 d^2 - 1) J_1^2 (\gamma d) - \tfrac{1}{2}\gamma^2 d^2 J_0^2 (\gamma d)]
+ 2{\cdot}61 \times 10^{-7} l d \sqrt{f} [52 \times 10^{29} (h^2/d) f^{-4} - 1] J_1^2 (\gamma d)\} \qquad [53]$$

where, $\gamma = \gamma_p$

$$Q^{EH} = 81{\cdot}64 \times 10^{29} f^{-3/2} h^2 l [\{2 - \gamma^2 a^2\} (2/\pi\gamma a) \cos^2 (\gamma a - 3\pi/4)
- 2 J_1^2 (\gamma d) + (2\gamma - 2) J_0^2 (\gamma d)]
+ 1{\cdot}57 f^{5/2} l \langle (2a/\pi\gamma) \cos^2 (\gamma a - 3\pi/4) - d^2 \{J_1^2 (\gamma d)
- [2 J_1 (\gamma d) J_0 (\gamma d)/(\gamma d)] + J_0^2 (\gamma d)\}\rangle
\div \{120{\cdot}31 \times 10^{-39} [J_1^2 (\gamma d) - (2/\pi\gamma a)(1 - \tfrac{1}{2}\gamma^2 a^2) \cos^2 (\gamma a - 3\pi/4)]
- 52{\cdot}2 \gamma^2 d l \times 10^{-39} \langle J_0^2 (\gamma d) + J_1^2 (\gamma d)/(\gamma^2 d^2)
- 2 J_0 (\gamma d) J_1 (\gamma d)/(\gamma d)\rangle\} \qquad [54]$$

where, $\gamma = \gamma_{eh}$

$$Q^{HE} = 179 \times 10^8 l f^{-1} [(\tfrac{1}{2}\gamma^2 a^2 - 1)(2/\pi\gamma a) \cos^2 (\gamma a - 3\pi/4) + J_1^2 (\gamma d)]
\div \{694{\cdot}22 \times 10^{22} h^2 f^{-7/2} [(\tfrac{1}{2}\gamma^2 a^2 - 1)(2/\pi\gamma a) \cos^2 (\gamma a - 3\pi/4)
+ J_1^2 (\gamma d)] + 2{\cdot}61 \times 10^{-7} l d f^{1/2} [52 \times 10^{29} h^2 f^{-4} d^{-1} - 1] J_1^2 (\gamma d)\}$$

where, $\gamma = \gamma_{he}$ [55]

NUMERICAL CALCULATIONS

Fig. 2 shows the variation of guide wavelength λ_g and Q of the resonator with respect to the frequency of excitation. Fig. 3 shows the variation of Q with respect to the length of the resonator for different frequencies of excitation. The following values of σ have been used.

$$\sigma_e = 3.54 \times 10^7 \ \Omega/m$$

$$\sigma_w = 5.8 \times 10^7 \ \Omega/m$$

The results given are for E mode of the first order. Higher order E modes are found to have very high attenuation and very low values of Q and the results are not reported. Further numerical computations are under progress and the results will be reported elsewhere.

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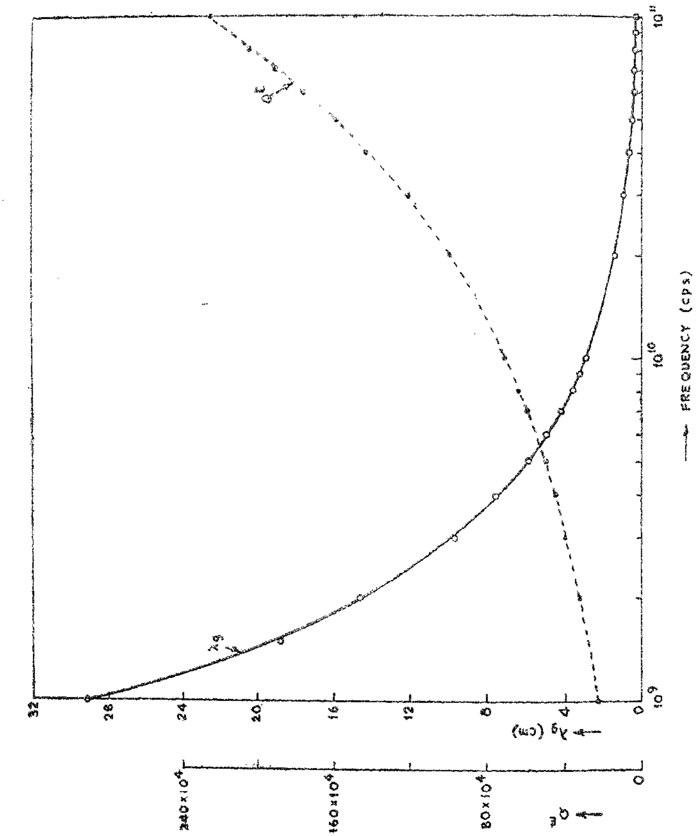

FIG. 2

Variation of guide wavelength and Q factor of E mode with frequency of excitation of the resonator.

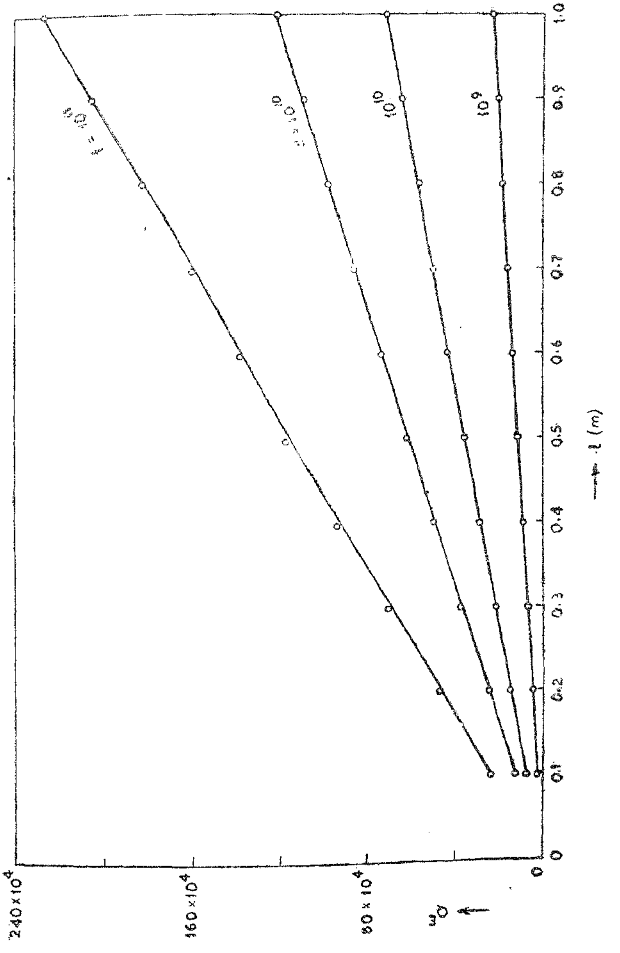

FIG. 3

Variation of Q (E-mode) with respect to the length of the resonator at different frequencies of excitation.

APPENDIX A1

The solution of the wave equation in cylindrical coordinate system (Fig. 1) lead to the z- component of E in the case of E mode and z- component of H in the case of H mode as follows:

$$E_z = [A\, H_n^{(1)}(\gamma_e\, \rho) + B\, H_n^{(2)}(\gamma_e\, \rho)] \cos n\,\theta \exp(-j\,h\,z) \quad [A.1]$$

$$H_z = [C\, H_n^{(1)}(\gamma_h\, \rho) + D\, H_n^{(2)}(\gamma_h\, \rho)] \sin n\,\theta \exp(\ \ j\,h\,z) \quad [A.2]$$

where, the radial propagation constant γ is related to the axial propagation constant h as follows

$$\gamma^2 = (2\,\pi/\lambda_0)^2 - h^2 \quad [A.3]$$

where, λ_0 is the free space wave length.

As the cyclic variability of E_z and H_z can be made to remain the same as θ changes from θ to $\theta + 2\pi n$ (n in an integer), by suitably adjusting the mode of excitation, eq. [A.1] and eq. [A.2] are reduced to

$$E_z = [A\, H_1^{(1)}(\gamma_e\, \rho) + B\, H_1^{(2)}(\gamma_e\, \rho)] \cos\theta \exp(-j\,h\,z) \quad [A.4]$$

$$H_z = [C\, H_1^{(1)}(\gamma_h\, \rho) + D\, H_1^{(2)}(\gamma_h\, \rho)] \sin\theta \exp(-j\,h\,z) \quad [A.5]$$

Assuming that the conductivity of the wire $\sigma_w = \infty$, and applying the boundary conditions on the surface ($\rho = d$) of the wire, $E_z = 0$ in the case of E mode and $E_\theta = 0$ in the case of H mode, the following relations between the coefficients A and B and C and D are obtained.

$$A = -B\,x \quad [A.6]$$

$$C = -D\,y \quad [A.7]$$

where,

$$x = \frac{H_1^{(2)}(\gamma_e\, d)}{H_1^{(1)}(\gamma_e\, d)}$$

and

$$y = \frac{H_1^{(2)'}(\gamma_h\, d)}{H_1^{(1)'}(\gamma_h\, d)}$$

Using Maxwell's equations and the relations [A.1], [A.2], [A.6], [A.7], the field components (equations 1 and 2) are obtained. The time variation of the field vectors is assumed as $\exp(j\,\omega\,t)$

APPENDIX A2

If
$$a = R_e(A)$$
$$a' = I_m(A)$$
$$b = R_e(B)$$
$$b' = I_m(B)$$
$$c = R_e(C)$$
$$c' = I_m(C)$$
$$d = R_e(D)$$
$$d' = I_m(D)$$
$$\chi = 2(a + j a')$$
$$\chi' = [(b - a) + j(b' - a')]$$
$$\psi = 2(c + j c')$$
$$\psi' = [(d - c) + j(d' - c')]$$

then,
$$-B[x H_1^{(1)}(\gamma_e \rho) - H_1^{(2)}(\gamma_e \rho)] = [\chi J_1(\gamma_e \rho) + \chi' H_1^{(2)}(\gamma_e \rho) \quad [A.8]$$

$$-B[x H_1^{(1)'}(\gamma_e \rho) - H_1^{(2)'}(\gamma_e \rho)] = [\chi J_1'(\gamma_e \rho) + \chi' H^{(2)'}(\gamma_e \rho)] \quad [A.9]$$

$$D[y H_1^{(1)}(\gamma_h \rho) - H_1^{(2)}(\gamma_h \rho)] = -[\psi J_1(\gamma_h \rho) + \psi' H_1^{(2)}(\gamma_h \rho)] \quad [A.10]$$

$$D[y H_1^{(1)'}(\gamma_h \rho) - H_1^{(1)'}(\gamma_h \rho)] = -[\psi J_1'(\gamma_h \rho) + \psi' H_1^{(2)'}(\gamma_h \rho)] \quad [A11]$$

APPENDIX A3

$$x = \frac{H_1^{(2)}(\gamma_e d)}{H_1^{(1)}(\gamma_e d)} = \frac{J_1(\gamma_e d) - j Y_1(\gamma_e d)}{J_1(\gamma_e d) + j Y_1(\gamma_e d)}$$

But $J_1(\gamma_e d) = 0$ for resonance condition

$$\therefore x = 1 \quad [A.12]$$

$$y = \frac{H_1^{(2)'}(\gamma_h d)}{H_1^{(1)'}(\gamma_h d)} = \frac{J_0(\gamma_h d) - j\,Y_0(\gamma_h d) - [J_1(\gamma_h d)/\gamma_h d] + j\,[Y_1(\gamma_h d)/\gamma_h d]}{J_0(\gamma_h d) + j\,Y_0(\gamma_h d) - [J_1(\gamma_h d)/\gamma_h d] - j\,[Y_1(\gamma_h d)/\gamma_h d]}$$

But
$$J_0(\gamma_h d) - \frac{J_1(\gamma_h d)}{\gamma_h d} = 0$$

$$\therefore y = -1 \qquad [A.13]$$

APPENDIX A4

When the argument $\gamma_e a$ is large

$$J_0(\gamma_e a) = \sqrt{\frac{2}{\pi\,\gamma_e a}} \cos\left(\gamma_e a - \frac{\pi}{4}\right)$$

$$J_0(\gamma_e a) = \sqrt{\frac{2}{\pi\,\gamma_e a}} \cos\left(\gamma_e a - \frac{3\pi}{4}\right)$$

$$J_2(\gamma_e a) = \sqrt{\frac{2}{\pi\,\gamma_e a}} \left[\frac{2}{\gamma_e a} \cos\left(\gamma_e a - \frac{3\pi}{4}\right) - \cos\left(\gamma_e a - \frac{\pi}{4}\right)\right]$$

If γ_e in large,

$$\gamma_e a >> \frac{2\,\gamma_e d^2}{a}$$

$$\gamma_e a > \frac{1}{a}$$

$$\frac{2}{\gamma_e a} \cos\left(\gamma_e a - \frac{3\pi}{4}\right) \cos\left(\gamma_e a - \frac{\pi}{4}\right) << \cos^2\left(\gamma_e a - \frac{3\pi}{4}\right) \ \&\ \cos^2\left(\gamma_e a - \frac{\pi}{4}\right)$$

$$\frac{2\,\gamma_h a}{\pi} >> \frac{4}{\pi\,\gamma_h a}$$

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