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SURFACE WAVES ON A DIELECTRIC DISC BACKED BY A METALLIC DISC

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ABSTRACT

The characteristic equation for surface waves (E_0) *on a metal-disc backed dielectric disc is formulated and solved for radial and transverse propagation constants, attenuation constants and percentage reduction in phase velocity. Expressions for the power flow in the radial and transverse directions, and the division of power between the inside and outside the dielectric disc have also been derived. By using the perturbation technique, attenuation constant for* E_0 *wave in the dielectric disc has been calculated.*

1. INTRODUCTION

The present study is a continuation of the investigations[1-26] on surface wave phenomena and radiation from dielectric objects that are being conducted in the Indian Institute of Science for the last two decades. The present study is concerned with the study of the characteristics of surface waves (E_0 mode) excited by means of a suitable horn placed above a dielectric disc backed by a metallic disc. The launching cone is fed by a coaxial guide passing through the centre of the disc. This wave is essentially analogous to the radial form of 'Zenneck Wave'. The propagation of electromagnetic waves along interfaces between different media was a controversial subject which led to lengthy philosophical discussions about the existence and physical realisability of surface waves. Schelkunoff[27] in his discussion of the anatomy of surface waves has mentioned that Dr. James R. Wait has prepared a list of eleven wave types which have been mentioned by different authors as surface waves. Plane waves guided by a plane interface between an insulator and a good conductor were first studied by Uller[28]. Zenneck[29] recognised the importance of these studies on the propagation of electromagnetic waves along the earth. Zenneck's investigation concerned with the case where one-half space is a pure dielectric backed by a dielectric which is more or less conductive. An illuminating discussion of 'surface waves' is given in Barlow's[30] book. The present study has been motivated by the necessity of understanding certain phenomena in connection with the study of the dielectric disc as an aerial. The radiation characteristics of such an aerial is under investigation and the results will be reported elsewhere.

2. FIELD COMPONENTS

The field components for the different media for the metal-disc backed dielectric disc (σ_1, ϵ_1, μ_0) immersed in air (σ_0, ϵ_0, μ_0) and excited in E_0 surface wave mode are (Fig. 1)

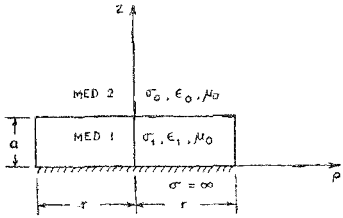

FIG. 1
Geometry of the Problem

Medium 1 : $0 < z \leq a, \ \rho \leq r$

$$E_{z1} = A\,[(j\,\gamma/(\sigma_1 + j\,\omega\epsilon_1)]\,H_0^{(2)}(-j\,\gamma\,\rho)\cos u_1 z$$
$$E_{\rho 1} - = A\,[(u_1/(\sigma_1 + j\,\omega\epsilon_1)]\,H_1^{(2)}(-j\,\gamma\,\rho)\sin u_1 z$$
$$H_{\phi 1} = A\,H_1^{(2)}(-j\,\gamma\,\rho)\cos u_1 z \qquad [1]$$

Medium 2 : $a \leq z, \ \rho > r$

$$E_{z2} = A\,(\gamma/\omega\epsilon_0)\exp(-u_2 z)\,H_0^{(2)}(-j\,\gamma\,\rho)$$
$$E_{\rho 2} = A\,(u_2/j\,\omega\,\epsilon_0)\exp(-u_2 z)\,H_1^{(2)}(-j\,\gamma\,\rho)$$
$$H_{\phi 2} = A\exp(-u_2 z)\,H_1^{(2)}(-j\,\gamma\,\rho) \qquad [2]$$

where, the time variation of the field quantities is assumed to be exp $(j\,\omega\,t)$ and

$$
\begin{aligned}
& u_1 = a_1 + j\,b_1 \\
& u_2 = a_2 - j\,b_2 \\
& \gamma = \alpha + j\,\beta \\
& u_2^2 = -\gamma^2 - k_2^2 \\
& k_2^2 = \omega^2\,\mu_0\,\epsilon_0 \\
& k_1^2 = -j\,\omega\,\mu_1\,(\sigma_1 + j\,\omega\,\epsilon_1) = -(\gamma^2 + u_1^2) = \omega^2\,\mu_1\,\epsilon_1 \\
& u_2^2 - u_1^2 = \omega^2\,\mu_0\,\epsilon_0\,(\epsilon_r - 1) \\
& \epsilon_r = \epsilon_1/\epsilon_0
\end{aligned}
\qquad [3]
$$

3. Boundary Conditions

The radial (ρ) component of the electric field and the azimuthal component (ϕ) of the magnetic field are continuous at $z=a$. Therefore, matching the impedances at the air-dielectric interface $(z=a)$, we obtain,

$$E_{\rho 1}/H_{\phi 1} = E_{\rho 2}/H_{\phi 2} \qquad [4]$$

4. Characteristic Equation

Using the appropriate field components [eqn. 1 and 2] and the impedance relation [eqn. 4], the following characteristic equation is obtained.

$$\frac{A\,[u_1/(\sigma_1 + j\,\omega\,\epsilon_1)]\;H_1^{(2)}(-j\,\gamma\,\rho)\sin u_1\,a}{A\,H_1^{(2)}(-j\,\gamma\,\rho)\cos u_1\,a}$$

$$= \frac{A\,[u_2/(j\,\omega\,\epsilon_0)]\,\exp(-u_2\,a)\,H_1^{(2)}(-j\,\gamma\,\rho)}{A\exp(-u_2\,a)\;H_1^{(2)}(-j\,\gamma\,\rho)}$$

which yields
$$-u_2 = (\epsilon_r)^{-1}\,u_1\tan(u_1\,a) \qquad [5]$$

The above equation [5] can be solved by plotting $(\epsilon_r)^{+1}\,u_1\tan u_1 a$ vs. u_1 as $f(a, \epsilon_r)$ and $[u_1^2 + (\omega^2/c^2)(\epsilon_r - 1)]^{1/2}$ vs. u_1 as $f(a, \epsilon_r)$. The values of u_1 satisfying the above equation for different values of a and ϵ_r are obtained from the intersections of the two sets of curves. The values of γ and u_2 can be found from the corresponding values of u_1. Assuming α to be very small β for different values of a and ϵ_r can be determined from the corresponding values of γ. The phase velocity $v_p\,(=\omega/\beta)$ of the surface waves and hence the percentage reduction in phase velocity $(c - v_p)/c$ % for different values of a and ϵ_r can be determined.

5. SOLUTION FOR a_1, b_1, a_2 and b_2

From the relations [eqn. 3] we obtain

$$u_2^2 - u_1^2 = (\omega^2/c^2)(\epsilon_r - 1)$$
$$a_2 b_2 = \alpha\beta$$
$$a_1 b_1 = -\alpha\beta \qquad [6]$$

which yield the following quartic equation in a_1

$$a_1^4 + a_1^2[(\omega^2/c^2)\,\epsilon_r + \alpha^2 - \beta^2] = \alpha^2\beta^2 \qquad [7]$$

Similarly,

$$a_2^4 - \alpha^2\beta^2 - a_2^2[(\omega^2/c^2)(\epsilon_r - 1) + a_1^2 - b_1^2] = 0 \qquad [8]$$

The solutions of eqn. [7] and eqn. [8] are respectively

$$a_1 = \left[\frac{-[(\omega^2/c^2)\,\epsilon_r + \alpha^2 - \beta^2] \pm \sqrt{\{[(\omega^2/c^2)\,\epsilon_r + \alpha^2 - \beta^2]^2 + 4\alpha^2\beta^2\}}}{2}\right]^{1/2} \qquad [9]$$

$$a_2 = \left[\frac{\{(\omega^2/c^2)(\epsilon_r - 1) + a_1^2 - b_1^2\} \pm \sqrt{\{[(\omega^2/c^2)(\epsilon_r - 1) + (a_1^2 - b_1^2)]^2 + 4\alpha^2\beta^2\}}}{2}\right]^{1/2} \qquad [10]$$

The values of b_1 and b_2 are determined by using the relations eqn. [6] appropriately in eqn. [9] and eqn. [10] respectively.

$$b_1 = -\alpha\beta \Bigg/ \left[\frac{-[(\omega^2/c^2)\epsilon_r + \alpha^2 - \beta^2] \pm \sqrt{\{[(\omega^2/c^2)\epsilon_r + \alpha^2 - \beta^2]^2 + 4\alpha^2\beta^2\}}}{2}\right]^{1/2} \qquad [11]$$

$$b_2 = \alpha\beta \Bigg/ \left[\frac{\{(\omega^2/c^2)(\epsilon_r - 1) + a_1^2 - b_1^2\} \pm \sqrt{\{[(\omega^2/c^2)(\epsilon_r - 1) + (a_1^2 - b^2)]^2 + 4\alpha^3\beta^2\}}}{2}\right]^{1/2} \qquad [12]$$

6. SOLUTION FOR α AND β

By adopting the same procedure as above, the following quartic equation for α is obtained.

$$\alpha^4 + \alpha^2(a_2^2 - b_2^2 + \omega^2\mu_0\epsilon_0) - a_2^2 b_2^2 = 0 \qquad [13]$$

The solution of equation [13] is

$$\alpha = \left[\frac{-(a_2^2 - b_2^2 + \omega^2\mu_0\epsilon_0) \pm \sqrt{[(a_2^2 - b_2^2 + \omega^2\mu_0\epsilon_0)^2 + 4a_2^2 b_2^2]}}{2}\right]^{1/2} \qquad [14]$$

The values for β are determined from the relation $\beta = a_2 b_2/\alpha$ and eqn. (14]

7. Phase Velocity

An accurate value of the phase velocity v_p is determined from the value of β obtained as above without placing restrictions on the value of α and is given by the relation

$$v_p = \frac{\omega}{a_2 b_2}\left[\frac{-(a_2^2 - b_2^2 + \omega^2 \mu_0 \epsilon_0) \pm \sqrt{[(a_2^2 - b_2^2 + \omega^2 \mu_0 \epsilon_0)^2 + 4 a_2^2 b_2^2]}}{2}\right]^{1/2} \qquad [15]$$

8. Power Flow

The total power flow in the radial (P_ρ) and transverse (P_z) directions consist of power flowing inside the disc in the ρ and z directions and power flowing outside the disc and is given by the following relations

$$P_\rho = \tfrac{1}{2} Re \int_{\phi=0}^{2\pi} \int_{z=a}^{\infty} E_{z2} H^*_{\phi 2}\, r\, d\phi\, dz + \tfrac{1}{2} Re \int_{\phi=0}^{2\pi} \int_{z=0}^{a} E_{z1} H^*_{\phi 1}\, r\, d\phi\, dz \qquad [16]$$

$$P_z = \tfrac{1}{2} Re \int_{\phi=0}^{2\pi} \int_{\rho=r_1}^{r} E_{\rho 2} H^*_{\phi 2}\, \rho\, d\phi\, d\rho + \tfrac{1}{2} \int_{\phi=0}^{2\pi} \int_{\rho=r}^{\infty} E_{\rho 1} H^*_{\phi 1}\, \rho\, d\phi\, d\rho \qquad [17]$$

where the asterisk represents the complex conjugate and the field components are maximum values in time and r is the radius of the disc, r_1 is the outer radius of the coaxial guide passing through the centre of the disc. This coaxial guide forms a part of the launching device which will be described in a later paper. The thickness of the dielectric disc is represented by a. Substituting proper field components from equations [1] and [2] in equations [16] and [17] and integrating, we obtain

$$P_\rho = \frac{\pi A^2 r}{\omega \epsilon_0} Re\,[\gamma H_0^{(2)}(-j\gamma\rho) H_1^{(1)}(j\gamma^*\rho) \int_{z=a}^{\infty} dz]$$

$$+ \frac{\pi A^2 \gamma}{\omega \epsilon_1} Re\,[\gamma H_0^{(2)}(-j\gamma\rho) H_1^{(1)}(j\gamma^*\rho) \int_{z=0}^{a} \cos^2 u_1 z dz]$$

$$= \frac{\pi A^2 r}{\omega \epsilon_0} Re\,[\gamma H_0^{(2)}(-j\gamma\rho) H_1^{(1)}(j\gamma^*\rho)(d-a)]$$

$$+ \frac{\pi A^2 r}{\omega \epsilon_1} Re\left[\gamma H_0^{(2)}(-j\gamma\rho) H_1^{(1)}(j\gamma^*\rho)\left(\frac{a}{2} + \frac{\sin 2 u_1 a}{4 u_1}\right)\right] \qquad [18]$$

where,

$$[H_1^{(2)}(-j\gamma\rho)]^* = H_1^{(1)}(j\gamma^*\rho) \qquad (\textit{see}\ \text{Appendix A.I})$$

and the integral $\int\limits_{z=a}^{\infty}$ has been replaced by the integral $\int\limits_{z=a}^{d}$ where, d represents the distance in the z direction within which most of the power is located. This relation can be utilised to determine the constant percentage power contour. The computation is under progress and will be reported elsewhere.

$$P_z=\frac{\pi A^2}{\omega \epsilon_0} Re \int\limits_{\rho=r_1}^{r} \frac{u_2}{j} H_1^{(2)}(-j\gamma\rho)\, H_1^{(1)}(j\gamma^*\rho)\,\rho\, d\rho$$

$$-\frac{\pi A^2}{\omega \epsilon_1} Re \left[\frac{u_1 \sin 2u_1 z}{2j} \int\limits_{\rho=r}^{\infty} H_1^{(2)}(-j\gamma\rho)\, H_1^{(1)}(j\gamma^*\rho)\,\rho\, d\rho\right] \qquad [19]$$

$$=\frac{\pi A^2}{\omega \epsilon_0} Re \left[\frac{u_2}{j}\frac{r}{\gamma^2-\gamma^{*2}}\{j\gamma^* H_0^{(1)}(j\gamma^* r)\, H_1^{(2)}(-j\gamma r)\right.$$

$$+j\gamma H_0^{(2)}(-j\gamma r)\, H_1^{(1)}(j\gamma^* r)\}$$

$$-\frac{u_2}{j}\frac{r_1}{\gamma^2-\gamma^{*2}}\{j\gamma^* H_0^{(1)}(j\gamma^* r_1)\, H_1^{(2)}(-j\gamma r_1)$$

$$\left.+j\gamma H_0^{(2)}(-j\gamma r_1)\, H_1^{(1)}(j\gamma^* r_1)\}\right]$$

$$-\frac{\pi A^2}{\omega \epsilon_1} Re \left[-\frac{u_1 \sin 2u_1 z}{2j}\frac{r}{\gamma^2-\gamma^{*2}}\left\{j\gamma^* H_0^{(1)}(j\gamma^* r)\, H_1^{(2)}(-j\gamma r)\right.\right.$$

$$\left.\left.+j\gamma H_0^{(2)}(-j\gamma r)\, H_1^{(1)}(j\gamma^* r)\right\}\right] \qquad [20]$$

since at $\rho=\infty$, all the Hankel functions vanish, the integrals in eqn. [19] have been evaluated by using the following relation

$$\int^{z} c_\nu(kz)\, \bar{c}_\nu(lz)\, z dz=[z/(k^2-l^2)]\, \{l\,\bar{c}_{\nu-1}(lz)\, c_\nu(kz)-k\, c_{\nu-1}(kz)\, \bar{c}_\nu(lz)\},\quad k\neq l$$

where, $z=l$ and $k=-j\gamma$ and $l=j\gamma^*$

9. EVALUATION OF P_ρ AND P_z

Making small argument approximations (see Appendix A.2) eqn. [18] and [20] reduce respectively to

$$P_\rho=\frac{4\pi A^2 r}{\omega \epsilon_0}\left[-(d-a)\left\{\frac{\alpha(\alpha q+p\beta)}{\alpha^2+\beta^2}+\frac{\beta(\alpha p-q\beta)}{\alpha^2+\beta^2}\right\}\right]$$

$$+\frac{4\pi A^2 r}{\omega\epsilon_1}\left[-\frac{a}{2}\left\{\frac{\alpha(\alpha q+p\beta)}{\alpha^2+\beta^2}+\frac{\beta(\alpha p-q\beta)}{\alpha^2+\beta^2}\right\}\right]$$

$$+\frac{4\pi A^2 r}{\omega\epsilon_1}\left[-\left(\left\{\frac{\alpha(\alpha p-q\beta)}{\alpha^2+\beta^2}-\frac{\beta(\alpha q+p\beta)}{\alpha^2+\beta^2}\right\}\times\right.\right.$$

$$\left\{\frac{(a_1\cos 2aa_1\sinh 2ab_1-b_1\sin 2aa_1\cosh 2ab_1)}{4(a_1^2+b_1^2)}\right\}+\left\{\frac{\alpha(\alpha q+p\beta)}{\alpha^2+\beta^2}\right.$$

$$\left.\left.\left.+\frac{\beta(\alpha p-q\beta)}{\alpha^2+\beta^2}\right\}\times\left\{\frac{a_1\sin 2aa_1\cosh 2ab_1+b_1\cos 2aa_1\sinh 2ab_1}{4(a_1^2+b_1^2)}\right\}\right)\right] \quad [21]$$

and

$$P_z=\frac{\pi A^2}{\omega\epsilon_0}\left[\frac{2b_2}{\pi^2}\left(\frac{2p\,\alpha\beta+q(\alpha^2-\beta^2)}{\alpha\beta(\alpha^2+\beta^2)}-\frac{2p'\alpha\beta+q(\alpha^2-\beta^2)}{\alpha\beta(\alpha^2+\beta^2)}\right)\right]$$

$$-\frac{\pi A^2}{\omega\epsilon_1}\left[-\left(\frac{b_1}{\pi^2}\;\frac{\sin 2a_1 z_1\cosh 2b_1 z}{\alpha\beta}\;\frac{2p\,\alpha\beta+q(\alpha^2-\beta^2)}{\alpha^2+\beta^2}\right.\right.$$

$$\left.\left.+\frac{a_1}{\pi^2}\;\frac{\cos 2a_1 z\sinh 2b_1 z}{\alpha\beta}\;\frac{2p\,\alpha\beta+q(\alpha^2-\beta^2)}{\alpha^2+\beta^2}\right)\right] \quad [22]$$

where

$$p=\tfrac{1}{2}\ln(0.89\,r)^2(\alpha^2+\beta^2)$$

$$q=arc\tan\beta/\alpha$$

$$p'=\tfrac{1}{2}\ln(0.89\,r_1)^2(\alpha^2+\beta^2)$$

But expressions for P_ρ and P_z (eqns. 18 and 20) reduce to the following by making large argument approximations (see Appendix A.2)

$$P_\rho=\frac{\pi A^2 r}{\omega\epsilon_0}\;\frac{2}{\pi\rho}\;(d-a)\;\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\exp(-2\alpha\rho)$$

$$+\frac{\pi A^2 r}{\omega\epsilon_1}\;\frac{2}{\pi\rho}\exp(-2\alpha\rho)\left[\frac{a}{2}\;\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\right.$$

$$+\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\;\frac{a_1\sin 2aa_1\cosh 2ab_1+b_1\cos 2aa_1\sinh 2ab_1}{4(a_1^2+b_1^2)}$$

$$\left.+\frac{\alpha}{\sqrt{(\alpha^2+\beta^2)}}\;\frac{a_1\cos 2aa_1\sinh 2ab_1-b_1\sin 2aa_1\cosh 2ab_1}{4(a_1^2+b_1^2)}\right] \quad [23]$$

$$P_z=\frac{\pi A^2}{\omega\epsilon_0}\Bigg[-\frac{2}{\pi r}\;\frac{2b_2 r}{4\alpha\beta}\;\frac{\beta}{\sqrt{\alpha^2+\beta^2}}\exp(-2\alpha r)$$

$$+\frac{2}{\pi r_1}\;\frac{2b_2 r_1}{4\alpha\beta}\;\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\exp(-2\alpha r_1)\Bigg]$$

$$-\frac{\pi A^2}{\omega\epsilon_1}\;\frac{r}{4\alpha\beta}\;\frac{2}{\pi r}\exp(-2\alpha r)\Bigg[\Bigg(\frac{b_1\cos 2a_1 z\sinh 2b_1 z}{2}$$

$$-\frac{a_1\sin 2a_1 z\cosh 2b_1 z}{2}\Bigg)\frac{\alpha}{\sqrt{(\alpha^2+\beta^2)}}-\Bigg(\frac{a_1\cos 2a_1 z\sinh 2b_1 z}{2}$$

$$+\frac{b_1\sin 2a_1 z\cosh 2b_1 z}{2}\Bigg)\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\Bigg] \quad [24]$$

10. DIVISION OF POWER

The power flow inside (P_{in}) and outside (P_{out}) the dielectric disc takes place both in the ρ direction as well as in the z direction which can be symbolised as

$$P_{in}=P_{in}^{\rho}+P_{in}^{z} \quad [25]$$

$$P_{out}=P_{out}^{\rho}+P_{out}^{z} \quad [26]$$

which for small argument approximations reduce to

$$P_{in}=\frac{\pi A^2 r}{\omega\epsilon_1}\;\frac{a}{2}\Bigg[-4\Bigg\{\frac{\alpha(\alpha q+p\beta)}{\alpha^2+\beta^2}+\frac{\beta(\alpha p-q\beta)}{\alpha^2+\beta^2}\Bigg\}\Bigg]+\frac{\pi A^2 r}{\omega\epsilon_1}\Bigg[-4\Bigg(\Bigg\{\frac{\alpha(\alpha p-q\beta)}{\alpha^2+\beta^2}$$

$$-\frac{\beta(\alpha q+p\beta)}{\alpha^2+\beta^2}\Bigg\}\Bigg\{\frac{a_1\cos 2a a_1\sinh 2a b_1-b_1\sin 2a a_1\cosh 2a b_1)}{4(a_1^2+b_1^2)}\Bigg\}$$

$$+\Bigg\{\frac{\alpha(\alpha q+p\beta)}{\alpha^3+\beta^2}+\frac{\beta(\alpha p-q\beta)}{\alpha^2+\beta^2}\Bigg\}\times$$

$$\times\Bigg\{\frac{a_1\sin 2a a_1\cosh 2a b_1+b_1\cos 2a a_1\sinh 2a b_1}{4(a_1^2+b_1^2)}\Bigg\}\Bigg]$$

$$-\frac{\pi A^2}{\omega\epsilon_1}\Bigg[\frac{b_1}{\pi^2}\;\frac{\sin 2a_1 z\cosh 2b_1 z}{\alpha\beta}\cdot\frac{2p\alpha\beta+q(\alpha^2-\beta^2)}{\alpha^2+\beta^2}$$

$$+\frac{a_1}{\pi^2}\;\frac{\cos 2a_1 z\sinh 2b_1 z}{\alpha\beta}\cdot\frac{2p\alpha\beta+q(\alpha^2-\beta^2)}{\alpha^2+\beta^2}\Bigg]$$

$$-\frac{\pi A^2}{\omega \epsilon_1}\left[-\left\{\frac{b_1}{\pi^2}\frac{\sin 2a_1 z \cosh 2b_1 z}{\alpha\beta}\cdot\frac{2p'\alpha\beta+q(\alpha^2-\beta^2)}{\alpha^2+\beta^2}\right.\right.$$

$$\left.\left.+\frac{a_1}{\pi^2}\frac{\cos 2a_1 z \sinh 2b_1 z}{\alpha\beta}\frac{2p'\alpha\beta+q(\alpha^2-\beta^2)}{\alpha^2+\beta^2}\right\}\right] \qquad [27]$$

$$P_{\text{out}}=\frac{\pi A^2 r}{\omega \epsilon_0}(d-a)\left[-4\left\{\frac{\alpha(\alpha q+p\beta)}{\alpha^2+\beta^2}+\frac{\beta(\alpha p-q\beta)}{\alpha^2+\beta^2}\right\}\right]$$

$$+\frac{\pi A^2}{\omega \epsilon_0}\left[-\frac{2b_2}{\pi^2}\frac{2p\alpha\beta+q(\alpha^2-\beta^2)}{\alpha\beta(\alpha^2+\beta^2)}\right] \qquad [28]$$

For large argument approximations equations [25] and [26] reduce to

$$P_{\text{in}}=\frac{\pi A^2 r}{\omega \epsilon_1}\frac{2}{\pi\rho}\exp(-2\alpha\rho)\left[\frac{a}{2}\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\right.$$

$$+\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\frac{a_1 \sin 2aa_1 \cosh 2ab_1+b_1\cos 2aa_1 \sinh 2ab_1}{4(a_1^2+b_1^2)}$$

$$\left.+\frac{\alpha}{\sqrt{(\alpha^2+\beta^2)}}\frac{a_1\cos 2aa_1 \sinh 2ab_1-b_1\sin 2aa_1\cosh 2ab_1}{4(a_1^2+b_1^2)}\right]$$

$$-\frac{\pi A^2}{\omega \epsilon_1}\left[\left(\frac{a_1\cos 2a_1 z\sinh 2b_1 z}{2}+\frac{b_1\sin 2a_1 z\cosh 2b_1 z}{2}\right)\times\right.$$

$$\times\frac{r}{4\alpha\beta}\frac{2}{\pi r}\frac{\beta}{\sqrt{(\alpha^2+\beta^3)}}-\left(\frac{b_1\cos 2a_1 z\sinh 2b_1 z}{2}\right.$$

$$\left.\left.-\frac{a_1\sin 2a_1 z\cosh 2b_1 z}{2}\right)\frac{r}{4\alpha\beta}\frac{2}{\pi r}\frac{\alpha}{\sqrt{(\alpha^2+\beta^2)}}\right]\exp(-2\alpha r)$$

$$-\frac{\pi A^2}{\omega \epsilon_1}\left[\left(\frac{b_1\cos 2a_1' z\sinh 2b_1 z}{2}-\frac{a_1\sin 2a_1 z\cosh 2b_1 z}{2}\right)\times\right.$$

$$\times\frac{a_1}{4\alpha\beta}\frac{2}{\pi r_1}\frac{\alpha}{\sqrt{(\alpha^2+\beta^2)}}-\left(\frac{a_1\cos 2a_1 z\sinh 2b_1 z}{2}\right.$$

$$\left.\left.+\frac{b_1\sin 2a_1 z\cosh 2b_1 z}{2}\right)\frac{r_1}{4\alpha\beta}\frac{2}{\pi r_1}\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\right]\exp(-2\alpha r_1) \qquad [29]$$

$$P_{out}=\frac{\pi A^2 r}{\omega\epsilon_0}(\alpha-a)\frac{2}{\pi\rho}\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\exp(-2\alpha\rho)$$

$$+\frac{\pi A^2}{\omega\epsilon_0}\left\langle\left(\frac{a_2 r}{4\alpha\beta}\cdot\frac{\alpha}{\sqrt{(\alpha^2+\beta^2)}}\cdot\frac{2}{\pi r}+\frac{b_2 r}{4\alpha\beta}\cdot\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\cdot\frac{2}{\pi r}\right)\right.$$

$$\left.-\frac{a_2 r}{4\alpha\beta}\cdot\frac{\alpha}{\sqrt{(\alpha^2+\beta^2)}}\cdot\frac{2}{\pi r}-\frac{b_2 r}{4\alpha\beta}\cdot\frac{\beta}{\sqrt{(\alpha^2+\beta^2)}}\cdot\frac{2}{\pi r}\right)\Big\rangle\exp(-2\alpha r) \qquad [30]$$

10. ATTENUATION CONSTANT

The attenuation constant α derived by using perturbation technique is given by the relation

$$\alpha=-(P_L/2\,P_\rho) \qquad [31]$$

where, P_L is the total power lost per unit length and P_p represents the power flow in the ρ direction as a surface wave. Assuming that there is no loss of power by radiation in the transverse direction and that the power in the transverse is concentrated within a distance d, the only loss is the dielectric loss in the material of the disc. In this paper, we shall consider only the dielectric loss and ignore the radiation loss. The attenuation constant α in eq. [31] can be put in a more convenient form by using the Poynting's theorem which states the energy balance relation as follows :

$$\int_v \vec{J}\cdot\vec{E}\,dv+\int_v\left(\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}+\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}\right)dv+\int_{A\ \text{(cross section perpendicular to the direction of flow)}}(\vec{E}\times\vec{H})\cdot\vec{n}\,da=0$$

where, the Poynting vector is $\vec{S}\triangleq\vec{E}\times\vec{H}$ and the current density $\vec{J}=\sigma\vec{E}$. The energy stored in the volume of the dielectric being zero, the energy balance equation reduces to

$$-\sigma\int_v E^2\,dv=\int_A\vec{S}\cdot\vec{n}\,da=\psi \qquad [32]$$

$$\frac{d\psi}{d\rho}=\frac{d}{d\rho}\int_A\vec{S}\cdot\vec{n}\,da=-\sigma\int_A E^2\,da \qquad [33]$$

$$\therefore|\alpha|=\left|\frac{1}{\psi}\frac{d\psi}{d\rho}\right|\triangleq\frac{\sigma_1\int_{z=0}^{a}\int_{\phi=0}^{2\pi}E^2\,r\,d\phi\,dz}{\left(\left|\int_{z=0}^{a}\int_{\phi=0}^{2\pi}E_{z1}H^*_{\phi1}\,r\,d\phi\,dz+\int_{z=a}^{\infty}\int_{\phi=0}^{2\pi}E_{z2}H^*_{\phi2}\,r\,d\phi\,dz\right|\right)}$$

where, $|E|^2 = |E_z|^2 + |E_\rho|^2$. So, α reduces to

$$|\alpha| = \sigma_1 \frac{\int_0^a |E_{z1}|^2 dz + \int_a^\infty |E_{z2}|^2 dz}{\left| \int_0^a E_{z1} H^*_{\phi 1} dz + \int_a^\infty E_{z2} H^*_{\phi 2} dz \right|} \qquad [34]$$

where, σ_1 is the conductivity of the dielectric disc.

Using appropriate field components, performing the integrations and simplifying eqn. [39] reduces to

$$\alpha = \sigma_1 \left(\frac{A^2 \gamma^2}{\sigma_1^2 + \omega^2 \epsilon_1^2} \{H_0^{(2)}(-j\gamma\rho)\}^2 \left\{ \frac{\sin 2u_1 a}{4u_1} + \frac{a}{2} \right\} + \frac{A^2 \gamma^2}{\omega^2 \epsilon_0^2} \{H_0^{(2)}(-j\gamma\rho)\}^2 \frac{\exp(-2u_2 a)}{2u_2} \right)$$

$$\div \left| A^2 \frac{\gamma(d-a)}{\omega \epsilon_0} H_0^{(2)}(-j\gamma\rho) H_1^{(1)}(j\gamma^*\rho) + j \frac{A^2 \gamma}{\sigma_1 + j\omega\epsilon_1} H_0^{(2)}(-j\gamma\rho) H_1^{(1)}(j\gamma^*\rho) \left(\frac{\sin 2u_1 a}{4u_1} + \frac{a}{2} \right) \right|$$

$$= \sigma_1 \cdot \frac{\frac{\gamma}{\sigma_1^2 + \omega^2 \epsilon_1^2} \left(\frac{\sin 2u_1 a}{4u_1} + \frac{a}{2} \right) [H_0^{(2)}(-j\gamma\rho)]^2 + \frac{\gamma}{\omega^2 \epsilon_0^2} \frac{\exp(-2u_2 a)}{2u_2} [H_0^{(2)}(-j\gamma\rho)]^2}{\left| \frac{d-a}{\omega\epsilon_0} H_0^{(2)}(-j\gamma\rho) H_1^{(1)}(j\gamma^*\rho) + \frac{j}{\sigma_1 + j\omega\epsilon_1} H_0^{(2)}(-j\gamma\rho) H_1^{(1)}(j\gamma^*\rho) \left(\frac{\sin 2u_1 a}{4u_1} + \frac{a}{2} \right) \right|}$$

$$= \frac{A + jB}{(C^2 + D^2)^{1/2}} \qquad [35]$$

$$\therefore |\alpha| = \left(\frac{A^2 + B^2}{C^2 + D^2} \right)^{1/2} \text{ by small argument approximations} \qquad [36]$$

where

$$A = \sigma_1 \left[\frac{a_2 b_2}{\beta(\sigma_1^2 + \omega^2 \epsilon_1^2)} \frac{1}{\pi^2} \{2a(q^2 - p^2) + (q^2 - p^2) x_1 + 2pq x_2\} + \frac{2 a_2 b_2 (q^2 - p^2)}{\pi^2 \beta} \frac{x_3}{\omega^2 \epsilon_0^2} + \frac{4pq}{\pi^2} \frac{a_2 b_2}{\omega^2 \epsilon_0^2} \frac{x_4}{\beta} + \frac{1}{\pi^2} \frac{\beta}{\sigma_1^2 + \omega^2 \epsilon_1^2} \{4apq - (q^2 - p^2) x_2 + 2pq x_1\} \right.$$

$$- \frac{2\beta}{\omega^2 \epsilon_0^2} \frac{q^2 - p^2}{\pi^2} x_4 + \frac{4pq}{\pi^2} \frac{\beta}{\omega^2 \epsilon_0^2} x_3 \Bigg]$$

$$B = \sigma_1 \Bigg[\frac{\beta}{\sigma_1^2 + \omega^2 \epsilon_1^2} \frac{1}{\pi^2} \{2a (q^2 - p^2) + x_1 (q^2 - p^2) + 2pq x_2\}$$

$$+ \frac{2(q^2 - p^2)}{\pi^2} \frac{\beta}{\omega^2 \epsilon_0^2} x_3 + \frac{4pq}{\pi^2} \frac{\beta}{\omega^2 \epsilon_0^2} x_4$$

$$- \frac{1}{\pi^2 \beta} \frac{a_2 b_2}{\sigma_1^2 + \omega^2 \epsilon_1^2} \{4apq - (q^2 - p^2) x_2 + 2pq x_1\}$$

$$+ \frac{a_2 b_2}{\omega^2 \epsilon_0^2} \frac{2(q^2 - p^2)}{\pi^2 \beta} x_4 - \frac{4pq}{\pi^2} \frac{a_2 b_2}{\beta \omega^2 \epsilon_0^2} x_3 \Bigg]$$

$$C = \frac{4}{\pi^2 r} \left[\frac{d-a}{\omega \epsilon_0} \frac{\beta (p\beta^2 + q a_2 b_2)}{a_2^2 b_2^2 + \beta^4} + \frac{a}{2} x_5 + \frac{x_5 x_1}{4} + \frac{x_2 x_8}{4} \right]$$

$$D = \frac{4}{\pi^2 r} \left[x_5 x_7 - \frac{d-a}{\omega \epsilon_0} \frac{(p a_2 b_2 - q \beta^2) \beta}{a_2^2 b_2^2 + \beta^4} + \frac{a}{2} x_8 + x_6 x_8 \right]$$

$$p = \frac{1}{2} ln (0.89 r^2) \left(\frac{a_2^2 b_2^2 + \beta^4}{\beta^2} \right)$$

$$q = arc \tan (\beta^2 / a_2 b_2)$$

$$x_1 = \frac{a_1 \sin 2aa_1 \cosh 2ab_1 + b_1 \cos 2aa_1 \sinh 2ab_1}{a_1^2 + b_1^2}$$

$$x_2 = \frac{a_1 \cos 2aa_1 \sinh 2ab_1 - b_1 \sin 2aa_1 \cosh 2ab_1}{a_1^2 + b_1^2}$$

$$x_3 = \frac{a_2 \cosh 2aa_2 \cos 2ab_2}{a_2^2 + b_2^2} + \frac{b_2 \sinh 2aa_2 \sin 2ab_2}{a_2^2 + b_2^2}$$

$$x_4 = \frac{b_2 \cosh 2aa_2 \cos 2ab_2}{a_2^2 + b_2^2} - \frac{a_2 \sinh 2aa_2 \sin 2ab_2}{a_2^2 + b_2^2}$$

$$x_5 = \frac{(p\sigma_1 a_2 b_2/\beta) + p\omega\epsilon_1\beta + q\omega\epsilon_1 (a_2 b_2/\beta) - q\sigma_1\beta}{[\sigma_1 (a_2 b_2/\beta) + \omega\epsilon_1\beta]^2 + [\omega\epsilon_1 a_2 b_2/\beta) - \sigma_1\beta]^2}$$

$$x_8 = \frac{(q\sigma_1 a_2 b_2 + q\omega\epsilon_1\beta^2 - p\omega\epsilon_1 a_2 b_2 + p\sigma_1\beta^2)\beta}{(\sigma_1 a_2 b_2 + \omega\epsilon_1\beta)^2 + (\omega\epsilon_1 a_2 b_2 - \sigma_1\beta^2)^2}$$

For large arrangement approximations eqn. [35] reduces to

$$|\alpha| = \left(\frac{E^2 + F^2}{G^2 + H^2}\right)^{1/2} \qquad [37]$$

where,

$$E = \left[\frac{\sigma_1}{\sigma_1^2 + \omega^2 \epsilon_1^2} \left\{\left(\frac{a_2 b_2}{\beta} y_3 - \beta y_4\right)\right\} + \sigma_1 \left\{\left(\frac{a_2 b_2}{\beta} \frac{x_9}{\omega^2 \epsilon_0^2} - \frac{\beta x_{10}}{\omega^2 \epsilon_0^2}\right)\left(\frac{a_2 b_2}{\beta} y_1 + \beta y_2\right) - \left(\frac{a_2 b_2}{\beta} x_{10} + \beta x_9\right)\left(\frac{a_2 b_2}{\beta} y_2 - \beta y_1\right)\right\}\right]$$

$$F = \frac{\sigma_1}{\sigma_1^2 + \omega^2 \epsilon_1^2} \left(\frac{a_2 b_2}{\beta} y_4 + \beta y_3\right) + \sigma_1 \left\{\left(\frac{a_2 b_2}{\beta} x_{10} + \beta x_9\right)\left(\frac{a_2 b_2}{\beta} y_1 + \beta y_2\right) + \left(\frac{a_2 b_2}{\beta} y_2 - \beta y_1\right)\left(\frac{a_2 b_2}{\omega^2 \epsilon_0^2} x_9 - \frac{\beta x_{10}}{\omega^2 \epsilon_0^2}\right)\right\}$$

$$y_1 = \frac{2}{\pi \rho \left[(a_2^2 b_2^2/\beta^2) + \beta^2\right]} \left\{\sinh 2\rho \frac{a_2 b_2}{\beta} \cos 2\rho\beta - \cosh 2\rho \frac{a_2 b_2}{\beta} \cos 2\rho\beta\right\}$$

$$y_2 = \frac{2}{\pi \rho \left[(a_2^2 b_2^2/\beta^2) + \beta^2\right]} \left\{\cosh 2\rho \frac{a_2 b_2}{\beta} \sin 2\rho \beta - \sinh 2\rho \frac{a_2 b_2}{\beta} \sin 2\rho\beta\right\}$$

$$y_3 = \left(\frac{x_1}{4} + \frac{a}{2}\right)\left(\frac{a_2 b_2}{\beta} y_1 + \beta y_2\right) - \frac{x_2}{4}\left(\frac{a_2 b_2}{\beta} y_2 - \beta y_1\right)$$

$$y_4 = \frac{x_2}{4}\left(\frac{a_2 b_2}{\beta} y_1 + \beta y_2\right) + \left(\frac{x_1}{4} + \frac{a}{2}\right)\left(\frac{a_2 b_2}{\beta} y_2 - \beta y_1\right)$$

$$x_9 = \frac{x_3}{2} - \frac{x_6}{2}$$

$$x_{10} = \frac{x_4}{2} - \frac{x_7}{2}$$

$$x_6 = \frac{a_2 \sinh 2aa_2 \cos 2ab_2 + b_2 \cosh 2aa_2 \sin 2ab_2}{a_2^2 + b_2^2}$$

$$x_7 = \frac{b_2 \sinh 2aa_2 \cos 2ab_2 - a_2 \cosh 2aa_2 \sin 2ab_2}{a_2^2 + b_2^2}$$

The experimental verification of the theory is under progress

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APPENDIX A.1

$$H_p^{(2)} = J_p(z) - i\, Y_p(z)$$

$$= \frac{J_p(z)\,(\sin \pi p - i \cos \pi p) + i\, J_{-p}(z)}{\sin \pi p}$$

$$\text{since } Y_p(z) = \frac{J_p(z) \cos \pi p - J_{-p}(z)}{\sin \pi \rho}$$

Using exponential functions.

$$H_p^{(2)}(z) = -i \frac{J_p(z) \exp(i p \pi) - J_{-p}(z)}{\sin \pi p}$$

$$[H_p^{(2)}(z)]^* = i \frac{J_p(z^*) \exp(-ip\pi) - J_{-p}(z^*)}{\sin \pi p}$$

But $\quad H_p^{(1)}(z^*) = J_p(z^*) + i\, y_p(z^*)$

$$= i \frac{J_p(z^*) \exp(i \pi \rho) - J_{-p}(z^*)}{\sin \pi \rho}$$

So $\quad [H_p^{(2)}(z)]^* = H_p^{(1)}(z^*)$

Hence $\quad [H_1^{(2)}(z)]^* = H_1^{(1)}(z^*)$

APPENDIX A 2

Small argument approximations

$$H_0^{(1)}(j\,\gamma^* r) = 1 + j\,(2/\pi)\, ln\, 0.89\, j\, \gamma^* r = j\,(2/\pi)\, ln\, 0\;89\, \gamma^* r$$

$$H_0^{(2)}(-j\,\gamma\, r) = 1 - j\,(2/\pi)\, ln\,(-0.89\, j\, \gamma\, r) = -j\,(2/\pi)\, ln\, 0.89\, \gamma\, r$$

$$H_1^{(1)}(j\,\gamma^* r) = -\,2/\pi \gamma^* r$$

$$H_1^{(2)}(-j\,\gamma\, r) = -2/\pi \gamma\, r$$

Using the relations

$$ln\,(x + jy) = ln\, s + j\,(\theta + 2\,\pi\, k) = ln\, s + j\,\theta$$

where $s=\sqrt{(x^2+y^2)},\ \theta=\text{arc tan } y/x$

k is an integer or zero

$\gamma^*=\alpha-j\beta$

So, $H_0^{(1)}(j\gamma^* r)=j(2/\pi)(p-jq)$

where, $p=\frac{1}{2}\,ln\,(0\ 89\,r)^2(\alpha^2+\beta^2)$

$q=\text{arc tan }\beta/\alpha$

Large argument approximations :

$$H_p^{(1)}(z)\sim[\sqrt{(2/\pi z)}]\exp\{j[z-(p+\tfrac{1}{2})\pi/2]\}$$

$$H_p^{(2)}(z)\sim[\sqrt{(2/\pi z)}]\exp\{-j[z-(p+\tfrac{1}{2})\pi/2\}$$

which yield

$$H_0^{(2)}(z)\sim[\sqrt{(2/\pi z)}]\exp\{-j(z-\pi/4)\}$$

$$H_1^{(1)}(z)\sim[\sqrt{(2/\pi z)}]\exp\{j(z-3\pi/4)]$$