

# A CALCULATION OF THE VISCOUS ENERGY DISSIPATION IN A CONVERGENT FLOW AND ITS APPLICATION TO THE PROCESS OF EMULSIFICATION

BY S. V. SUBRAMANYAM

(Department of Physics, Indian Institute of Science, Bangalore-12, India)

(Received ; November 15, 1968)

## ABSTRACT

*The dispersion of one liquid in the other as fine globules requires energy (i) to form the extra interfacial area and (ii) to overcome the viscous resistance during the disintegration of the liquid into globules via jets. The interfacial energy increases in direct proportion to the throughput of the disperse phase while much larger energies are required to overcome the viscous dissipation. The latter has been calculated first for the two dimensional wedge flow in such limiting cases as low Re, high Re, boundary layer flow etc., and the results are later extended to three dimensional axisymmetric flow. It has been found that the energy of viscous dissipation varies as the square of the throughput and accounts for most of the power consumption in high speed practical emulsifiers. There is favourable agreement with the industrial data available.*

## 1. INTRODUCTION

In this paper, we wish to discuss the role of interfacial tension and viscosity in the process of emulsification. The formation of an emulsion requires the dispersion of one liquid as fine droplets in another liquid. This means that the surface area of the liquid to be dispersed has to be increased enormously and the process requires a definite amount of energy. It has been customary to consider this as the sole external energy required to form an emulsion<sup>1,2</sup>. But in practice amounts of energy many orders of magnitude larger than this are needed to form emulsions in ordinary devices, like homogenisers. It appears likely that in high speed practical emulsifiers, viscous energy terms come into play prominently<sup>3</sup> and so it is necessary to study in detail the two types of energy consumption, namely, surface energy and viscous energy.

From a knowledge of the size of the drops and the surface forces, the surface energy could easily be calculated. If the bulk liquid is divided into droplets of radius  $a$ , the work done per unit time against the interfacial tension  $\gamma$ , when an amount  $Q$  of the bulk phase is dispersed per unit time, is

$$W_{\gamma} = 3 Q \gamma / a. \quad [1]$$

As an illustration, we may consider the power required to emulsify an oil (with interfacial tension  $\gamma \approx 1$  dyne/cm) at a rate of about 100 gal/hour. If the droplets are about  $1 \mu$  in radius, about  $5 \cdot 10^{-4}$  H.P. would be required to create the new interface. But in practice, a homogeniser of this capacity would consume about 1 H.P., which is several order of magnitudes larger. Also eq [1] suggests that the power requirement is proportional to the emulsion throughput, while in practice the power increases at a somewhat faster rate<sup>4</sup>, as some higher power of  $Q$ .

In an earlier paper<sup>5</sup>, considering a very simplified model of emulsification system, applicable to homogenisers, we were able to show that substantially large amounts of power are spent as viscous dissipation and that this power consumption increases as  $Q^2$ . But these calculations which were just an extension of Hagen-Poiseuille type of solutions, had the obvious limitation of assuming Laminar flow. It is the purpose of this paper to give a more reliable estimate of the viscous dissipation in convergent flows.

A direct calculation of the viscous energy of a fluid flowing through a converging axisymmetric nozzle is quite difficult because of the following reasons. Firstly the flow will have to be studied in several distinct regions like far off from the apex, near the apex, along the wall surface etc., wherein the different hydrodynamic approximations of low Reynolds number, high Reynolds number, Boundary layer flow etc., will have to be used. Then the viscous energy in each such flow will have to be calculated to find the total resistance to the flow. Because all these regions overlap on each other, matching of the solutions causes certain difficulties. Secondly the flow near the apex of the cone is not clearly known. Hence the flow pattern will have to be arbitrarily assumed in order to make the calculation. This can be either a pure sink flow with radial velocity distribution or a vortex flow as was assumed by Ackeberg.

In view of these difficulties, it appears best to solve the two dimensional case first. Here the complete solutions of the Navier-Stokes equations are known and the calculations can be performed in detail. The three dimensional case will be taken up later, in Section 3, using the above results for guidance.

## 2. VISCOUS ENERGY DISSIPATION IN TWO DIMENSIONAL WEDGE FLOW

The complete solution of the Navier-Stokes equations for the wedge flow is known<sup>6, 7, 8</sup> and these can be used to calculate the viscous energy in full detail for the two dimensional flow. Hence the present section is concerned with the two dimensional analysis.

We shall first summarize the wedge flow solutions needed for our calculations. Using  $(r, \theta, z)$  coordinate system, the  $z$ -axis being along the line of intersection of the two planes, we can write the radial flow solution

$$u_r = \nu F(\theta)/r, \quad u_\theta = 0. \quad [2]$$

where  $\nu$  is the kinematic viscosity of the liquid. The total volume flux per unit distance perpendicular to the flow plane is equal to

$$Q = \int_{-\alpha}^{\alpha} u_r r d\theta = \nu \int_{-\alpha}^{\alpha} F(\theta) d\theta \quad [3]$$

where  $\alpha$  is the semi-wedge angle. (See Fig. 1a).

The conditions at the walls  $\theta = \pm \alpha$  give

$$F(\alpha) = F(-\alpha) = 0 \quad [4]$$

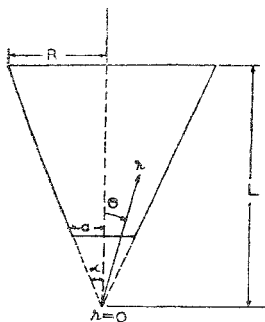


FIG. 1a

Two dimensional wedge flow,  $(r, \theta, z)$  co-ordinate system.  $z$ -axis is the line of intersection of the two planes.

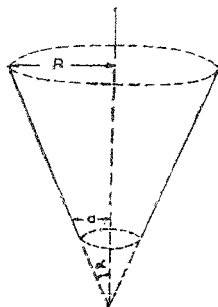
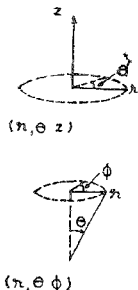


FIG. 1b

Geometry of an axis-symmetric flow.

On substitution of the expressions for  $u_r$  and  $u_\theta$  from (2) in the equations of motion,

$$\begin{aligned} -(\nu^2/r^3) F^2 &= -(1/\rho) (\partial p/\partial r) + \nu^2 F''/r^3 \\ 0 &= -(1/\rho) (\partial \rho/\partial \theta) + 2\nu^2 F'/r^2 \end{aligned}$$

and eliminating  $p$  between the two equations, we get

$$F'' + F^2 + 4F + K = 0 \quad [5]$$

$$\text{or } \theta = (3/2)^{1/2} \int \frac{dF}{(H - 3KF - 6F^2 - F^3)^{1/2}}, \quad [6]$$

where  $H$  and  $K$  are the constants to be evaluated.

For a converging channel,  $F > 0$ . If  $e_1$ ,  $e_2$  and  $e_3$  are the roots of the cubic equation

$$H - 3KF - 6F^2 - F^3 = 0,$$

then the roots are real when  $e_1 \neq 0$ ,  $0 \neq e_2 \neq e_3$ .

The solution [6] can be written in terms of the elliptical integrals

$$\theta = \left( \frac{1}{m} \right) dn^{-1} \left[ \left( \frac{e_2 - e_3}{F - e_3} \right)^{\frac{1}{2}}, k \right] \quad [7]$$

$$\text{or } F = 2[m^2(k^2 - 2) - 1] + 6m^2(1 - k^2)dn^{-2}(m\theta, k) \quad [8]$$

where  $m$  and  $k$  are constants to be evaluated. One can introduce  $R_o$  depending on the velocity along the axial streamline.

Then

$$\begin{aligned} R_o &= F = e_2 = u_o r / \nu \\ m^2 &= (1 + R_o/2)/(1 - 2k^2). \end{aligned} \quad [9]$$

Insertion of  $F(\pm \alpha) = 0$  yields a transcendental determination of  $k$ ,

$$dn^2(m\alpha, k) = 1 - R_o/e_3 \quad [10]$$

and hence

$$Sn^2(m\alpha, k) = \frac{R_o(1 - 2k^2)}{2k^4[3k^2 - 3 + (R_o/2)(k^2 - 2)]} \quad [11]$$

For extremely large  $R_o$ ,

$$Sn^2(m\alpha, k) \approx (1 - 2k^2)/k^2(k^2 - 2) \quad [12]$$

The value of  $k$  can be found by solving either of the equations [11] or [12]. This value of  $k$  can be substituted into the eq. [8] to determine the velocity profile.

We use the above solution of the wedge flow problem<sup>6, 7, 8</sup> to calculate the viscous dissipation. Since numerical computation is quite involved, we shall consider some of the limiting cases and evaluate the viscous energy.

### 2.1 Flows at low $Re$ , small $\alpha$ :

In this case, we get

$$F = R_o(1 - \beta^2/\alpha^2) \quad [13]$$

the familiar parabolic distribution. The factor  $R_0$  can be replaced in terms of the volume flow rate  $Q$  by using [3], i.e.,

$$Q/\nu = \int_{-\alpha}^{\alpha} F(\theta) d\theta = 4\alpha R_0/3$$

Hence the velocity distribution is governed by the equation

$$u_r = (3Q/4\alpha r) [1 - (\theta^2/\alpha^2)] \quad [14]$$

Then a calculation of viscous energy is straight forward and one gets

$$E = \int_{\theta=-\alpha}^{\alpha} \int_{r=r_1}^{r_2} r d\theta dr \eta \left[ 2 \left( \frac{\partial u_r}{\partial r} \right)^2 + 4 \frac{u_r^2}{r^2} + \frac{1}{r^2} \left( \frac{\partial u_r}{\partial \theta} \right)^2 \right] \quad [15]$$

$$= \frac{\eta Q^2}{4} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) 3 \left( \frac{12\alpha^2 + 5}{5\alpha^3} \right). \quad [16]$$

Here  $\eta$  is the viscosity of the liquid, ( $\nu = \eta/\rho$ ).

For  $\alpha \rightarrow 0$ , the geometry approaches that of flow between parallel walls. For such a case,

$$r_1 = \frac{aL}{R-a}, \quad r_2 = \frac{RL}{R-a}, \quad \alpha = \frac{R-a}{L},$$

where  $R$  and  $a$  are the radii as the inlet and outlet of the tube and  $L$  its length. Then

$$E = 3\eta Q^2 L (R+a)/4 R^2 a^2 \quad [17]$$

For flow between parallel walls,  $a \rightarrow R$  and we get back the well known expression<sup>10</sup>.

For a typical case of  $\alpha = 6^\circ$  or  $\pi/30$  radians.

$$E = (\eta Q^2/4) [1/r_1^2 - 1/r_2^2] 2700.$$

## 2.2 Flows at large $Re$ ; ( $\alpha R_0^{1/2}$ large):

In this case,

$$F = R_0 [3 \tanh^2 \{ (R_0/2)^{1/2} (\alpha - \theta) + \tanh^{-1} \sqrt{(2/3)} \} - 2] \quad [18]$$

where  $R_0$  can be found from

$$\begin{aligned} Q/\nu &= \int_{-\alpha}^{\alpha} F(\theta) d\theta \\ &= R_0 \left[ 2\alpha - (2/R_0)^{1/2} \frac{\tanh(\sqrt{2}R_0)\alpha}{1 + 0.815 \tanh(\sqrt{2}R_0)\alpha} \right] \end{aligned} \quad [19]$$

$$= R_0 2\alpha \text{ for large } R_0 \quad [20]$$

Then the viscous energy dissipation can easily be calculated.

$$E = \frac{\eta Q^2}{4} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) \left[ \frac{0.4383 R_0^{1/2}}{\alpha^2} + \frac{6}{\alpha} - \frac{6.927}{\alpha^2 R_0^{1/2}} \right] \quad [21]$$

For the typical case of  $\alpha = 6^\circ = \pi/30$  radian,  $Q = 5$  cc/sec,  $\nu \approx 10^{-2}$  C.G.S. units,

$$\begin{aligned} R_0 &= Q/2\nu\alpha \approx 2377, \\ E &= (\eta Q^2/4) [1/r_1^2 - 1/r_2^2] 1957 \end{aligned} \quad [22]$$

As the  $Re$  goes on increasing, the flow pattern in the wedge also undergoes a continuous change. Whereas at low  $Re$ , the flow was parabolic, at high  $Re$ , the flow near the axis of the wedge remains almost uniform and the viscous effects which determine the velocity profile predominate only near the walls. A part of the viscous dissipation develops in the boundary layer region also. The angle in which the boundary layer occurs is approximately given by  $3(\nu/Ur)^{1/2}$  or  $3R_0^{-1/2}$ . The contribution from the boundary layer region also has been separately calculated and this forms only a fraction of the total dissipation.

The form of the above expression [21] is slightly different from the corresponding one at low  $Re$  [16]. This is because of the different flow patterns and the different limiting cases in which the two are applicable. But for typical values of the parameters, the two are of the same order of magnitude. The details of the flow pattern at small and large  $Re$  do have interesting differences. But the viscous dissipation is the integrated effect over the whole flow pattern and this apparently is not greatly altered. Hence a calculation of viscous energy at low  $Re$  can probably be used over a wider range of  $Re$  to a good degree of approximation. We may expect a similar situation in the three dimensional axisymmetric flow also.

### 2.3 Flow at finite $Re$ , Inertial effects neglected:

The estimation of viscous energy at any specific  $Re$  involves a good deal of numerical computation. The values of  $m$  and  $k$  are to be found by solving the equations [9] and [11] and then the velocity profile can be obtained from the equations [8]. From this velocity profile the dissipation energy can be calculated from the integration of [15]. However it has just now been shown that the viscous energy is independent of the magnitude of  $Re$  to the first approximation. This seems to imply that we can safely neglect the contribution from the inertial terms in the equations of motion. Then the equation determining  $F$  would be from [5]

$$F'' + 4F + K = 0.$$

$$\text{Using the conditions } Q = \nu \int_{-\alpha}^{\alpha} F(\theta) d\theta, F(\pm\alpha) = 0 \quad [24]$$

we get

$$F = \frac{Q}{\nu} \frac{\cos 2\theta - \cos 2\alpha}{\sin 2\alpha - 2\alpha \cos 2\alpha} \quad [25]$$

or

$$u_r = \frac{Q}{r} \frac{\cos 2\theta - \cos 2\alpha}{\sin 2\alpha - 2\alpha \cos 2\alpha} \quad [26]$$

Then the expression for viscous energy becomes

$$E = \frac{\eta Q^2}{2} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) \frac{12\alpha \cos^2 2\alpha - 12 \cos 2\alpha \sin 2\alpha + \frac{1}{2} \sin 4\alpha + 10\alpha}{(2\alpha \cos 2\alpha - \sin 2\alpha)^2} \quad [27]$$

For small  $\alpha$ ,

$$E = \frac{\eta Q^2}{4} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) \frac{3}{\alpha^3} \quad [28]$$

For  $\alpha = 6^\circ = \pi/30$  radians,

$$E = \frac{\eta Q^2}{4} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) 2700 \quad [28a]$$

The three different cases of evaluating the viscous energy lead to almost the same results, namely the equations [16], [21] and [27]. So, as was remarked earlier, in any practical case, the energy required can be found by using any one of these. Also the functional form of the energy is very similar to that in the three dimensional case, as will be shown later.

#### 2.4 Boundary layer calculations in a wedge flow :

It is not necessary here to go into the details of the boundary layer flow in a wedge, as the solution in no way differs from the full solution obtained previously at high  $Re$ , i.e., [20]<sup>8</sup>. One can choose the co-ordinates ( $x, z$ ),  $x$ -axis along the boundary layer and  $z$  perpendicular to it. Then if the velocity at infinity is  $u = -U_0(l/x)$ ,  $U_0$  being the speed at  $x=l$ , then the solution satisfying the boundary conditions is

$$u = -\frac{U_0 l}{x} \left[ 3 \tanh^2 \left\{ \left( \frac{U_0 l}{2\nu} \right)^{1/2} \left( \frac{z}{x} \right) + 1.146 \right\} - 2 \right] \quad [29]$$

This agrees with equation [18] if ( $z/x$ ) is replaced by  $(\alpha - \theta)$ . The viscous energy dissipation is then found by integrating over the boundary layer thickness, ( $\theta$  varying from  $\theta - \delta'$  to  $\alpha$ ,  $\delta' \sim 3/R_0^{1/2}$ ).

Hence

$$E = \frac{\eta Q^2}{4} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) \frac{3}{\alpha^2} \left[ \frac{0.0294}{R_0^{1/2}} \left( \frac{Q}{\alpha\nu} \right) - \frac{6.616}{R_0^{1/2}} \right] \quad [30]$$

which is a fraction of the total dissipation [Eq. 21]. Even though the boundary layer effects are important in determining the local conditions of flow, the energy dissipated in this region is small compared to the total dissipation.

This completes our calculation of the two dimensional problem. We shall apply these results to the question of emulsification in a later section.

### 3. VISCOUS INCOMPRESSIBLE FLOW INSIDE A CONE

In order to extend the two dimensional wedge flow calculations to the three dimensional case, it becomes necessary to understand the hydrodynamic details of the flow inside a cone. An attempt along these directions has been made by several workers<sup>6, 11, 12</sup>. Ackerberg<sup>12</sup> has summarised the details of the problem and these will be used for the calculation of energy in the present case.

#### 3.1 Stokes region ;

In the two dimensional problem of the flow through a wedge, the full equations of motion are satisfied by a radial flow solution<sup>13</sup>. And so a natural extension would be to look for a radial flow solution inside a cone also. Harrison<sup>6</sup> succeeded in obtaining such a solution when the contribution from the inertial terms is neglected. By an iterative procedure Ackerberg<sup>12</sup> was able to include the inertial terms as successive correction factors. In the problem we have a steady, axisymmetric converging motion of an incompressible viscous fluid inside an infinite right circular cone [fig. 1b]. A spherical polar coordinate system  $(r, \theta, \phi)$  can be used with the velocity components  $u_r, u_\theta$  and  $u_\phi$ . Axial symmetry allows us to drop out the  $\phi$  terms. If the fluid has density  $\rho$  and viscosity  $\eta$ , ( $\nu = \eta/\rho$ ). Ackerberg was able to show that

$$\begin{aligned} u_r &= -(\nu^2/A\xi^2) [f_0'(\mu) + f_1'(\mu)/\xi + \dots] \\ u_\theta &= [\nu^2] \{A\xi^2(1-\mu^2)^{1/2}\} [f_1/\xi + \dots] \end{aligned} \quad [31]$$

where  $f_0(\mu) = (1/3) B(\mu - \beta)^2(\mu + 2\beta)$

$$f_1(\mu) = (1/36) B^2(1 - \mu^2)(\mu - \beta)^2 \{2\mu - (5\beta^2 - 3)/\beta\}$$

$$B = 3/[(1 - \beta)^2(1 + 2\beta)] \quad [32]$$

$$\xi = r\nu/A, \quad \mu = \cos \theta, \quad \beta = \cos \alpha, \quad 2\pi A = \text{volume flow rate.}$$

The boundary conditions at the wall surface and the volume flow rate condition have been used. From such a calculation, it can clearly be shown that Harrison's solution which consisted only of the  $f_0$  term described purely radial flow. The presence of the  $f_1, f_2, \dots$  terms deviated the streamline from purely radial flow towards the wall. For the case of a nonnewtonian



fluid, it has recently been shown that such a bending of the streamlines should finally result in a vortex flow near the apex<sup>14</sup>. This has not been explicitly observed in any of the experiments so far.

From the expression of the velocity components, the energy of dissipation can be calculated. Retaining only the leading terms, we have

$$E = \frac{4\pi\rho\nu^4}{A} \frac{B^2}{3} \left[ \left( \frac{1}{\xi_2^3} - \frac{1}{\xi_1^3} \right) \left( \frac{16}{5}\beta^5 - 7\beta^4 + \frac{4}{3}\beta^3 + 4\beta^2 - \frac{23}{15} \right) + \frac{B}{24} \left( \frac{1}{\xi_2^4} - \frac{1}{\xi_1^4} \right) \left( -\frac{9}{5}\beta^7 + \frac{56}{5}\beta^5 - 42\beta^3 + \frac{224}{5}\beta^2 - \frac{96}{5} + \frac{7}{\beta} \right) \right] \quad [33]$$

The complete expression can be found elsewhere<sup>15</sup>.

For small angular cones,  $\xi$  will have to be considerably large for Stokes flow to be valid. (Ackerberg showed that if  $\beta=0.866$ , i.e.,  $\alpha=30^\circ$ ,  $\xi \approx 2$  for Stokes assumption to hold). For a typical case of  $\beta \approx 0.90$ ,  $B \approx 107$ ,

$$E = \frac{4\pi\rho\nu^4}{A} \frac{B^2}{3} \left[ \left( \frac{1}{\xi_2^3} - \frac{1}{\xi_1^3} \right) (-0.0238) + \frac{B}{24} \left( \frac{1}{\xi_2^4} - \frac{1}{\xi_1^4} \right) (-0.0024) \right]$$

In terms of  $(r, \beta)$ , eq. [33] can be written as

$$E = 4\pi\eta A^2 \left( \frac{1}{r_2^3} - \frac{1}{r_1^3} \right) \frac{3(48\beta^3 - 9\beta^2 - 46\beta - 23)}{15(1-\beta)^2(1+2\beta)^2} - 4\pi\eta A^2 \left( \frac{1}{r_2^4} - \frac{1}{r_1^4} \right) \frac{3(9\beta^3 + 45\beta^2 + 79\beta + 35)}{40\beta(1-\beta)(1+2\beta)^3} \quad [34]$$

If  $L$  is the length of the cone,  $R, a$  the two radii, and if the apex angle  $\alpha$  is small, i.e.  $\beta \approx 1 - \delta$ ,  $\delta$  small, then [34] reduces to

$$E_0 \sim \frac{8\eta L Q^2 (R^2 + a^2 + Ra)}{3\pi R^3 a^3} (1 + 3\delta) \quad [35]$$

Except for the  $\delta$  term, this expression can also be derived by an extension of the Hagen-Poiseuille flow pattern<sup>15</sup>.

### 3.2 Flow pattern near the apex of the cone :

The above calculations hold good only in the Stokes region, i.e., regions of low  $Re$  or large  $\xi = r\nu/A$ . For typical cases of the throughput ( $2\pi A$ ) 0.1 cc/sec, 1 cc/sec, 10 cc/sec, the assumption requires a minimum distance of the efflux from the apex to be 3, 30 or 300 cm respectively. For distances smaller than this, the Stokes assumption is not valid and the results may deviate considerably.

This necessitates more detailed calculations to find the flow pattern near the apex of the cone. Here since  $\xi$  is small, the local Reynolds number ( $1/\xi$ ) will be large enough to permit a boundary layer to occur. Hence two distinct regions of flow appear—one along the axis of the cone conforming to the boundary conditions at the axis and the other along the wall with the conditions at the wall satisfied. The two flows merge with each other at the edge of the boundary layer, the thickness of which can be taken as the viscous length.

For the boundary layer calculations to be valid, the local  $Re$  should be at least of the order of  $10^2$  and for the typical cases of the throughput mentioned earlier, *i.e.*, 0.1, 1.0, 10.0 cc/sec,  $r$  should be less than 0.01, 0.1, or 1 cm respectively. This leaves out a wide region between the Stokes flow and the boundary layer flow, where flow is not easy to comprehend.

Several more difficulties crop up in an analysis of the core flow and the boundary layer flow, such as the evaluation of certain constants of integration. In the analogous case of the two dimensional wedge flow, the full solutions of the equations are already available and so it is easier to extend the calculations of viscous dissipation near the efflux by a comparison with the two dimensional case.

It has been shown in an earlier section that the viscous energy of the fluid flowing in a convergent channel can be estimated in several limiting cases, namely (a) flow at low  $Re$  (b) flow at large  $Re$ , (c) flow at a finite  $Re$  with inertial terms neglected and (d) boundary layer flow. The viscous energy in the first three cases is nearly the same. The boundary layer flow is confined to a relatively small volume and so in this region the dissipation is small compared to that in the outer regions of the flow. From these results, it is clear that the viscous energy does not, to the first order, depend on the magnitude of the  $Re$  and so any of the limiting cases, as in (a) to (c) can be used for an estimation of the viscous dissipation.

It is quite likely that a similar situation exists for the three dimensional flow in a cone. Here there are mainly two regions of flow, (a) Stokes flow region where the Reynolds number is small and (b) the flow near the apex where the flow pattern is not clearly defined. Of course there is the intermediate region between these two. The boundary layer region which is present, in addition, does not significantly contribute to the viscous energy as in the two dimensional case. The flow pattern near the apex can be either a radial flow as in the Stokes region or a vortex flow. If we assume it to be a sink flow with radial streamlines, then results very similar to those in Stokes region are obtained.

Although a determination of the complete flow pattern near the point-apex of the cone is quite complicated, in practice we observe a potential sink flow  $U\alpha 1/r^2$ . This is because of the finite dimensions of the efflux hole in homogenisers.

As was shown previously, the viscous energy does not, to the first order, depend on the magnitude of the  $Re$  and so the expression [34] or [35] can be used to find the viscous energy dissipation.

#### 4. RESULTS AND DISCUSSION

In this paper, the power required to disperse a liquid as globules in another liquid is calculated. Because of the increased area of the dispersed phase, a definite amount of energy is required to overcome these interfacial forces and this energy increases in direct proportion to the throughput of the disperse phase. But in practice, comparatively larger energies are required in the dispersion and in fact the energy increases much faster than the throughput.

The viscous resistance to the flow appears to be the dominant factor in the energy requirements of the dispersion. This contribution has been calculated, first in the two dimensional wedge flow and then in the axisymmetric converging flow.

In the two dimensional wedge flow, the full solutions of the Navier-Stokes equations are available for the cases of low  $Re$ , large  $Re$ , finite  $Re$  with inertial effects neglected and boundary layer flow. The power required to overcome the viscous resistance in the boundary layer region is relatively small as the boundary layer occupies only a fraction of the total volume and becomes significant at very high  $Re$ . The other three cases give nearly the same results probably because the viscous resistance is a bulk property and does not very much depend on the details of the flow pattern. Hence the calculated viscous dissipation at low  $Re$  is practically the same as the energy requirement at high  $Re$  also. We expect this to be true in three dimensional case as well.

The viscous dissipation in the flow through an axisymmetric converging channel is later calculated with the Stokes flow pattern. To a first order of magnitude, these results will be applicable even at high Reynolds numbers.

The present model gives correct orders of powers required in practical emulsification. If there are 100 nozzles in parallel, the viscous power required to emulsify 100 gallons of liquid per hour will be  $\sim 200$  HP if  $a \sim 1 \mu$  and  $\sim \frac{1}{4}$  HP if  $a \sim 10 \mu$ . These figures compare very well with the data in industrial practice. The power required for creating new interface amounts to  $6 \times 10^{-4}$  HP if  $a = 1 \mu$  and  $6 \times 10^{-5}$  HP if  $a = 10 \mu$ . Also the viscous power varies as the square of the throughput showing the predominance of these forces over the interfacial forces in industrial machinery. Typical values of the viscous power required to be overcome are shown in fig. (2) together with the industrial requirements. For a machine like a homogeniser, the geometry of which has been used in the present calculations, the agreement is quite good.

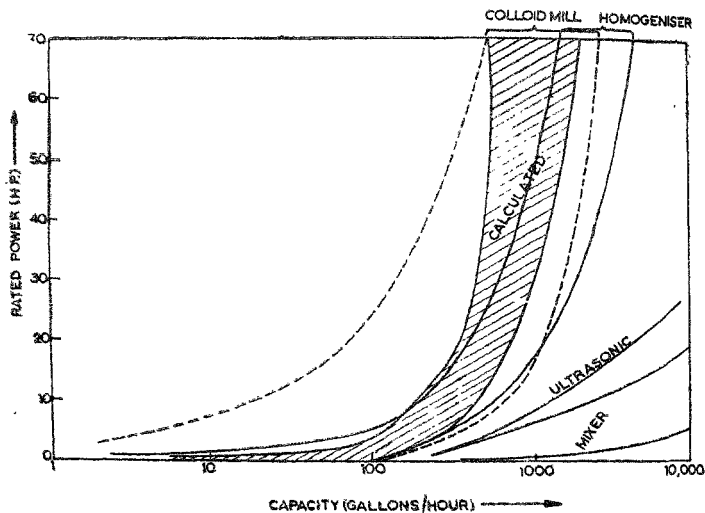


FIG. 2

Power requirement of emulsifying machinery, compared with the calculated values

#### 5. ACKNOWLEDGEMENTS

The author is thankful to Prof. R. S. Krishnan for his encouragement, Dr. E. S. Raja Gopal for many useful discussions and the University Grants Commission for the award of a Junior Research Fellowship.

#### 6. REFERENCES

1. Freundlich, H. . . . . In 'Emulsion Technology', Ed. by H. Bennett, Chem. Pub. Co Inc., Brooklyn, New York, 1946.
2. Sutherland, G. M. . . . . 'Introduction of Emulsions' Chem. Pub. Co., New York, 1946.
3. Gopal, E. S. R. . . . . in 'Emulsion Science', Ed. P. Sherman Academic Press, London, 1968.
4. Griffin, W. C. . . . . 'in Encyclopedia of Chemical Technology' Ed. R. E. Kirk and D. F. Othmer, Interscience, New York, 8, 1965.

5. Subramanyam, S. V. and Raja Gopal, E. S. . . . . *Kolloidzeitschrift*, 1966, **210**, 80.
6. Harrison, W. J. . . . . *Proc. Camb. phil. Soc.*, 1920, **19**, 307.
7. Millsaps, K. and Pöhlhausen, K. . . . . *J. aeronaut Sci.*, 1953, **20**, 187.
8. Rosenhead, L. . . . . in 'Laminar Boundary Layers', Ed. Rosenhead, L. Oxford, Clarendon Press, 1963.
9. Pai, S. L., . . . . . 'Viscous Flow Theory I—Laminar Flow' D. Van Nostrand Co. Inc., New York, 1966.
10. Duncan, W. J., Thom, A. S. and Young, A. D. . . . . 'An Elementary treatise on the mechanics of fluids' Edward Arnold Pub. Ltd., London, 1962.
11. Bond, W. N. . . . . *Phil Mag*, 1925, (6), **50**, 1058,
12. Ackerberg, R. C. . . . . *J. Fluid Mech.*, 1965, **21**, 47.
13. Goldstein, S. . . . . 'Modern Developments in Fluid Mechanics' Vol. 1, 2. Oxford, Clarendon Press, 1938.
4. Ramacharyalu, N. . . . . *Z. angew. Math. Mech.*, 1967, **47**, 9.
15. Subramanyam, S. V. . . . . *Ph. D. Thesis, Indian Institute of Science* 1968.