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Resonantly shunted piezoelectric layers as passive vibration control devices†

S. B. KANDAGAL, SUNETRA SARKAR AND KARTIK VENKATRAMAN*
Aeroservoelasticity Laboratory, Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560 012, India.
email: kartik@aero.iisc.ernet.in; Phones: 91-80-3942419, 3942420, 3942874; Fax: 91-80-3600134.

Abstract

The effect of resonant shunting on the vibration behaviour of a duralumin cantilever beam is experimentally investigated with reference to the reduction of response amplitude and additive damping and the change in resonance frequency. The overall reduction in tip amplitude is around 4% for a peizoceramic layer with electromechanical coupling coefficient (k_{31}) equal to 0.30. However, higher values (k_{31} = 0.36, typically applied in beams and rods) of electromechanical coupling coefficient result in significantly higher levels of reduction of vibration amplitude with a change in natural frequency from short circuit to open circuit value. A 20–30% reduction in response amplitude and 8–10% change in natural frequency (open circuit to short circuit) is possible when the planar electromechanical coupling coefficient (k_p, typically applied in discs and plates) is 0.6–0.65.

Keywords: Passive vibration control, piezoelectric elements and shunting.

1. Introduction

Piezoceramics are transformers that convert mechanical energy into electrical energy and *vice versa*. When they are bonded to a structure, the mechanical strain energy generated in the piezoceramic is converted to electrical voltage in the poling direction of the piezoceramic device (Fig. 1). This voltage or electrical energy can be dissipated or shunted to another frequency band using electrical networks connected to the terminals of piezoceramic, thereby controlling the mechanical energy of motion of the structure. If the electrical network contains electrical energy sources, then we term it as an active network, and the control scheme as an active control one. If there are no energy sources, then the network is known as a passive network, and the control scheme is a passive control one. The present work will be concerned with the latter.

Passive vibration absorbers or controllers are well known in vibration engineering.[1,2] Conventional passive controllers cannot be tuned to different operating regimes such as changes in external excitation frequency or amplitude, and even change in system parameters and hence the move towards active vibration control devices. However, piezoceramic materials with tunable passive electrical networks can alleviate some of these shortcomings. The tunable passive electrical networks connected to the piezoceramic can modify the frequency selective vibration transmission properties of the structure itself.

Electrical passive shunting of piezoceramics has been investigated in the recent past.[3–5] These studies have focused on experimental investigation of the additive damping and change in resonance

*Author for correspondence.
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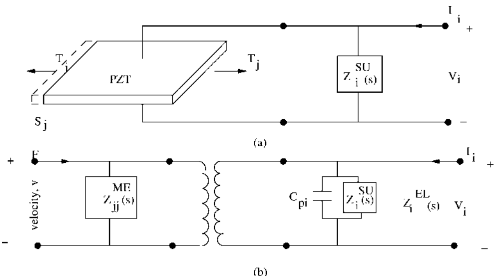

FIG. 1. (a) Physical model of uniaxial shunted piezoelectric and its (b) network analog.

frequency. The analytic vibration models represent the damping and stiffness due to electrical shunting of the piezoceramic as a complex frequency-dependent modulus similar to that used in viscoelastic solids.[3] The optimum shunting parameters for the piezoceramic vibration absorber is also derived and experimentally verified.

The present work investigates experimentally the variation of damping with resonantly shunted piezoceramics.

2. Modeling of shunted piezoelectric materials

The constitutive equations of a linear piezoelectric material can be written as:[3]

$$\begin{bmatrix} \mathbf{q} \\ \varepsilon \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \mathbf{D} \\ \mathbf{D}^{\mathrm{T}} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{\Phi} \\ \sigma \end{bmatrix} \tag{1}$$

where

$$\begin{aligned} \mathbf{q} &= [q_1, q_2, q_3]^T \\ \varepsilon &= [\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{23}, 2\varepsilon_{13}, 2\varepsilon_{12}]^T \\ \mathbf{\Phi} &= [\Phi_1, \Phi_2, \Phi_3]^T \\ \sigma &= [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}]^T \end{aligned} \tag{2}$$

$$\mathbf{E} = \begin{bmatrix} \varepsilon_{01} & 0 & 0 \\ 0 & \varepsilon_{01} & 0 \\ 0 & 0 & \varepsilon_{03} \end{bmatrix} \tag{3}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \tag{5}$$

Representing these equations in terms of voltage and current, whhich are defined as

$$\mathbf{v}_i = \int_0^{L_i} \Phi_i d\mathbf{x}_i, \qquad \mathbf{i}_i = \int_{A_i} \mathrm{q}_i d\mathbf{a}_i\,. \tag{6}$$

Assuming that the field within and electrical charge on the surface are uniform for the piezoelectric material, the relations (6), in the Laplace domain, become :

$$\mathbf{v}(s) = \mathbf{L.}\,\Phi\,(s), \qquad \mathbf{i}(s) = s\,\mathbf{A}(s) \tag{7}$$

where $\mathbf{L}$ is a diagonal matrix of the lengths of the piezoceramic patch in the i^{th} direction, $\mathbf{A}$, the diagonal matrix of the areas of surfaces perpendicular to the i^{th} direction, and s, the Laplace parameter. By taking the Laplace transform of eqn (1) and using eqns (7) to eliminate Φ and $\mathbf{q}$, the general equation for a piezoelectric in terms of the external current input and applied voltage is obtained as:

$$\begin{bmatrix} \mathbf{i} \\ \varepsilon \end{bmatrix} = \begin{bmatrix} s\mathbf{AEL}^{-1} & s\mathbf{AD} \\ \mathbf{D}^T\mathrm{L}^{-1} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \sigma \end{bmatrix} \tag{8}$$

The generalized compliance matrix in the upper left partition is diagonal and the elements of this partition have the form,

$$\mathbf{A}_i\mathbf{E}_i/\mathbf{L}_i = C_{pi} \tag{9}$$

where C_{pi} is the capacitance between the surfaces perpendicular to the i^{th} direction at constant stress. By grouping these into $\mathbf{C}_p$, the constituent equation becomes,

$$\begin{bmatrix} \mathbf{i} \\ \varepsilon \end{bmatrix} = \begin{bmatrix} s\mathbf{C}_p & s\mathbf{AD} \\ \mathbf{D}^T\mathrm{L}^{-1} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \sigma \end{bmatrix} = \begin{bmatrix} \mathbf{Y}^D(s) & s\mathbf{AD} \\ \mathbf{D}^T\mathrm{L}^{-1} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \sigma \end{bmatrix} \tag{10}$$

where $\mathbf{Y}^{\mathrm{D}}(s)$ is the open circuit admittance of the piezoelectric (due to the inherent capacitance with free mechanical boundary conditions). For shunted piezoelectric applications, a passive electrical circuit is connected between the surface electrodes (Fig. 1). Since the circuit is placed across the electrodes, it appears in parallel to the inherent piezoelectric capacitance in that direction. The admittances add in parallel. Hence the governing constitutive equation (10) becomes,

$$\begin{bmatrix} \mathbf{i} \\ \varepsilon \end{bmatrix} = \begin{bmatrix} \mathbf{Y}^{EL} & s\mathbf{AD} \\ \mathbf{D}^T \mathrm{L}^{-1} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \sigma \end{bmatrix} \tag{11}$$

where

$$\mathbf{Y}^{EL} = \mathbf{Y}^{D} + \mathbf{Y}^{SU}. \tag{12}$$

The externally applied current, **i**, is the sum of the currents flowing through the shunting impedance, the inherent piezoelectric capacitance, and the piezoelectric transformer. The shunting admittance matrix is assumed to be diagonal and frequency dependent with the form,

$$\mathbf{Y}^{SU}(s) = \begin{bmatrix} \mathrm{Y}_1^{SU}(s) & 0 & 0 \\ 0 & \mathrm{Y}_2^{SU}(s) & 0 \\ 0 & 0 & \mathrm{Y}_3^{SU}(s) \end{bmatrix} \tag{13}$$

The voltage appearing across the electrodes can be estimated from eqn (11), which will be

$$\mathbf{v} = (\mathbf{Z}^{EL})\mathbf{i} - (\mathbf{Z}^{EL} s\mathbf{AD})\sigma \tag{14}$$

where $\mathbf{Z}^{EL}$ is the electrical impedance matrix and is equal to $(\mathbf{Y}^{EL})^{-1}$. The strains in terms of stress and input current can be obtained by substituting eqn (14) in eqn (11),

$$\varepsilon = [\mathbf{C} - \mathbf{D}^T \mathbf{L}^{-1} \mathbf{Z}^{EL} s\mathbf{AD}]\,\sigma + [\mathbf{D}^T \mathbf{L}^{-1} \mathbf{Z}^{EL}]\,\mathbf{i} \tag{15}$$

This governing equation for the shunted piezoelectric gives the strain for a given applied stress and forcing current. The shunted piezoelectric compliance can be defined from eqn (15), as

$$\mathbf{C}^{SU} = [\mathbf{C}^{E} - \mathbf{D}^T \mathbf{L}^{-1} \mathbf{Z}^{EL} s\mathbf{AD}] \tag{16}$$

It is to be noted that the short and open circuit electrical impedances with constant stress will be $\mathbf{Z}^{E}(s) = 0$ and $\mathbf{Z}^{D}(s) = (\mathbf{C}_p s)^{-1}$, respectively

$$s\mathbf{L}^{-1} E\mathbf{A} = \mathbf{C}_p s\,. \tag{17}$$

With these, eqn (16) can be written as

$$\mathbf{C}^{SU} = [\mathbf{C}^{E} - \mathbf{D}^T \bar{\mathbf{Z}}^{EL} (\varepsilon^T)^{-1} \mathbf{D}], \tag{18}$$

where the nondimensional electrical impedance matrix is defined as

$$\bar{\mathbf{Z}}^{EL} = \mathbf{Z}^{EL}(\mathbf{Z}^{D})^{-1} = (s\mathbf{C}_p + \mathbf{Y}^{SU})^{-1} s\mathbf{C}_P\,. \tag{19}$$

Since $\mathbf{Z}^{EL}$ is diagonal, the electrical contribution to the compliance can be written as the summation of electrical impedances,

$$\mathrm{C}^{SU} = \left[\mathrm{C}^{E} - \sum_{i=1}^{3} \left[\bar{\mathbf{Z}}_i^{EL} \left(\frac{\mathrm{D}^T}{\mathrm{D}_i} E_i^T \right) \right] \right] = \left[\mathrm{C}^{E} - \sum_{i=1}^{3} \bar{\mathbf{Z}}_i^{EL} \mathrm{M}_i \right], \tag{20}$$

where $\mathbf{D}_i$ denotes the i^{th} row of $\mathbf{D}$ and for piezoceramic $\mathbf{M}_i$ have the form

$$\mathbf{M}_1 = \frac{1}{\varepsilon_1^T}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{15}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{M}_2 = \frac{1}{\varepsilon_1^T}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{15}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

$$\mathbf{M}_3 = \frac{1}{\varepsilon_3^T}\begin{bmatrix} d_{31}^2 & d_{31}^2 & d_{31}d_{33} & 0 & 0 & 0 \\ d_{31}^2 & d_{31}^2 & d_{31}d_{33} & 0 & 0 & 0 \\ d_{31}d_{33} & d_{31}d_{33} & d_{33}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

The above equations constitute a general expression for the compliance matrix of a piezoelectric element. Equation (20) can be simplified further, when the piezoelectric element is loaded uniaxially with either a normal or shear stress and only one pair of electrodes is necessary to provide an external electric field with components in only one direction. For loading in the j^{th} direction and the field in i^{th} direction the term in the compliance matrix will be

$$c_{jj}^{SU} = c_{jj}^{E} - \bar{Z}_i^{EL}(d_{ij})^2 / \varepsilon_i^T. \tag{23}$$

The electromechanical coupling coefficient is defined as the ratio of the peak energy stored in the capacitor to the peak energy stored in the material strain with the piezoelectric electrodes open. It represents the percentage of mechanical strain energy which is converted into electrical energy and *vice versa*. The electromechanical coupling coefficient, k_{ij}, can be represented as

$$k_{ij} = d_{ij}/\sqrt{c_{jj}\,\varepsilon_i^T}. \tag{24}$$

Hence, the compliance C_{jj}^{SU} is obtained by substituting eqn (24) in eqn (23),

$$c_{jj}^{SU} = c_{jj}^{E}\,\left[1 - k_{ij}^2\,\bar{Z}_i^{EL}\right]. \tag{25}$$

From eqn (25), it can be observed that the compliance of the shunted piezoelectric is equal to the short-circuit compliance of the piezoelectric material modified by a nondimensional term which depends on the electrical shunting circuit and the material's electromechanical coupling coefficient. Substituting $\bar{Z}^{EL} = 1$, for the open circuit case, to get the shunted mechanical compliance as

$$c_{jj}^{D} = c_{jj}^{E}\,\left[1 - k_{ij}^2\right]. \tag{26}$$

Equation (25) gives the change in mechanical properties of the piezoceramic as the boundary conditions are changed from open circuit to short circuit. Similarly, an analogous expression can be thought for the change in the inherent capacitance of the piezoelectric as the mechanical boundary conditions are changed.

$$c_{pi}^{CR} = c_{pi}^{CS}\ [1-k_{ij}^2]. \tag{27}$$

The mechanical impedance of the shunted piezoceramic can be obtained in the nondimensional form by using eqns (25) and (26). For uniaxial loading in the j^{th} direction, the mechanical impedance of the piezoelectric can be expressed as a function of Laplace parameter, s, as

$$Z_{jj}^{ME}(s) = \frac{A_j}{C_{pi}L_j s}\cdot \tag{28}$$

The final expression for the nondimensional mechanical impedance, which is defined as the ratio of the shunted mechanical impedance to the open circuit impedance, for the shunted piezoelectric can be derived using eqns (28) and (26) as

$$\bar{Z}_{jj}^{ME}(s) = \frac{Z_{jj}^{SU}(s)}{Z_{jj}^{D}(s)} = \frac{(1-k_{ij}^2)}{[1-k_{ij}^2\bar{Z}_i^{EL}(s)]}\cdot \tag{29}$$

When the shunted piezoelectric is coupled to the structure, the nondimensional mechanical impedance, $\bar{Z}_{jj}^{ME}$ can become complex and frequency dependent since it depends on the complex frequency-dependent electrical impedance. Since impedance is primarily stiffness, it can be represented as a complex modulus.

$$\bar{Z}_{jj}^{ME}(s) = \bar{E}_{jj}(\omega)\,[1+i\eta_{jj}(\omega)] \tag{30}$$

where $\bar{E}$ is the ratio of shunted stiffness to open circuit stiffness of the piezoelectric and η the material loss factor. This leads to frequency-dependent equations for the complex modulus of the shunted piezoelectric. The loss factor η and modulus $\bar{E}$ can be expressed as

$$\eta_{jj}(\omega) = \frac{Im\,[\bar{Z}^{ME}(s)]}{Re\,[\bar{Z}^{ME}(s)]}, \quad \bar{E}_{jj}(\omega) = Re[\bar{Z}^{ME}(s)]. \tag{31}$$

The above two equations give us the indication of damping and stiffness variation with electrical shunting, respectively. In the case of electrical shunting of the piezoceramic, a resonant circuit is created by shunting the inherent capacitance of the piezoelectric with a resistor and inductor in series forming an RLC circuit. A resonant circuit connected in parallel with the piezoceramic is shown in Fig. 2. The mechanical analog of an inductor, capacitor, and a resistor is a mass, spring, and damper, respectively. Thus the resonant electrical circuit behaves very similar to a classical tuned mass vibration absorber. However, one has to keep in mind that the effectiveness of connecting a resonantly shunted piezoceramic to a vibrating system with the intent of controlling its vibration will be limited by the electromechanical efficiency of the piezoceramic, k_{ij}. A high value of k_{ij} would imply a more effective resonantly shunted piezoceramic tuned vibration absorber. With an inductor and a resistor in parallel with the piezoelectric's inherent capacitance, the total electrical impedance can be written as

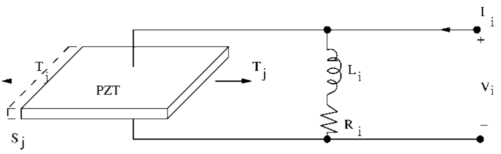

FIG. 2. Resonantly shunted piezoelectric.

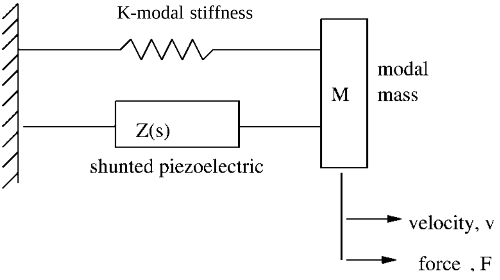

FIG. 3. Single-degree-of-freedom of system model with shunted piezoelectric element in parallel with the system modal mass.

$$Z_i^{SU}(s) = L_i s + R_i, \quad \bar{Z}_i^{EL}(s) = \frac{L_i C_{pi}^{CS} s^2 + R_i C_{pi}^{CS} s}{L_i C_{pi}^{CS} s^2 + R_i C_{pi}^{CS} s + 1} \tag{32}$$

where L_i is the shunting inductance and R_i, the shunting resistance. Substituting this relation in eqn (29), and simplifying to get nondimensional impedance of a resonantly shunted piezoelectric,

$$\bar{Z}_{jj}^{RSP}(s) = 1 - k_{ij}^2 \left(\frac{\delta^2}{\gamma^2 + \delta^2 r\gamma + \delta^2} \right) \tag{33}$$

where $\omega_e = 1 / \sqrt{(L_i C_{pi}^{CR})}$ = electrical resonant frequency, and $\delta = \omega_e/\omega_n$ = nondimensional tuning ratio.

The above equation gives the effective mechanical impedance of a piezoelectric element shunted by a resonant circuit. Frequency tuning parameter, δ, refers to the frequency to which the electrical circuit is tuned, and the damping parameter, r, is indicative of the damping in the shunting circuit.

As the stiffness of the piezoelectric material is frequency-dependent, maximizing the loss factor of the piezoelectric material does not necessarily maximize the loss factor of the total structural system of which the piezoelectric patch is a part. In order to accurately model the system, modal damping as a function of frequency or shunting parameters (such as resistance), this frequency-dependent stiffness must be carried through the calculations. In order to achieve this, the dynamics of the host structure is modeled by a single vibration mode. The piezoceramic is then coupled in parallel to this one degree-of-freedom (DOF) system as shown in Fig. 3.

Following the 1-DOF system modeling techinque, the modal velocity of the vibrating system with resonantly shunted piezoelectric can be expressed in the Laplace domain as,

$$v(s) = \frac{F(s)}{Ms + (K/s) + Z_{jj}^{RSP}(s)} \tag{34}$$

where Ms and K/s are, respectively, the impedance associated with the modal mass and stiffness of the host vibrating system and Z_{jj}^{RSP}, the modal impedance associated with the resonantly shunted piezoelectric. After reduction and nondimensionalization, an expression for the position transfer function of a mechanical system with RLC in parallel with the base stiffness and a force acting on the mass can be found from the above equation as:

$$\frac{x}{x^{ST}} = \frac{(\delta^2+\gamma^2)+\delta^2 r\gamma}{(1+\gamma^2)\,(\delta^2+\gamma^2+\delta^2 r\gamma)+K_{ij}^2\,(\gamma^2+\delta^2 r\gamma\ )}\ . \tag{35}$$

The generalized electromechanical coupling coefficient, K_{ij}, is defined as :

$$K_{ij}^2 = \left(\frac{K_{pzt}}{K+K_{pzt}}\right)\frac{k_{ij}^2}{1-k_{ij}^2}. \tag{36}$$

The optimal tuning parameters for the transfer function are to be found by identifying the magnitudes of the transfer functions that correspond to $r=0$ (short circuit) and $r=\infty$ (open circuit). From eqn (35), for $r = 0$, we have

$$\frac{x}{x^{ST}} = \frac{(\delta^2-g^2)}{(1-g^2)\,(\delta^2-g^2)-K_{ij}^2 g^2}\,, \tag{37}$$

and for $r = \infty$

$$\frac{x}{x^{ST}} = \frac{1}{(1+K_{ij}^2)-g^2}\,. \tag{38}$$

The two transfer functions (eqns (37) and (38)) are equated and a quadratic expression is obtained, which can be used to get the invariant frequency points.

$$g^4 - g^2\,[(1+K_{ij}^2)+\delta^2] + [(\delta^2/2)\,(2+K_{ij}^2)] = 0\,. \tag{39}$$

The solution and simplification of the above equation gives the optimum tuning parameter δ^{opt} and the optimal circuit damping r^{opt} as :

$$\delta^{opt} = \sqrt{1+\mathrm{K}_{ij}^2}\ ; \tag{40}$$

$$r^{opt} = \sqrt{2}\,\frac{K_{ij}}{(1+K_{ij}^2)}\,. \tag{41}$$

The effect of various circuit resistor values at optimal tuning is shown in Fig. 4. The response is similar to that of a vibration absorber. As the damping parameter is increased, the two distinct system modes coalesce into a single mode that converges to the system response with open-circuit piezoelectric as the damping parameter approaches infinity.

3. Experimental set-up

In order to investigate the properties of the resonantly shunted piezoceramic, dynamic tests were conducted on a duralumin cantilever beam specimen with surface-bonded piezoceramic devices. The cantilever beam was 166 mm long, 30.5 mm wide and 0.9 mm thick. The schematic diagram of the beam and piezoceramic is shown in Fig. 5. The top and bottom piezoceramic devices were

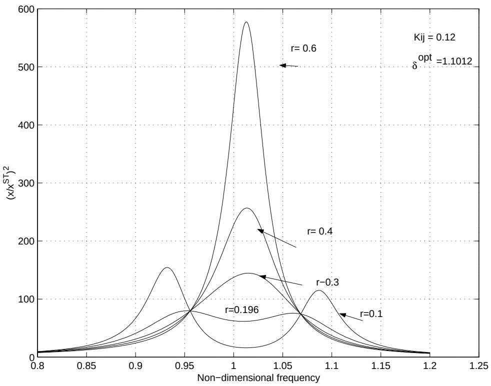

FIG. 4. Response of 1-DOF system containing LRC (δ^{opt} = 1.1012).

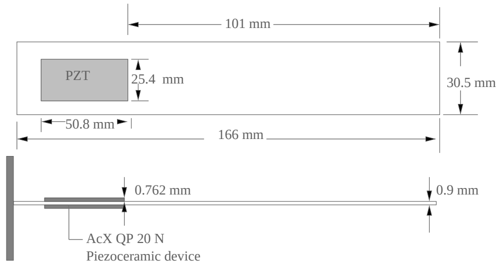

FIG. 5. Beam with PZT.

QP20N of Active Control eXperts Inc., USA. The piezoceramic patches were attached to the beam with a very thin layer of epoxy. The material properties of the beam and piezoceramic are listed in Table I.

The top PZT was shorted for all the experiments, whereas the bottom PZT was used for passive shunting experiments. The shunting inductance was varied between 0 and 200 H. The natural frequencies of the beam were found using impulse excitation technique consisting of instrumented impulse hammer PCB Piezotronics 208A03, B&K 4344 accelerometer, and B&K 2635 charge amplifier. The experimental set-up is shown in Fig. 6. The beam was excited at its first resonance frequency using Derritron VP2MM exciter, 25 W Derritron power amplifier and signal generator from AD 3525 analyzer. Input force is measured using B&K 8200 force transducer and amplified by B&K 2626 conditioning amplifier. The acceleration response of the beam was picked up at the tip by B&K 4344 accelerometer and amplified by B&K 2635 charge amplifier. These signals were acquired by National Instruments ATMIO 16 data acquistion card in LabView (Ver. 5.0) software. Constant input force level was ensured for each of the resonating shunting cases.

Damping was estimated from the energy dissipated in one cycle. This is given by the area enclosed within the force versus displacement curve for the vibrating system with piezo bonded to it, that is,

$$U = \oint F dx = \int_0^{2\pi/\Omega} F v dt \tag{42}$$

Table I
Material specifications

Material	Duralumin	PZT
Length (mm)	166	50.8
Width (mm)	30.5	25.4
Thickness (mm)	0.9	0.762
E (GPa)	70	69
Capacitance (μf)	–	0.09
Density (kg/m^3)	2700	7700
Coupling coefficent (k_{31})	–	0.3

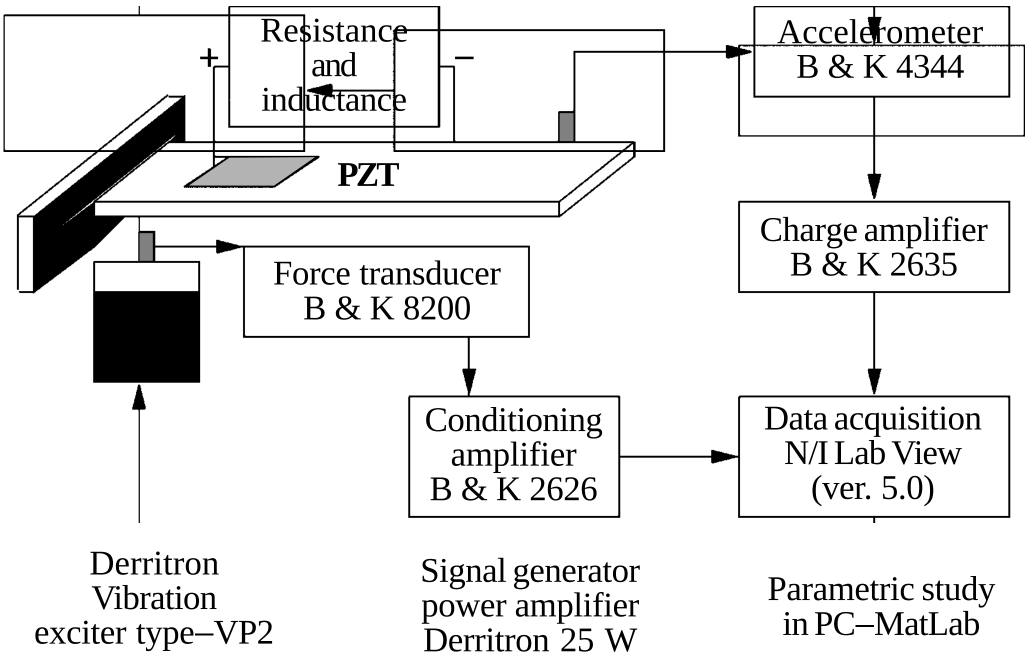

FIG. 6. Experimental set-up.

where Ω is the excitation frequency and v the velocity. Velocity and displacement signals were obtained by integrating successively the acceleration signal using a high-pass filter implemented in Matlab/Simulink (Ver. 5.3 (R11)). A Butterworth filter of order 16 with cutoff frequency at 12.75 Hz was used. A high-pass filter is needed to eliminate the DC component in the integrated signal. The cutoff frequency was selected on the basis that the phase and amplitude distortion in the measured and filtered values of acceleration are minimal. To compare the damping performance of shunted PZT, an equivalent damping coefficient C_{eqv} is determined for different cases.[7] Thus,

$$C_{eqv} = \frac{U}{\pi \Omega X_0^2} \tag{43}$$

where U is the energy dissipated given by eqn (42) and X_o the displacement amplitude. C_{eqv} is evaluated for different inductive shunting values. The damping coefficient ζ is evaluated as:

$$\zeta = \frac{C_{eqv}}{2\omega_n}\,. \tag{44}$$

4. Discussion

The base damping for the first mode with and without PZT are 0.0162 and 0.0097, respectively. The piezoceramic material, being brittle, has less effect in enhancing damping as an unconstrained damping layer.[8] In the present investigation, the base damping is referred to as the damping of the beam with PZT shorted for all the cases. The inductive shunting experiments are done for the beam with 0–200 H. The effect of inductance is studied. The enhancement of damping in the present case is mainly attributed to the inherent resistance of the inductance coil and is not significant.

The value of the electromechanical coupling coefficient, k_{ij}, given by the manufacturer is equal to 0.3. As stated in an earlier study,[9] this value is quite small to observe a significant shift in the frequencies between open- and short-circuit terminations of the PZT. The generalized electromechanical coupling coefficient K_{ij}, evaluated from eqn (36), and using the value of K_{ij} = 0.3, is 0.12. However, the experimentally evaluated[9] value was found to be 0.035. This large deviation is attributed to the effect of the bonding layer on the PZT's ability to transform mechanical energy into electrical energy. The optimal values of the inductance and resistance needed for the resonant shunting (RLC circuit across beam), with this modified K_{ij}, are listed in Table II. These are basically calculated from eqns (38) and (39) with the corrected generalized electromechanical coupling coefficient. Note that the optimal inductance needed would be 600 H and the optimal resistance needed is 3800 Ω. These values differ significantly from those calculated using the theoretically estimated value of K_{ij} = 0.12. The analytically simulated frequency response of the piezo-shunted duralumin beam with these theoretical values of optimal inductance and resistance is shown in Fig. 4.

The experimental simulations confirm the discrepancy stated above. Only the inductance was varied. There was no shift in the resonance frequency for different values of inductive shunting. This too confirms the fact stated above, namely that K_{ij} is quite low. The tip response amplitude, shown in Fig. 7, reduces to an extent of 4%, is basically due to the addition of inductance (equivalent to the addition of mass) and the damping is due to inherent resistance of the inductance coil which is 110 Ω for every 10 henry.

5. Conclusions

The effect of resonant-shunting of a piezoceramic bonded to a duralumin cantilever beam is investigated with reference to its vibration behaviour, namely, to its reduction of tip response amplitude and additive damping, and change in its resonance frequency. The overall reduction in tip amplitude is around 4% for a piezoceramic layer with electromechanical coupling coefficient (k_{31}) equal to 0.30. However, higher values (k_{31} = 0.36, typically applied in beams and rods) of electromechanical coupling coefficient result in significantly higher levels of reduction of vibration amplitude with a change in natural frequency from short- to open-circuit value. A reduction of

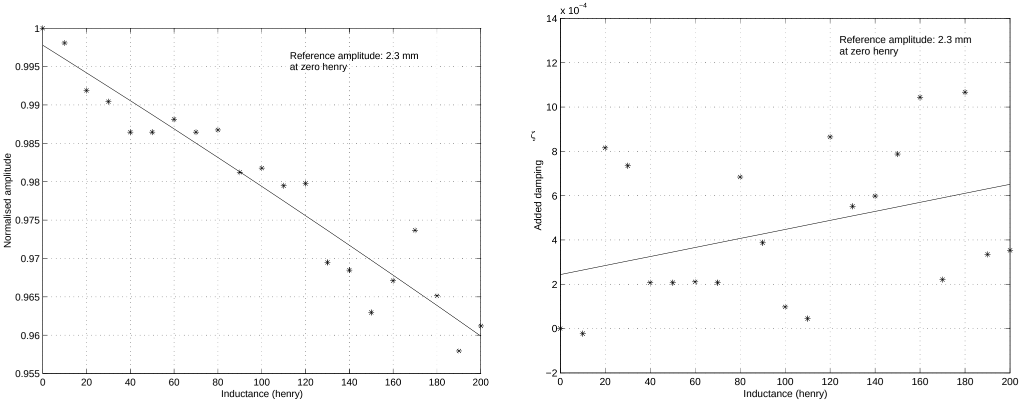

FIG. 7. Response of beam with PZT (inductive shunted).

Table II
Experimental tuning parameters for resistor and resonant shunting

Cantilever beam	
Natural frequencies without PZT (I, II modes)	16.8 and 115 Hz
Natural frequencies with PZT (I, II modes)	22.5 and 96 Hz
Natural frequency (shorted)	22.5 Hz
Natural frequency (open)	22.5 Hz
Damping ratio (without PZT)	0.0097
Damping ratio (with PZT, ref: short)	0.0162
Generalized coupling coefficient (estimated) K_{ij}	0.12
Generalized coupling coefficient (measured) K_{ij}	0.035
Inductance tuning	
Optimal inductance	L = 600 H
Optimal resistance	R = 3800 Ω
Optimal frequency tuning	1.1012
Optimal dissipation tuning	0.05

20–30% in response amplitude and a change in natural frequency of 8–10% (open and short-circuit) are possible when the planar electromechanical coupling coefficient (k_p, typically applied in discs and plates) is 0.6–0.65.

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Nomenclature

q : electrical displacement (charge/area)
R : generic resistance
L : generic inductance

d_{ij} : piezoelectric material constant
$\mathbf{\Phi}$: vector of electric fields (volts/meter)
$\mathbf{i}$: vector of external applied current (amps)
ε : strain
$\mathbf{v}$: voltage
$\mathbf{C}$: compliance matrix
$\mathbf{E}$: piezoceramic dielectric constant matrix
E : elastic modulus of the material
$\mathbf{A}$: diagonal matrix of cross-sectional areas of piezoelectric bar
s : Laplace variable
C_{pi} : inherent capacitance of piezoelectric in the i^{th} direction
$\mathbf{Y}_c^D$: open-circuit admittance of the piezoelectric
$\mathbf{Y}^{SU}$: shunting admittance
$\mathbf{Y}$: electrical admittance of the piezoelectric
$\mathbf{Z}$: electrical impedance of piezoelectric (shunting impedance in parallel to the inherent capacitance)
$\bar{\mathbf{Z}}_{jj}^{ME}$: effective mechanical impedance of shunted piezoelectric
δ : ω_e/ω_n, resonant shunted piezoelectric frequency parameter
η : loss factor
σ : material stress
γ : complex nondimensional frequency, s/ω_n
ρ : nondimensional resistance (or frequency)
ω_n : natural frequency of the structure (rad/s)
ω_e : resonant shunted piezoelectric electrical resonant frequency (rad/s)
r : dissipation tuning parameter ($RC_{pi}\omega_n$)
g : ω/ω_n, real nondimensional frequency ratio
d_{31} : piezoelectric constant (strain/field)
K : modal stiffness of the beam
K_{pzt} : stiffeness due to PZT bonded to the structure.[6]
M : modal mass of the beam

Subscripts

pi : piezoelectric

Superscripts

E : value taken at constant field (short circuit)
D : value taken at constant electrical displacement (open circuit)
RSP : relates to resonant shunting
SU : shunted value
CR : value at constant strain
CS : value at constant stress
T : transpose of a matrix or vector