HEAT TRANSFER FOR THE FLOW OF A POWER-LAW FLUID IN A CURVED PIPE

By K. Kanaka Raju and (Mrs.) S. L. Rathna
(Department of Applied Mathematics, Indian Institute of Science, Bangalore-12, India)

[Received: July 7, 1969]

Abstract

In this paper we have studied the heat transfer in the flow of a power law fluid in a curved pipe of circular cross-section. Following Dean, we assumed the curvature of the pipe to be small and evaluated the temperature distribution upto the first order in the parameter L, *which is the ratio of the radius of pipe to that of the coil. It is found that the fluid is heated throughout relative to the wall upto a critical Prandtl number and for higher Prandtl numbers relatively cooled and heated regions develop. The physical reasons for such a behaviour are analysed. By the study of the variation of this Prandtl number with the flow behaviour index* n, *we conclude that dilatant fluids* ($n > 1$) *are suitable for more efficient working of heat exchangers. The Nusselt number is not affected by curvature upto the first order of* L.

1. Introduction

Owing to industrial applications, heat transfer phenomena in non-Newtonian fluids are of great importance. Metzner[1] and Skelland[2] have reviewed in detail the heat transfer problems and their applications. As emphasised by Fraas and Ozisik[3] the heat transfer in curved pipes or helical coils play a fundamental role in various heat exchangers, heat engines and in many processing industries. But only a few investigations[4-10] of heat transfer phenomena of Newtonian fluids in curved pipes are available and this problem for non-Newtonian fluids therefore needs thorough investigation. It has been pointed out by Metzner[11] that many of the fluids used in industry as well as in biophysics sustain a power law stress of the form $\mu_p (e_{ij})^n$, where μ_p is the apparent viscosity and e_{ij} is the rate of strain tensor and n is the flow behaviour index. We concentrate on the heat transfer in power law fluids in this paper.

Hawes[4] has determined experimentally the behaviour of the velocity and temperature profiles in a curved pipe for a Newtonian fluid, and a theoretical approach to this problem has been recently attempted by Ozisik and Topakoglu[5]. From their analysis we find that there is a possibility that the fluid is divided into relatively heated and cooled regions depending on the magnitude of the Prandtl numbers. Mori and Nakayama[8-10] have studied the problem theoretically and experimentally for large Deans numbers for Newtonian fluids. Their simplified analysis shows that the flow field is approximately divided into a shear-free core region and the boundary layer along the wall. Moreover, in all the above investigations, the dissipative effects are neglected in the heat transfer problem. But this is not justified at higher Prandtl numbers and this plays a dominant role in determining the temperature profile as seen in §2.

Broadly, the procedure is as flows : the velocity fields in the flow is obtained first and then the variations of the temperature field due to convection and dissipation are calculated. We have already obtained[12] the velocity field due to pressure driven flow of a power law fluid in a curved pipe. Here we evaluate the temperature profiles up to the first order of the curvature ratio L, when the wall temperature around the periphery of any cross-section is uniform. The isotherms and the variation of temperature for $n=1$ (Newtonian), $n=0.8$ (Psuedo-plastic) and $n=1.2$ (dilatant) are discussed in detail in § 4. We find that irrespective of the nature of the fluid, there exists a critical Prandtl number below which the fluid is uniformly heated and beyond which there are relatively cooled and heated regions owing to increased convection. We have given a detailed explanation of the same phenomena.

The mean bulk temperature and the Nusselt number are also evaluated. As expected with the present approximation in L, they are not affected by the curvature of the pipe, since the flow field is taken only up to the first order.

2. Basic Equations and Formulation of the Problem

The constitutive equation for a power law fluid as given by Ostwald and generalised by Tomita[13] is

$$T_{ij} = -P\,\delta_{ij} + \mu_p \Theta\, E_{ij}\,, \qquad [2.1]$$

where T and E are the stress and the rate of strain tensors, p is the pressure, μ_p is a constant and

$$\Theta = \left| E_{11}^2 + E_{22}^2 + E_{33}^2 + 2\,E_{12}^2 + 2\,E_{23}^2 + 2\,E_{31}^2 \right|^{(n-1)/2} \qquad [2.2]$$

n being the flow behaviour index, $n=1$, $n > 1$ and $n < 1$ represent respectively the Newtonian, dilatant and psuedoplastic fluids.

Let the axially symmetric flow in the curved pipe of circular cross-section be generated by a constant applied pressure gradient along the axis. Let a and b denote the radii of cross-section and coil, respectively and $L=a/b$, the curvature ratio. Following Dean[14] we use the polar coordinate system (R, θ) in the cross-section situated in a meridian plane with azimuthal angle ϕ. The element of arc length can be written in this coordinate system as

$$(dS)^2 = (dR)^2 + (R\,d\theta)^2 + \{(b + R\sin\theta)\,d\phi\}^2. \qquad [2.3]$$

Let $U(R, \theta)$, $V(R, \theta)$, $W(R, \theta)$ be the velocity components in the directions of R, θ, ϕ and $T(R, \theta)$ be the temperature at any point.

We assume that the curvature of the pipe is small, so that L is small and the simplified equations of the motion are

$$\rho\left(U\frac{\partial U}{\partial R} + \frac{V}{R}\frac{\partial U}{\partial \theta} - \frac{V^2}{R} - \frac{W^2\sin\theta}{b}\right) = \frac{\partial}{\partial R}T_{RR} + \frac{1}{R}\frac{\partial}{\partial \theta}T_{R\theta} + \frac{T_{RR} - T_{\theta\theta}}{R} - \frac{T_{\phi\phi}\sin\theta}{b}, \qquad [2.4]$$

$$\rho\left(U\frac{\partial V}{\partial R} + \frac{V}{R}\frac{\partial V}{\partial \theta} + \frac{UV}{R} - \frac{W^2\cos\theta}{b}\right) = \frac{\partial}{\partial R}T_{R\theta} + \frac{1}{R}\frac{\partial}{\partial \theta}T_{\theta\theta} + \frac{2}{R}T_{R\theta} - \frac{T_{\phi\phi}\cos\theta}{b}, \qquad [2.5]$$

$$\rho\left(U\frac{\partial W}{\partial R} + \frac{V}{R}\frac{\partial W}{\partial \theta}\right) = \frac{\partial}{\partial R}T_{R\phi} + \frac{1}{R}\frac{\partial}{\partial \theta}T_{\theta\phi} + \frac{T_{R\phi}}{R} + \frac{2\,T_{R\phi}\sin\theta}{b} - \frac{1}{b}\frac{\partial P}{\partial \phi}. \qquad [2.6]$$

The equation of continuity is

$$\frac{\partial U}{\partial R} + \frac{U}{R} + \frac{1}{R}\frac{\partial V}{\partial \theta} = 0. \qquad [2.7]$$

and the energy equation is

$$\rho\,C_p\left[U\frac{\partial T}{\partial R} + \frac{V}{R}\frac{\partial T}{\partial \theta}\right] = k\left[\frac{\partial^2 T}{\partial R^2} + \frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{1}{R}\frac{\partial T}{\partial R} + \frac{\sin\theta}{b}\frac{\partial T}{\partial R} + \frac{\cos\theta}{bR}\frac{\partial T}{\partial \theta}\right] + \Phi, \qquad [2.8]$$

where C_p is the specific heat, k is the thermal conductivity, Φ is the dissipation function given by

$$\Phi = T_{ij}\,E_{ij}. \qquad [2.9]$$

The stress components are given by

$$\left.\begin{aligned}
T_{RR} &= -P + 2\,\mu_p\,\Theta\,(\partial U)/\partial R\,, \\
T_{\theta\theta} &= -P + 2\,\mu_p\,\Theta\left(\frac{1}{R}\;\frac{\partial V}{\partial\theta} + \frac{U}{R}\right), \\
T_{\phi\phi} &= -P + 2\,\mu_p\,\Theta\left(\frac{U\sin\theta + V\cos\theta}{b}\right), \\
T_{\theta\phi} &= \mu_p\,\Theta\left(\frac{1}{R}\;\frac{\partial W}{\partial\theta} - \frac{W\cos\theta}{b}\right), \\
T_{\phi R} &= \mu_p\,\Theta\left(\frac{\partial W}{\partial R} - \frac{W\sin\theta}{b}\right), \\
T_{R\theta} &= \mu_p\,\Theta\left(\frac{\partial V}{\partial R} - \frac{V}{R} + \frac{1}{R}\;\frac{\partial V}{\partial\theta}\right).
\end{aligned}\right\} \qquad [2.10]$$

We introduce the following non-dimensional variables u, v, w, $\bar{T}$, $\bar{\Theta}$, p by substituting

$G = (1/b)\;(\partial P/\partial\phi) =$ Constant axial force due to pressure gradient.

$$K = \left(\frac{G}{2^{(n+1)/2}\,\mu_p}\right)^{1/n} \qquad [2.11]$$

$$R_e = \frac{\rho\, a^{1+2/n}}{\mu_p\, K^{n-2}\, 2^{(n-3)/2}\,(n+1)} = \text{Generalised Reynolds number} \qquad [2.12]$$

$$\left.\begin{aligned}
U &= R_e\,K\,a^{1+(1/n)}\,u, \\
V &= R_e\,K\,a^{1+(1/n)}\,v, \\
W &= \bar{w}_0\,(w_0 + L\,w_1), \quad \text{where } \bar{w}_0 = \frac{-nKa^{1+(1/n)}}{(n+1)} \\
P &= 2^{(n-1)/2}\,\mu_p\,K^n\,a\,p \\
R &= a\,r \\
\Theta &= 2^{(n-1)/2}K^{n-1}\,a^{(n-1)/2}\,\bar{\Theta}
\end{aligned}\right\} \qquad [213]$$

and $\bar{T}=(T-T_w)/T_c$; $T_c=T_1\sim T_2$= difference between the inlet and outlet temperatures, T_w the constant wall temperature. The equations [2.4] to [2.8] in non-dimensional form are solved for the velocity profile upto the first order of the curvature ratio L in an earlier investigation[32]. As in the case of Newtonian fluids here also the main flow in curved pipes is accompanied by the secondary flow, due to shear, curvature and non-Newtonian effects. The velocity components are given by

$$u=L\sin\theta\,[A+B\,r^{s-1}+C\,r^{(2n+2)/n}-D\,r^{(3n+3)/n}], \qquad [2.14]$$

$$v=L\cos\theta\,[A+s\,B\,r^{s-1}+\frac{3n+2}{n}\,C\,r^{(2n+2)/n}-\frac{4n+3}{n}\,D\,r^{(3n+3)/n}] \qquad [2.15]$$

$$w=[1-r^{1+(1/n)}]+L\,w_1 \qquad [2.16]$$

where

$$w_1=\sin\theta\left[\frac{R_e^2}{120\,n}\,r^{1/n}\left\{30\,A\,[1-r^{(n+1)/n}]+\frac{60\,B\,(n+1)^2}{(ns+1)(ns+2+n)}\left(1-r^{(ns+1)/n}\right)\right.\right.$$

$$\left.+5\,C\,[1-r^{(3n+3)/n}]-3\,D\,[1-r^{(4n+4)/n}]\right\}-(1-r)$$

$$\left.+\frac{3n^2+7n+4}{2n\,(3n+1)}(1-r^{1/n})+\frac{3n^2-5n-4}{2n\,(3n+1)}[1-r^{2+(1/n)}]\right], \qquad [2.17]$$

$$s=\frac{n+1}{2n}+\frac{\sqrt{(\sqrt{17}\,n-1)^2+2n\,(\sqrt{17}-1)}}{2n},$$

$$A=\frac{n^3\{ns\,(21\,n^3+53\,n^2+38\,n+8)-(60\,n^4+185\,n^3+200\,n^2+92\,n+15)\}}{12\,(n+1)\,(1-s)\,(2n+1)\,(3n+1)\,(4n^2+9n+3)\,(n^2+4n+1)},$$

$$B=\frac{n^3\,(13\,n^3+31\,n^2+23\,n+5)}{4(1-s)(2n+1)(3n+1)(n^2+4n+1)(4n^2+9n+3)},$$

$$C=\frac{n^4}{4\,(n+1)\,(3n+1)(n^2+4n+1)},$$

and

$$D=\frac{n^4}{12\,(n+1)\,(2n+1)\,(4n^2+9n+3)}. \qquad [2.18]$$

The non-dimensional form of the energy equation is

$$R_e\left[u\frac{\partial T}{\partial r}+\frac{v}{r}\frac{\partial T}{\partial \theta}\right]=\frac{1}{P_r}\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+L\left(\sin\theta\frac{\partial T}{\partial r}\right.\right.$$

$$\left.\left.+\frac{\cos\theta}{r}\frac{\partial T}{\partial \theta}\right)\right]+\frac{4(n+1)}{n}\beta\left[r^{(n+1)/n}-Lnr\left(\frac{\partial w_1}{\partial r}-w_0\sin\theta\right)\right] \quad [2.19]$$

where T now stands for $\tilde{T}$. Here,

$$\left.\begin{aligned} P_r &= \frac{\rho c_p}{k} K a^{2+(1/n)} = \text{the generalised Prandtl number} \\ E_c &= \frac{K^2 n^2}{(n+1)^2}\frac{a^{2+(2/n)}}{c_p T_c} = \text{the generalised Eckert number} \\ \beta &= E_c/R_e \end{aligned}\right\} \quad [2.20]$$

The boundary conditions being

$$T=0 \text{ on } r=1. \quad [2.21]$$

3. Solution of the Energy Equation

Upto first order of L, taking

$$T=T_0(r,\theta)+L\,T_1(r,\theta), \quad [3.1]$$

substituting for velocity components from [2.14]—[2.18] and separating the various order terms, [2.19] reduces to

$$\frac{\partial^2 T_0}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 T_0}{\partial \theta^2}+\frac{1}{r}\frac{\partial T_0}{\partial r}=4\left(1+\frac{1}{n}\right)\beta P_r r^{1+(1/n)}, \quad [3.2]$$

and

$$\frac{\partial^2 T_1}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 T_1}{\partial \theta^2}+\frac{1}{r}\frac{\partial T_1}{\partial r}=R_e P_r\left(u\frac{\partial T_0}{\partial r}+\frac{v}{r}\frac{\partial T_0}{\partial \theta}\right)$$

$$+4(n+1)\beta P_r r\left(\frac{\partial w_1}{\partial r}-w_0\sin\theta\right)-\sin\theta\frac{\partial T_0}{\partial r} \quad [3.3]$$

with boundary conditions

$$T_0=0;\ T_1=0 \text{ on } r=1 \quad [3.4]$$

Solving we obtain

$$T_0 = \frac{4\,\beta\, P_r\, n\,(n+1)}{(3\,n+1)^2}\,(1 - r^{(3n+1)/n}) \tag{3.5}$$

while

$$T_1 = \frac{4\,\beta\, P_r}{(3\,n+1)}\, r\, F(r)\sin\theta\,, \tag{3.6}$$

where

$$\begin{aligned} F(r) &= A_1\,(1-r^{(n+1)/n}) + A_2\,(1-r^{(2n+2)/n}) + A_3\,(1-r^{(3n+1)/n}) \\ &+ A_4\,(1-r^{(4n+4)/n}) + A_5\,(1-r^{(5n+3)/n}) + A_6\,(1-r^{(5n+5)/n}) \\ &+ A_7\,(1-r^{(6n+4)/n}) + A_8\,(1-r^{(ns+n+2)/n}) + A_9\,(1-r^{(ns+2n+1)/n}), \end{aligned} \tag{3.7}$$

and the constants $A_1, A_2, \cdots, A_9$ are given by

$$A_1 = \frac{(3\,n+4)\,(n+1)}{2\,(3\,n+1)} - \frac{R_e^2}{120}\left[30\,A + 5\,C - 3\,D + \frac{60\,B\,(n+1)^2}{(ns+1)\,(ns+2+n)}\right]$$

$$A_2 = \frac{R_e^2\,(n+2)\,(3n+1)}{16\,(2n+1)}\,A\,,$$

$$A_3 = \frac{(n+1)}{2\,(3n+1)\,(5n+1)}\,[2\,R_e\,P_r\,A\,n^2 - (11\,n^2 + 13\,n + 4)]\,,$$

$$A_4 = \frac{R_e^2\,(3\,n+4)\,(3\,n+1)}{192\,(3\,n+2)}\,C\,,$$

$$A_5 = \frac{R_e\,P_r\,n^2\,(n+1)}{(7n+3)\,(5n+3)}\,C\,,$$

$$A_6 = -\frac{R_e^2\,(4\,n+5)\,(3\,n+1)}{200\,(7\,n+5)}\,D\,,$$

$$A_7 = -\frac{R_e\,P_r\,n^2\,(n+1)}{8\,(2n+1)\,(3n+2)}\,D\,,$$

$$A_8 = \frac{R_e^2\,(n+1)^3\,(3\,n+1)\,(ns+2)}{2\,(ns+1)\,(ns+3\,n+2)\,(ns+2+n)^2}\,B,$$

$$A_9 = \frac{R_e\,P_r\,n^2\,(n+1)}{(ns+4\,n+1)\,(ns+2\,n+1)}\,B,$$

It is to be noted that the coefficients A_3, A_5, A_7 and A_9 depend on Pr and consequently represent convective contribution to temperature and the rest on dissipative contribution. This fact plays an important role later in the discussion.

4. Discussion of the Result

The solution of the energy equation [2.19] consists of two parts. T_0 gives the temperature profile on neglecting the curvature, while T_1 is the contribution to the temperature distribution due to the curvature of the pipe. We note that for the Newtonian case, T_0 reduces to the solution obtained by Schlichting[15].

(*a*) *Newtonian fluid* ($n=1$) : The isotherms for a Newtonian fluid ($n=1$) are plotted in figure 1. Taking $Re=63.3$, $L=\frac{1}{3}$ (same as used by Dean), $Pr=10^3$, we notice that the isotherms are similar to the ones given by Hawes[4] based on his experimental investigations. The curve $\bar{T}=0$ divides the cross-section into two domains in each of which the isotherms form closed curves. The domain towards the inner side of the curvature represents cooled region, the maximum relative drop in temperature being 0.3729, while the off side represents the heated region, the maximum rise in temperature being 1.275. This asymmetry is due to the centrifugal force which tends to push the fluid elements to the off side. Secondly, the separating isotherm $\bar{T}=0$ occurs almost in the same domain where the fluid elements detach from the off side boundary and flow into the interior.

Figure 2 gives the variation of temperature for a Newtonian fluid for different Prandtl numbers. It is noticed that for Prandtl numbers beyond critical value, the isotherms divide the cross section into cooled and heated regions, whereas for lower Prandtl numbers, the fluid is heated uniformly throughout the cross-section. It is evident from the energy equation that the increase of Prandtl number can be thought of as an apparent increase in Reynolds number and hence that of Deans number. According to McConalogue and Srivastava[16], it is seen that with the increase of Deans number the secondary flow field can be broadly classified into shear free mid-region and off side attached boundary layer region, and towards the inside the fluid elements flow into the interior from the boundary. The axial velocity profiles become more oval shaped with the centre shifting towards the off side, upto a critical position as the Deans number increases. The off side region is highly sheared and almost negligible shear on the inside. Consequently, with the increase of Prandtl number, the fluid elements convect more heat from the shear free mid-region and comparatively shear free inside region, thus producing cooled region on the inside and heated region on the off side.

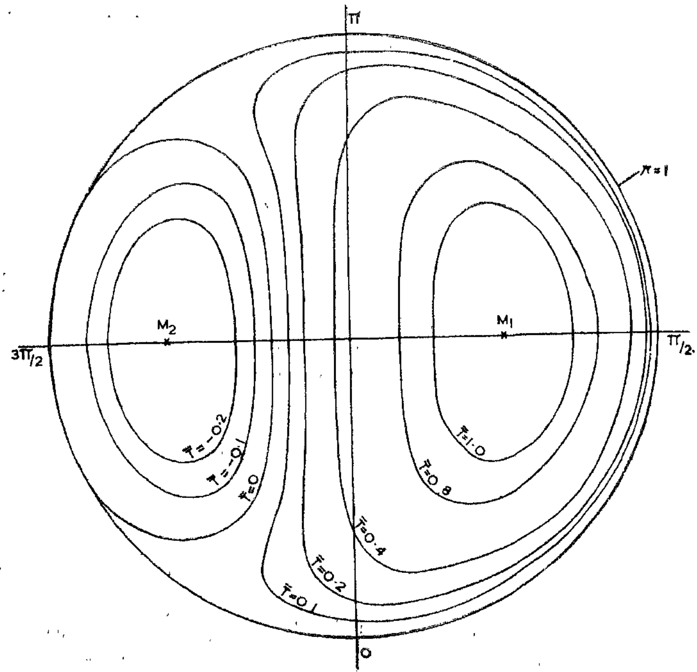

FIG. 1

Isotherms For Newtonian Fluids

This effcct is due to the behaviour of the flow field and consequently needs a critical Prandtl number. For lower Prandtl numbers the shear free region is very less and the convective effects are smaller. Therefore the fluid is uniformly heated throughout and no such separation takes place. The figure 1 confirms this pattern and from numerical computations we find that the critical Prandtl number is about 460. The order of magnitude of this Prandl number agrees well with the experiments of [7] and as discussed in [5].

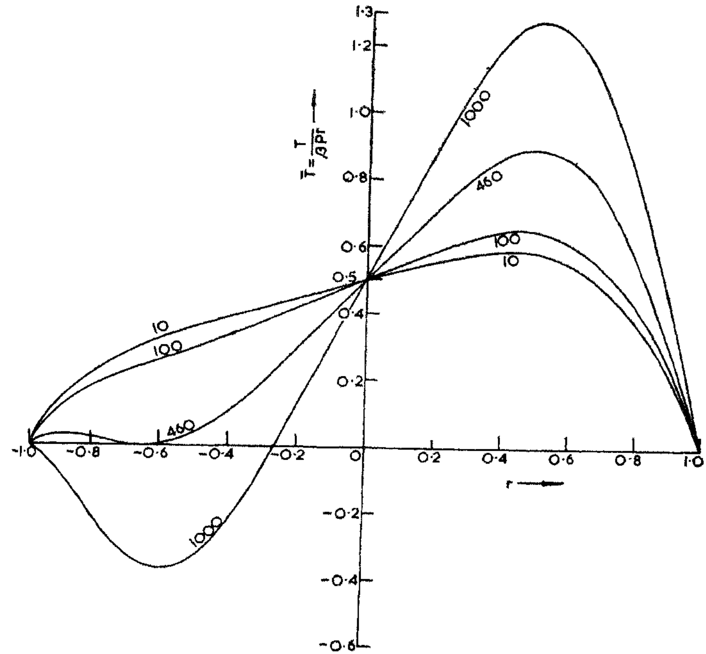

FIG. 2

Variation of Temperature for Newtonian Fluid for different Prandtl Numbers, Critical Prandtl Number Being 460

(*b*) *Non-Newtonian fluids* : A comparison of isotherms for Newtonian fluids ($n=1$), dilatant fluids ($n=1.2$) and psuedoplastic fluids ($n=0.8$) is given in figure 3. The general pattern of the isotherms for non-Newtonian fluids is similar to that of Newtonian fluids.

The behavior of non-Newtonian power law fluids can easily be understood in terms of heat balance. Psuedoplastic fluids with $n < 1$ sustain less strain and dilatant fluids with $n > 1$ sustain more strain than Newtonian fluids. Consequently, for psuedoplastic fluids both heat generation as well as convective effects are less and for dilatant fluids they are more. So one expects weaker

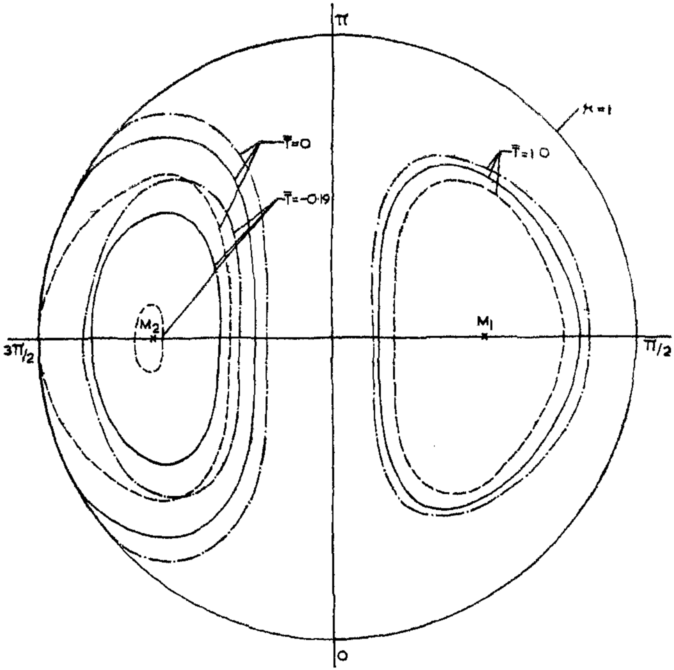

FIG. 3

Comparison of the isotherms for Newtonian ($n=1$ ——), Dilatant ($n=1.2$ — . —) and Pseudoplastic Fluids ($n=0.8$ — —)

separation of cooler and hotter regions for the former, while it is stronger for the latter. This is confirmed in figure 3. As in the case of Newtonian fluids discussed earlier, we can find the critical Prandtl numbers for non-Newtonian fluids also. The following table gives relative maximum and minimum temperatures and the critical Prandtl numbers for different values of n.

n	Maximum relative cooling, ($Pr=10^3$)	Maximum relative heating, ($Pr=10^3$)	Critical Prandtl number
0.8	−0.20	1.11	570
1.0	−0.37	1.28	460
1.5	−0.51	1.41	380

Finally figure 4 gives the variation of temperatures for Newtonian and non-Newtonian fluids for a fixed Prandtl number, 10^3. From these results we conclude that the dilatant fluids should be a better material for the efficient working of a heat exchanger.

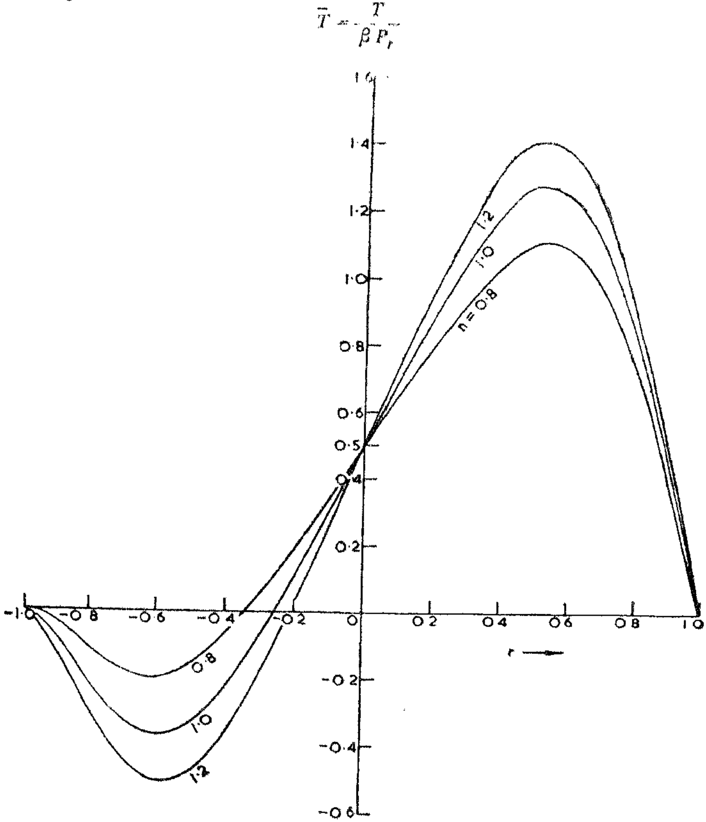

FIG. 4

Variation of Temperature for different values of n when $\theta = \pm\pi/2$

5. NUSSELT NUMBER

The non-dimensional mean bulk temperature using [3.5] and [3.6] with [3.7] is given by

$$\bar{T}_v = \frac{4\beta Pr\, n(n+1)(4n+1)}{(3n+1)^2(5n+1)} + \frac{T_w}{T_c}. \qquad [5.1]$$

The mean heat flux per unit length along the cross-section of the pipe in dimensional form is

$$q = -\frac{k}{2\pi a}\int_0^{2\pi}\left(\frac{\partial T}{\partial R}\right)_{R=a} a\, d\theta. \qquad [5.2]$$

Then the Nusselt number is defined to be

$$Nu = \frac{q\, 2a}{k\, \bar{\bar{T}}_v}. \qquad [5.3]$$

Using [5.1] and [5.2] along with [3.5], [3.6] and [3.7] the Nusselt number is found to be

$$Nu = \frac{1}{G(n) + d\, H(n)}, \qquad [5.4]$$

where

$$\left.\begin{aligned} G(n) &= \frac{n(4n+1)}{(3n+1)(5n+1)}, \\ H(n) &= \frac{(3n+1)}{4(n+1)} \\ d &= T_w / 4\beta\sigma\, T_c. \end{aligned}\right\} \qquad [5.5]$$

It is evident that up to the first order of the L the Nusselt number is not affected by the curvature as in the case of rate of flow discussed by Dean. However, we find that Nusselt number increases with decreasing value of n, which is qualitatively same as in the other investigations of heat transfer phenomena[17].

ACKNOWLEDGEMENT

The authors wish to thank Prof. P. L. Bhatnagar and Dr. C. Devanathan for the helpful discussions in the preparation of this paper.

REFERENCES

1. A. B. Metzner .. .. 'Advances in Heat transfer', Vol. 2, Ed. J.P. Hartnett and T. F. Irvine Jr. Academic Press, 1965
2. A. H. P. Skelland .. .. 'Non-Newtonian flow and heat transfer', John Wiley and Sons, Inc, New York, 1967.
3. A. P. Fraas and M N. Ozisik .. 'Heat Exchanger Design'. John Wiley and Sons, Inc., New York, 1965.
4. W. B. Hawes .. .. *Trans Instn.chem.Engrs.*, 1932, **10**, 161.
5. M. N. Ozisik and H. C. Topakoglu *J. Heat Transfer.* 1968, **90**, 313.
6. V. Kubair and N. R. Kuloor .. *Int.J.Heat Mass Transfer*, 1966, **9**, 63.
7. R. A. Seban and E. F. McLaughlin *Ibid*, 1963, **6**, 387.
8. Y. Mori and W. Nakayama .. *Ibid*. 1965, **8**, 67.
9. ———— .. .. .. *Ibid*, 1967, **10**, 37.
10. ———— .. .. .. *Ibid*, 1967, **10**, 681.
11. A. B. Metzner .. .. .. Handbook of fluid dynamics, ed. V.L.Streeter, McGrawHill Book Co., New York, 1961.
12. S. L. Rathna .. .. .. Proceedings of Conference on Fluid Mechanics, Bangalore, 1967, 378.
13. Y. Tomita .. .. .. *Bull J.S.M.E.*, 1959, **2**, 469.
14. W. R. Dean .. .. .. *Phil. Mag.*, 1927, **4**, 208.
15. H. Schlichting .. .. 'Boundary Layer Theory' McGraw Hill Book Co., New York, 1967.
16. D. J. McConalogue and R. S. Srivastava. *Proc.R.Soc.*, 1968. **307A**, 37.
17. M. J. Shah, E. E. Petersen and A. Acrivos. AICHE Jl., 1962, **8**, 542,