INVESTIGATIONS ON CORRUGATED SURFACE WAVE LINE

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Abstract

The characteristic equation for a surface wave line loaded with uniformly spaced thin metallic discs excited in E_0 mode has been derived. The effects of spacing and groove depth on field distributions have been studied. Considering the existence of only the fundamental mode, the radial field distribution curves as a function of groove depth and spacing show the surface wave nature of the disc-loaded structure. Solution of the characteristic equation shows the existence of pass and stop-bands which have been represented graphically in the form of dispersion diagrams.

Introduction

Theoretical and experimental investigations on corrugated surface wave structure by several authors[1-11] have emphasised essentially on the surface wave character of such structures when the spacing between discs is very small compared to the wavelength of excitation and groove depth. The present work has been motivated with the object of studying not only the surface wave character of the structure but also to find out the characteristics of the type of wave that can exist when the restriction on the small spacing is removed. This investigation forms a part of the programme of surface wave work[12-17] on guiding or radiating structures that is being carried out since the last few years. The paper presents a report on the derivation of the field components taking into account the existence of the space harmonics. Treating the problem by boundary value methods, the characteristic equation involving the fundamental as well as the space harmonics has been derived. The characteristic equation has been solved graphically and the roots of the equations thus obtained have been checked with the computer solution. The field distribution diagrams drawn by utilsing the roots of these equations and the field components show the surface wave decay in the radial directions and the decay is found to be independent of the longitudinal coordinate Dispersion diagrams for various spacings have also been drawn.

Field Components

The field components for the cylindrical surface wave as derived from wave equation in cylindrical coordinate systems $(\rho,\ \phi,\ z)$ are as follows:

$$E_z = C\, H_0^{(1)}\,(j\gamma\,\rho)\,\exp\,(j\,\beta\,z)$$

$$E_\rho = (C\,\beta/\gamma)\,H_0^{(1)\prime}\,(j\,\gamma\,\rho)\,\exp\,(j\,\beta\,z)$$

$$E_\phi = (C\,k^2/\omega\,\mu\,\gamma)\,H_0^{(1)\prime}\,(j\,\gamma\,\rho)\,\exp\,(j\,\beta\,z) \qquad [1]$$

where,

Time dependence has been assumed to be $\exp\,(\ \ j\omega t)$, $\beta^2 = \gamma^2 + k^2$, $k\ \ \omega\sqrt{\mu_0\,\epsilon_0}$), and the Hankel functions H's have been used to satisfy the condition at infinity.

The field expressions given in equation [1] are for a uniform smooth cylindrical rod. By taking into consideration the periodicity of the structure (Fig. 1) the field components in region (1) are

$$E_z^{(1)} = \sum_{m=-\infty}^{\infty} C_m\,H_0^{(1)}\,(j\,\gamma_m\,\rho)\,\exp\,(j\,\beta_m\,z)$$

$$E_\rho^{(1)} = \sum_{m=-\infty}^{\infty} C_m\,(\beta_m/\gamma_m)\,H_0^{(1)\prime}\,(j\,\gamma_m\,\rho)\,\exp\,(j\,\beta_m\,z)$$

$$H_\phi^{(1)} = \sum_{m=-\infty}^{\infty} C_m\,(k^2/\omega\,\mu\,\gamma_m)\,H_0^{(1)\prime}\,(j\,\gamma_m\,\rho)\,\exp\,(j\,\beta_m\,z) \qquad [2]$$

where,

$\beta_m = \beta_0 + (2\,\pi\,m/L)$ and $m = 0,\ \pm 1,\ \pm 2,\ \ldots$

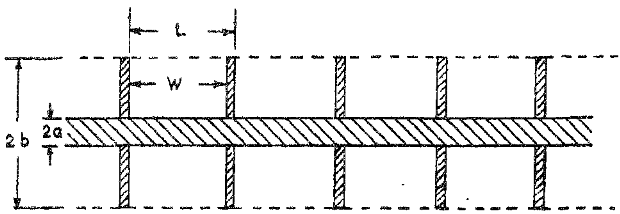

Fig. 1
Sketch of corrugated structure.

The effect of periodicity has been taken into account by introducing the z-dependence as $\exp j\,[\beta + (2\pi m/L)\,z]$ which is a consequence of Floquet's theorem.

A single section of the structure composed of any two adjacent discs and the central rod can be regarded as a radial transmission line shorted at one end. In this case a TEM wave can be supported provided the spacing between the discs is very small compared to the wavelength. The field components for the radial transmission line are

$$E_z = A\,[H_0^{(1)}(k\rho)\,H_0^{(2)}(ka) - H_0^{(1)}(ka)\,(H_0^{(2)}(k\rho)]\,\exp.\,(j\beta_0 nL)$$

$$H_\phi = Aj\,[\sqrt{(\epsilon/\mu)}]\,[H_0^{(1)\prime}(k\rho)\,H_0^{(2)}(ka) - H_0^{(1)}(ka)\,H_0^{(2)\prime}(k\rho)]\,\exp(j\beta_0 nL)\quad [3]$$

where $n = 0, 1, 2\ldots$ represents the particular groove from $z = 0$.

CHARACTERISTIC EQUATION

The variation of the axial electric field $E_z(b, z)$ for the n-th groove over its mouth $\rho = b$ is[11]

$$E_z(b, z) = B\exp(j\beta_0 nL)/\sqrt{[1-(2z_n/W)^2]} \quad [4]$$

where, $z_n = z_0 - nL$ is the distance from the centre of the n-th groove and the excitation constant is represented by B. For each section at the mouth of the groove $(\rho = b)$ and over the range $-\infty \leq z \leq \infty$ of the structure the following boundary condition holds good,

$$E_z^{(1)}(\rho = b) = \sum_{m=-\infty}^{\infty} C_m H^{(1)}(j\gamma_m b)\exp(j\beta_m z)$$

$$= \begin{cases} B\exp(jn\beta_0 L)/\sqrt{[1-(2z_n/W)^2]}, & |z_n| < W/2 \\ 0 & ,\ W/2 < |z_n| < L/2 \end{cases} \quad [5]$$

The value of the amplitude constant C_m is found from equation [5] by multiplying equation [5] by $\exp(-j\beta_m z)$ and integrating over the limits $nL - (W/2)$ to $nL + W/2$ as follows :

$$C_m = [(\pi WB)/2L]\;J_0(\beta_m W/2)/[H_0^{(1)}(j\gamma_m b)] \quad [6]$$

The amplitude constant A is expressed in terms of the constant B by matching the average value of E_z in the two regions at the mouth of the groove $\rho = b$. This gives,

$$A = (B\pi/4j)\,[F_0(kb)]^{-1} \quad \ldots \quad [7]$$

where,

$$F_r(kb) = J_0(ka)\,Y_r(kb) - Y_0(ka)\,J_r(kb)$$

By matching the azimuthal component of the magnetic field H_ϕ in the two regions at $\rho = b$ after substituting the values of A and C_m in the equations [3] and [2] respectively, the following characteristic equation is obtained.

$$\frac{k W}{L} \sum_{m=-\infty}^{\infty} \frac{J_0(\beta_m W/2)\sin(\beta_m W/2)}{Y_m(\beta_m W/2)} \frac{K_1(Y_m b)}{K_0(Y_m b)} = -\frac{F_1(kb)}{F_0(kb)} \qquad [8]$$

where

$$F_1(kb) = J_0(ka)\,Y_1(kb) - Y_0(ka)\,J_1(kb)$$

$$F_0(kb) = J_0(ka)\,Y_0(kb) - Y_0(ka)\,J_0(kb)$$

Solution of the Characteristic Equation

Assuming that the structure is such that it supports only the fundamental mode the characteristic equation [8] reduces to

$$\frac{2k}{L} \frac{J_0(\beta_0 W/2)\sin(\beta_0 W/2)}{\beta_0 \gamma_0} \frac{K_1(\gamma_0 b)}{K_0(\gamma_0 b)} = -\frac{F_1(k\,b)}{F_0(k\,b)} \qquad [9]$$

By assuming that β_0 and γ_0 are real, the above characteristic equation is solved by successive bisection method to yield the roots of the characteristic equation. Some of the roots are tabulated below. It is to be noted that the roots have been calculated to seven decimal placcs and they are approximated to two places in the table for convenience.

Table 1

Roots of the characteristic equation [9]

W in cms. \ b in cms.	1 8	2	2 2	2.4	3 4	3 6
2	1 96	2 17	2.84	5.66	1.97	2.14
.4	0 96	2.18	2 83	4.87	1.97	2.16
.6	1.96	2.16	2.70	4.08	1.97	2,14
.8	1 96	2.12	2.54	3.50	1.96	2.11
1	1.96	2.09	2.39	3 08	1 96	2.07
1.2	1.96	2.05	2.27	2.76	1.96	2.04
1.4	1 96	2.02	2.17	2.52	1.96	2 01
1.6	1 95	2.00	2.09	2.33	1.96	1 99

It it noticed that the characteristic equation has roots only for particular combinations of b and W.

EFFECT OF HARMONICS

In equation [8] the fundamental mode corresponds to $n = 0$, the next term of interest is the one corresponding to $n = -1$. Considering only upto the first backward space harmonic $n = -1$, the characteristic equation [8] reduces to

$$\frac{2k}{L}\left[\frac{J_0(\beta_0 W/2)\sin(\beta_0 W/2)}{\beta_0\gamma_0}\frac{K_1(\gamma_0 b)}{K_0(\gamma_0 b)} + \frac{J_0(\beta_{-1} W/2)\sin(\beta_{-1} W/2)}{\beta_{-1}\gamma_{-1}}\frac{K_1(\gamma_{-1} b)}{K_0(\gamma_{-1} b)}\right] = -\frac{F_1(kb)}{F_0(kb)} \qquad [10]$$

It is found on computing the roots of this equation that for very small spacings they are the same as that of equation [8]. The field distributions considering the fundamental and the first backward space harmonic are

$$E_z = C_0 H_0^{(1)}(j\gamma_0\rho)\exp(j\beta_0 z) + C_{-1} H_0^{(1)}(j\gamma_{-1}\rho)\exp(j\beta_{-1} z)$$

$$E_\rho = C_0(\beta_0/\gamma_0) H_0^{(1)'}(j\gamma_0\rho)\exp(j\beta_0 z) + C_{-1}(\beta_{-1}/\gamma_{-1}) H_0^{(1)'}(j\gamma_{-1}\rho)\exp(j\beta_{-1} z)$$

$$H_\phi = C_0(k^2/\omega\mu\gamma_0) H_0^{(1)'}(j\gamma_0\rho)\exp(j\beta_0 z) + C_{-1}(k^2/\omega\mu\gamma_{-1}) H_0^{(1)'}(j\gamma_{-1}\rho)\exp(j\beta_{-1} z) \qquad [11]$$

where,

$$C_0 = \frac{\pi WB}{2L}\frac{J_0(\beta_0 W/2)}{H_0^{'(1)}(j\gamma_0 b)}$$

$$C_{-1} = \frac{\pi WB}{2L}\frac{J_0(\beta_{-1} W/2)}{H_0^{'(1)}(j\gamma_{-1} b)}$$

The modulus of the field components, for example,

$$|E_z| = \{[C_0 H_0^{(1)}(j\gamma_0\rho)\cos(\beta_0 z) + C_{-1} H_0^{(1)}(j\gamma_{-1}\rho)\cos(\beta_{-1} z)]^2 + [C_0 H_0^{(1)}(j\gamma_0\rho)\sin(\beta_0 z) + C_{-1} H_0^{(1)}(j\gamma_{-1}\rho)\sin(\beta_{-1} z)]^2\}^{1/2} \qquad [12]$$

shows that the radial decay is a function of z

FIELD DISTRIBUTION AND DISPERSION DIAGRAMS

A large number of field distribution diagrams for different combinations of spacing and grooe depth have been computed from equation [8]. Some of the results of the radial field decay are shown in Figs. 2—5. A dispersion diagram also has been computed and is represented in Fig. 6.

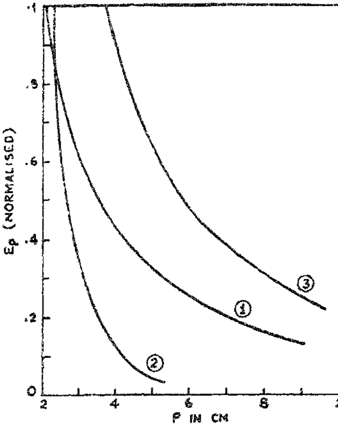

Fig. 2

Field distribution diagrams, W=0.2 cm.
(i) b=1.8 cm., (ii) b=2 cm., (iii) b=3.4 cm.

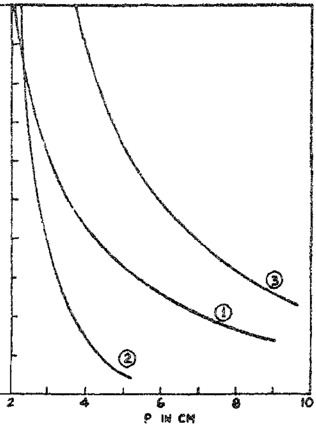

Fig. 3

Field distribution diagrams, W=0.4 cm.
(i) b=1.8 cm., (ii) b=2 cm., (iii) b=3.4 cm

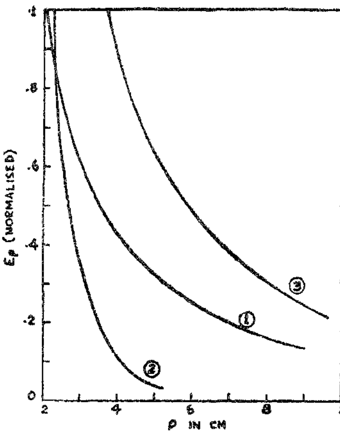

Fig 4

Field distribution diagrams, W=0.6 cm.
(i) b=1.8 cm., (ii) b–2 cm., (iii) b=3.4 cm.

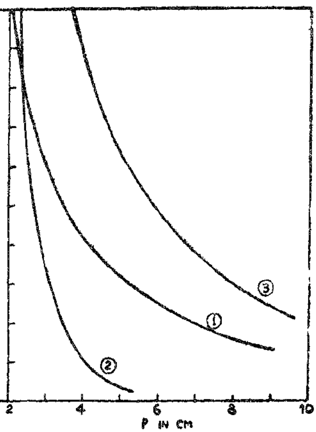

Fig. 5

Field distribution diagrams, W'=0 8 cm.
(i) b=0.8 cm., (ii) b=2 cm., (iii) b=3.4 cm.

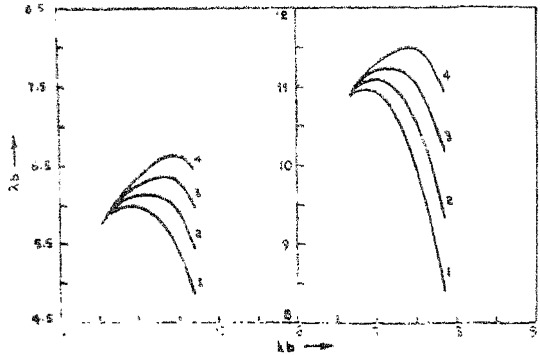

FIG. 6

Dispersion diagrams, λb vs kb
(i) $W=1$ cm., (ii) $W=1.2$ cm., (iii) $W=1.4$ cm., (iv) $W=1.6$ cm.

Further theoretical and experimental work on the propagation characteristics of disc-loaded structure excited by E_0 wave are under progress.

CONCLUSION

The above [illegible] lead to the following conclusions.

1. The radial field decay curves show that the structure can support strongly bound surface wave for only certain combinations of spacing and groove depth.
2. The dispersion diagram reveals the existence of pass and stop bands which are generally the characteristics of a periodic structure

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