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ON COMPRESSIBLE LAMINAR BOUNDARY LAYER FLOW OVER A SEMI-INFINITE FLAT PLATE WITH VARIABLE FLUID PROPERTIES

By B. C. PANDEY*

(Department of Applied Mathematics, Indian Institute of Science, Bangalore-12, India) (Received. January 3, 1969)

ABSTRACT

We have considered the effect of the variation of fluid properties, such as temperature dependence of the viscosity coefficient, on the boundary layer solution. To get over the difficulty of large computational work we have developed a series solution by splitting up the flow variables into sets of universal functions which are independent of the law governing the viscosity-temperature relation and also of the surface and free stream conditions. We have also given tables for the universal functions and have considered some examples.

1. INTRODUCTION

In this $p = w_c$ have node a study of the affield of the variation of fluid properties on the boundary layer solution, taking the flow over a semi-infinite flat plate at zero angle of incidence. The surface temperature of the plate has been assumed to be constant and the effect of pressure-gradient is neglected.

*Present Address :

Department of Mathematics, Birla Institute of Technology, Dhanbad, Sindri, Bihar.

If the product of viscosity and density $(\overline{\mu} \cdot \overline{\rho})$ is not constant, the velocity and temperature fields are coupled and the solutions of the equations governing the flow depend on the prescribed free-stream and surface conditions and on the parameter occurring in the viscosity-temperature relation. Hence, under such conditions, a large number of numerical solutions are required to study the characteristics of the flow.

In order to get over this difficulty of large computational work, we have developed a Görtler type series solution by splitting up the normalised stream function f and the normalised temperature θ into sets of universal functions which are independent of the law governing the viscosity-temperature relation and also of the surface and free stream conditions. The surface and free-stream parameters, and the parameter associated with the viscositytemperature relation appear as coefficients of the universal functions into which f and θ have been split up. The equations for universal functions have been integrated for Prandtl number 0.7 and the flow characteristics can be easily computed for a fairly good range of surface and free-stream conditions and for the parameter occuring in the viscosity temperature relation from these solutions. To illustrate the procedure we have drawn graphs corresponding to a few sets of parameters. The tables of the universal functions are given so that they might be used for computing the flow characteristics corresponding to other sets of parameters.

2. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

The non-dimensionalised equations governing the two-dimensional compressible steady boundary layer flow are

$$\frac{\partial}{\partial \bar{x}}(\bar{\rho}\,\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{\rho}\,\bar{v}) = 0, \qquad [2.1]$$

$$\vec{\rho} \, \overline{u} \, \frac{\partial \vec{u}}{\partial \vec{x}} + \vec{\rho} \, \overline{v} \, \frac{\partial \vec{u}}{\partial \vec{y}} = \frac{\partial}{\partial \vec{y}} \left(\vec{u} \, \frac{\partial \vec{u}}{\partial \vec{y}} \right), \qquad [2.2]$$

$$\overline{\rho}\,\overline{u}\,\frac{\partial\,\overline{T}}{\partial\,\overline{x}} + \overline{\rho}\,\overline{\sigma}\,\frac{\partial\,\overline{T}}{\partial\,\overline{y}} = (\gamma - 1)\,M_{\sigma}^{2}\,\overline{\mu}\,\left(\frac{\partial\,\overline{u}}{\partial\,\overline{y}}\right)^{2} + \frac{\partial}{\partial\,\overline{y}}\left(\frac{\overline{\mu}}{\sigma}\,\frac{\partial\,\overline{T}}{\partial\,\overline{y}}\right), \qquad [2.3]$$

and the equation of state for air obeying the perfect gas law is

$$\vec{p} = \vec{\rho} \cdot \vec{T}$$
 [2.4]

where

$$\overline{x} = (x/L), \quad \overline{y} = (y/L) \sqrt{R_{\bullet}}, \quad \overline{u} = (u/U_{\bullet}), \quad v = (v/U_{\bullet}) \sqrt{R_{\bullet}},$$

$$\tilde{\vec{p}} = (p/p_{\bullet}), \quad \tilde{p} = (p/p_{\bullet}), \quad \tilde{\mu} = (\mu/\mu_{\bullet}), \quad \tilde{T} = (T/T_{\bullet}),$$

 $c_p/c_p = \gamma = \text{constant}, \quad M_{\bullet} = \text{free stream Mach number},$
 $R_{\bullet} = \text{free stream Reynolds number}, \quad \sigma = \text{Prandtl number},$
 $L = \text{characteristic length}; \text{ and the variable with suffix ∞ denotes is value in free stream.}$

The boundary conditions are

(i) at
$$\overline{y} = 0$$
; $\overline{u} = 0$, $\overline{o} = 0$, and $\overline{T} = \overline{T}_w = \text{constant}$
(ii) at $\overline{y} = \infty$, $\overline{u} = 1$, $\overline{T} = 1$ [2.5]

In terms of the similarity variables defined by

$$\begin{split} \xi &= \frac{1}{x}, \quad \eta = \frac{1}{\sqrt{(2\xi)}} \int_{0}^{y} \vec{p} \, d\vec{y}, \\ \psi &= \sqrt{(2\xi)} \, f(\eta), \quad \vec{T} = \vec{T}(\eta) \end{split} \tag{2.6}$$

where the stream function ψ is such that

$$\overline{\rho}\,\overline{u} = \frac{\partial\psi}{\partial\overline{y}}, \quad \overline{\rho}\,\overline{u} = -\frac{\partial\psi}{\partial\overline{x}},$$

the equations [2.2] and [2.3] reduce to

$$(\vec{\rho}, \vec{\mu}, f')' + ff' = 0$$
 [2.7]

$$\left(\frac{\overline{\rho}}{\sigma}\frac{\overline{\mu}}{T'}\right)' \cdot f\overline{T}' + (\gamma - 1) M_{\bullet}^2 \overline{\rho} \overline{\mu} f''^2 = 0 \qquad [2.8]$$

where dash denotes differentiation with respect to η . In terms of the normalised temperature θ defined by

$$\theta = \frac{\overline{T}-1}{\overline{T}_{w}-1}, \quad i.e., \quad \overline{T} = (\overline{T}_{w}-1) \quad \theta + 1 = \epsilon \quad \theta + 1,$$

$$\epsilon = (\overline{T}_{w}-1) \neq 0, \qquad [2.9]$$

where

the equation [2.8] reduces to

$$\left(\frac{\bar{\rho}\,\bar{\mu}}{\sigma}\,\theta'\right)' + f\theta' + \frac{(\gamma-1)\,M_{*}^{2}}{\bar{T}_{\mu}-1}\,(\bar{\rho}\,\bar{\mu})\,f''=0 \qquad [2.10]$$

B. C. PANDEY

• and the boundary conditions [2.5] now reduce to

(i)
$$f(0) = f'(0) = 0$$
; $\theta(0) = 1$, (ii) $f'(\infty) = 1$; $\theta(\infty) = 0$ [2.11]

We assume that
$$\hat{\overline{\rho}} \ \overline{\mu} = 1 + a_1 \ \overline{T} + a_2 \ \overline{T}^2$$

= $A + \epsilon \ B \ \theta + \epsilon^2 \ a_2 \ \theta^2$. [2.12]

where

$$A = 1 + a_1 + a_2$$
, $B = a_1 + 2a_2$,

and the constants a_1 and a_2 can be determined by fitting the quadratic expression [2.12] with either the power-law for viscosity-temperature relation or with the Sutherland viscosity law.

We expand f and θ in powers of ϵ as follows :

$$f = f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots$$
, [2.13]

$$\theta = \theta_0 + \epsilon \ \theta_1 + \epsilon^2 \ \theta_2 + \cdots \qquad [2.14]$$

Substituting [2.12], [2.13] and [2.14] in [2.7] and [2.19] and by comparison of coefficients of equal powers of ϵ , we obtain the following recursive system of equations, of which all but the first are linear :

$$A f_0''' + f_0 f_0'' = 0, \qquad [2.13]$$

$$\frac{A}{\sigma} \theta_0'' + f_0 \theta_0' + \frac{A(\gamma - 1) M_{\bullet}^2}{T_w - 1} f_0''^2 = 0, \qquad [2.16]$$

$$A f_1''' + f_0 f_1'' + f_0'' f_1 = -- B (f_0''' \theta_0 + f_0'' \theta_0'), \qquad [2.17]$$

$$\frac{A}{\sigma}\theta_{1}^{\prime\prime}+f_{0}\theta_{1}^{\prime}=-\frac{B}{\sigma}(\theta_{0}^{\prime2}+\theta_{0}\theta_{0}^{\prime\prime})-\theta_{0}^{\prime}f_{1}-\frac{(\gamma-1)M_{*}^{2}}{\bar{T}_{w}-1}(B\theta_{0}f_{0}^{\prime\prime2}+2Af_{0}^{\prime\prime}f_{1}^{\prime\prime}),$$
[2.18]

$$A f_{2}^{\prime\prime\prime} + f_{0} f_{2}^{\prime\prime} + f_{0}^{\prime\prime} f_{2} = -f_{1} f_{1}^{\prime\prime} - (B \theta_{1}^{\prime} + 2 a_{2} \theta_{0} \theta_{0}^{\prime} f_{0}^{\prime\prime} - B (\theta_{0}^{\prime} f_{1}^{\prime\prime} + \theta_{0} f_{1}^{\prime\prime\prime}) - (B \theta_{1} + a_{2} \theta_{0}^{2} f_{0}^{\prime\prime\prime})$$

$$(2.19)$$

$$\frac{A}{\sigma}\theta_2^{\prime\prime}+f_0\theta_2^{\prime}=-\frac{B}{\sigma}\left(2\theta_1^{\prime}\theta_0^{\prime}+\theta_0\theta_1^{\prime\prime}+\theta_1\theta_0^{\prime\prime}\right)-\frac{a_2}{\sigma}\left(2\theta_0\theta_0^{\prime2}+\theta_0^2\theta_0^{\prime\prime}\right)$$

The boundary conditions [2.11] reduce to

$$f_{j}(0) - f_{j}'(0) = 0, \quad (j = 0, 1, 2, \cdots)$$

$$\theta_{0}(0) = 1; \quad \theta_{j}(0) = 0, \quad (j - 1, 2, \cdots)$$

$$f_{0}'(\infty) = 1, \quad f_{j}'(\infty) = 0 \quad (j = 1, 2, \cdots)$$

$$\theta_{1}(\infty) = 0 \quad (j = 0, 1, 2, \cdots).$$

[2.21]

3. Splitting of f and θ into Universal Functions

We find that the equations [2.15] to [2.20] contain coefficients which are dependent on the free stream and plate conditions and on the fluid properties. We split them into a set of recursive system of equations determining the universal functions which are independent of the surface and free-stream conditions and also of the law governing the viscosity-temperature relation.

We set

$$\overline{\eta} = \eta / \sqrt{A} , \qquad [3.1]$$

$$F_i - (f_i / \sqrt{A}), \quad (i = 0, 1, 2, \cdots)$$
 [3.2]

$$\theta_0 = \theta_{00} + [(\gamma - 1) M_{*}^2 / \bar{T}_w - 1] \theta_{01} , \qquad [3.3]$$

$$F_{1} = (B/A) F_{10} + (B/A) [(\gamma - 1) M_{\bullet}^{2}/T_{w} - 1] F_{11}, \qquad [3.4]$$

$$\theta_{1} = \frac{B}{A} \theta_{10} + \frac{B}{A} \left(\frac{(\gamma - 1)M_{*}^{2}}{\overline{T}_{w} - 1} \theta_{11} + \frac{B}{A} \left\{ \frac{(\gamma - 1)M_{*}^{2}}{\overline{T}_{w} - 1} \right\}^{2} \theta_{12}, \quad [3.5]$$

The equations [2.15] and [2.16] under the transformations [3.1] to [3.3] reduce to

$$F_0'' + F_0 F_0'' = 0, \qquad [3.6]$$

$$(1/\sigma) \,\theta_{00}^{\prime\prime} + F_0 \,\theta_{00}^{\prime} = 0\,, \qquad [3.7]$$

$$(1/\sigma) \theta_{01}^{\prime\prime} + F_0 \theta_{01}^{\prime} = -F_0^{\prime\prime \, z}, \qquad [3.8]$$

where

$$' \equiv d/d\eta$$
,

and the corresponding boundary conditions [2.21] become

(i) $F_0(0) = F'_0(0) = 0; \quad \theta_{00}(0) = 1, \quad \theta_{01}(0) = 0,$ (ii) $F'_0(\infty) = 1; \quad \theta_{00}(\infty) = \theta_{01}(\infty) = 0.$ [3.9]

B. C. PANDEY

The equations [2.17] and [2.18] under the transformations [3.1] to [3.5] reduce to

$$F_{10}^{\prime\prime\prime} + F_0 F_{10}^{\prime\prime} + F_0^{\prime\prime} F_{10} = -(F_0^{\prime\prime\prime} \theta_{00} + F_0^{\prime\prime} \theta_{00}^{\prime}), \qquad [3.10]$$

$$F_{11}^{\prime\prime\prime} + F_0 F_{11}^{\prime\prime} + F_0^{\prime\prime} F_{11} = -(F_0^{\prime\prime\prime} \theta_{01} + F_0^{\prime\prime} \theta_{01}^{\prime}), \qquad [3.11]$$

$$(1/\sigma)\theta_{10}^{\prime\prime} + F_0 \theta_{10}^{\prime} = -(1/\sigma) \left(\theta_{00}^2 + \theta_{00} \theta_{00}^{\prime\prime}\right) - F_{10} \theta_{00}^{\prime}, \qquad [3.12]$$

$$\begin{aligned} (1/\sigma) \,\theta_{11}^{\prime\prime} + F_0 \,\theta_{11}^{\prime} &= -(1/\sigma) \, (2 \,\theta_{00} \,\theta_{01} + \theta_{00} \,\theta_{01}^{\prime\prime} + \theta_{01} \,\theta_{00}^{\prime\prime}) \\ &- (\theta_{00}^{\prime} \,F_{15} + \theta_{01}^{\prime} \,F_{10} + \theta_{00} \,F_{00}^{\prime\prime\prime} + 2 \,F_0^{\prime\prime} \,F_{10}^{\prime\prime}), \end{aligned} \tag{3.13}$$

$$(1/\sigma) \theta_{12}'' + F_0 \theta_{12}'' = -(1/\sigma) (\theta_{01}^2 + \theta_{01} \theta_{01}'') - (\theta_{01}' F_{11} + \theta_{01} F_{01}''^2 + 2F_0'' F_{11}''), \quad [3.14]$$

and the corresponding boundary conditions [2.21] become

(i)
$$F_{ij}(0) = F'_{ij}(0) = 0,$$

(ii) $F'_{ij}(\infty) = 0,$
(j=0, 1) [3.15]

(iii)
$$\theta_{1j}(0) = 0$$

(iv) $\theta_{1j}(\infty) = 0$ } $(j=0, 1, 2)$ [3.16]

The equations [2.19] and [2.20], with the help of the relations [3.1] to [3.5] and the relations

$$F_{2} = \left(\frac{B}{A}\right)^{2} F_{20} + \left(\frac{B}{A}\right)^{2} \frac{(\gamma - 1)M^{2}}{(\bar{T}_{w} - 1)} F_{21} + \left(\frac{B}{A}\right)^{2} \left\{\frac{(\gamma - 1)M^{2}}{(\bar{T}_{w} - 1)}\right\}^{2} F_{22} + \frac{a_{2}}{A} F_{23}$$
$$+ \frac{a_{2}}{A} \frac{(\gamma - 1)M^{2}}{(\bar{T}_{w} - 1)} F_{24} + \frac{a_{2}}{A} \left\{\frac{(\gamma - 1)M^{2}}{(\bar{T}_{w} - 1)}\right\}^{2} F_{25}, \qquad [3.17]$$

and

•

$$\theta_{2} = \left(\frac{B}{A}\right)^{2} \theta_{26} + \left(\frac{B}{A}\right)^{2} \frac{(\gamma - 1)M_{-}^{2}}{(\bar{T}_{w} - 1)} \theta_{21} + \left(\frac{B}{A}\right)^{2} \left\{\frac{(\gamma - 1)M_{-}^{2}}{(\bar{T}_{w} - 1)}\right\}^{2} \theta_{22} + \left(\frac{B}{A}\right)^{2} \left\{\frac{(\gamma - 1)M_{-}^{2}}{(\bar{T}_{w} - 1)}\right\}^{2} \theta_{23} + \frac{a_{2}}{A} \theta_{24} + \frac{a_{2}}{A} \frac{(\gamma - 1)M_{-}^{2}}{(\bar{T}_{w} - 1)} \theta_{25} + \frac{a_{2}}{A} \left\{\frac{(\gamma - 1)M_{-}^{2}}{(\bar{T}_{w} - 1)}\right\}^{2} \theta_{26} + \frac{a_{2}}{A} \left\{\frac{(\gamma - 1)M_{-}^{2}}{(\bar{T}_{w} - 1)}\right\}^{4} \theta_{27}, \qquad [3.18]$$

reduce to

$$F_{20}^{\prime\prime\prime} + F_0 F_{20}^{\prime\prime} + F_0^{\prime\prime} F_{20}^{\prime} - -F_{10} F_{10}^{\prime\prime} - \theta_{10} F_0^{\prime\prime\prime} - \theta_{10}^{\prime} F_0^{\prime\prime} - \theta_{00}^{\prime} F_{10}^{\prime\prime} - \theta_{00} F_{10}^{\prime\prime\prime} - \theta_{00} F_{10}^{\prime\prime\prime} - g_{00} F_{10}^{\prime\prime} - g_{00} F_{10}^{$$

$$F_{22}^{\prime\prime\prime} + F_0 F_{22}^{\prime\prime} + F_0^{\prime\prime} F_{22}^{\prime\prime} - F_{11} F_{11}^{\prime\prime} - \theta_{12}^{\prime} F_0^{\prime\prime} - \theta_{12} F_0^{\prime\prime\prime} - \theta_{01}^{\prime} F_{11}^{\prime\prime} - \theta_{01} F_{11}^{\prime\prime\prime} , [3.21]$$

$$F_{23}^{\prime\prime\prime} + F_0 F_{23}^{\prime\prime} + F_0^{\prime\prime} F_{23}^{\prime} - \theta_{00}^2 F_0^{\prime\prime\prime} - 2 \theta_{00} \theta_{00}^{\prime} F_0^{\prime\prime}, \qquad [3.22]$$

$$F_{24}^{\prime\prime\prime} + F_0 F_{24}^{\prime\prime} + F_0^{\prime\prime} F_{24}^{\prime} - 2 \theta_{00} \theta_{01} F_0^{\prime\prime\prime} - 2 (\theta_{00} \theta_{01}^{\prime} + \theta_{01} \theta_{00}^{\prime}) F_0^{\prime\prime}, \qquad [3.23]$$

$$F_{25}^{\prime\prime\prime} + F_0 F_{25}^{\prime\prime} + F_0^{\prime\prime} F_{25} + -\theta_{01}^2 F_0^{\prime\prime} - 2\theta_{01} \theta_{01}^{\prime} F_0^{\prime\prime}, \qquad [3.24]$$

$$(1/\sigma) \theta_{20}^{\prime\prime} + F_0 \theta_{20}^{\prime} - (2/\sigma) \theta_{10}^{\prime} \theta_{00}^{\prime} - (1/\sigma) \theta_{10}^{\prime\prime} \theta_{00}^{\prime} - (1/\sigma) \theta_{10} \theta_{00}^{\prime\prime} - (1/\sigma) \theta_{10} \theta_{00}^{\prime\prime} - \theta_{10}^{\prime} \theta_{00}^{\prime\prime} - \theta_{10}^{\prime} \theta_{00}^{\prime} - (1/\sigma) \theta_{10}^{\prime\prime} \theta_{00}^{\prime\prime} - (1/\sigma) \theta_{00}^{\prime\prime} - (1/\sigma) \theta_{00}^{\prime\prime} - (1/\sigma) \theta_{00$$

$$(1/\sigma) \theta_{21}'' + F_0 \theta_{21}' - (2/\sigma) (\theta_{10}' \theta_{01}' + \theta_{11}' \theta_{00}') - (1/\sigma) (\theta_{10}'' \theta_{01} + \theta_{11}'' \theta_{00}) - (1/\sigma) (\theta_{10} \theta_{01}' + \theta_{11} \theta_{00}') - (\theta_{10}' F_{11} + \theta_{11}' F_{10}) - (\theta_{00}' F_{21} + \theta_{01}' F_{20}) - \theta_{10} F_{0}''^{-2} - 2F_{0}'' F_{10}'' \theta_{00} - F_{10}'^{-2} - 2F_{0}'' F_{20}'',$$

$$(3.26)$$

$$(1/\sigma) \theta_{22}'' + F_0 \theta_{22}'' - (2/\sigma) \theta_{11}' \theta_{01} - (2/\sigma) \theta_{12}' \theta_{00} - (1/\sigma) (\theta_{11}'' \theta_{01} + \theta_{12}'' \theta_{00}) - (1/\sigma) (\theta_{11} \theta_{01}' + \theta_{12} \theta_{00}'') - \theta_{11}' F_{11} - \theta_{12}' F_{10} - \theta_{00}' F_{22} - \theta_{01}' F_{21} - \theta_{11} F_{02}'' - 2 F_{10}' F_{11}'' - 2 (F_{10}'' \theta_{01} + F_{11}'' \theta_{00}) F_{0}'' - 2 F_{0}'' F_{21}'',$$

$$[3.27]$$

$$(1/\sigma) \theta_{23}'' + F_0 \theta_{23}'' = -(2/\sigma) \theta_{12}' \theta_{01} - (1/\sigma) \theta_{12}'' \theta_{01} - (1/\sigma) \theta_{12} \theta_{01}'' - 2F_0'' F_{11}'' \theta_{01} - F_{11}'' - 2F_0'' F_{22}'',$$

$$(1/\sigma) \theta_{12}'' + F_0 \theta_{12}' + F_0'' + F_0''$$

$$(1/\sigma) \,\theta_{24}'' + F_0 \,\theta_{24}'' = -(2/\sigma) \,\theta_{00} \,\theta_{00}'' - (1/\sigma) \,\theta_{00}^2 \,\theta_{00}'' - \theta_{00}' F_{23}, \qquad [3.29]$$

$$(1/\sigma) \; \theta_{25}'' + F_0 \; \theta_{25}'' = -(2/\sigma) \; (\theta_{01} \; \theta_{00}'' + 2 \; \theta_{00} \; \theta_{00}' \; \theta_{01}'') \\ -(1/\sigma) \; (2 \; \theta_{00} \; \theta_{00}'' \; \theta_{01} + \theta_{00}^2 \; \theta_{01}'') - \theta_{00}' \; F_{24} \\ -\theta_{01}' \; F_{23} - \theta_{00}^2 \; F_{00}''' - 2 \; F_{00}'' \; F_{23}'', \qquad [3.30]$$

$$(1/\sigma) \,\theta_{26}'' + F_{6} \,\theta_{26}' = -(2/\sigma) \,(2\,\theta_{01}\,\theta_{00}'\,\theta_{01}'\,\theta_{00}'\,\theta_{01}') \\ -(1/\sigma) \,(\theta_{01}^{2}\,\theta_{00}'\,+2\,\theta_{00}\,\theta_{01}\,\theta_{01}') \\ -\theta_{01}'\,F_{24} - \theta_{00}'\,F_{25} - 2\,\theta_{00}\,\theta_{01}\,F_{0}''\,^{2} - 2\,F_{0}''\,F_{24}',$$

$$[3.31]$$

$$(1/\sigma) \theta_{27}'' + F_0 \theta_{27}' = -(2/\sigma) \theta_{01} \theta_{01}' - (1/\sigma) \theta_{01}^2 \theta_{01}'' - \theta_{01}' F_{25}'' - \theta_{01}^2 F_0'' - 2F_0'' F_{25}'', \qquad [3.32]$$

and the corresponding boundary conditions [2.21] become

(i)
$$F_{2J}(0) = 0$$
, $F'_{2J}(0) = 0$
(ii) $F'_{2J}(\infty) = 0$
(iii) $\theta_{2J}(0) = 0$
(iv) $\theta_{2I}(\infty) = 0$
 $j = 0, 1, 2, 3, 4, 5, 6, 7$ [3.33]

4. DISCUSSION AND RESULTS

Using the solution of the equation [3.6] from [3], we have integrated the equations [3.7], [3.8], [3.10] to [3.14] under the boundary conditions [3.9], [3.15] and [3.16] for $\sigma = 0.7$ on the Elliott 803 computer at Hindustan Aeronautics Ltd., Bangalore. In order to illustrate the application of the universal functions given in tables 1 to 7, we have determined the coefficients a_1 and a_2 occuring in the relation [2.12] by fitting it with the power-law connecting the viscosity and temperature for the exponent $\omega = 0.5$ and the values for a_1 and a_2 so obtained are

$$a_1 = 0.635296$$

 $a_2 = -0.561750$, [4.1]

Using [4.1] we have drawn the profiles for different surface and free-stream conditions indicated in the figures 1 to 4. Figure 1 shows the velocity distribution for two sets of parameters

(i)
$$\frac{T_w - T_o}{T_o} = 0.1$$
, $(\gamma - 1) M_w^2 = 0.9$
(ii) $\frac{T_w - T_o}{T_o} = 0.2$, $(\gamma - 1) M_o^2 = 1.6$ [4.2]

We observe that the velocity corresponding to the set (ii) of [4.2] is greater. than that corresponding to the set (i) of [4.2] at all points. We also find that the velocity corresponding to each of the set (1) and (ii) of [4.2] is greater than that corresponding to Blasius solution.



FIG. 1 Velocity Profiles

Figure 2 shows the distribution of $f''(\overline{\eta})$ for the parameters

$$\frac{T_w - T_o}{T_o} = 0.1, \ (\gamma - 1) \ M_o^2 = 0.9$$
 [4.3]

The profile for $f''(\bar{\eta})$ corresponding to the Blasius equation is also drawn for sake of comparison. We observe that corresponding to the set of parameters [4.3], $f''(\bar{\eta})$ attains its maximum value at a point close to the plate and not on the plate itself as in case of Blasius profile. This difference is attributed to the dependence of $f''(\bar{\eta})$ on free-stream Mach number, freestream temperature, plate temperature and coefficients occuring in viscositytemperature relation.

In Figure 3 we observe that the temperature θ first rises, attains a maximum value close to the plate and then asymptotically attains its free stream value near the baundary layer edge.



F16. 2





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Temperature Gradient

Figure IV shows the distribution of temperature gradient $\theta'(\overline{\eta})$ across the boundary layer. We find that $\theta'(\overline{\eta})$ is equal to zero at $\overline{\eta} = 0.8$ which is in conformity with the occurrence of the maximum value of $\theta(\overline{\eta})$ at $\overline{\eta} = 0.8$ as shown in figure 3. $\theta'(\overline{\eta})$ with its greatest positive value on the plate $\overline{\eta} = 0.8$ gradually decreases, changes its sign on crossing $\overline{\eta} = 0.8$, attains minimum value at $\overline{\eta} = 2$ and finally asymptotically approaches zero near the boun ary layer edge.

The tables 1 to 7 for the universal functions θ_{00} , θ_{01} , F_{10} , F_{11} , θ_{10} , θ_{11} , θ_{12} can be easily utilised for computing the flow characteristics upto first order approximation for a fairly good range of free-stream and surface conditions and for laws governing viscosity-temperature relation to which the quadratic expression [2.12] can be fitted with.

$\overline{\eta}$	θ ₀₀	θ'_{00}	θώο

0	1	41393019	0
.1	.95860755	41390752	.00068001
.2	.91722303	41374884	.00271986
.3	.87586692	41331844	.00611252
.4	.\$3457289	41248176	.01084179
.5	.79338844	41110657	.01687568
.6	.75237380	- 40906480	.02416326
.7	.71160177	40623462	03262909
.8	.67115694	40250308	.04217097
.9	.63113460	39776892	.05265685
1.0	.59163948	39194566	.06392359
1.1	.55278407	38496458	.07577831
1.2	.51468681	37677772	.08800017
1.3	.47746960	36736046	.10034366
1.4	.44125571	35671370	.11254560
1.5	.40616693	34486522	.12433164
1.6	.37232094	33187044	.13542610
1.7	.33982837	31781196	-14556217
1.8	.305790 39	30279827	.15449104
1.9	.27929617	28696138	.16199280
2.0	.25142054	27045339	.16788608
2.1	.22522228	25344235	.17203538
2.2	.20074285	23610731	.17435665
2.3	.17800547	21863301	.17482023
2.4	.15701474	20120438	.17345218
2.5	.13775709	18400107	.17033114
2.6	.12020136	16719243	.16558362
2.7	.10430027	15(93302	.15937762
2.8	.08999191	13535892	.15191325
2.9	.07720183	12058495	.14341362

TABLE 1

B. C. PANDEY

TABLE 1-(contd)

$\overline{\eta}$	θοο	θ'_{00}	$\theta_{ao}^{''}$
·			
3.0	.06584520	10670289	.13411468
3.1	.05582925	09378074	.12425534
3.2	.04705559	08186281	.11406824
3.3	.03942251	07097077	.10377241
3.4	.03282723	06110539	.09356631
3.5	.02716782	05224880	.08362345
3.6	.02234499	04436714	.07408934
3.7	.01826352	03741351	.06508002
3.8	.01483332	03133084	.05668221
3.9	.01197049	02605482	.04895452
4.0	.00959780	02151659	.04192986
4.1	.00764501	01764517	03561833
4.2	.00604897	01436956	.03001069
4.3	.00475358	01162050	.02508193
4.4	.00370956	00933188	.02079488
4.5	.00287396	- 00744176	.01710365
4.6	.00220989	00589310	.01395669
4.7	.00168574	00463419	.01129950
4.8	.00127495	00361880	.00907696
4.9	.00095527	00280618	.00723510
5.0	.00070820	00216087	.00572256
5.1	.00051853	00165235	.00449152
52	.00037403	00125469	.00349840
5.3	.00026467	00094609	.00270417
5.4	.00018246	00070842	.00207443
5.5	.00012112	00052675	.00157934
5.6	.00007567	00038894	.00119338
5.7	.00004222	00028519	.00089498
5.8	.00001776	00020765	.00066619
5.9	.00000000	00015014	.00049219

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	i) ₀₁	θ'_{ot}	θ_{01}''
0	0	.17295104	15436691
.1	.01652305	.15750614	15460136
2.	0.3149987	.14202171	15510729
.3	.04492563	.12648556	15558666
.4	.05679580	.11091515	15574374
.5	.06710904	.09535713	15529138
.6	.07587009	.07988623	15395764
.7	.08309251	.06460316	15149654
.8	.08880107	.04963143	14770066
.9	.09303388	.03511283	14241232
1.0	.09584389	.02120191	13553746
1.1	.09729987	.00805901	12705529
1.2	.09748657	00415737	11702354
1.3	.09650424	01529867	10558353
1.4	.09446707	02523439	09295459
1.5	.09150110	03385948	07942439
1.6	.08774134	04110063	06533624
17	.08332838	04692064	05106643
1.8	.07840460	05132076	03700262
1.9	.07311029	95434060	02352068
2,0	.06757999	05605555	01095966
2.1	.06193912	05657276	+.00039507
2.2	.05630100	05602388	.01033114
2.3	.05076490	05455860	.01870712
2.4	.04541451	05233649	.02546425
2.5	.04031742	04951931	.03061638
2.6	.03552524	04626421	.03424030
2.7	.03107427	04271797	.03646483
2.8	.02698692	03901264	.03745133
2.9	.02327320	03526276	.03738595
3.0	.01993265	03156389	.03646335

TABLE 2

B. C. PANDEY

TABLE 2-(conctd)

η	θοι	θ'_{01}	θ_{01}^{\prime}
3.1	.01695616	02799243	.03486940
3.2	.01432796	02460646	.03278170
3.3	.01202731	02144725	.03035893
3.4	.01003007	01854127	.02773825
3.5	.00831016	01580238	.02503430
3.6	.00684059	01353412	.02233932
3.7	.00559447	01143185	.01972479
3.8	.00454572	00958474	.01724346
3.9	.00366954	00797752	.01493185
4.0	.00294284	00659197	.01281284
4.1	.05234444	00540817	.010.09810.
4.2	.00185518	00440548	.009:9026
4.3	.00145801	00356336	.00763554
4.4	.00113784	00286194	.00637442
4.5	.00088157	00228247	.00524429
4.6	.00067786	00180758	.00428009
4.7	.00051710	00142149	.00346559
4.8	.00039109	~.00111005	.00278412
4.9	.00029302	00086080	.00221928
5.0	.00021722	~.00066286	.00175537
5.1	.00015906	00050687	.00137778
5.2	.00011473	00038488	.00107315
5.3	.00008117	00029022	.00082952
5.4	.00005596	00021731	.00063634
5.5	.00003715	00016169	.00048447
5.6	.00002320	00011931	.00036608
5.7	.00001293	00008748	.00027454
5.8	.00000544	00006370	.00020436
5.9	.00000000	00004606	.00015098

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7]	<i>F</i> ₁₀	F_{10}	F_{10}''	F ₁₀ "
0	()	0	35029192	.19438162
.1	00171902	03405505	33076556	.19699554
.2	00674528	06613622	31073626	.20430074
.3	01487812	09617115	-,28977627	.21544995
.4	02590766	12404899	26754481	.22952782
.5	03961138	14962950	24379563	.24552973
.6	05575168	17275293	21838557	.26236478
.7	07407447	19325073	19128270	.27386401
.8	09430880	21095703	16257242	.29381331
.9	11616779	22572053	13245988	.30599653
1.0	13935065	23741640	10126783	.31425581
1.1	16354590	24595763	06942783	.51755709
1.2	18843570	25130524	03746529	.31506331
1.3	21370105	25347557	00597730	.30520064
1.4	23902777	25255108	.03439520	.29071751
1.5	26411285	24867317	.05300044	.26871933
1.6	28867095	24205130	.07921116	.24068646
1.7	31244058	23295395	.10246036	.20745632
1.8		22170174	.12227415	.17017881
1.9	35672122	20865668	.13829954	.13024687
2.0	37687454	19420918	.15032408	.08919831
2.1		17876315	.15828656	.04861396
2.2	41260731	16272105	.86227710	.01000681
2.3	42806704	14646895	.16252736	02527528
2.4	44190601	13036347	.15939216	05613556
2.5	45415517	11472074	.15332358	08178341
2.6	46487451	09980802	.14484115	10176117
2.7	47414818	08583872	.13449996	11594757
2.8	48207891	07297042	.12285945	12452922
2.9	.48878232	06130580	.11045564	12795217

TABLE 3

B. C. PANDEY

TABLE 3-(contd)

$\overline{\eta}$	F ₁₀	F'_{10}	<i>F</i> ["] ₁₀	F ₁₀ ""
3.0	49438185	05089633	.09777780	12686201
3.1	49900357	04174766	.08525121	.12203491
3.2	50277221	03382684	.07322606	11430952
3.3	,50580765	02706984	.06197259	10452638
3.4	50822204	02138978	.05168188	09347545
3.5	51011801	01668435	.04247674	08185926
3.6	51158763	01284295	.03439042	07026568
3.7	51271160	00975273	.02743714	05915847
3.8	51355945	00730329	.02156382	04887412
3.9	51419014	00539068	.01669177	03963178
4.0	51465228	00391984	.01272163	03154632
4.1	51498593	00280615	.00954307	02464774
4.2	51522299	00197624	.00704263	01889764
4.3	51938850	00136784	.00511019	~.01421329
4.4	51550221	00092941	.00364314	.010479()
4.5	51557867	00061900	.00254495	00756805
4.6	51562920	00040332	.00174937	00534733
4.7	51566165	00025643	.00117511	.00369017
4.8	51568210	00015855	.00077120	.00248157
4.9	51569447	- 00009485	.00049309	.00162095
5.0	51570182	00005449	.00030596	00102372
5.1	51570590	00002969	.00018310	00062014
5.2	51570807	00001502	.00010465	00035567
5.3	51570911	00000679	.00005618	00018852
5.4	51570958	00000246	.00002733	00008745
5.5	51570972	00000041	.00001100	00002956
5.6	51570964	.0000035	.00000236	00004:099
5.7	51570967	.00000046	.00000173	00001498
5.8	- 51570965	.00000027	.00000321	00001935
. 12:2 95217	51570961	.00000000	.00000288	00001606

η	F ₁₁	F' 11	$F_{11}^{\prime\prime}$	F'''
0	0	0	.01827934	08121781
. 1	.00007816	.00143387	.01051845	07400224
.2	.00026211	.00212782	.00348284	06666854
.3	.00048151	.00215537	-,00280393	05899391
.4	.00067053	.00159338	00829987	05083212
.5	.00078326	00052351	01295250	04211854
.6	.00076420	00096716	01670714	03287309
.7	.00057887	00278632	01951553	02320157
.8	.00019920	00483751	02134424	01329158
.9	00039307	00702199	02218243	00340246
1.0	00120635	00924135	02204819	.00615133
1.1	00223933	01140061	02099293	.01502343
1.2	00348152	01341168	01910313	.02286465
1.3	00491412	01519685	01649902	.02935359
1.4	00651121	01669193	01333014	.03422848
1.5	00824122	01784887	00976793	.03731340
1.6	01006869	01863745	-,00599599	.03853822
1.7	01195603	01904597	00219894	.03794764
1.8	01385541	01908077	.00144910	.03569316
1.9	.01576048	01876462	.00479510	.03204423
2.0	01760786	01813420	.00771567	.02731338
2.1	01937843	01723677	.01082296	.02187621
2.2	02104814	01612650	.01196710	.01611235
2.3	02259856	01486068	.01323543	.01037962
2.4	02401700	01349611	.01394885	.00498691
2.5	02529629	01208612	.01415574	.00017619
2.6	02643431	01067807	.01392470	00388680
2.7	-,02743332	00931181	.01333661	06711376
2.8	02829914	00801879	.01247717	00948692
2.9	02904030	00682190	.01143038	01104675

TABLE 4

B. C. PANDEY

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TABLE 4-(contd)

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			And a second sec	r 40.
$\overline{\eta}$	<i>F</i> ₁₁	<i>F</i> ′ ₁₁	<i>F</i> '' ₁₁	<i>F</i> ₁₁ "
3.0	02966723	00573595	.01027344	01187735
3.1	03019145	00476851	.00907324	01209046
3.2	03062494	00392105	.00788438	01181090
3.3	03097955	00319022	.00674862	01116429
3.4	03126664	00256911	.00569541	01026769
3.5	03149672	00204846	.00471310	+.00922357
3.6	03167932	00161762	.00390077	
3.7	03182286	00126543	.00317010	.00701167
3.8	03193465	00098086	.00254730	00595716
3.9	03202093	00075347	.00202482	.0+498454
4.0	03208692	00057369	.00159287	00411219
4.1	03213696	00043298	.00124056	00334804
4.2	03217457	00032395	.00095682	.06269212
4.3	03220260	00024027	.00073100	00213925
4.4	03222328	00017665	.00055328	00168065
4.5	03223844	00012873 •	.00051494	0%130595
4.6	03224943	00009295	.06030805	,00100397
4.7	03225733	00006649	.00022706	.00076372
4.8	03226297	00004710	.00016567	00057494
4.9	03226692	00003301	.00011977	00042836
5.0	03226969	00002288	.00008579	~.00031589
5.1	03227160	00001565	.00006087	~ .00023053
5.2	03227289	00001055	.00004278	00016650
5.3	03227376	00000698	.00002978	00011899
5.4	03227433	00000451	.00002052	00008415
5.5	03227469	00000281	.00001400	00005888
5.6	03227491	00000166	.0! 000946	60604075
5.7	00227504	00000089	.00000632	00002790
5.8	03227510	00000037	.00000422	0001906
5.9	03227511	00000000.	.00000331	00001533

η	θ ₁₆	θ'_{10}	θ_{10}''
θ	0	.74610695	
.1	.06974531	.65011353	92114643
.2	.13027321	.56163953	84943845
.3	.18230158	.48002027	78390017
.4	.22648653	.40468238	72367747
.5	.26343091	.33513301	66801035
.6	.29369197	.27095133	61621533
.7	.31778817	.21178171	56767060
.8	.33620543	.15732784	52181021
.9	.3494\.278	.10734741	47812071
1.0	.35781751	.06164673	43614366
1.1	.36186974	.02007494	39548248
1.2	.36196623	01748262	35581066
1.3	.35850409	05111182	31588091
1.4	.35187264	08087819	27853551
1.5	.34245541	10683624	24071245
1.6	.33063052	12903925	20344886
1.7	.31677062	14754907	16687816
1.8	.30124121	16244536	13122188
1.9	.28439851	17383326	09677493
2,0	.25658677	18185225	06388498
2.1	.24813463	18667520	03292790
2.2	.22935124	18851488	00428089
2.3	.21052282	18762016	.02170455
2.4	.19190900	18427274	.04472836
2.5	.17373975	17878088	.06456190
2.6	.15621343	17147148	.08106330
2.7	.13949486	16268090	.09418587
2.8	.12371539	15274533	.10397893
2.9	.10897300	14199844	.11058294

TABLE 5

B. C. PANDET

TABLE 5 -(contd)

$\overline{\eta}$	$\theta_{\mu 0}$	θ'_{10}	$\theta_{10}^{\prime\prime}$
3.0	.09533403	13072770	.11421865
3.1	.08283500	11923713	.11517209
3.2	.07148575	10777157	.11377709
3.3	.06127261	09654786	.11039746
3.4	.05216210	08574572	.10540855
3.5	.04410470	07550744	09918147
3.6	.03703831	06593896	Padden , ,
3.7	.03089213	05711233	.08439592
3.8	.02558973	04906895	.07644914
3.9	.02105180	04182349	06847503
3.0	.01719874	03533681	0.006 0.00
4.1	.01395272	02967706	.05321409
4.2	.01123921	02471029	.046207.61
4.3	.00898825	02041795	.03973633
4.4	.00713508	01674360	.03385260
. 4.5	.00562099	01362729	.028576; 8
4.6	.00439308	01100813	.023947+T
4.7	.00340483	00882627	.01982636
4.8	.00261524	00702448	.016299. 8
49	.00198905	00554930	.01328679
5.0	.00149614	00435170	.01073987
5.1	.00111102	00338756	005(0920
5.2	.00081222	00261776	.00684462
5.3	.00058211	00200815	.00539748
5,4	.00040624	00152929	.00422201
5.5	.00027278	00115616	.00327611
56	.00017220	00086774	.00252194
5.7	.00009696	00064655	.00192606
5.8	.00004114	00047826	.00145942
5.9	.0000000	00035122	.60109720

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			en la classical. Bendros per la fillar capacita encompositamentas
77	θ_{ot}	θ'_{ot}	θ_{GI}^{\sim}
0	0	.01200758	.230295#2
. 1	.00227609	.03277859	.18615828
.2	.00641839	.04942431	. 14756998
.3	.01204000	.06243692	.11330775
.4	.01879745	.07219588	.08234110
.5	.02638037	.07898733	.05383429
.6	.03450316	.08302387	.02715449
.7	.04289865	.08446545	.00188386
.8	.05131307	.08344142	02217198
.9	.05950879	-08007278	04498515
1.0	.06725491	.07449399	06631913
1.1	.07433943	.06687230	08576121
1.2	.08056836	.05742303	10277232
1.3	.08577212	.04641907	11675232
1.4	.08981137	.03419321	12711719
1.5	.09258287	.02113251	13337825
1.6	.09402427	.00766504	13521529
1.7	.09411728	00575994	13253291
1.8	.09288859	01869636	12549215
1.9	.09040825	03072729	11451370
2.0	.08678549	04148978	10024944
2.1	.08216228	05069553	08352916
2.2	.07670504	05814518	06528830
2.3	.07059536	06373479	04648764
2.4	.06406824	06651986	02884108
2.5	.05712294	07201627	00828072
2.6	.05032030	06117589	.16457600
2.7	.04498794	04650836	.10307320
2.8	.04060564	04279565	.00292632
2.9	,03635810	04199054	.01278524
3.0	.03223652	04031495	.:02035345

TABLE 6

B. C. PANDEY

TABLE 6 - (contd)

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$\overline{\eta}$	θ10.	θ' 19	$\theta_{10}^{\prime\prime}$
3.1	.02831669	03799102	.02578547
3.2	.02465314	03522130	.02931281
3.3	.02128136	03218271	.03121082
3.4	.01822058	02902370	.03177002
3.5	.01547662	02586380	.03127466
3.6	.01304475	02279513	.02998721
3.7	.01091230	01988500	.02813969
3.8	.00906093	01717927	.02592919
3.9	.00746870	01470586	.02351748
4.0	.00611157	01247821	.02103289
4.1	.00496480	01049846	.01857356
4.2	.00400084	00876029	.01621144
4.3	.00320512	00725130	.01399647
4.4	.00254651	00595507	.01196047
4.5	.00200766	00485270	.01012084
4.6	.00157019	00392419	.00848368
4.7	.00121772	00314934	.00704659
4.8	.00093587	00250854	.00580100
4.9	.00071218	00198323	.00473413
5.0	.00053596	00155631	.00383052
5.1	.00039817	00121228	.00307336
5.2	.00029121	00093737	.00244543
5.3	.00020880	00071948	.00192984
5.4	.00014577	00054821	.00151060
5.5	.00009791	00041467	.00117293
5.6	.00006184	00031138	.00090347
5.7	.00003484	00023212	.00069039
5.8	.00001479	00017178	.00052341
5.9	.0000000	00012620	.00039371

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η	θ_{ii}	θ_{10}	$\theta_{10}^{''}$
0	0	.00108388	01201757
.1	.00005675	.00013163	00719227
.2	.00004083	00038484	00327469
.3	00000849	00054904	00012926
.4	00005966	00043359	+.00232833
.5	00008809	00010474	+.00414370
.6	00007560	.00037446	.00533685
.7	00001025	.00094223	.00591581
.8	.00011376	.00153757	.00589055
.9	.00029620	.00210112	.00528570
1.0	.00053106	.00257715	.00415113
1.1	.00080706	.00291650	.00256848
1.2	.00110847	.00307983	.00065256
1.3	.00141626	.00304077	00145286
1,4	.00170950	.00278842	00358503
1.5	.00196700	.00232849	00557651
1.6	.00216899	.00168301	00727163
17	.00229861	.00088829	00854278
1.8	.00234323	00000855	00930353
1.9	.00229527	00095419	00951662
2,0	.00215259	00189411	00919549
2.1	.00191834	00277750	00839929
2.2	.00160042	00356134	00722262
2.3	.00121049	00421327	00578186
2.4	.00077657	00444453	00443203
2.5	.00026067	00582305	00188273
2.6	00020677	00273123	.04614743
2.7	00026415	.00123829	.02563733
2.8	00008926	.00173071	00628651
2.9	.00005474	.00117205	00491885
3.0	.00014938	.00074088	00374802

TABLE 7

TABLE 7-(contd)

3.1.00020638.00041588 00277425 3.2 .00023549.00017936 00198581 3.3 .00024459.00001319 00136349 3.4 .00023995 00009811 00052494 3.5 .00022636 00016767 00052494 3.6 .00020744 00020632 0000700 3.8 .00016341 00022370 .0000700 3.8 .00016341 00022370 .00004903 3.9 .00014143 00021443 .00013000 4.0 .00012073 00019881 .00017757 4.1 .00010178 00017971 .00020255 4.3 .00006995 00013859 .00020255 4.4 .00305709 00013859 .00017362 4.5 .00004612 00010068 .00017362 4.6 .00002288 00008425 .00015490 4.7 .00002288 00005711 .00011882 4.9 .00001773 00002339 .00008314 5.1 .0000159 00002339 .00006885 5.2 .00001551 00001830 .00004568 5.4 .00000551 0000193 .00002288 5.7 .00000265 0000193 .00002288 5.7 .00000265 00000631 .00001733 5.8 .0000041 00000353 .0001054	η	θ ₁₂	θ'12	F''12
3.2 $.00023549$ $.00017936$ 00198581 3.3 $.00024459$ $.00001319$ 00136349 3.4 $.00023995$ 00009811 00088442 3.5 $.00022636$ 00016767 00052494 3.6 $.0002744$ 0002632 0000700 3.8 $.00016381$ 00022273 00007700 3.8 $.00016341$ 00022370 $.00004903$ 4.0 $.00012073$ 00019881 $.00017757$ 4.1 $.000101273$ 00017971 $.00020102$ 4.2 $.00008483$ 00017971 $.00020152$ 4.3 $.00006995$ 00013859 $.0002255$ 4.4 $.00305709$ 00013859 $.00017362$ 4.5 $.00004612$ 00010068 $.00017362$ 4.6 $.00003689$ 00008425 $.00015490$ 4.7 $.00002288$ 00005711 $.00011820$ 4.9 $.00001773$ 00002339 $.00008314$ 5.1 $.00001551$ 00001830 $.00004568$ 5.2 $.00000551$ 00001830 $.00002288$ 5.3 $.00000551$ 0000193 $.00002288$ 5.4 $.00000255$ 0000193 $.00002288$ 5.7 $.00000265$ 00000834 $.00002288$ 5.7 $.00000265$ 00000353 $.00001733$ 5.8 $.00000041$ 00000353 $.0000154$	3.1	.00020638	.00041588	00277425
3.3 $.00024459$ $.00001319$ 00136349 3.4 $.00023995$ 00009811 00088442 3.5 $.00022636$ 00016767 00052494 3.6 $.00020744$ 0002632 00026253 3.7 $.00018583$ 00022273 00007700 3.8 $.00016341$ 00022370 $.00004903$ 3.9 $.00014143$ 00021443 $.00013000$ 4.0 $.00012073$ 00019881 $.00017757$ 4.1 $.00010178$ 00017971 $.00020255$ 4.3 $.00006995$ 00013859 $.00020255$ 4.4 $.00305709$ 00013859 $.00019024$ 4.5 $.00004612$ 00010068 $.00017362$ 4.6 $.00003689$ 00008425 $.00015490$ 4.7 $.00002288$ 00005711 $.00011682$ 4.9 $.00001773$ 00002339 $.00008314$ 5.1 $.00001551$ 00001830 $.00004568$ 5.2 $.00000551$ 00001830 $.00004568$ 5.4 $.00000551$ 0000193 $.00002288$ 5.5 $.00000265$ 00000834 $.00002288$ 5.7 $.00000265$ 00000834 $.00002288$ 5.7 $.00000265$ 00000631 $.00001733$ 5.8 $.00000041$ 00000353 $.0000154$	3.2	.00023549	.00017936	00198581
3.4 $.00023995$ 00009811 00088442 3.5 $.00022636$ 00016767 00052494 3.6 $.00020744$ 0002632 00026253 3.7 $.00018583$ 00022273 00007700 3.8 $.00016341$ 00022370 $.00004903$ 3.9 $.00014143$ 00021443 $.00013000$ 4.0 $.00012073$ 00019881 $.00017757$ 4.1 $.0001178$ 00017971 $.00020255$ 4.3 $.00006995$ 00013859 $.00020255$ 4.4 $.00305709$ 00011890 $.00019024$ 4.5 $.00004612$ 00010068 $.00017362$ 4.6 $.00003689$ 00008425 $.00015490$ 4.7 $.00002288$ 00005711 $.00011682$ 4.9 $.00001773$ 00002924 $.00009919$ 5.0 $.00001356$ 00003722 $.00003144$ 5.1 $.00000551$ 00001830 $.000025638$ 5.2 $.00000551$ 00001830 $.00002288$ 5.4 $.00000551$ 0000193 $.00002288$ 5.5 $.00000265$ 0000193 $.00002288$ 5.7 $.00000265$ 00000834 $.00002288$ 5.7 $.00000265$ 00000631 $.00001733$ 5.8 $.00000041$ 00000353 $.0000154$	3.3	.00024459	.00001319	00136349
3.5 $.00022636$ 00016767 00052494 3.6 $.00020744$ 00020632 00026253 3.7 $.00018583$ 00022273 00007700 3.8 $.00016341$ 00022370 $.00004903$ 3.9 $.00014143$ 00021443 $.00013000$ 4.0 $.00012073$ 00019881 $.00017757$ 4.1 $.00011078$ 00017971 $.00020102$ 4.2 $.00008483$ 00015917 $.00020255$ 4.3 $.00006995$ 00013859 $.00020255$ 4.4 $.00305709$ 00011890 $.00017362$ 4.5 $.00004612$ 00010068 $.00017362$ 4.6 $.00003689$ 00008425 $.00015490$ 4.7 $.00002288$ 00005711 $.00011682$ 4.9 $.00001773$ 00004632 $.00009919$ 5.0 $.00001356$ 00003722 $.00008314$ 5.1 $.00000551$ 00001830 $.00004568$ 5.4 $.00000551$ 00001830 $.00002288$ 5.4 $.00000255$ 0000193 $.00002288$ 5.5 $.00000265$ 00000834 $.00002288$ 5.7 $.00000265$ 00000834 $.00002288$ 5.7 $.00000066$ 00000631 $.00001733$ 5.8 $.00000041$ 00000353 $.0000154$	3.4	.00023995	00009811	00088442
3.6 $.00020744$ 00020632 00026253 3.7 $.00018583$ 00022273 00007700 3.8 $.00016341$ 00022370 $.00004903$ 3.9 $.00014143$ 00021443 $.00013000$ 4.0 $.00012073$ 00019881 $.00017757$ 4.1 $.0001178$ 00017971 $.00020102$ 4.2 $.00008483$ 00015917 $.90020752$ 4.3 $.00006995$ 00013859 $.00020255$ 4.4 $.00305709$ 00011890 $.00019024$ 4.5 $.00004612$ 00010068 $.00017362$ 4.6 $.00003689$ 00008425 $.00015490$ 4.7 $.00002288$ 00005711 $.00011682$ 4.9 $.00001773$ 00004632 $.00009919$ 5.0 $.00001356$ 00003722 $.00008314$ 5.1 $.00000551$ 00001830 $.00004568$ 5.4 $.00000551$ 00001830 $.00002288$ 5.4 $.00000265$ 0000193 $.00002288$ 5.7 $.00000265$ 00000834 $.00002288$ 5.7 $.00000169$ 00000834 $.00002288$ 5.7 $.000000265$ 00000631 $.00001733$ 5.8 $.00000041$ 00000353 $.0000154$	3.5	.00022636	00016767	00052494
3.7.00018583 00022273 00007700 3.8 .00016341 00022370 .00004903 3.9 .00014143 00021443 .00013000 4.0 .00012073 00019881 .00017757 4.1 .00010178 00017971 .00020102 4.2 .00008483 00015917 .00020255 4.3 .00006995 00011890 .00019024 4.5 .00004612 0001068 .00017362 4.6 .00003689 00008425 .00015490 4.7 .00002288 00006972 .00013561 4.8 .00002288 00006972 .0001361 4.9 .00001773 00002924 .00009919 5.0 .00001356 00003722 .00008314 5.1 .00000551 00002339 .00006885 5.2 .00000551 00001830 .00002909 5.6 .00000265 0000193 .00002909 5.6 .00000169 00000631 .0000288 5.7 .00000066 00000631 .00001733 5.8 .00000041 00000353 .0001054	3.6	.00020744	00020632	00026253
3.8.00016341 00022370 .00004903 3.9 .00014143 00021443 .00013000 4.0 .00012073 00019881 .00017757 4.1 .0001178 00017971 .00020102 4.2 .00008483 00015917 .00020255 4.3 .00006995 00011890 .00019024 4.5 .00004612 00010068 .00017362 4.6 .00003689 00008425 .00015490 4.7 .00002921 00006972 .00013561 4.8 .00001773 00004632 .00009919 5.0 .00001356 00003722 .00008314 5.1 .00000551 00002339 .00006885 5.2 .00000551 00001830 .00002909 5.6 .00000265 0000193 .00002909 5.6 .00000169 00000834 .00002288 5.7 .00000265 00000353 .00001733 5.8 .00000265 00000353 .00001733	3.7	.00018583	00022273	00007700
3.9 $.00014143$ 00021443 $.00013000$ 4.0 $.00012073$ 00019881 $.00017757$ 4.1 $.0001178$ 00017971 $.00_020102$ 4.2 $.00008483$ 00015917 $.90020752$ 4.3 $.00006995$ 00013859 $.00020255$ 4.4 $.0005709$ 00011890 $.00019024$ 4.5 $.00004612$ 00010068 $.00017362$ 4.6 $.00003689$ 00008425 $.00015490$ 4.7 $.00002921$ 00006972 $.00013561$ 4.8 $.00002288$ 00005711 $.00011682$ 4.9 $.00001773$ 00004632 $.00009919$ 5.0 $.00001356$ 00003722 $.00008314$ 5.1 $.00000551$ 00001830 $.00004568$ 5.4 $.00000255$ 00001420 $.00003633$ 5.5 $.00000265$ 0000193 $.00002909$ 5.6 $.00000169$ 00000834 $.00002288$ 5.7 $.00000096$ 00000631 $.00001733$ 5.8 $.00000041$ 00000353 $.0000154$	3.8	.00016341	00022370	.00004903
4.0 $.00012073$ 00019881 $.00017757$ 4.1 $.00010178$ 00017971 $.00_020102$ 4.2 $.00008483$ 00015917 $.90020752$ 4.3 $.00006995$ 00013859 $.00020255$ 4.4 $.00035709$ 00011890 $.00019024$ 4.5 $.00004612$ 00010068 $.00017362$ 4.6 $.00003689$ 00008425 $.00015490$ 4.7 $.00002921$ 00006972 $.00013561$ 4.8 $.00002288$ 00005711 $.00011682$ 4.9 $.00001773$ 00004632 $.00009919$ 5.0 $.00001356$ 00003722 $.00008314$ 5.1 $.00001023$ 00002964 $.00006885$ 5.2 $.00000551$ 00001830 $.00004568$ 5.4 $.0000265$ 0000193 $.00002909$ 5.6 $.00000169$ 00000834 $.00002288$ 5.7 $.00000066$ 00000631 $.00001733$ 5.8 $.00000041$ 00000353 $.0000154$	3.9	.00014143	00021443	.00013000
4.1.00010178 00017971 .00020102 4.2 .00008483 00015917 .90020752 4.3 .00006995 00013859 .00020235 4.4 .00005709 00011890 .00019024 4.5 .00004612 00010068 .60017362 4.6 .00003689 00008425 .00015490 4.7 .00002921 00006972 .00013561 4.8 .00002288 00006972 .0001682 4.9 .00001773 00004632 .00009919 5.0 .00001356 00003722 .00008314 5.1 .000001023 00002964 .00006885 5.2 .00000551 00001830 .00004568 5.4 .0000265 00001420 .00003633 5.5 .00000265 00000834 .00002288 5.7 .00000169 00000834 .00002288 5.7 .00000169 00000834 .00002288 5.7 .0000006 00000631 .00001733 5.8 .00000041 00000353 .0000154	4.0	.00012073	00019881	.00017757
4.2.0000848300015917.90020752 4.3 .0000699500013859.00020255 4.4 .0030570900011890.00019024 4.5 .0000461200010068.00017362 4.6 .0000368900008425.00015490 4.7 .0000292100006972.00013561 4.8 .000022880000632.0000919 5.0 .0000177300004632.0000919 5.0 .0000135600003722.00008314 5.1 .0000102300002964.00006885 5.2 .0000055100001830.00004568 5.4 .000026500001420.00003633 5.5 .000002650000193.00002909 5.6 .0000016900000834.00002288 5.7 .0000006631.00001733 5.8 .000004100000353.0000154	4.1	.00010178	00017971	.00020102
4.3.00006995 00013859 .00020255 4.4 .00305709 00011890 .00019024 4.5 .00004612 00010068 .00017362 4.6 .00003689 00008425 .00015490 4.7 .00002921 00006972 .00013561 4.8 .00002288 00005711 .00011682 4.9 .00001773 00004632 .00009919 5.0 .00001356 00003722 .00008314 5.1 .00001023 00002399 .00006885 5.2 .00000759 00001830 .00004568 5.4 .00000390 00001420 .00003633 5.5 .00000265 0000193 .000022909 5.6 .00000169 00000834 .00002288 5.7 .00000096 00000631 .00001733 5.8 .00000041 00000353 .0000154	4.2	.00008483	00015917	.00020752
4.4.0000570900011890.00019024 4.5 .0000461200010068.00017362 4.6 .0000368900008425.00015490 4.7 .0000292100006972.00013561 4.8 .0000228800005711.00011682 4.9 .0000177300004632.00009919 5.0 .0000135600003722.00008314 5.1 .0000102300002964.00006885 5.2 .0000055100001830.00004568 5.4 .000026500001420.00003633 5.5 .000002650000193.00002909 5.6 .00001690000631.00001733 5.8 .0000004100000631.00001733 5.9 .000000000000353.0001054	4.3	.00006995	00013859	· .00020255
4.5.0000461200010068.00017362 4.6 .0000368900008425.00015490 4.7 .0000292100006972.00013561 4.8 .0000228800005711.00011682 4.9 .0000177300004632.00009919 5.0 .0000135600003722.00008314 5.1 .0000102300002964.00006885 5.2 .0000075900001830.00004568 5.4 .000026500001420.00003633 5.5 .000002650000193.00002909 5.6 .000016900000834.00002288 5.7 .000009600000631.00001733 5.8 .000004100000353.0000154	4.4	.00005709	00011890	.00019024
4.6.00003689 00008425 .00015490 4.7 .00002921 00006972 .00013561 4.8 .00002288 00005711 .00011682 4.9 .00001773 00004632 .00009919 5.0 .00001356 00003722 .00008314 5.1 .00001023 00002964 .00006885 5.2 .00000759 00001830 .00004568 5.4 .00000351 00001420 .00003633 5.5 .00000265 0000193 .00002909 5.6 .00000169 00000834 .00002288 5.7 .00000096 00000631 .00001733 5.8 .00000041 00000353 .0000154	4.5	.00004612	00010068	.00017362
4.7.00002921 00006972 .00013561 4.8 .00002288 00005711 .00011682 4.9 .00001773 00004632 .00009919 5.0 .00001356 00003722 .0000885 5.1 .00001023 00002964 .00006885 5.2 .00000759 00002339 .00005638 5.3 .00000551 00001830 .00004568 5.4 .00000265 0000193 .00002909 5.6 .00000169 00000834 .00002288 5.7 .0000096 00000631 .00001733 5.8 .00000041 00000474 .00001377 5.9 .0000000 00000353 .0000154	4.6	.00003689	00008425	.00015490
4.8 .00002288 00005711 .00011682 4.9 .00001773 00004632 .00009919 5.0 .00001356 00003722 .0000885 5.1 .00001023 00002339 .00005638 5.2 .00000551 00001830 .00004568 5.4 .0000265 0000193 .00002999 5.6 .00000265 0000193 .0000299 5.6 .00000169 0000834 .00002288 5.7 .0000096 0000631 .00001733 5.8 .00000041 00000353 .0000157	4.7	.00002921	00006972	.00013561
4.9.00001773 00004632 .000091919 5.0 .00001356 00003722 .00008314 5.1 .00001023 00002964 .00006885 5.2 .00000759 00002339 .00005638 5.3 .00000551 00001830 .00004568 5.4 .00000265 00001420 .00003663 5.5 .00000265 0000193 .00002909 5.6 .00000169 00000834 .00002288 5.7 .00000096 00000631 .00001783 5.8 .00000041 00000353 .0000154	4.8	.00002288	00005711	.00011682
5.0 .00001356 00003722 .00008314 5.1 .00001023 00002964 .00006885 5.2 .00000759 00002339 .00005638 5.3 .00000551 00001830 .00004568 5.4 .00000265 0000193 .00002909 5.6 .00000169 0000834 .00002288 5.7 .00000096 0000631 .00001733 5.8 .00000041 00000353 .0000154	4.9	.00001773	00004632	.00009919
5.1 .00001023 00002964 .00006885 5.2 .00000759 00002339 .00005638 5.3 .00000551 00001830 .00004568 5.4 .00000265 0000193 .00002909 5.6 .00000169 0000834 .00002288 5.7 .00000096 00000631 .00001733 5.8 .00000041 00000353 .0000154	5.0	.00001356	00003722	.00008314
5.2 .00000759 00002339 .00005638 5.3 .00000551 00001830 .00004568 5.4 .00000265 00001420 .00002099 5.6 .00000169 0000834 .00002288 5.7 .00000096 00000631 .00001733 5.8 .00000041 00000353 .0000154	5.1	.00001023	00002964	.00006885
5.3 .00000551 00001830 .00004568 5.4 .00000390 00001420 .00003663 5.5 .00000265 00001093 .00002909 5.6 .00000169 00000834 .00002288 5.7 .00000096 00000631 .00001783 5.8 .00000041 00000353 .0000154	5.2	.00000759	00002339	.00005638
5.4 .00000390 00001420 .00903663 5.5 .00000265 00001093 .00002909 5.6 .00000169 00000834 .00002288 5.7 .00000096 00000631 .00001783 5.8 .00000041 00000474 .00001377 5.9 .00000000 00000353 .00001054	5.3	.00000551	00001830	.00004568
5.5 .00000265 00001093 .00002909 5.6 .00000169 00000834 .00002288 5.7 .00000096 00000631 .00001783 5.8 .00000041 00000474 .00001377 5.9 .0000000 00000353 .00001054	5.4	.00000390	00001420	.00003663
5.6 .00000169 00000834 .00002288 5.7 .00000096 00000631 .00001783 5.8 .00000041 00000474 .00001377 5.9 .00000000 00000353 .00001054	5.5	.00000265	00001093	.00002909
5.7 .00000096 00000631 .00001783 5.8 .00000041 00000474 .00001377 5.9 .00000000 00000353 .00001054	5.6	.00000169	00000834	.00002288
5.8 .00000041 00000474 .00001377 5.9 .00000000 00000353 .00001054	5.7	.00000096	00000631	.00001783
5.9 .0000000000000353 .00001054	5.8	.00000041	00000474	.00001377
	5.9	.00000000	00000353	.00001054

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