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Short Communication

Phase-matched relativistic second harmonic generation in a plasma

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Abstract

An intense laser radiation $(\omega_1, \vec{k}_1)$ propagating through a plasma at an angle to the density ripple $(0_1, \vec{k}_0)$ produces current and density perturbation at $(\omega_1, \vec{k}_1 + \vec{k}_0)$. The density perturbation combines with the oscillatory velocity at $(\omega_1, \vec{k}_1)$ to produce nonlinear current at $(2\omega_1, (2\vec{k}_1 + \vec{k}_0))$, driving a second harmonic electromagnetic radiation. For a specific ripple wave number $k_0 \approx k_{0c}$, the phase-matching conditions for the second harmonic process are satisfied, leading to resonant enhancement in the energy conversion efficiency. The efficiency of the process is sensitive to the angle between the density ripple and the incident laser and beam energy.

Keywords: Harmonic generation, laser–plasma interaction, relativistic effects.

1. Introduction

Harmonic and subharmonic generation in plasmas has been an area of considerable interest for more than four decades [1]–[7]. It acts as a valuable diagnostics in short pulse laser–plasma interaction experiments and in frequency up-conversion devices. In this process, two (or more) photons of energy $\hbar\omega_1$ and momentum $\hbar\vec{k}_1$ combine to produce a photon of second (or higher) harmonic radiation of energy $\hbar\omega_2$ and momentum $\hbar\vec{k}_2$, where, $(\omega_1, \vec{k}_1)$ and $(\omega_2, \vec{k}_2)$ satisfy the linear dispersion codection for electromagnetic waves. The energy and momentum conservation in a second harmonic process is subject to the following conditions

$$\left.\begin{aligned} \omega_2 &= 2\omega_1 \\ \vec{k}_2 &= 2\vec{k}_1 \end{aligned}\right\} \tag{1}$$

In an overdense or inhomogeneous plasma, second harmonic can be generated by exciting an electron plasma wave either by linear mode conversion near the critical layer or by decay instability [7], [8]. The density oscillations arising from the Langmuir wave combines with the oscillatory velocity caused by the laser to produce a second harmonic, giving rise to second harmonic radiation. At higher laser intensities, relativistic effects contribute to harmonic generation as the electron oscillatory velocity contains all the harmonics. Park *et al*. [3] present an account of relativistic third harmonic generation by linearly and circularly polarized light. Esarey *et al*. [4] deal with the effects of diffraction and phase detuning on relativistic harmonic generation by intense laser beams. Wilks *et al*. [5] describe odd harmonic generation by an ultraintense laser beam incident upon a *sharp vacuum–overdense plasma*. Liu *et al*. [6] have observed third harmonic generation from a hydrogen gas with a range of laser intensities.

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In an underdense plasma, no significant density oscillation is induced by the laser that could beat with the oscillatory electron velocity due to the laser to produce a second harmonic. Further, condition (1) does not hold in a dispersive medium like a plasma, and the process is nonresonant as the momentum $\hbar\vec{k}_2$ of second harmonic photon is larger than twice the momentum of a fundamental photon, and thus is low in efficiency. The process can be resonant if an additional momentum $\hbar\vec{k}_0 = \hbar\vec{k}_2 - 2\hbar\vec{k}_1$ is provided. Rax and Fisch [9] studied resonant third harmonic generation by an intense laser in a plasma based on a resonant density-modulation scheme. Shibu and Tripathi [10] have developed a nonlocal theory for the above process. Agrawal *et al*. [11] have studied resonant second harmonic generation of a millimeter wave in a plasma filled waveguide in the presence of a helical magnetic wiggler. Weissman *et al*. [12] have studied second harmonic generation in Bragg-resonant quasi-phase-matched periodically segmented waveguides. Ding *et al*. [13] have developed a theory for quasi-phase-matched backward second and third harmonic generation in a periodically doped semiconductor.

In this paper, we propose a mechanism of providing the momentum by a density ripple, which acts as a virtual photon of quantum energy 0 and momentum $\hbar\vec{k}_0$, where $\vec{k}_0$ is the wave vector of the density ripple. The physics of the process could be understood as follows: The laser induces oscillatory velocity $\vec{v}_1(\omega_1,\vec{k}_1)$ of electrons which beat with the density ripple $n_0(0,\vec{k}_0)$ to produce a current $n_0\vec{v}_1$ and a density perturbation $n_1(\omega_1,\vec{k}_1+\vec{k}_0)$. n_1 beats with $\vec{v}_1(\omega_1,\vec{k}_1)$ to produce a nonlinear transverse current at $(2\omega_1, 2\vec{k}_1+\vec{k}_0)$, giving rise to second-harmonic radiation. For resonant excitation, the second-harmonic field must satisfy the electromagnetic dispersion relation $\omega_2^2 = \omega_P^2/\gamma + k_2^2c^2$, where ω_p is the plasma frequency and γ, the relativistic factor. Using $\omega_2 = 2\omega_1$, $k_2^2 = |2\vec{k}_1+\vec{k}_0|^2$, and $\omega_1^2 = (\omega_p^2/\gamma)+k_1^2c^2$ in the dispersion relation, one obtains the value of $\vec{k}_0$ required for the phase matching: $k_0 = \frac{1}{2}\left[-4k_1\cos\theta \pm \sqrt{16k_1^2\cos^2\theta + 12\omega_p^2/(\gamma c^2)}\right]$, where θ is the angle between $\vec{k}_0$ and $\vec{k}_1$ (c. f. Fig.1). Figure 2 displays the behavior of k_0 with ω_1 for different values of θ and γ.

2. Nonlinear current density

Consider the propagation of an intense laser in a plasma of density n_0^0. The fields of the laser are given as,

$$\vec{E}_1 = \hat{x}E_1 e^{-i(\omega_1 t - k_1 z)},$$

$$\vec{B}_1 = \frac{c}{\omega_1}\vec{k}_1 \times \vec{E}_1, \tag{2}$$

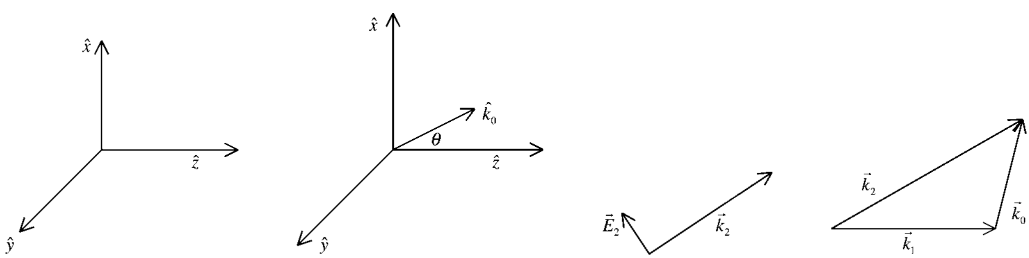
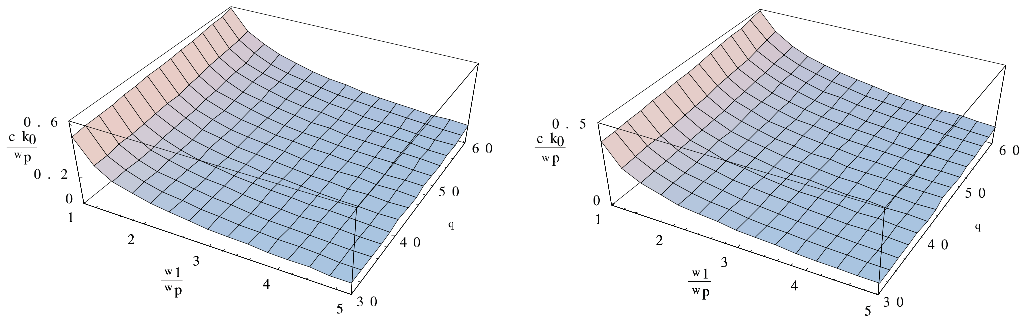


FIG. 1. Schematic of the process.



(a) (b)

FIG. 2. Variation of normalized density ripple wave number ck_o/ω_p with normalized plasma frequency ω_1/ω_p and angle between incident laser and density ripple θ for (a) γ = 1.2 and (b) γ = 1.4.

where $k_1 \approx (\omega_1/c)\left[1-\omega_p^2/(\gamma\omega_1^2\right]^{1/2}$, $\omega_p^2 = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency, e and m are the electronic charge and rest mass, respectively. γ will be defined later.

The plasma also contains a density ripple such that

$$n_0 = n_0 \exp(i\vec{k}_0.\hat{x}), \tag{3}$$

whose wave vector $\vec{k}_0$ makes an angle θ with the z-axis. The equation of motion for electron momentum, $\vec{p}$, $d\vec{p}/dt = -e\vec{E}_1-(e/c)\vec{v}_1 \times \vec{B}_1$ can be resolved into x and z components as,

$$\frac{dp_x}{dt} = -\frac{eE_1}{\omega_1}\frac{d}{dt}\sin(\omega_1 t - k_1 z), \tag{4}$$

and

$$\frac{dp_z}{dt} = -ek_1 v_{1x} E_1 \cos(\omega_1 t - k_1 z)/\omega_1. \tag{5}$$

From eqn (4), we get,

$$p_x = -eE_1 \sin(\omega_1 t - k_1 z)/\omega_1. \tag{6}$$

Equation for electron energy (γmc^2) is

$$mc^2\frac{d\gamma}{dt} = -e\vec{E}.\vec{v}_1 = -ev_{1x}E_1\cos(\omega_1 t - k_1 z), \tag{7}$$

where

$$\gamma^2 = 1+(p_x^2+p_z^2)/(m^2c^2). \tag{8}$$

γ is also $1/\sqrt{1-v_1^2/c^2}$.

Multiplying eqn (5) by ω_1/k_1 and substracting from (7), we get

$$mc^2\gamma - (\omega_1/k_1)p_z = \text{constant} = mc^2\gamma_0 - (\omega_1/k_1)p_{0z}, \tag{9}$$

where p_{0z} is the initial value of p_z and γ_0, the initial value of γ; $\gamma_0^2 = 1 + p_{0z}^2/(m^2c^2)$. Using eqns (6) and (9) in (7) and on simplification one obtains γ as

$$\gamma \approx \frac{1}{1-\eta_0^2}\left[-\eta_0^2 + \sqrt{1+(1-\eta_0^2)a_0^2}\right], \tag{10}$$

where $a_0^2 = (eE_1/m\omega_1 c)^2$ and $\eta_0 = ck_1/\omega_1$ are the plasma refractive indices.

The x and z components of electron velocity can be written as

$$v_{1x} = \frac{p_x}{m\gamma} = \frac{eE_1}{mi\omega_1\gamma} e^{-i(\omega_1 t - k_1 z)}, \tag{11}$$

and

$$v_{1z} = \frac{p_z}{m\gamma} = \eta_0 c \frac{(\gamma - 1)}{\gamma}, \tag{12}$$

respectively. In solving the above equations, we have assumed electron collision frequency,

$$\nu << \omega_1.$$

$\vec{v}_1$, in conjunction with the density ripple n_0, produces a density perturbation $n_1^{'} = n_1^{'} e^{-i[\omega_1 t - (\vec{k}_1 + \vec{k}_0).\hat{x}]}$. This is obtained by solving the equation of continuity $\partial n_1^{'}/\partial t + \nabla . n_0 \vec{v}_1 = 0$,

$$n_1^{'} = \frac{(\vec{k}_0 . \vec{v}_1) n_0}{2\omega_1}, \tag{13}$$

where we have used the identity $\mathrm{Re}\,\vec{A}.\mathrm{Re}\,\vec{B} = \frac{1}{2}\,\mathrm{Re}\left[\vec{A}.\vec{B} + \vec{A}.\vec{B}^*\right]$.

$\vec{v}_1$ and $\vec{B}_1$ also exert a ponderomotive force on the electrons,

$$\vec{F}_p = -\frac{e}{2c}\vec{v}_1 \times \vec{B}_1. \tag{14}$$

The oscillatory velocity $\vec{v}_2$ of electrons due to $\vec{F}_p$ is

$$\vec{v}_2^{'} \cong \hat{z}\frac{ek_1\vec{v}_1.\vec{E}_1}{4mi\omega_1^2} e^{-i(2\omega_1 t - 2k_1 z)}. \tag{15}$$

The nonlinear component of the second harmonic current density $\vec{J}_2 = \vec{J}_2 e^{-i[2\omega_1 t - (2\vec{k}_1 + \vec{k}_0).\hat{x}]}$ due to beating of $\vec{v}_1$ with $\vec{n}_1^{'}$ and $\vec{v}_2^{'}$ with n_0 is,

$$\vec{J}_2^{NL} = -\frac{1}{2}n_1^{'} e\vec{v}_1 - \frac{1}{2}n_0 e\vec{v}_2^{'}$$

$$\approx -\frac{n_0 e^2 E_1 v_{1x}}{4mi\omega_1^2}\left[\hat{x}\frac{k_{0x}}{\gamma}+\hat{z}\frac{k_1}{2}\right]. \tag{16}$$

The linear component of the second harmonic current density due to self-consistent electric field $\vec{E}_2 = \vec{E}_2 e^{-i[2\omega_1 t-(2\vec{k}_1+\vec{k}_0).\hat{x}]}$ is given by,

$$\vec{J}_2^L = -\frac{n_0^0 e^2 \vec{E}_2}{2mi\gamma\omega_1}. \tag{17}$$

Using (16) and (17) in the Poisson equation, one obtains the second harmonic density perturbation $n_2 = n_2 e^{-i[2\omega_1 t-(2\vec{k}_1+\vec{k}_0).\hat{x}]}$:

$$n_2 = -\frac{1}{e}\frac{\nabla.\vec{J}_2}{2i\omega_1}, \tag{18}$$

where $\vec{J}_2 = \vec{J}_2^L + \vec{J}_2^{NL}$.

3. Second harmonic field

The wave equation for the second harmonic field is written as:

$$\nabla^2\vec{E}_2 - \nabla(\nabla.\vec{E}_2) = -\frac{8\pi}{c^2} i\omega_1\vec{J}_2 - \frac{4\omega_1^2}{c^2}\vec{E}_2. \tag{19}$$

From the Poisson equation we get,

$$\nabla.\vec{E}_2 = -4\pi e n_2$$

$$\approx \frac{\pi n_0 e^2 E_1 v_{1x}}{2m\omega_1^3\varepsilon_2}\left[k_1^2+\frac{k_{0x}^2}{\gamma}+\frac{1}{2}k_{0z}k_1\right], \tag{20}$$

where $\varepsilon_2 \approx 1-\omega_P^2/\omega_1^2$.

Using (20) in (19), and replacing ∇^2 by $-(2\vec{k}_1+\vec{k}_0)^2$, we obtain

$$\vec{E}_2 = \frac{1}{D_2}\frac{\pi n_0 e^2 E_1 v_{1x}}{m\omega_1}\vec{F}, \tag{21}$$

where $D_2 = 4\omega_1^2 - \omega_P^2/\gamma - (2\vec{k}_1+\vec{k}_0)^2 c^2$, and

$$\vec{F} = \frac{c^2}{\omega_1^2\varepsilon_2}\left[\left(k_1^2+\frac{k_{0x}^2}{\gamma}+\frac{k_{0z}k_1}{2}\right)\left(2\vec{k}_1+\vec{k}_0\right)\right]+2\left(\hat{x}\frac{k_{0x}}{\gamma}+\hat{z}\frac{k_1}{2}\right).$$

Equation (21) is valid when D_2 is not exactly zero, and the length of the plasma is greater than $1/|k_2-(2k_1+k_0)|$. When $D_2=0$, one may write the harmonic field as $E_2 = A_2(z)e^{-i(2\omega_1 t-k_2 z)}$. Then eqn (21) takes the form

$$2ik_2 \frac{\partial A}{\partial z} = -\frac{8\pi i \omega_1}{c^2} J^{NL} \text{ or } A = -\frac{8\pi i \omega_1}{c^2 2ik_2} J^{NL} z .$$

The second harmonic Poynting vector can be written as

$$\vec{P}_2 = \frac{c}{8\pi} \vec{E}_2^* \times \vec{H}_2 \approx \frac{c^2}{8\pi} \frac{(2\vec{k}_1 + \vec{k}_0)}{\omega_2 D_2^2} \frac{\pi^2 n_0^2 e^4 E_1^2 v_{1x}^2}{m^2 \omega_1^2} \left|\vec{F}\right|^2 . \tag{22}$$

Using eqn (2), we get the Poynting vector for the fundamental wave as

$$\vec{P}_1 = \frac{c}{8\pi} \vec{E}_1^* \times \vec{H}_1 \approx \frac{c^2 \vec{k}_1 E_1^2}{8\pi \omega_1} . \qquad 23)$$

From (22) and (23) we get,

$$\left|\frac{\vec{P}_2}{P_1}\right| \approx \frac{1}{32} \left(\frac{\omega_p}{\omega_1}\right)^4 \left(\frac{n_0}{n_0^0} \frac{v_{1x}}{c}\right)^2 \left|\vec{G}\right| , \tag{24}$$

where $D_{20} = \dfrac{D_2}{\omega_1^2}, \vec{G} = \dfrac{\left|\vec{F}\right|^2 c^2}{\omega_1^2} \dfrac{(2\vec{k}_1 + \vec{k}_0)}{k_1 \left|D_{20}\right|^2}$.

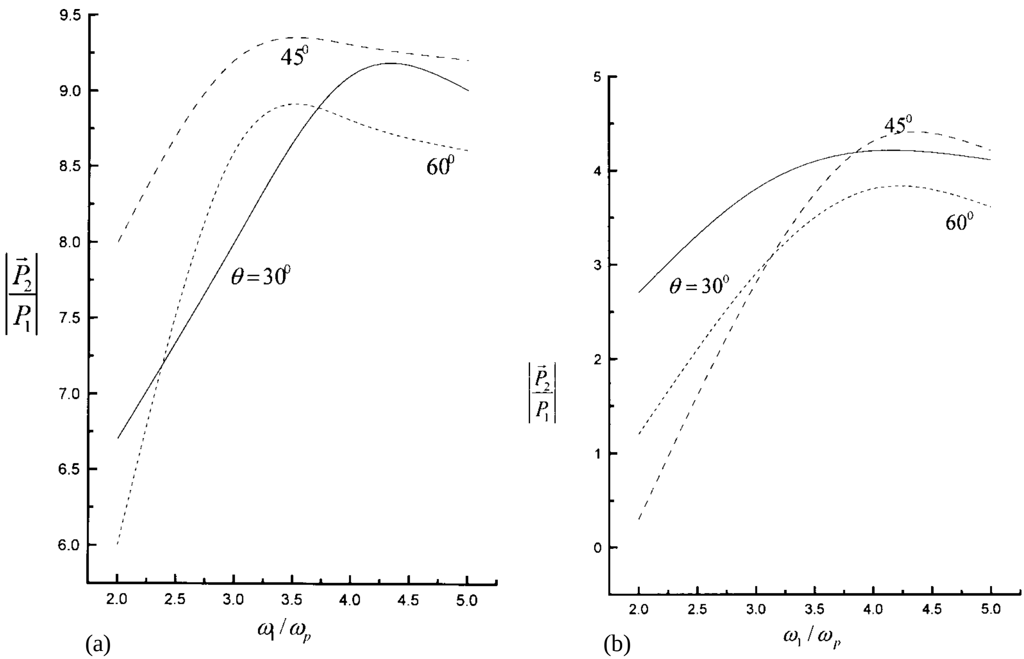


Fig. 3. Variation of efficiency of the process $|\vec{P}_2/P_1|$ with ω_1/ω_p for different values of θ. The parameters are ω_1= 1.06 x 10^{15} rad/*s*, n^0/n_0^0=3%, and (a) γ = 1.2 and (b) γ = 1.4.

Figure 3 shows the variation of $|\vec{P}_2/P_1|$ with ω_1/ω_p for different values of θ and γ. The maximum efficiency of the process is 9% at $\omega_1/\omega_p = 4$, $\theta = 45^o$, $\gamma = 1.2$ and n^0 / n_0^0 ~3%.

4. Discussion

A density ripple in a plasma could be properly employed for resonant second harmonic generation. The process is efficient even in the underdense region. The density ripple could be excited by an acoustic wave launched externally or produced internally via parametric instability of the laser [14]. Typically, the pulse length is smaller than the ion acoustic wavelength, so the density modulation can be considered as static on the time scale of laser pulse dynamics. Alternatively, a density ripple can also be produced by laser irradiation of a periodically doped semiconductor [13]. An efficiency of ~ 9% can be achieved for using a 1.06 μ Nd:YAG laser of intensity ~ 10^{18} W/ cm^2 in a plasma of density ~10^{20} cm^{-3} with a density ripple of 3%. Laser propagation in a plasma is influenced by a number of nonlinear effects, e.g. self-focusing, parametric instabilities and charged particle acceleration. Hence, the estimate of harmonic generation is only an upper bound on it. The above analysis is valid when the length of the plasma is greater than $1/|k_2 - (2k_1 + k_0)|$.

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