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Identification of unstable transfer model with a zero by optimization method

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Abstract

A method is proposed for closed loop identification of an unstable first-order plus time delay with a zero transfer function model using the step response of a PI or PID-controlled system. A standard optimization method is used to estimate the model parameters to match the closed loop response of the model with that of the actual response. A simple method is proposed for initial guesses of the transfer function model parameters (time delay, time constant, gain and the value of zero). The method is applied to unstable transfer function models with a positive or negative zero, and to a CSTR to identify an unstable FOPTD (first-order plus time delay) with a zero. It gives good results in all the simulation case studies considered. The effect of measurement noise and controller settings on the identification of transfer function model is studied.

Keywords: Unstable FOPTD system with a zero, PI/PID controller, optimization method, identification.

1. Introduction

Transfer function models are used to design PI/PID controllers [1], [2]. Closed loop identification method is preferred over the open loop method since the former is insensitive to disturbances, and is essential to identify the transfer function model of an unstable system. Kavdia and Chidambaram [3] and Srinivas and Chidambaram [4] have proposed closed loop methods for identifying unstable first-order plus time delay (FOPTD) without any zero using the response of a proportional controller. Since the proportional controller introduces an offset in the response, the method is not employed in chemical plants. In addition, in the case of certain parameter values (for example, when the ratio of time delay to time constant is greater than 0.7) of unstable FOPTD systems, proportional controller alone cannot stabilize the processes. Recently, Pramod and Chidambaram [5] have proposed a closed loop method for identifying an unstable FOPTD model without any zero using the step response of a PID-controlled systems response. There are systems which are to be modeled as unstable FOPTD with a zero: $k_p\,(1-\tau_N\,s)\,e^{-Ls}/(\tau\,s-1)$. Here, k_p is the process steady-state gain, τ_N, the time constant of the numerator, τ, the time constant of the system and L, the time delay. τ_N, τ and L have units of time. Transfer function model can be obtained by linearizing the dynamic mathematical model or by using the servo response of the closed loop system. The zero may be positive or negative depending on the process behavior. Such transfer functions are reported for systems such as chemical reactors [6]. The presence of a zero in the transfer function model introduces overshoot or inverse response (depending on whether the zero is negative or positive) behaviour [7]. If the system has an unstable pole, then both the initial undershoot and overshoot are further increased. In the present work, a closed

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loop identification method based on a standard optimization technique is proposed. The method is applied to three transfer function models and to a chemical reactor problem.

2. Proposed method

The response ($y_{p,cl}$) of the closed system using a PI or PID controller is obtained for a known magnitude of change in the set point. The transfer function of the process is assumed as $k_p\,(1 - \tau_N\, s)\, e^{-Ls}/(\tau\, s - 1)$. From the closed loop response, the initial guess values for the model parameters (k_p, τ_N, τ and L) are calculated (see next section). Using this model and same controller settings, the closed loop response for the same step magnitude is obtained by simulation. Let the response be denoted by $y_{m,cl}$. The final model parameters are obtained by minimizing the sum of the squared difference between $y_{p,cl}$ and $y_{m,cl}$. The objective function is then the sum of $(y_{p,cl} - y_{m,cl})^2$ over several points of the response. The MATLAB optimization routine *leastsq* is used in the present work. This method employs Levenberg–Marquadrt algorithm.

A major criticism in using any optimization method is the selection of initial guess values of the model parameters for which a procedure is suggested in the present work for model parameters k_p, τ_N, τ and L: (i) The time delay is assumed to be that of the closed loop response. (ii) The guess value for the time constant of the process is assumed as half of the effective time constant (τ_e) of the closed loop response. The value of τ_e is assumed as $t_s/4$, where τ_s is the settling time of the closed loop response (to reach 98% of the steady-state value and to remain within the limit). Hence, the initial guess for τ is given by $t_s/8$. (iii) The guess value for k_p is calculated from simple proportional controller formulae for unstable FOPTD system [8] as $(\tau/L)^{0.5}/k_c$, where L is the guess value for the time delay and k_c, the proportionality constant of the PID controller used.

The initial guess value for τ_N is selected as that of the closed loop system. The guess for τ_N for the closed loop system is selected as follows: (i) If the response shows an inverse behaviour (similar to that shown in Fig. 1), then the guess value for τ_N is selected as the time duration in which the inverse response is observed in the output. This assumption follows from the approximation exp $(-\tau_N s) = 1-\tau_N s$. In case no inverse response is noticed, then the open loop transfer function must contain a negative zero $(1+\tau_N s)$. In such cases, the initial guess for τ_N is assumed to be the delay or 0.

To evaluate the proposed method, transfer function model with known model parameters is considered with suitable PI/PID controller and the closed loop response obtained. Using the proposed method and the data of the closed loop response, model parameters are estimated and the closed loop response of the identified model is compared with the original system for the same PID settings.

For the system whose transfer function is not known, closed loop servo response should be obtained with a suitable PID controller. Employing the proposed method and the data of closed loop response, model parameters are estimated. A PID controller is designed for the identified model. This PID controller will be implemented on the original system and also on the identified model and the servo responses compared.

Table I
Effect of initial guess values of the model parameters on the number of iterations and computational time for case study 1

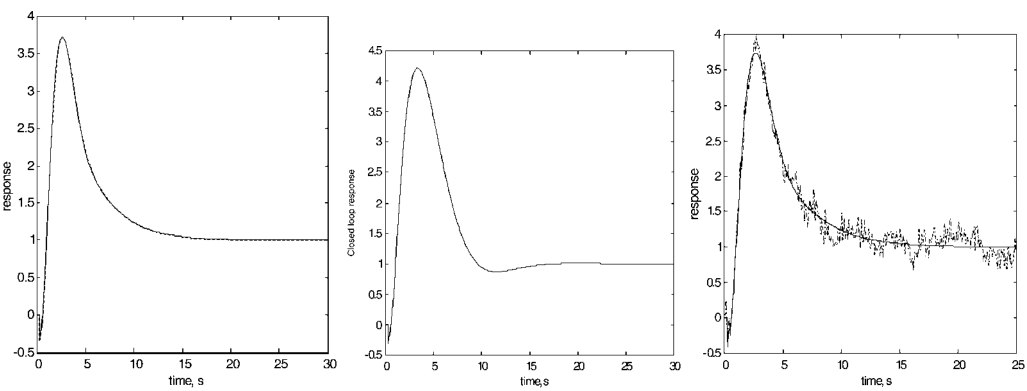
Parameter	For guesses by the present method	For guess variation in k_p		For guess variation in τ		For guess variation in L		For guess variation in τ_N	
		+10%	−10%	+10%	−10%	+10%	−10%	+10%	−10%
Number of iterations	69	77	69	126	62	69	132	70	71
Computational time	9.9	11.3	10.2	19.1	9.18	10.16	19.39	10.27	10.71

+10%: The initial guess values used are 1.1 times that of the values of the proposed method.

3. Simulation results

3.1. *Case study 1*

Let us consider the process $(1 - 0.25\ s)\ e^{-0.25s} / (s-1)$. The PI settings used are k =1.43 and τ_I=15. The closed loop response for a unit step change in set point is obtained by the *Simulink* package and the response is shown in Fig. 1(a). As discussed in the previous section, the initial guess values for the model parameters are obtained from Fig. 1(a) as: $L = 0.25$, $\tau_N = 0.25$, $k_p = 1.9$ and $\tau = (15/8) = 1.85$. MATLAB routine *leastsq* is used. The final converged parameters are $L = 0.25$, $\tau_N = 0.25$, $k_p = 1$ and $\tau = 1$. The computational time on Pentium III PC (933MHz) is 9.9 s and the number of iterations is 69. The closed loop response of this identified model with the same PI settings, shown in Fig. 1(a) along with that of the actual process, has very good matching. The initial guess values are changed by 10% of the proposed method considering one parameter at a time. The model parameters converge to the same values. The computational time required and the number of iterations for the perturbed initial guesses are listed in Table I. The computational time and the number of iterations using the proposed method for the initial guesses are less compared to the ±10% perturbed values of initial guesses of the proposed method. This is particularly seen in the case of increased initial guess values (+10%) for process gain and time



(a) (b) (c)

FIG. 1. Identification of transfer function model for case study 1 with (a) PI controller, (b) PID controller and (c) noise corrupted output. Solid: actual process, dot: identified model.

Table II
Effect of changing controller settings on the identification of transfer function model

Transfer function model	PID settings		Initial guess values				Final converged parameters			
	k_c	τ_I	k_p	τ	τ_N	L	k_p	τ	τ_N	L
Case	1.43	15	1.9	2.5	0.25	0.25	1	1	0.25	0.25
study 1	1.29	18	2.58	2.8	0.33	0.25	1	1	0.25	0.25
Case	1.87	8.6	1.69	2.5	0.0	0.25	0.99	0.98	–0.25	0.27
study 2	1.68	10.32	1.88	2.5	0.0	0.25	0.99	0.98	–0.25	0.25

constant and decreased values (–10%) of time delay. However, the model parameters converge to the same values.

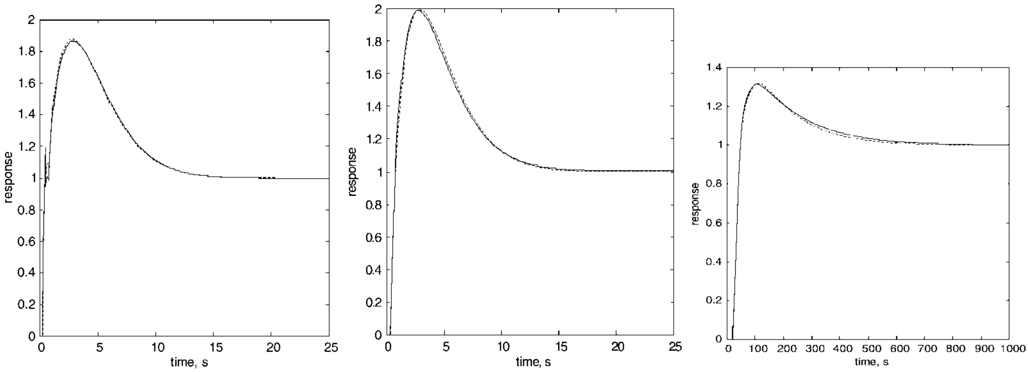
As stated earlier, the initial guess is important for any optimization method. To show the importance of the proposed method, the initial guess values of the parameter were deviated significantly (each value is twice that of the value of the proposed method). The optimization method did not converge. Hence, it is better to use the guess values of the present method which offer guaranteed convergence.

The same process with different PI controller settings [k_c = 1.2852, τ_I = 18] is considered. The closed loop response of the process is shown in Fig. 1(b). The initial guess values for the model parameters are obtained from Fig. 1(b) as τ_d = 0.25, τ_N = 0.33, k_p = 2.58 and τ = (20/8) = 2.5. The optimization method (*leastsq*) converged to the same parameters (τ_d = 0.25, τ_N = 0.25, k_p = 1 and τ = 1). The computational time on Pentium III PC is 14.39 s and the number of iterations is 84. The closed loop response of this identified model with the same PI settings, shown in Fig. 1(b) along with that of the original process, offers very good matching. The identified model is robust to the controller settings (Table II).

The effect of measurement noise on the model parameters is evaluated by adding white noise with noise power 0.001 and the sample time 0.1 s to the process output. The corrupted output (refer Fig. 1(c)) is used for feedback control action and for model identification. To get the initial guesses, a smooth curve is first drawn. As discussed earlier, the initial guess values of the model parameters are noted from the smoothed curve as τ_d = 0.25, τ_N = 0.25, k_p = 2.32 and τ = (20/8) = 2.5. The final identified parameters obtained from the optimization method are τ_d = 0.2535, τ_N = 0.2626, k_p = 1.0009 and τ = 1.0272. These parameters are close to those obtained without measurement noise. The computational time on Pentium III PC is 13.2 s and the number of iterations is 94 (to Table III). The proposed method hence is robust to the measurement noise.

3.2. *Case study 2*

Let us consider the process (1+0.25 s) $e^{-0.25s}$/(s–1). The PI settings used are k_c = 1.87 and τ_I = 8.6. The closed loop response for a unit step change in set point is obtained by *Simulink* package and the response is shown in Fig. 2(a). As discussed in the previous section, the initial guess values for the model parameters obtained from Fig. 2(a) are: L = 0.25, τ_N = 0.0, k_p =1.69 and τ = (20/8) = 2.5. MATLAB routine *leastsq* is used. The final converged parameters are L = 0.2692, τ_N =



(a) (b) (c)

FIG. 2. Identification of transfer function model for (a) case study 2, (b) case study 3 and (c) the reactor problem. Solid: actual process, dot: identified model.

-0.2516, k_p= 0.9957 and τ = 0.9784, the computational time is 11.1 s and the number of iterations is 70. The closed loop response of this identified model with the same PI settings shown in Fig. 2(a) along with that of the actual process offers very good matching. The initial guess values are changed by 10% of the proposed method considering one parameter at a time. The model parameters converge to the same values. The identified model is the same even if the controller settings are changed (refer to Table II). The effect of white noise in the process output on identification of transfer function model is evaluated and the converged parameters are listed in Table III.

3.3. *Case study 3*

Let us consider the process (1+0.25 s) $e^{-0.25s}$/[(s-1) (0.1 s +1)]. The PID settings used are k_c = 1.8515, τ_I = 9.817 and τ_D = 0.1233. A first-order filter time constant 0.135 is also used in series with the controller. The closed loop response for a unit step change in set point is obtained by *Simulink* package and the response is shown in Fig. 2(b). As discussed in the previous section, the initial guess values for the model parameters obtained from Fig. 2(b) are: L = 0.25, τ_N = 0.0, k_p = 1.71 and τ = (20/8) = 2.5. MATLAB routine *leastsq* is used. The final converged parameters are L = 0.3266, τ_N = -0.2135, k_p = 1.0002 and τ = 1.0087. The computational time is 12.19 *s* and the number of iterations is 76. The closed loop response of this identified model with the same PID settings, shown in Fig. 2(b) along with that of the actual process, offers very good matching. The effect of white noise in the process output on identification of transfer function model is evaluated and the converged parameters are listed in Table III.

3.4. *Case study 4: Application to a chemical reactor problem*

We consider an isothermal CSTR with the reaction rate given by $[-k_1\, c/(1+k_2\, c)^2]$. The nonideal mixing is described by Cholette's model. Here n is the fraction of the reactant feed that enters the zones of the perfect mixing and m the fraction of the total volume of the reactor where reaction occurs [i.e. $(1-m)$ fraction of the volume is a dead zone]. Liou and Chien [9] give the transient equation for the reactor as:

Table III
Effect of noise on the identification of transfer function model

Transfer function model	Without noise						With noise					
	Final converged values				No	Time	Final converged values				No	Time
	k_p	L	τ_N	τ			k_p	L	τ_N	τ		
Case study 1	1	0.25	0.250	1	69	9.9	1	0.254	0.264	1.02	94	13.2
Case study 2	1	0.25	–0.25	1	70	11.1	1	0.22	–0.21	1.04	196	31.7
Case study 3	0.99	0.32	–0.21	1	76	12.2	1	0.29	–0.17	1.04	260	41.8
Chemical reactor	4.23	20.3	–10.6	175	110	64.5	4.2	25.2	–17.5	198.4	134	78.81

L, τ_N and τ are in s.

$$dc/dt = (nq/mv)(c_f - c) - [k_1\, c/(1+k_2\, c)^2] \quad (1)$$
$$n\, c + (1-n)\, c_f = c_e \quad (2)$$
$$\text{at } t = 0,\ c = c_o \quad (3)$$

Here, c and c_e are, respectively, the concentrations of the reactant in the well-mixed reactor zone and the exit stream. The controlled variable is c_e and the manipulated variable is the feed concentration c_f. For the present simulation study, we consider $n = m = 0.75$, $k_1 = 10\ s^{-1}$; $k_2 = 10\ (mol/l)^{-1}$; V = 1 l.

This particular rate form $[-k_1\, c/(1 + k_2\, c)^2]$ has been extensively studied [10] and its applicability to heterogeneous and enzyme-catalyzed reactions has been demonstrated. For $c_f = 3.288$ mol/l, we get $c_e = 1.8$ mol/l and $c = 1.304$ mol/l, and the linearization of the nonlinear equations around these nominal operating points gives the transfer function model as $\Delta c_e(s)/\ \Delta c_f(s) = 2.21\ (1 + 11.133\ s)\ e^{-20s}/(98.3\ s–1)$. We have assumed a measurement delay of 20 s in the derivation of the above transfer function model. A PID controller with the settings $k_c = 1.477$, $\tau_I = 229.1$ and $\tau_D = 9.56$ and a first-order filter (time constant = 10.67) is considered. The closed loop servo response of the nonlinear equation for a step change in c_e from 1.8 to 1.9 is obtained as shown in Fig. 2(c) (in terms of deviation variable). Employing the method discussed earlier, the initial guesses for the model parameters are obtained from the response as $L = 20$, $\tau_N = 0$, $k_p = 1.311$ and $\tau = (600/8) = 75$. The *leastsq* MATLAB routine gives the final converged parameters as $L = 20.254$, $\tau_N = -10.55$, $k_p = 4.23$ and $\tau = 175.4$. The computational time on Pentium III PC is 64.5 s and the number of iterations is 110. The closed loop response of this identified model with the same PID and filter settings gives very good comparison with that of the actual response as shown in Fig. 2(c). The initial guess values are changed by 10% of the proposed method considering one parameter at a time. The optimization routine gives the same final converged parameters. The effect of white noise on the process output on identification of transfer function model is evaluated and the converged parameters are listed in Table III.

In all the above case studies, the closed loop time constant is obtained from the response as $(t_s/4)$. It is assumed that the closed loop time constant is equal to twice that of open loop time constant, i.e. $\tau_c = 2\tau$ and hence $\tau = \tau_c/2 = t_s/8$. For higher order systems, $\tau_c = n\tau$, where n is more than two. If we take $n = 3$, then $\tau = t_s/12$. The value of n depends on the controller system design. Initial guess values of τ also affect the initial guess values of k_p. If we take $t_s/12$ rather than $t_s/8$ for the guess value of τ, for some cases during the iterative process (unconstrained optimization), this value goes to the negative side and reverts to the positive side thereby taking longer time to converge. Therefore, we recommend the initial guess for τ to be $t_s/8$. By simulation, it has also been observed that depending on the case study the parameter that is insensitive to initial guesses keeps changing.

4. Conclusions

A method is proposed to identify an unstable first-order plus time delay model with a zero using the PI or PID-controlled step response data. The identification is carried out by a standard optimization routine. The initial guesses for the model parameters are obtained from the closed loop initial delay, settling time, the controller gain and the time during which an inverse response is obtained. The proposed method can be used regardless of the type of closed loop response (such as over or under damped) in the identification step. The proposed method gives model parameters whose response matches well with that of the system. The effect of changing controller settings on identification of the transfer function model is also studied. The proposed method is robust to measurement noise and change in controller settings. If the initial guess is selected arbitrarily, the optimization routine does not converge. The guess values by the present method give guaranteed convergence. Simulation studies of the proposed method of three unstable transfer function models and also of the nonlinear CSTR control problem show that the identified models are in good agreement with the original processes.

References

1. K. J. Astrom, and T. Hagglund, *PID controllers: Theory, design and tuning,* ISA Pub., Research Triangle Park, NC, USA (1995).

2. M. Chidambaram, *Applied process control,* Allied Publishers, New Delhi, p.7 (1998).

3. M. Kavdia and M. Chidambaram, On-line controller tuning for unstable systems, *Comp. Chem. Engng,* **20**, 301–305 (1996).

4. N. Srinivas and M. Chidambaram, Comparison of on-line controller tuning methods for unstable systems, *Process Control Quality*, **8**, 177–183 (1996).

5. S. Pramod and M. Chidambaram, Closed loop identification by optimization method for tuning PID controllers, *Indian Chem. Engr,* **43**, 90–94 (2001).

6. M. Chidambaram and G. P. Reddy, Nonlinear control of systems with input and output and multiplicity, *Comp. Chem. Engng,* **20**, 295–299 (1997).

7. C. C. Goodwin, S. F. Grace and M. E. Salgado, *Control system design*, Prentice-Hall, pp. 216–218 (2001).

8. A. M. Depaor and M. O'Malley, Controllers of Ziegle–Nichols type for unstable processes with time delay, *Int. J. Control*, **41**, 1025–1036 (1989).

9. C. T. Liou and Y. S. Chien, The effect of non-ideal mixing on input multiplicity in a CSTR, *Chem. Engng Sci.*, **46**, 2113–2116 (1991).

10. V. R. Kumar and B. D. Kulkarni, On the operation of a bistable CSTR: A strategy employing stochastic resonance, *Chem. Engng Sci.*, **49**, 2709–2713 (1994).