A THEORETICAL ANALYSIS OF THE SOURCE-EXCITED ELECTROMAGNETIC FIELDS

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ABSTRACT

The paper presents an analysis of the source-excited fields. The method is applied to the case of a wave guide formed by two parallel plate dielectrics. The condition for the existence of surface wave, leaky wave and guided wave is discussed.

INTRODUCTION

Electromagnetic waves guided or radiated by a physical structure satisfy Maxwell's equations, boundary conditions, Sommerfeld's radiation condition at infinity and launching conditions. The sub-division of a dynamic electromagnetic field into guided waves, surface waves of the forward and backward types, leaky waves and radiated waves, is convenient but arbitrary as electromagnetic waves manifest themselves in these categories depending on the location and nature of the poles of the integrand of a contour integral which also has branch points. The value of this type of integral is expressed as the sum of branch-cut integrals and residues at poles of the integrand. The source-excited fields are written in terms of such integrals, so that the branch-cut integral accounts for radiated waves and the residues for the discrete spectrum which consists of waves guided along an interface. The present study is an attempt to explain the existence of these waves from the point of view of the contribution due to poles and branch-cut integrations in the integral representation of the electromagnetic field. Special emphasis, however, is placed on the evaluation of the field by the saddle point method as this approach dispenses with the branch-cut which in most cases cannot be chosen uniquely The saddle-point method is used in solving problems on ground wave propagation and problems which take into account the presence of the source.

SOURCE-FREE FIELD

In a source free region consisting of a perfect homogeneous and isotropic medium, the electromagnetic field is given by the solution of the wave equation

$$(\nabla^2 + k^2)\,\psi = 0 \qquad [1]$$

where ψ is the eigenfunction corresponding to the eigenvalue k. The eigenfunction is expanded in a series of some suitable set of orthogonal functions. The coefficients of this expansion are determined by imposing appropriate boundary conditions on the surfaces. The orthogonality properties of the field components expressed in terms of mode functions, are the direct consequences of the orthogonality properties of the eigenfunctions. For non-degenerate eigenfunctions, the following orthogonality conditions hold:

$$\int_A \psi_p \psi_q \, d A = 0 \quad p \neq q$$

$$\int_A \nabla \psi_p \cdot \nabla \psi_q \, d A = 0 \quad p \neq q, \ (p, q = 1, 2, \ldots)$$

with the following normalisation condition

$$\int_A \nabla \psi_p \cdot \nabla \psi \, d A = 1$$

The orthogonal set of mode functions is complete in the sense that it allows an arbitrary E or H field within a waveguide to be expanded as

$$\mathbf{E} = \sum_p A_p \left[\mathbf{E}'_{tp} + \mathbf{i}_z E'_z\right] + \sum_p B_p \mathbf{E}''_{tp} \qquad [2]$$

where the single primed and double primed quantities indicate E-mode and H-mode functions respectively and the coefficients A_p and B_p are given by

$$A_p = \frac{\int_A \mathbf{E}\cdot\mathbf{E}'^{*}_{tp} \, d A}{\int \mathbf{E}'_{tp}\cdot\mathbf{E}'^{*}_{tp} \, d A} \qquad B_p = \frac{\int_A \mathbf{E}\cdot\mathbf{E} \, d A}{\int_A \mathbf{E}''_{tp}\cdot\mathbf{E}''^{*}_{tp} \, d A} \qquad [3]$$

and

$$\mathbf{H} = \sum B_p \left(\mathbf{H}''_{tp} + \mathbf{i}_z \mathbf{H}''_{zp}\right) + \sum A_p \mathbf{H}'_{tp} \qquad [4]$$

So, from a knowledge of the transverse component of E over cross-section A of the guide, the amplitude of the modes, and from the eigenvalues, the characteristics of the modes can be determined. The problem is then solved in terms of the eigenfunctions and eigenvalues. Thus the concept of eigenfunctions and eigenvalues is mathematically useful and has yielded valuable information about the characteristics and classification of modes[1] in conventional closed metallic guides, where, no significant error is introduced by not applying the radiation condition at infinity. The launching condition, however, determines the field configuration. No dynamic electromagnetic field can exist in the absence of a physical source. In the case of source-free solutions, the source is regarded as being situated at $z \to -\infty$, when $+z$ is the direction of propagation. But for open waveguides, the expansion of the field only in a discrete spectrum of waves, is not complete and the expansion must be supplemented by a radiation field. This approach is essentially necessary for the understanding of the different aspects of electromagnetic waves.

SOURCE-EXCITED FIELD

When an electric source $J(\mathbf{r})$ is distributed throughout a finite volume V, the electric field $\mathbf{E}(\mathbf{r}')$ at any point of observation $\mathbf{r}'$ is expressed by the volume integral

$$\mathbf{E}(\mathbf{r}') = \iiint_V \Gamma^{(1)}(\mathbf{r}, \mathbf{r}') \cdot J(\mathbf{r})\, dv \qquad [5]$$

where, the dyadic green's function $\Gamma^{(1)}$ of the first kind is given by[2]

$$\Gamma^{(1)}(\mathbf{r}, \mathbf{r}') = i\,\omega\,\mu\,[(\mathbf{i}_z \cdot \mathbf{i}_z - (1/k^2)\,\nabla\,\nabla')\,G^{(1)}(\mathbf{r}, \mathbf{r}') + \nabla_t\,\nabla'_t\,H^{(1)}(\mathbf{r}, \mathbf{r}') + \mathbf{i}_z \times \nabla\,\mathbf{i}_z \times \nabla'\,H^{(2)}(\mathbf{r}, \mathbf{r}')] \qquad [6]$$

where the scalar Green's function $G^{(1)}$ is

$$G^{(1)}(\mathbf{r}, \mathbf{r}') = -\sum_p \{(k_p^2/2\,i\,h_p')\,\psi_p(x, y)\,\psi_p(x', y')\,e^{i\,h_p'\,|z-z'|}\}/2\,i\,h_p'$$

$$H^{(1)}(\mathbf{r}, \mathbf{r}) = -\sum_p \{\psi_p(x, y)\,\psi_p(x', y')\,e^{i\,h_p\,|z-z'|}\}/2\,i\,h_p'$$

$$H^{(2)}(\mathbf{r}, \mathbf{r}) = -\sum \{\phi_p(x, y)\,\phi_p(x', y')\,e^{i\,h_p''\,|z-z'|}\}/2\,i\,h_p''$$

$$\nabla_t = \mathbf{i}_x(\partial/\partial x) + \mathbf{i}_y(\partial/\partial y)$$

ψ_p is the eigenfunction corresponding to E-modes and ϕ_p is the eigenfunction corresponding to H-modes. Primed operators operate on primed quantities only. The function $G^{(1)}$ obeys the following inhomogeneous equation and satisfies the appropriate boundary conditions also.

$$(\nabla^2 + k_0^2)\,G^{(1)}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r}, \mathbf{r}') \qquad [8]$$

Here δ is the Dirac delta function.

POLE WAVES AND BRANCH-CUT WAVES

When an interface separating any two media is excited by a line source parallel to one of the axes, say y, the interface being parallel to the yz plane, the problem of finding the field at or away from the interface is formulated in terms of the Green's function

$$G(x, z) = \int_{-\infty}^{+\infty} f(h)\,e^{-px}\,e^{ihz}\,dh \qquad [9]$$

where the transverse (x) and longitudinal (z) propagation constants are p and h respectively. They are related as follows

$$h^2 = p^2 + k_0^2 \qquad [10]$$

k_0 being the free space wave number of plane waves.

The above integral (equation 9) which represents a continuous spectrum, is evaluated by integrating along the contour shown in Fig. 1. This contour lies on the top or proper leaf of the two-sheeted Riemannian plane so that the waves given rise to by the poles satisfy the radiation condition *i.e.*, $Re\,p > 0$ for all such waves. The integral along the infinite semi-circle vanishes and the integral of equation [9] becomes

$G(x, z) = 2\pi i \Sigma$ Residues at poles included by the contour – Branch-cut integral [11]

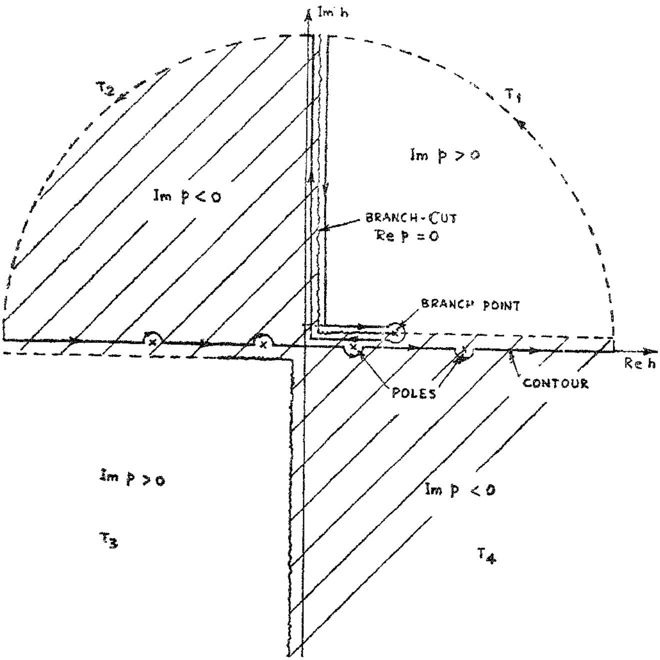

FIG. 1
Top Leaf of h-Plane

The branch-cut integral represents a continuous spectrum of waves which are not bound to the interface and hence accounts for radiated waves. The waves due to the residues at the poles may be considered as those due to a discrete spectrum. The waves in a conventional HSP guide may be considered analogous to pole waves. But the field in an open guide is represented by the sum of the contributions due to the branch-cut integral and the residues at the poles.

STEEPEST DESCENT METHOD OF EVALUATION OE INTEGRAL

To avoid the difficulty of choosing a uneque branch-cut, the saddle point method which is sometimes also called the steepest descent method, is used. The solution of the integral of equation [9] is arrived at by transforming the h-plane representation into a τ-plane representation by the transformation $h = k_0 \sin \tau$ and $p = -ik_0 \cos \tau$. Here $\tau = \xi + i\eta$ represents a complex plane. This transformation maps the quadrants of the h-plane into infinite strips as shown in Fig. 2. The original contour along the real axis of the h plane maps onto the contour along

$$\xi = -(\pi/2), +\infty > \eta \geqslant 0;\ -(\pi/2) \leqslant \xi \leqslant +(\pi/2),\ \eta = 0;\ \xi$$
$$= +(\pi/2), 0 \geqslant \eta > -\infty$$

If this is deformed into the steepest descent path (SDP) given by

$$\cos(\xi - \theta) \cosh \eta = 1 \qquad [12]$$

and passing through the saddle point $\tau = \theta$, the integration of equation [9] along the SDP gives a space wave $G_s(r\ \theta)$ at a point of observation (r, θ) $\theta \neq \pm\pi/2$ and a field close to the interface $G_s(z)$ for $\theta \approx \pm\pi/2$ in an asymptotic form when $k_0 r >> 1$ and $k_0|z| >> 1$ together with terms like G_p due to the residues at the poles between the original contour and the SDP.

It may be noted that if the SDP captures any poles on the bottom leaf of the h-plane, these give rise to what are called leaky waves

Thus $G(x, z) = G_s(r,\ \theta)$ or $G_s(z)$

$$+ \sum_{p=1}^{n} G_p(r,\ \theta) \qquad [13]$$

Equation [13] is invalid when a pole occurs near the saddle-point. In such a case a slightly different method leads to an additional term expressed in terms of the complementary error function along with terms of the type shown in equation [13].

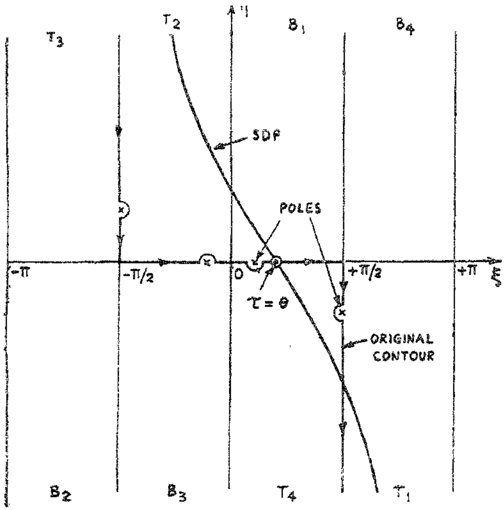

FIG. 2
τ-Plane: Streepest-Descent-Path

PARALLEL PLANE DIELECTRIC WAVEGUIDE

The steepest descent method of field evaluation has been used to study the propagation characteristcs of waves excited in a parallel plane dielectric waveguide excited by a two dimensional delta function source. The fundamental equation which needs solution in this case is

$$E_y = \int_{-\infty}^{+\infty} iq \exp\left(-p\left\{x-(a+d)\right\}\right) \exp(ihz)\, dh/\pi X$$

where

$$X = \exp(pa)\left\{(p+q)^2 \exp(qd) - (p-q)^2 \exp(-qd)\right\}$$
$$- \exp(-pa)\left[p^2 - q^2\right]\left[\exp(qd) - \exp(-qd).\right]$$
$$p = \pm\sqrt{(h^2 - k_0^2)}$$
$$q = \pm\sqrt{(h^2 - k^2)}$$

h – propagation constant in the direction of propagation z.

k_0 – Free space wave number

k – Wave number in the dielectric

a – Half the spacing between the plates

d – Thickness of the plates

Equation 14 gives the field outside the guide.

Conclusions

Some of the results obtained from the solution of this equation lead to the following conclusions :

1. For smaller thicknesses there is only one mode with the surface wave character for spacings ranging from 1 λ to 6 λ.
2. There is a particular type of mode which results in a wave travelling at an angle to the surfaces Such modes do not exist for small spacings. The number of such modes increases with spacing.
3. Leaky wave modes are significant only near the source for spacings of the order of 1 λ to 2 λ.

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