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TRANSPORT PROCESSES IN TWO-COMPONENT PLASMA ON THE BASIS OF LANDAU EQUATION

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ABSTRACT

In this paper, transport processes in a fully ionized plasma, governed by the kinetic equation proposed by Landau, have been investigated by expanding the distribution functions of ions and electrons in terms of generalized Hermite Polynomials, following Grad. From the resulting transport equations, expressions for viscosity, thermal conductivity, diffusivity, electrical conductivity and Townsend coefficients in the presence of a constant, uniform, strong magnetic field, are aeduced. These expressions have similar forms to those obtained earlier by the same procedure by Devanathan, Raghavachar and Ram Babu on the basis of Fokker-Planck equation. This is as expected since Landau equation, in principle, is another version of the Fokker-Planck equation, taking into account small simultaneous particle interactions. Using the expression for electrical conductivity, the decay length of disturbances in the stellar photosphere, like regions of turbulence, is calculated and is found to increase both with the decrease in number density and increasing temperature, thereby providing an efficient mechanism for coronal heating. Further, the ratio of thermal conductivity to electrical conductivity has linear dependence on temperature in agreement with the Wiedemann-Franz law, although the slopes in the two cases are different. The other transport coefficients show the same behaviour as in earlier investigations.

1. INTRODUCTION

The behaviour of ionized gases is described by a system of kinetic equations:

$$\frac{\partial f_\alpha}{\partial t} + v_{\alpha i}\frac{\partial f_\alpha}{\partial x_i} + \frac{F_{\alpha i}}{m_\alpha}\frac{\partial f_\alpha}{\partial v_{\alpha i}} = \left(\frac{\delta f_\alpha}{\delta t}\right)_c \qquad [1.1]$$

where f_α, $\mathbf{v}_\alpha$, $\mathbf{F}_\alpha$ and m_α are respectively the distribution function, velocity, external force and the mass of the α-*th* particle. With the collision term as described by the *B-G-K* model[1] of the Boltzmann collision integral, Devanathan, Uberoi and Bhatnagar[2] have studied transport processes in a collision-dominated multicomponent plasma, based on the procedure developed by Grad[3]. But, for a fully ionized plasma, small simultaneous collisions dominate its behaviour as pointed out in detail by Jeans[4] and Spitzer *et al*[5]. Devanathan, Raghavachar and Ram Babu[6] have studied small random fluctuations by using the Fokker-Planck equation. To simplify the extreme mathematical complexity of the Fokker-Planck equation, various models for it have been proposed[7,8]. Recently, we[9] investigated the model given by Dougherty and Watson[8]. According to this model we found that the ratio of the thermal conductivity to viscosity was $5k/3m$ as compared to the value $5k/2m$ of the *B-G-K* model, owing to the fact that the diffusion term in the collision part is underestimated in the model. Consequently, a better representation of small, simultaneous collision effects is sought for through a relatively simple collision term first given by Landau[10]:

$$\left(\frac{\delta f_\alpha}{\delta t}\right)_c = \sum_{s=1}^{N} 2\pi A_{\alpha s}\frac{\partial}{\partial v_{\alpha i}}\int\left(\frac{f_s}{m_\alpha}\frac{\partial f_\alpha}{\partial v_{\alpha j}} - \frac{f_\alpha}{m_s}\frac{\partial f_s}{\partial v_{sj}}\right)U_{ij}\,d\mathbf{v}_s \qquad [1.2]$$

where

$$A_{\alpha s} = \frac{e_\alpha^2 e_s^2 \ln \Lambda_\alpha}{m_\alpha}, \qquad [1.3]$$

$$\Lambda_{\alpha s} = \frac{3}{2 z_\alpha z_s e^3}\left[\frac{k^3 T_\alpha^3 T_s^3}{\pi(N_\alpha T_s + N_s T_\alpha)(T_\alpha + T_s)^2}\right]^{\frac{1}{2}},$$ the ratio of Coulomb cut-off parameters,

and $$U_{ij} = \frac{\delta_{ij}}{|\mathbf{v}_\alpha - \mathbf{v}_s|} - \frac{(v_{\alpha i} - v_{si})(v_{\alpha j} - v_{sj})}{|\mathbf{v}_\alpha - \mathbf{v}_s|^3}, \qquad [1.5]$$

in Cartesian tensor notation. This equation has been used by Braginskii[11] to calculate the transport properties for a two-component assembly using the Chapman-Enskog-Hilbert method. Recently, Srivastava[12] has applied Grad's method to electron component of Landau equation, neglecting ion dynamics, in order to study viscosity and heat conductivity.

In the present paper, the investigation of transport processes is generalized to a two-component assembly of fully ionized gas on the basis of the Landau equation, using the Grad's method. The transport equations are obtained in §2. And in the next article, the expressions for viscosity, thermal conductivity, diffusivity, electrical conductivity and Townsend coefficients, in the presence of a constant and uniform strong magnetic field are derived.

2. Transport Equations for Non-Equilibrium Phenomena

The notations and the procedure of obtaining the closed form of transport equations are the same as in the reference [6]. Since, we are interested only in simple situations, we have derived equations for density, mean velocity, stresses and heat flux only. Some of these are recorded below:

$$\frac{\partial N_\alpha}{\partial t} + \frac{\partial}{\partial x_i}(N_\alpha u_{\alpha\alpha i}) = 0, \tag{2.1}$$

$$\frac{1}{N_\alpha}\frac{\partial}{\partial t}(N_\alpha u_{\alpha\alpha p}) + \frac{1}{N_\alpha}\frac{\partial}{\partial x_i}(N_\alpha P_{\alpha ip}) - \frac{1}{m_\alpha}(e_\alpha E_p + F_p) - \frac{e_\alpha}{cm_\alpha}\epsilon_{pjk} H_k u_{\alpha\alpha j}$$

$$= \sum_{s=1}^{2} \frac{8 A_{\alpha s} N_s}{3 M_{\alpha s} c_{\alpha s}^5}\left(\frac{\pi}{2}\right)^{1/2} \Big[\tfrac{7}{10} c_{\alpha s}^2 (u_{ssp} - u_{\alpha\alpha p}) + \tfrac{3}{10}(u_{\alpha\alpha p} P_{sii} - u_{ssp} P_{\alpha ii})$$

$$+ \tfrac{6}{5}(u_{\alpha\alpha i} P_{sip} - u_{ssi} P_{\alpha ip}) - \tfrac{6}{5}(u_{\alpha\alpha p} P_{spp} - u_{ssp} P_{\alpha pp})$$

$$+ \tfrac{4}{5}(S_{sppp} - S_{\alpha ppp}) - \tfrac{9}{10}(S_{sp} - S_{\alpha p})\Big], \tag{2.2}$$

$$\frac{1}{N_\alpha}\frac{\partial}{\partial t}(N_\alpha P_{\alpha pp}) + \frac{1}{N_\alpha}\frac{\partial}{\partial x_i}(N_\alpha S_{\alpha ipp}) - \frac{2}{m_\alpha}(e_\alpha E_p + F_p)\, u_{\alpha\alpha p} - \frac{2 e_\alpha}{cm_\alpha}\epsilon_{pjk} H_k P_{\alpha pj}$$

$$= \sum_{s=1}^{2} \frac{8 A_{\alpha s} N_s}{m_\alpha c_{\alpha s}^3}\left(\frac{\pi}{2}\right)^{1/2}\left[\frac{2}{15} u_{\alpha\alpha i} u_{ssi}\left(1 + \frac{m_\alpha}{M_{\alpha s}}\frac{c_\alpha^2}{c_{\alpha s}^2}\right) + \frac{11}{5} c_{\alpha s}^2\right.$$

$$+ \frac{1}{15} c_s^2 + \frac{2 m_\alpha}{3 M_{\alpha s}}\frac{c_\alpha^2 c_s^2}{c_{\alpha s}^2} - \frac{2 m_\alpha}{M_{\alpha s}} c_\alpha^2 + \frac{2 u_{\alpha\alpha p} u_{ssp}}{5}\left(1 + \frac{m_\alpha}{M_{\alpha s}}\frac{c_\alpha}{c_{\alpha s}} + \frac{5 m_\alpha}{3 M_{\alpha s}}\frac{c_s}{c_{\alpha s}}\right)$$

$$+ \frac{1}{15} P_{\alpha ii}\left(1 - \frac{3 m_\alpha}{M_{\alpha s}}\frac{c_\alpha^2}{c_{\alpha s}^2}\right) + \frac{1}{15} P_{sii}\left(1 - \frac{3 m_\alpha}{M_{\alpha s}}\frac{c_\alpha c_s}{c_{\alpha s}^2}\right)$$

$$\left. + \frac{1}{5} P_{spp}\left(\frac{m_\alpha}{M_{\alpha s}}\frac{c_\alpha^2}{c_{\alpha s}^2} - 1\right) - \frac{1}{5} P_{\alpha pp}\left(1 + \frac{2 m_\alpha}{M_{\alpha s}}\frac{c_\alpha^2}{c_{\alpha s}^2} + \frac{4 m_\alpha}{M_{\alpha s}}\frac{c_s^2}{c_{\alpha s}^2}\right)\right] \tag{2.3}$$

and

$$\frac{1}{N_\alpha}\frac{\partial}{\partial t}(N_\alpha P_{\alpha pq}) + \frac{1}{N_\alpha}\frac{\partial}{\partial x_i}(N_\alpha S_{\alpha ipq}) - \frac{e_\alpha}{m_\alpha}(E_p u_{\alpha\alpha q} + E_q u_{\alpha\alpha p})$$

$$- \frac{1}{m_\alpha}(F_p u_{\alpha\alpha q} + F_q u_{\alpha\alpha p}) - \frac{e_\alpha H_k}{cm_\alpha}(\epsilon_{pjk} P_{\alpha jq} + \epsilon_{qjk} P_{\alpha jp})$$

$$= \sum_{s=1}^{2} \frac{8 A_{\alpha s} N_s}{3 m_\alpha c_{\alpha s}^3}\left(\frac{\pi}{2}\right)^{1/2}\left[\frac{4}{5} P_{spq}\left(\frac{m_\alpha}{M_{\alpha s}}\frac{c_\alpha^2}{c_{\alpha s}^2} - 1\right) - \frac{4}{5} P_{\alpha pq}\left(1 + \frac{2 m_\alpha}{M_{\alpha s}}\frac{c_\alpha^2}{c_{\alpha s}^2}\right.\right.$$

$$\left.\left. + \frac{4 m_\alpha}{M_{\alpha s}}\frac{c_s^2}{c_{\alpha s}^2}\right) + (u_{\alpha\alpha p} u_{ssq} + u_{\alpha\alpha q} u_{ssp})\left(\frac{m_\alpha}{M_{\alpha s}}\frac{c_s^2}{c_{\alpha s}^2} - \frac{m_\alpha}{5 M_{\alpha s}}\frac{c_\alpha^2}{c_{\alpha s}^2} - 2\right)\right] \quad [2.4]$$

where $$M_{\alpha s} = \frac{m_\alpha m_s}{m_\alpha + m_s}, \quad c_\alpha = \left(\frac{k T_{\alpha\alpha}}{m_\alpha}\right)^{1/2} \quad [2\ 5]$$

and $$c_{\alpha s} = (c_\alpha^2 + c_s^2)^{1/2}. \quad [2\ 6]$$

3. STATIONARY NON-EQUILIBRIUM PROCESSES

In this section, some important non-equilibrium processes are considered and the expressions for the various transport coefficients for macroscopically neutral two-component plasma are deduced.

Considering the Lorentz problem, from the momentum equation [2.2] the following expressions for the electrical conductivity σ and the generalized diffusion coefficients σ_α and σ_β are easily derived:

$$\sigma = (1/\Delta)(e_\beta N_\beta - e_\alpha N_\alpha)^2, \quad [3\ 1]$$

$$\sigma_\alpha = (N_\alpha/\Delta)(e_\beta N_\beta - e_\alpha N_\alpha), \quad [3.2]$$

$$\sigma_\beta = (N_\beta/\Delta)(e_\alpha N_\alpha - e_\beta N_\beta) \quad [3.3]$$

where

$$\Delta = \frac{32}{3}\frac{m_\alpha N_\alpha N_\beta A_{\alpha\beta}}{M c_{\alpha\beta}{}^3}\left(\frac{\pi}{2}\right)^{1/2}. \quad [3\ 4]$$

Because of the density dependence of σ, as suggested by Lighthill[13], it is of great importance to study the variation with number density of the characteristic decay distance Z_0, as given by Alfven[14]:

$$Z_0 = \left(\frac{H_0 \lambda^2}{c^2 \pi^{1/2}}\right)\cdot\frac{\sigma}{\rho^{1/2}}. \quad [3.5]$$

The values of Z:

$$Z = \sigma \rho^{-1/2}, \quad \rho = m_e N_e + m_p N_p , \tag{3.6}$$

corresponding to fully ionized hydrogen plasma, with electron and proton species at equal temperature and equal number density, are tabulated in Table I. Also, the variation of σ with N and T is shown in Table II.

TABLE I

Values of $\text{Log}_{10} Z$

T °K \ N cm^{-3}	10^3	10^6	10^9	10^{12}	10^{15}
10^2	19.6371	18.2796	16.9783		
10^3	21.0422	19.6371	18.2796	16.9783	
10^4	22.4597	21.0422	19.6371	18.2796	19.9783
10^5	23.8888	22 4597	21.0422	19.6371	18.2796
10^6	25.3342	23.8964	22.4686	21.0531	19.6608

TABLE II

Values of $\text{Log}_{10} \sigma$

T °K \ N cm^{-3}	10^3	10^6	10^9	10^{12}	10^{15}
10^2	9.2587	9.3896	9 5898		
10^3	10.6537	10.7587	10.8914	11.0899	
10^4	12.0712	12.1535	12 2586	12.3896	12.5899
10^5	13.5004	13.5713	13.6537	13.7597	13.8914
10^6	14 9460	15.0076	15.0792	15.1644	15.2723

It is found that the value of Z increases with decreasing number density and also with increasing temperature. Thus one can conclude that the dependence of the electrical conductivity on number density helps efficient heating in the Coronal region. This result is analogous to the one found by Howe[15] on the basis of σ derived for the *B-G-K* model.

The first Townsend coefficient[16] corresponding to the electron component is

$$A = \sigma_\alpha \, k \, T_{\alpha\alpha} / N_\alpha \, \sigma \tag{3 7}$$

and thus depends linearly on temperature, and is inversely proportional to number density. This corresponds to the expression obtained by Druyvesteyn[17]. The Townsend coefficient associated with temperature gradients is given by

$$k\,\sigma_\alpha\,/\,\sigma \qquad [3.8]$$

and is hence independent of both the temperature and the number density. The values of A are given in Table III.

TABLE III

Values of $\mathrm{Log}_{10}\,A$

T°K \ Ncm^{-3}	10^3	10^6	10^9	10^{12}	10^{15}
10^2	−5.8425	−2.8425	0.1575	3.1575	6.1575
10^3	−6.8425	−3.8425	−0.8425	2.1575	5.1575
10^4	−7.8425	−4.8425	−1.8425	1.1575	4.1575
10^5	−8.8425	−5.8425	−2.8425	0.1575	3.1575
10^6	−9.8425	−6.8425	−3.8425	−0.8425	2.1575

In order to study the coefficients of viscosity, diffusivity and thermal conductivity, choose z-axis to be in the direction of the primitive magnetic field. Replacing the heat flux tensors by its equivalent lower order moments in the stress equations [2.3] and [2.4], and concentrating on the gradient dependence of the stresses, we obtain :

$$P_{\alpha 33} = -\mu^{(0)}_{\alpha 33}\, e_{\alpha 33} - \mu^{(1)}_{\alpha 33}\, \nabla\, T_{\alpha\alpha} - \mu^{(2)}_{\alpha 33}\, \nabla\, N_\alpha - \mu^{(3)}_{\alpha 33}\,(e_{\alpha 11} + e_{\alpha 22} + e_{\alpha 33}) \qquad [3.9]$$

where

$$\mu^{(0)}_{\alpha 33} = \frac{2\,k\,T_{\alpha\alpha}}{m_\alpha\,\Delta_1}, \qquad [3.10]$$

$$\mu^{(1)}_{\alpha 33} = \frac{k}{m_\alpha\,\Delta_1}(u_{\alpha\alpha 1},\ u_{\alpha\alpha 2},\ 3u_{\alpha\alpha 3}) - \frac{5\,k\,\Delta_2}{m_\alpha\,\Delta_1\,(\Delta_1 + 3\,\Delta_2)}(u_{\alpha\alpha 1},\ u_{\alpha\alpha 2},\ u_{\alpha\alpha 3}), \qquad [3.11]$$

$$\mu^{(2)}_{\alpha 33} = \frac{T_{\alpha\alpha}}{N_\alpha}\,\mu^{(1)}_{\alpha 33}, \qquad [3.12]$$

$$\mu^{(3)}_{\alpha 33} = \frac{1}{2}\,\mu^{(0)}_{\alpha 33}\,\frac{\Delta_1 - 2\,\Delta_2}{\Delta_1 + 3\,\Delta_2}, \qquad [3.13]$$

$$\Delta_1 = \frac{56 A_{\alpha\alpha} N_\alpha \pi^{1/2}}{5\, m_\alpha\, c_\alpha^3} + \frac{8\, A_{\alpha\beta} N_\beta}{5\, m_\alpha\, c_{\alpha\beta}^3}\left(\frac{\pi}{2}\right)^{1/2}\left(1 + \frac{2m}{M_{\alpha\beta}}\;\frac{c_\alpha^2}{c_{\alpha\beta}^2} + \frac{4\, m_\alpha}{M_{\alpha\beta}}\;\frac{c_\beta^2}{c_{\alpha\beta}^2}\right) \quad [3.14]$$

$$\Delta_2 = \frac{32\, A_{\alpha\alpha} N_\alpha\, \pi^{1/2}}{15\, m_2\, c_\alpha^3} + \frac{8\, A_{\alpha\beta} N_\beta}{15\, m_\alpha\, c_{\alpha\beta}^3}\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{3\, m_\alpha}{M_{\alpha\beta}}\;\frac{c_\alpha^2}{c_{\alpha\beta}^2} - 1\right). \quad [3.15]$$

Similarly,

$$\begin{pmatrix} P_{\alpha 23} \\ P_{\alpha 13} \end{pmatrix} = -\mu_{\alpha 3}^{(0)} \begin{pmatrix} e_{\alpha 23} \\ e_{\alpha 13} \end{pmatrix} - \mu_{\alpha 3}^{(1)}\, \nabla\, T_{\alpha\alpha} - \mu_{\alpha 3}^{(2)}\, \nabla\, N_\alpha \quad [3.16]$$

where

$$\mu_{\alpha 3}^{(0)} = \frac{2\, k\, T_{\alpha\alpha}}{m_\alpha\, [(16/9)\; \Delta_1^2 + w_\alpha^2]} \begin{pmatrix} (4/3)\, \Delta_1 & -w_\alpha \\ w_\alpha & (4/3)\, \Delta_1 \end{pmatrix}, \quad [3.17]$$

$$\mu_{\alpha 3}^{(1)} = \frac{1}{2\, T_{\alpha\alpha}}\; \mu_{\alpha 3}^{(0)} \begin{pmatrix} 0 & u_{\alpha\alpha 3} & u_{\alpha\alpha 2} \\ u_{\alpha\alpha 3} & 0 & u_{\alpha\alpha 1} \end{pmatrix}, \quad [3.18]$$

and

$$\mu_{\alpha 3}^{(2)} = (T_{\alpha\alpha}/N_\alpha)\; \mu_{\alpha 3}^{(1)}, \quad w_\alpha = (e_\alpha\, \mathrm{H}_0/cm_\alpha)\,. \quad [3.19]$$

And

$$P_{\alpha 12} = -\mu_{\alpha 12}^{(0)} \begin{Bmatrix} e_{\alpha 11} \\ e_{\alpha 12} \\ e_{\alpha 22} \end{Bmatrix} - \mu_{\alpha 12}^{(1)}\, \nabla\, T_{\alpha\alpha} - \mu_{\alpha 12}^{(2)}\, \nabla\, N_\alpha \quad [3.20]$$

where

$$\mu_{\alpha 12}^{(0)} = \frac{3\, k\, T_{\alpha\alpha}}{2\, m_\alpha\; \Delta_3}\, (-w_\alpha\,,\;\; \Delta_1\,,\;\; w_\alpha)\,, \quad [3.21]$$

$$\mu_{\alpha 12}^{(1)} = \frac{k}{4\, m_\alpha\; \Delta_1}\, (\, \Delta_1\, u_{\alpha\alpha 2} - 2\, w_\alpha\, u_{\alpha\alpha 1}\,,\;\; \Delta_1\, u_{\alpha\alpha 1} + 2\, w_\alpha\, u_{\alpha\alpha 2}\,,\;\; 0), \quad [3.22]$$

$$\mu_{\alpha 12}^{(2)} = \frac{T_{\alpha\alpha}}{N_\alpha}\, \mu_{\alpha 12}^{(1)}\,, \quad [3.23]$$

$$\Delta_3 = \Delta_1^2 + 3\, w_\alpha^2 \;\;\text{and}\;\; \Delta_4 = \Delta_1^2 + (3/2)\, w_\alpha^2\,. \quad [3.24]$$

Finally

$$P_{\alpha 11} = -\mu_{\alpha 1}^{(0)} \begin{Bmatrix} e_{\alpha 11} \\ e_{\alpha 12} \\ e_{\alpha 22} \end{Bmatrix} - \mu_{\alpha 1}^{(1)}\, \nabla\, T_{\alpha\alpha} - \mu_{\alpha 1}^{(2)} \nabla\, N_\alpha - \mu_{\alpha 33}^{(3)}\, (e_{\alpha 11} + e_{\alpha 22} + e_{\alpha 33}) \quad [3.25]$$

where

$$\mu_{\alpha 1}^{(0)} = \frac{k\, T_{\alpha\alpha}}{m_\alpha\; \Delta_1\; \Delta_3}\, (2\; \Delta_4,\;\; 3\, w_\alpha\; \Delta_1,\;\; 3\, w_\alpha^2)\,, \quad [3.26]$$

$$\mu_{\alpha 1}^{(1)} = \frac{k}{m_\alpha \Delta_3}\left[\frac{3 w_\alpha}{2}(u_{\alpha\alpha 2}, u_{\alpha\alpha 1}, 0) + \frac{\Delta_4}{\Delta_1}(3 u_{\alpha\alpha 1}, u_{\alpha\alpha 2}, u_{\alpha\alpha 3}) + \frac{3 w_\alpha^2}{2 \Delta_1}(u_{\alpha\alpha 1}, u_{\alpha\alpha 2}, u_{\alpha\alpha 3})\right] - \frac{5 k \Delta_2}{m_\alpha \Delta_1 (\Delta_1 + 3 \Delta_2)}(u_{\alpha\alpha 1}, u_{\alpha\alpha 2}, u_{\alpha\alpha 3}) \quad [3.27]$$

and

$$\mu_{\alpha 1}^{(2)} = \frac{T_{\alpha\alpha}}{N_\alpha} \mu_{\alpha 1}^{(1)}, \quad [3\ 28]$$

with similar expression for $P_{\alpha 22}$. The diagonal terms of the stress tensor depend also on the dilatation terms, in agreement with references [6, 9, 13] Also, the stresses in the plane containing the magnetic field are coupled by the field and [3.15] gives the viscosity in this plane as :

$$\frac{2 k T_{\alpha\alpha}}{m_\alpha \Delta_1}\left(1 - \frac{4 \Delta_1^2 + 9 w_\alpha^2}{16 \Delta_1^2 + 9 w_\alpha^2}\right). \quad [3\ 29]$$

The viscosity coefficients corresponding to $P_{\alpha 12}$, $P_{\alpha 11}$ and $P_{\alpha 22}$ are given by

$$\frac{2 k T_{\alpha\alpha}}{m_\alpha \Delta_1}\left(1 - \frac{(1/4) \Delta_1^2 + 3 w_\alpha^2}{\Delta_1^2 + 3 w_\alpha^2}\right) \quad [3.30]$$

and

$$\frac{2 k T_{\alpha\alpha}}{m_\alpha \Delta_1}\left(1 - \frac{(3/2) w_\alpha^2}{\Delta_1^2 + 3 w_\alpha^2}\right). \quad [3\ 31]$$

Thus the anisotropy due to the magnetic field is evident. The coefficients of viscosity in the plane perpendicular to the magnetic field are less than those in the plane containing the field.

Concentrating on the temperature and density gradients of the heat flux vector, we obtain

$$S_{\alpha 3} = - K_{\alpha 3}^{(0)} \frac{\partial T_{\alpha\alpha}}{\partial x_3} - K_{\alpha 3}^{(1)} \frac{\partial N_\alpha}{\partial x_3}, \quad [3.32]$$

where

$$K_{\alpha 3}^{(0)} = \frac{k^2 T_{\alpha\alpha}}{m_\alpha^2 \Delta_{\alpha 9}}, \quad [3.33]$$

$$\Delta_{\alpha 9} = \frac{\Delta_{\alpha 5} \Delta_{\alpha 6} - \Delta_{\alpha 7} \Delta_{\alpha 8}}{\Delta_{\alpha 5} + \Delta_{\alpha 6} - \Delta_{\alpha 7} - \Delta_{\alpha 8}}, \quad [3\ 34]$$

$$K_{\alpha 3}^{(1)} = \frac{T_{\alpha\alpha}}{2 N_\alpha} K_{\alpha 3}^{(0)}, \quad [3.35]$$

$$\Delta_{\alpha 5} = -\frac{72}{35} B_{\alpha\alpha} + 8 B_{\alpha\beta} \left[-\frac{256}{105} + \frac{2}{15} \frac{m_\alpha}{M_{\alpha\beta}} + \frac{52 m_\alpha}{63 M_{\alpha\beta}} \frac{c_\alpha^2}{c_{\alpha\beta}^2} + \frac{1}{5} \frac{c_\beta^2}{c_\alpha^2} - \frac{62 m_\alpha}{M_{\alpha\beta}} \frac{c_\beta^2}{c_{\alpha\beta}^2} + \frac{2 m_\alpha}{M_{\alpha\beta}} \frac{c_\beta^2}{c_\alpha^2} - \frac{49 m_\alpha}{6 M_{\alpha\beta}} \frac{c_\beta^4}{c_\alpha^2 c_{\alpha\beta}^2} - \frac{4}{3} \frac{c_\beta^2 c_{\alpha\beta}^2}{c_\alpha^4} + \frac{4 c_\beta^4}{c_\alpha^4} - \frac{8}{15} \frac{c_{\alpha\beta}^2}{c_\alpha^2} \right], \qquad [3.36]$$

$$\Delta_{\alpha 6} = \frac{59}{16} B_{\alpha\alpha} + \frac{1}{20} B_{\alpha\beta} \left[\frac{304}{7} - 16 \frac{m_\alpha}{M_{\alpha\beta}} - 32 \frac{c_\beta^2}{c_\alpha^2} + 40 \frac{c_\beta^4}{c_\alpha^4} - \frac{39 m_\alpha}{7 M_{\alpha\beta}} \frac{c_\alpha^2}{c_{\alpha\beta}^2} - 44 \frac{m_\alpha}{M_{\alpha\beta}} \frac{c_\beta^2}{c_{\alpha\beta}^2} + 20 \frac{m_\alpha}{M_{\alpha\beta}} \frac{c_\beta^4}{c_\alpha^2 c_{\alpha\beta}^2} \right], \qquad [3.37]$$

$$\Delta_{\alpha 7} = \frac{21}{10} B_{\alpha\alpha} + 3 B_{\alpha\beta} \left[\frac{48}{7} - \frac{8}{5} \frac{m_\alpha}{M_{\alpha\beta}} - \frac{8}{5} \frac{c_\beta^2}{c_\alpha^2} + \frac{4 m_\alpha}{5 M_{\alpha\beta}} \frac{c_\beta^2}{c_{\alpha\beta}^2} + \frac{13 m_\alpha}{7 M_{\alpha\beta}} \frac{c_\alpha^2}{c_{\alpha\beta}^2} \right], \qquad [3.38]$$

$$\Delta_{\alpha 8} = \frac{256}{105} B_{\alpha\alpha} + \frac{8}{3} B_{\alpha\beta} \left[\frac{46}{35} + \frac{1}{5} \frac{m_\alpha}{M_{\alpha\beta}} + \frac{74 m_\alpha}{105 M_{\alpha\beta}} \frac{c_\alpha^2}{c_{\alpha\beta}^2} + \frac{8 c_\beta^4}{c_\alpha^4} + \frac{12}{5} \frac{c_\beta^2}{c_\alpha^2} + \frac{9 m_\alpha}{M_{\alpha\beta}} \frac{c_\beta^4}{c_\alpha^2 c_{\alpha\beta}^2} - \frac{2 m_\alpha}{M_{\alpha\beta}} \frac{c_\beta^2}{c_{\alpha\beta}^2} + \frac{3 m_\alpha}{M_{\alpha\beta}} \frac{c_\beta^2}{c_\alpha^2} - \frac{2 c_\beta^2 c_{\alpha\beta}^2}{c_\alpha^4} - \frac{4}{5} \frac{c_{\alpha\beta}^2}{c_\alpha^2} \right], \qquad [3.39]$$

$$B_{\alpha\alpha} = \frac{A_{\alpha\alpha} N_\alpha \pi^{1/2}}{m_\alpha c_\alpha^3} \qquad [3.40]$$

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$$B_{\alpha\beta} = \frac{A_{\alpha\beta} N_\beta c_\alpha^2}{m_\alpha c_{\alpha\beta}^5} \left(\frac{\pi}{2} \right)^{1/2}. \qquad [3.41]$$

he thermal conductivity coefficient K of the binary mixture is approximately iven by

$$K = m_\alpha N_\alpha K_{\alpha 3}^{(0)} + m_\beta N_\beta K_{\beta 3}^{(0)}. \qquad [3.42]$$

The ratio K/σ becomes completely independent of the number density owing to the fact that the plasma is macroscopically neutral. In general, K/σ depends on the temperatures of both the species. However, when both the species are at the same temperature, K/σ depends linearly on temperature, in agreement with the Wiedemann-Franz law[18], although the slopes in the two cases are different. This indicates that plasma has, even in this simple model, more complicated equation of state.

Finally, equations [3.10] and [3.33] show that the ratio of thermal conductivity to viscosity along the magnetic field is no longer a constant, thus indicating that the relatively simple Landau equation is a very good representation of the small, simultaneous collisional effects than any other model Fokker-Planck equation.

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