THE OSCILLATIONS OF A SPHEROID ALONG ITS AXIS IN MICROPOLAR FLUID

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Abstract

The present paper investigates the nature of the flow field when a spheroid is suspended in a infinitely extending micropolar fluid. This is a sequel to our earlier paper on a similar motion in an elastico-viscous fluid and was undertaken with a view to studying the effect of micro rotation on the flow field and to compare the flow behaviour of the above two fluids. The particular case of a sphere is studied in detail.

1 Introduction

The present investigation is a sequel to our earlier paper[1] on the oscillation of a spheroid along its axis in a non-Newtonian fluid. It was undertaken with a view: (i) to compare the stream function for the flow in the case of a micropolar fluid with that in the case of a non-Newtonian fluid, (ii) to study the nature of microrotation, and (iii) to examine the effect of varying Reynolds number on the flow. We have considered a a spheriod to be suspended in an infinitely extending micropolar fluid, whose constitutive equation was given by Eringen[2], and we have studied the flow induced when it performs small amplitude oscillations along its axis. The assumption that the amplitude of oscillation is small is generally the case in any experimental set-up and it introduces much simplification in our calculations. We are now able to express the stream function in terms of Bessel functions and Legendre polynomials, without the use of complicated spheroidal wave functions[3, 4].

2. Formulation of the problem

Let the spheroid be defined by the equation $R = a\ (1 + e \cos 0)$ in a spherical polar coordinate system (R, θ, ϕ) with origin at a focus and the axis of symmetry of the spheroid as the $\theta = 0$ axis. The spheroid oscillates along $\theta = 0$ about its mean position. Since the amplitude of oscillation is assumed to be small, we have taken Stokes' approximation to hold at all points within the fluid. Under this approximation, we can take the dependence of all velocity components and micro-rotation components on time T through a factor e^{inT}, omiting all powers and products of velocity and micro-rotation components.

The field equations of the micropolar fluids are given by the following partial differential equations :

Continuity equation :

$$\partial \rho/\partial t + \nabla \cdot (\rho \underline{v}) = 0, \qquad [2.1]$$

Momentum equation :

$$(\lambda_\nu + 2\mu_\nu + \kappa_\nu) \nabla \nabla \cdot \underline{v} - (\mu_\nu + \kappa_\nu) \nabla \times \nabla \times \underline{v} + \kappa_\nu \nabla \times \underline{\nu} - \nabla p + \rho f$$
$$= \rho [\partial \underline{v}/\partial t - \underline{v} \times (\nabla \times \underline{v}) + \tfrac{1}{2} \nabla \underline{v}^2], \qquad [2.2]$$

First stress moment equation :

$$(\alpha_\nu + \beta_\nu + \gamma_\nu) \nabla \nabla \cdot \underline{\nu} - \gamma_\nu \nabla \times \nabla \times \underline{\nu} + \kappa_\nu \nabla \times \underline{v} - 2\kappa_\nu \underline{\nu} + \rho \underline{l} = \rho j \dot{\underline{\nu}}, \qquad [2.3]$$

where $\underline{v}$ and $\underline{\nu}$ are the velocity vector and the micro-rotation vector respectively, ρ the density of the fluid, λ_ν, μ_ν, κ_ν are coefficients of viscosity and α_ν, β_ν, γ_ν are coefficients of gyro-viscosity ; $\underline{f}$ and $\underline{l}$ give the body force and body couple, respectively ; p is the isotropic pressure and the micro-inertial rotation is given by $\dot{\sigma}_k = j \dot{\nu}_k$, where j is a constant on the assumption of micro-isotropy.

Let u, v, w be the physical components of the velocity vector and ν_r, ν_θ, ν_ϕ those of the micro-rotation vector in the r, θ, ϕ directions, respectively. Then we have

$$\left.\begin{aligned} & u = u(r, \theta)\, e^{inT}, \; v = v(r, \theta)\, e^{inT}, \; w = 0, \\ & \nu_r = 0, \; \nu_\theta = 0, \; \nu_\phi = \nu_\phi(r, \theta)\, e^{inT}. \end{aligned}\right\} \qquad [2.4]$$

We non-dimensionalise the quantities involved by the relations :

$$u = a n \bar{u}, \; v = an \bar{v}, \; \nu_\phi = n \bar{\nu}_\phi, \; p = \rho a^2 n^2 \bar{p}, \; t = n^{-1} \bar{t}, \; r = a\bar{r}, \; n_2 = \mu_\nu a^2/\gamma_\nu,$$
$$n_3 = (\kappa_\nu a^2/\gamma_\nu), \; j = a^2 j_0, \; Re = \rho a^2 n/(\mu_\nu + k_\nu) = \text{Reynolds number} \qquad [2.5]$$

The non-dimensional form of the equations governing the motion are (on dropping bars) :

$$\frac{\partial u}{\partial r} + \frac{2u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{v \cot \theta}{r} = 0, \qquad [2.6]$$

$$\nabla^2 \vec{q} + n_3/(n_2 + n_3) \operatorname{curl} \vec{\nu} - Re \operatorname{grad} p = i\, Re\, \vec{q}, \qquad [2.7]$$

and

$$-\operatorname{curl}\operatorname{curl}\vec{\nu}+n_3\operatorname{curl}\vec{q}=\{2n_3+i\,Re\,j_0\,(n_2+n_3)\}\,\vec{\nu}, \qquad [2.8]$$

where

$$\vec{q}=u\,\hat{i}+v\,\hat{j} \quad \text{and} \quad \vec{\nu}=\nu_\phi\,\hat{k}, \qquad [2.9]$$

$\hat{i}$, $\hat{j}$, $\hat{k}$ being the unit vectors in the direction of r, θ, ϕ increasing respectively.

Eliminating $\vec{\nu}$ between equations [2.7] and [2.8],

we get the following equation to determine $\vec{q}$:

$$\nabla^4\vec{q}-a\,\nabla^2\vec{q}+b\,\vec{q}=c\operatorname{grad}p, \qquad [2.10]$$

where

$$a=i\,Re\,[1+j_0\,(n_2+n_3)]+\frac{n_3\,(2n_2+n_3)}{n_2+n_3},$$

$$b=[2n_3+i\,Re\,j_0\,(n_2+n_3)]\,i\,Re, \quad c=ib. \qquad [2.11]$$

Since a and b are constants, we can write [2.10] in the form

$$(\nabla^2+h^2)\,(\nabla^2+k^2)\,\vec{q}=c\operatorname{grad}p, \qquad [2.12]$$

where the operators are commutative and

$$h^2=-\left(\frac{a-\sqrt{(a^2-4b)}}{2}\right), \quad k^2=-\left(\frac{a+\sqrt{(a^2-4b)}}{2}\right)$$

are in general complex quantities.

We define a vector $\vec{q}_1$ such that

$$\vec{q}=\vec{q}_1+(c/h^2k^2)\operatorname{grad}p, \qquad [2.13]$$

then, provided $\nabla^2p=0$, we have

$$(\nabla^2+h^2)\,(\nabla^2+k^2)\,\vec{q}_1=0, \qquad [2.14]$$

with

$$\operatorname{div}\vec{q}_1=0. \qquad [2.15]$$

Let

$$(\nabla^2+k^2)\,\vec{q}_1=\vec{q}_2 \qquad [2.16]$$

then [2.15] states that

$$(\nabla^2+h^2)\,\vec{q}_2=0. \qquad [2.17]$$

Choose
$$\vec{q}_1 = \vec{q}_3 + 1/(k^2-h^2)\,\vec{q}_2$$

$$(\nabla^2+k^2)\,\vec{q}_1 = (\nabla^2+k^2)\,\vec{q}_3 + (\nabla^2+k^2)\,1/(k^2-h^2)\,\vec{q}_2,$$
$$= (\nabla^2+k^2)\,\vec{q}_3 + (\nabla^2+h^2+k^2-h^2)\,1/(k^2-h^2)\,\vec{q}_2$$
$$= (\nabla^2+k^2)\,\vec{q}_3 + \vec{q}_2,$$

in virtue of [2.17]. [2.16] implies that
$$(\nabla^2+k^2)\,\vec{q}_3 = 0. \tag{2.18}$$

∴ We can write
$$\vec{q} = (c/h^2 k^2)\ \text{grad}\ p + \vec{q}_3 + 1/(k^2-h^2)\,\vec{q}_2 \tag{2.19}$$

where
$$(\nabla^2+h^2)\,\vec{q}_2 = 0 \ \text{ and } \ (\nabla^2+k^2)\,\vec{q}_3 = 0, \tag{2.20}$$

$$\text{div}\,[\vec{q}_3 + 1/(k^2-h^2)\,\vec{q}_2] = 0 \ \text{ and } \ \nabla^2 p = 0. \tag{2.21}$$

The solution of the vector wave equation $(\nabla^2+l^2)\,\vec{q}=0$ is expressible in terms of the solution of the scalar wave equation $(\nabla^2+l^2)\,\psi=0$ as follows:

(i) $\vec{q} =$ grad ψ

(ii) $\vec{q} =$ curl $(\vec{a}\,\psi)$ [2.22]

where $\vec{a}$ is a constant unit vector.

and

(iii) $\vec{q} = (1/l)$ curl curl $(\vec{a}\,\psi)$.

From [2.22], we find that the most suitable form of $\vec{q}_2$ and $\vec{q}_3$ which are divergence-free and most suited to satisfy the boundary conditions are obtained when $\vec{a}$ is chosen as the constant unit vector along the axis of symmetry, *i.e.*
$$\vec{a} = (\cos\theta,\ -\sin\theta,\ 0) \tag{2.23}$$

and expression (iii) is taken. We then have
$$\vec{q}_3 = (1/k)\ \text{curl curl}\ (\vec{a}\,\psi_1),\quad \vec{q}_2 = (1/h)\ \text{curl curl}\ (\vec{a}\,\psi_2) \tag{2.24}$$

where

$$\psi_1 = \sum_{n=0}^{\infty} R_n(r)\, P_n(\cos\theta), \quad \psi_2 = \sum_{n=0}^{\infty} S_n(r)\, P_n(\cos\theta) \qquad [2.25]$$

$P_n(\cos\theta)$ being the Legendre polynomial of order n and

$$R_n(r) = \delta_n (kr)^n f_n(kr), \quad S_n(r) = \beta_n (hr)^n f_n(hr), \qquad [2.26]$$

where δ_n and β_n are constants and

$$f_m(\zeta) = \left(-\frac{1}{\zeta}\frac{d}{d\zeta}\right)^m \frac{\exp(-i\zeta)}{\zeta}. \qquad [2.27]$$

The boundary conditions to be satisfied are

$$u\cos\theta - v\sin\theta = U, \quad u\sin\theta + v\cos\theta = 0,$$
$$v_\phi = 0 \text{ or } r = 1 + e\cos\theta. \qquad [2.28]$$

p satisfies the equation $\nabla^2 p = 0$, and we choose

$$p = \sum_{n=0}^{\infty} \frac{F_n}{r^{n+1}} P_n(\cos\theta). \qquad [2.29]$$

The non-vanishing velocity and micro-rotation components are given by the relations

$$u = \frac{c}{h^2k^2}\frac{\partial p}{\partial r} - \frac{1}{kr\sin\theta}\frac{1}{r}\frac{\partial}{\partial\theta}\left\{r\sin\theta\left(\psi_{1r}\sin\theta + \frac{\psi_{1\theta}\cos\theta}{r}\right)\right\}$$
$$- \frac{1}{h(k^2-h^2)\, r\sin\theta}\frac{1}{r}\frac{\partial}{\partial\theta}\left\{r\sin\theta\left(\psi_{2r}\sin\theta + \frac{\psi_{2\theta}\cos\theta}{r}\right)\right\} \qquad [2.30]$$

$$v = \frac{c}{h^2k^2}\frac{1}{r}\frac{\partial p}{\partial\theta} + \frac{1}{kr\sin\theta}\frac{\partial}{\partial r}\left\{r\sin\theta\left(\psi_{1r}\sin\theta + \frac{\psi_{1\theta}\cos\theta}{r}\right)\right\}$$
$$+ \frac{1}{h(k^2-h^2)\, r\sin\theta}\frac{\partial}{\partial r}\left\{r\sin\theta\left(\psi_{2r}\sin\theta + \frac{\psi_{2\theta}\cos\theta}{r}\right)\right\}, \qquad [2.31]$$

$$v_\phi = \frac{n_2+n_3}{n_3}\frac{1}{[2n_3 + i\,Re\, j_0(n_2+n_3)]}\left[\left(\frac{n_3^2}{n_2+n_3} - i\,Re - k^2\right)(\text{curl}\,\vec{q}_3)\cdot\hat{k}\right.$$
$$\left. + \frac{1}{k^2-h^2}\left(\frac{n_3^2}{n_2+n_3} - i\,Re - h^2\right)(\text{curl}\,\vec{q}_2)\cdot\hat{k}\right] \qquad [2.32]$$

Substituting for p, ψ_1 and ψ_2 from [2.29] and [2.25] in [2.30]–[2.32] and thereafter in [2.28], we get three boundary conditions to be satisfied on $r=1+e\cos\theta$. These relations can be expanded in powers of $e\cos\theta$ ($e << 1$), so that we have

$$K_0(F_j, \delta_i, \beta_k)+K_1(F_j, \delta_i, \beta_k)\cos\theta+K_2(F_j, \delta_i, \beta_k)\cos^2\theta+\cdots=U,$$
$$L_0(F_j, \delta_i, \beta_k)+L_1(F_j, \delta_i, \beta_k)\cos\theta+L_2(F_j, \delta_i, \beta_k)\cos^2\theta+\cdots=0,$$
$$M_0(F_j, \delta_i, \beta_k)+M_1(F_j, \delta_i, \beta_k)\cos\theta+M_2(F_j\, \delta_i, \beta_k)\cos^2\theta+\cdots=0.$$

This gives a triple infinity of equations in the triple infinity of unknowns

F_j, δ_i, β_k; $i, j, k, = 1, 2, 3, \cdots$.

Namely,
$$K_0=U,\ K_1=0,\ K_2=0,\ \cdots,$$
$$L_0=L_1=L_2=\cdots=0,$$
$$M_0=M_1=M_2=\cdots=0. \qquad [2.33]$$

We have not calculated the actual expression for F_j, δ_i, β_k in the general case as they are extremely complicated.

3. Particular Cases

Case (i) : $e=0$, *a sphere*

We choose in this case

$$\psi_1=\delta_0 f_0(kr),\quad \psi_2=\beta_0 f_0(hr) \text{ and } p=F_1\cos\theta/r^2.$$

u, v and v_ϕ are given by

$$u=-\frac{2}{r^3}F_1\cos\theta\,\frac{c}{h^2k^2}-\frac{\delta_0\,2\cos\theta}{kr}\frac{d}{dr}f_0(kr)-\frac{\beta_0}{h(k^2-h^2)}\frac{2\cos\theta}{r}\frac{d}{dr}f_0(hr),$$

$$v=-\frac{1}{r^3}F_1\sin\theta\,\frac{c}{h^2k^2}+\frac{\delta_0\cdot\sin\theta}{k}\left(\frac{d^2}{dr^2}f_0(kr)+\frac{1}{r}\frac{d}{dr}f_0(kr)\right)$$
$$+\frac{\beta_0\sin\theta}{h(k^2-h^2)}\left(\frac{d^2}{dr^2}f_0(hr)+\frac{1}{r}\frac{d}{dr}f_0(hr)\right), \qquad [3.3]$$

and

$$v_\phi=\frac{n_2+n_3}{n_3}\cdot\frac{1}{2n_3+i\,Re\,j_0(n_2+n_3)}\left[\left(\frac{n_3^2}{n_2+n_3}-i\,Re-k^2\right)\frac{\delta_0}{kr^2}\left\{r\frac{d^3}{dr^3}f_0(kr)\right.\right.$$
$$\left.+2\frac{d^2}{dr^2}f_0(kr)-\frac{2}{r}\frac{d}{dr}f_0(kr)\right\}+\frac{1}{k^2-h^2}\left(\frac{n_3^2}{n_2+n_3}-i\,Re-h^2\right)\frac{\beta_0}{hr^2}\cdot$$
$$\left.\times\left\{r\frac{d^3}{dr^3}f_0(hr)+2\frac{d^2}{dr^2}f_0(hr)-\frac{2}{r}\frac{d}{dr}f_0(hr)\right\}\right] \qquad [3.4]$$

The boundary conditions to be satisfied are

$$u\cos\theta - v\sin\theta = U,\quad u\sin\theta + v\cos\theta = 0,\quad v_\phi = 0 \text{ on } r = 1, \tag{3.5}$$

Using recurrence relations

$$f_0''(\zeta) - (1/\zeta)\, f_0'(\zeta) = \zeta^2 f_2(\zeta)$$

and

$$f_0''(\zeta) + (2/\zeta)\, f_0'(\zeta) = -f_0(\zeta)$$

we find that the boundary conditions are satisfied, if

$$\frac{2}{3}\,\delta_0\, k f_0(k) + \frac{2}{3}\,\frac{\beta_0 h}{k^2 - h^2}\, f_0(h) = U,$$

$$\frac{3e F_1}{k^2 h^2} - \delta_0 k^3 f_2(k) - \frac{\beta_0 h^3 f_2(h)}{k^2 - h^2} = 0,$$

$$-\delta_0 k^2 \left(\frac{n_3^2}{n_2 + r_3} - i\,Re - k^2\right) f_1(k) - \frac{\beta_0 h^2}{k^2 - h^2}\left(\frac{n_3^2}{n_2 + r_3} - i\,Re - h^2\right) f_1(h) = 0. \tag{3.6}$$

Solving for δ_0, β_0, F_1, we have

$$\delta_0 = \frac{3}{2}\,\frac{U}{k f_0(k)\, B},\qquad \frac{\beta_0}{k^2 - h^2} = -\frac{3}{2}\,\frac{k}{h^2}\,\frac{UA}{f_0(k)\,B},$$

$$\frac{e F_1}{h^2 k^2} = \frac{1}{2}\,\frac{Uk}{B f_0(k)}\,[k f_2(k) - A h f_2(h)], \tag{3.7}$$

where

$$B = 1 - \frac{k}{h}\, A\, \frac{f_0(h)}{f_0(k)},$$

and

$$A = \frac{[n_3^2/(n_2 + r_3) - i\,Re - k^2)\, f_1(k)]}{[n_3^2/(n_2 + r_3) - i\,Re - h^2)\, f_1(h)]} \tag{3.8}$$

Case 2: *Spheroid with small ellipticity* $0 < e << 1$:

For a spheroid with small ellipticity, we take the zeroth order approximation to p, ψ_1 and ψ_2 to be the same as that in the case of the sphere and further consider terms of order e, neglecting terms of order e^2 and above. We choose

$$p = \frac{F_1 \cos\theta}{r^2} + e\left[\frac{F_0}{r} + F_2 \cdot \frac{3\cos^2\theta - 1}{2r^3}\right]$$

$$\psi_1 = \delta_0 f_0(kr) + e\,\delta_1 f_1(kr)\, kr\cos\theta$$

and $$\psi_2 = \beta_0 f_0 (hr) + e\, \beta_1 f_1 (hr)\, hr \cos\theta.$$

Expression for u, v and ν_ϕ are found using [2.30]–[2.32]. The boundary conditions [2.28] will be satisfied if

$$\frac{2}{3}\,\delta_0 k\, f_0(k) + \frac{2}{3}\,\frac{\beta_0 h}{k^2-h^2}\, f_0(h) = U,$$

$$\frac{3F_1 c}{h^2k^2} - \delta_0 k^3 f_2(k) - \frac{\beta_0}{k^2-h^2}\, h^3 f_2(h) = 0,$$

$$\frac{c}{h^2k^2}\left(-F_0 + \frac{3F_2}{2}\right) + \delta_1 k f_1'(k) + \beta_1 \frac{h f_1'(h)}{k^2-h^2} = 0$$

$$\frac{c}{h^2k^2}\left(9F_1 - \frac{15F_2}{2}\right) + \delta_0 k^3 [2f_2(k) + k f_2'(k)] + \delta_1 k [k f_1''(k) - f_1'(k)]$$
$$+ \frac{\beta_0 h^3}{k^2-h^2}[2f_2(h) + h f_2'(h)] + \frac{\beta_1 h}{k^2-h^2}[h f_1''(h) - f_1'(h)] = 0,$$

$$\frac{c}{h^2k^2}\left(-3F_1 - F_0 + \frac{9}{2}F_2\right) - \frac{\delta_0 k^3}{3}[2f_2(k) + k f_2'(k)] + \frac{2}{3}\,\delta_0 k^2 f_0'(k)$$
$$-\delta_1 k [k f_1''(k) + f_1'(k)] - \frac{\beta_0 h^3}{3}[2f_2(h) + h f_2'(h)] + \frac{2}{3}\,\frac{h^2\beta_0}{k^2-h^2} f_0'(h)$$
$$-\frac{\beta_1 h}{k^2-h^2}[h f_1''(h) + f_1'(h)] = 0,$$

$$\left(\frac{n_3^2}{n_2+n_3} - i\,Re - k^2\right)\delta_0 k^2 f_1(k) + \frac{1}{k^2-h^2}\left(\frac{n_3^2}{n_2+n_3} - i\,Re - h^2\right)\beta_0 h^2 f_1(h) = 0,$$

$$\left(\frac{n_3^2}{n_2+n_3} - i\,Re - k^2\right)[\delta_0 k^3 f_1'(k) + \delta_1 k^3 f_1'''(k) + 4k^2 f_1''(k) - 4k f_1'(k)]$$
$$+ \frac{1}{k^2-h^2}\left(\frac{n_3^2}{n_2+n_3} - i\,Re - h^2\right)[\beta_0 h^3 f_1'(h) + \beta_1 h^3 f_1'''(h) + 4h^2 f_1''(h)$$
$$-4h f_1'(h)] = 0$$

These seven equations have to be solved for the seven unknowns δ_0, F_1, β_0; δ_1, β_1, F_0, F_2. The expressions for δ_0, F_1, β_0 are the same as in the case of the sphere, while δ_1, β_1, F_0, F_2 specify the change in the velocity field and micro-rotation due to the ellipticity of the spheroid.

4 DISCUSSION OF THE RESULTS

We have studied in detail the stream function and micro-rotation in case (i) and have compared the results with those obtained in referesence 1. We can define a stream function Ψ for the motion by the relations

$$u = -\frac{1}{r^2 \sin\theta}\frac{\partial \Psi}{\partial \theta}, \quad v = \frac{1}{r \sin\theta}\frac{\partial \Psi}{\partial r} \tag{4.1}$$

Then

$$\Psi = \frac{U \sin^2\theta}{2 r B}\left[\left(-1 + \frac{3i}{k} + \frac{3}{k^2}\right) - 3 r^2 \left(\frac{i}{kr} + \frac{1}{k^2 r^2}\right)\exp[-ik(r-1)]\right]$$

$$- \frac{U \sin^2\theta}{2 r B} A \frac{k^2}{h^2}\exp[-i(h-k)]\left[\left(-1 + \frac{3i}{h} + \frac{3}{h^2}\right)\right.$$

$$\left. - 3r^2\left(\frac{i}{hr} + \frac{1}{h^2 r^2}\right)\exp[-ih(r-1)]\right].$$

Choosing h and k of the form

$$h = h_1 - i h_2, \quad k = k_1 - i k_2$$

where $h_1, h_2, k_1, k_2 > 0$ and U of the form

$U = \alpha \exp(i t)$ where α is real

$$\Psi = -\frac{\alpha \sin^2\theta}{2}\left[\left\langle\left(1 + \frac{3k_2}{k_1^2 + k_2^2} - \frac{3(k_1^2 - k_2^2)}{(k_1^2 + k_2^2)^2}\right)\frac{B_1}{r} + \left(\frac{3k_1}{k_1^2 + k_2^2} + \frac{6 k_1 k_2}{(k_1^2 + k_2^2)^2}\right)\frac{B_2}{r}\right.\right.$$

$$- 3[\exp -k_2(r-1)]\left\{\left(-\frac{k_2}{k_1^2 + k_2^2} + \frac{k_1^2 - k_2^2}{r(k_1^2 + k_2^2)^2}\right) B_1 \cos k_1 (r-1)\right.$$

$$-\left(\frac{k_1}{k_1^2 + k_2^2} + \frac{2 k_1 k_2}{(k_1^2 + k_2^2)^2 r}\right) B_2 \cos k_1 (r-1) + \left(-\frac{k_2}{k_1^2 + k_2^2} + \frac{k_1^2 - k_2^2}{r(k_1^2 + k_2^2)^2}\right) \times$$

$$\left.\left. \times B_2 \sin k_1 (r-1) + \left(\frac{k_1}{k_1^2 + k_2^2} + \frac{2 k_1 k_2}{r(k_1^2 + k_2^2)^2}\right) B_1 \sin k_1 (r-1)\right\}\right\rangle \cos t$$

$$+ \left\langle\left(\frac{3k_1}{k_1^2 + k_2^2} + \frac{6 k_1 k_2}{(k_1^2 + k_2^2)^2}\right)\frac{B_1}{r} - \left(1 + \frac{3k_2}{k_1^2 + k_2^2} - \frac{3(k_1^2 - k_2^2)}{(k_1^2 + k_2^2)^2}\right)\frac{B_2}{r}\right.$$

$$+ 3[\exp -k_2(r-1)]\left\{\left(-\frac{k_2}{k_1^2 + k_2^2} + \frac{k_1^2 - k_2^2}{r(k_1^2 + k_2^2)^2}\right) B_1 \sin k_1 (r-1)\right.$$

$$-\left(\frac{k_1}{k_1^2+k_2^2}+\frac{2\,k_1k_2}{(k_1^2+k_2^2)^2 r}\right) B_2 \sin k_1\,(r-1)-\left(-\frac{k_2}{k_1^2+k_2^2}+\frac{k_1^2-k_2^2}{r\,(k_1^2+k_2^2)^2}\right)\cdot$$

$$\times B_2 \cos k_1\,(r-1)-\left(\frac{k_1}{k_1^2+k_2^2}+\frac{2\,k_1\,k_2}{r\,(k_1^2+k_2^2)^2}\right) B_1 \cos k_1\,(r-1)\Big\rangle \sin t\Big]-$$

a similar expression where k_1, k_2, B_1, B_2 are replaced by h_1, h_2, C_1, C_2 respectively,

$$1/B=B_1+i\,B_2$$

and

$$(A/B)\,(k^2/h^2)\,\exp[-i\,(h-k)]=C_1+i\,C_2.$$

The expression for v_ϕ is

$$v_\phi=-\frac{n_2+n_3}{n_3}\cdot\frac{[n_3^2/(n_2+n_3)-i\,Re-k^2)]}{2n_3+i\,Re\,j_0\,(n_2+n_3)}\cdot\frac{3}{2}\,\frac{U}{r}\cdot\frac{k\,f_1\,(k)}{f_0\,(k)\,B}\left[\frac{f_1\,(kr)}{f_1\,(k)}-\frac{f_1\,(hr)}{f_1\,(h)}\right]$$

$$=-\frac{1}{r^4}\Big\langle\frac{\exp[-k_2\,(r-1)]}{(1+k_2)^2+k_1^2}\Big\{\Big\langle K_1\,[(1+k_2\,r)\,(1+k_2)+k_1^2\,r]$$

$$-K_2\,k_1\,(r-1)\Big\rangle\cos k_1\,(r-1)+\Big\langle K_2\,[(1+k_2\,r)\,(1+k_2)+k_1^2\,r]$$

$$+K_1\,k_1\,(r-1)\Big\rangle\sin k_1\,(r-1)\Big\}\Big\rangle\cos t$$

$$+\frac{1}{r^4}\Big\langle\frac{\exp[-k_2\,(r-1)]}{(1+k_2)^2+k_1^2}\Big\{\Big\langle K_2\,[(1+k_2\,r)\,(1+k_2)+k_1^2\,r]$$

$$+K_1\,k_1\,(r-1)\Big\rangle\cos k_1\,(r-1)-\Big\langle K_1\,[(1+k_2\,r)\,(1+k_2)+k_1^2\,r]$$

$$-K_2\,k_1\,(r-1)\Big\rangle\sin k_1\,(r-1)\Big\}\Big\rangle\sin t$$

$-$ a similar expression where k_1, k_2 are replaced by h_1, h_2,

and

$$K_1+i\,K_2=\frac{n_2+n_3}{n_3}\cdot\frac{(n_3^2/n_2+n_3)-i\,Re-k^2}{2\,n_3+i\,Re\,j_0\,(n_2+n_3)}\cdot\frac{3}{2}\,\alpha\,\frac{k\,f_1\,(k)}{B\,f_0\,(k)},$$

Numerical results have been computed for Reynolds numbers $Re=0.5$, $Re=8$ and $Re=50$, for a micropolar fluid with $n_2=n_3=1$. The results in the case of a Newtonian fluid are also given for the above values of the Reynolds number for comparison.

We note the following points:

(1) For large values of the Reynolds number, the magnitude of velocity components in the case of a micropolar fluids are almost identical with those in the case of a Newtonian fluid. As we move away from the sphere l_1^2 and l_2^2 for a micropolar fluid are slightly less than that for a Newtonian fluid, but the decrease is not as marked as in the case of Oldroyd or Rivlin-Ericksen fluids.

(2) The magnitude of the micro-rotation is of the order of 10^{-23} and 10^{-47} at $r=10$ for Reynolds number 8 and 50 respectively. This shows that when the Reynolds number is large, the micro-rotation is negligibly small at some distance away from the sphere. Even close to the sphere, namely $r=2.5$, the maguitu'e of the micro-rotation is of the order of 10^{-6} and 10^{-14} respectively in the above two cases. In effect, a micropolar fluid is almost indistinguishable from a Newtonian fluid at large Reynolds numbers. When the characteristic velocity of the fluid is large, the microrotations are suppressed and their effect is hardly perceptible.

(3) On the other hand, for small values of the Reynolds number, we notice that the magnitude of the u and v velocities in the case of a micropolar fluid differ markedly from those of a Newtonian fluid. Close to the sphere this deviation is prominent, whereas as we move away, the flow becomes more and more Newtonian. We observe that the presence of micro-rotations causes the u and v velocities to have a larger magnitude than in the Newtonian case. This is to be compared with that for a general non-Newtonian fluid, where the non-Newtonian normal stresses cause a decrease in the magnitude. The non-Newtonian stresses act in opposition to the forces causing the motion and tend to damp out the disturbance, whereas the micro-rotations act in conjunction, helping the disturbance to grow close to the sphere.

In the case of a spheroid with small ellipticity, the basic nature of flow is the same as in the case of a sphere, suitably modified by terms of order e. A detail study has been made in reference 1 for a non-Newtonian fluid.

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$u = \alpha \cos\theta\, l_1(r) \cos[t - \epsilon_1(r)]$, $v = -\alpha \sin\theta\, l_2(r) \cos[t - \epsilon_2(r)]$, $\nu_\theta = l_3(r) \cos[t - \epsilon_3(r)]$, $n_2 = n_3 = 1$

			l_1^2	$\tan \epsilon_1$	l_2^2	$\tan \epsilon_2$	l_3^2	$\tan \epsilon_3$
	Micropolar	2.5	0.171675	0.470798	0.237843×10^{-1}	37.373672	0.689634×10^{-3}	−5.347334
		5.0	$.874372 \times 10^{-2}$	1.723066	$.294976 \times 10^{-2}$	−.188333	$.984387 \times 10^{-6}$	.671224
		10.0	$.101553 \times 10^{-3}$	3.983673	$.356075 \times 10^{-4}$	9.067033	$.423822 \times 10^{-10}$	1.521945
Re = 0.5	Newtonian	2.5	.141639	.278212	$.179040 \times 10^{-1}$	5.673047		
		5.0	$.798036 \times 10^{-2}$	1.122190	$.218542 \times 10^{-2}$	−.076605	...	...
		10.0	$.986353 \times 10^{-4}$	2.346893	$.316925 \times 10^{-4}$	2.538100		
	Micropolar	2.5	$.199185 \times 10^{-1}$	.665878	$.890220 \times 10^{-2}$	.424547	$.746011 \times 10^{-6}$	−1.041784
		5.0	$.275448 \times 10^{-3}$	.649121	$.685195 \times 10^{-4}$	.639597	$.758509 \times 10^{-12}$	−2.016067
		10.0	$.430215 \times 10^{-5}$	.649784	$.107548 \times 10^{-5}$	.649784	$.234463 \times 10^{-23}$	59.042895
Re = 8	Newtonian	2.5	$.202673 \times 10^{-1}$	.658008	$.914632 \times 10^{-2}$	.392105		
		5.0	$.277197 \times 10^{-3}$	.641691	$.686203 \times 10^{-4}$	.626188	...	...
		10.0	$.432810 \times 10^{-5}$	.642857	$.108250 \times 10^{-5}$	.642857		
	Micropolar	2.5	$.745032 \times 10^{-2}$	.276823	$.184312 \times 10^{-2}$	.270922	$.523653 \times 10^{-14}$	1.193328
		5.0	$.116393 \times 10^{-3}$	.277266	$.290986 \times 10^{-4}$	.277269	$.139350 \times 10^{-28}$	1.484383
		10.0	$.181870 \times 10^{-5}$	.277265	$.454630 \times 10^{-6}$	.277265	$.171878 \times 10^{-47}$	.479206
Re = 50	Newtonian	2.5	$.744860 \times 10^{-2}$	.276570	$.184680 \times 10^{-2}$	.269362		
		5.0	$.116454 \times 10^{-3}$	.276923	$.291100 \times 10^{-4}$	.276923	...	...
		10.0	$.181960 \times 10^{-5}$	.276923	$.455000 \times 10^{-6}$	.276923		