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STABILITY OF AN ACCELERATED PLASMA SHELL IN THE PRESENCE OF MAGNETIC FIELD

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ABSTRACT

The stability of an accelerated cylindrical shell of conducting fluid in the presence of a magnetic field having axial and azimuthal components against small wave number axi-symmetric disturbances is studied. In the exploding case, it is seen that the system is overstable and the frequency of unbounded oscillations depends on the thickness, the rate of explosion and the initial magnetic field of the system. Further, we find that the presence of a volume current distribution reduces the growth rate of instability. In the imploding case there occurs pure instability, the growth rate of which increases logarithmically and does not depend on the initial magnetic field but depends on the thickness of the shell and on the rate of collapse.

1. INTRODUCTION

The considerable interest in the problem of stability of a collapsing or exploding cylindrical shell of conducting medium in the presence of a magnetic field originates in an effort to investigate the means of production of high magnetic fields which will lead towards the success of thermonuclear energy generation. A number of theoretical and experimental studies have been done recently in this subject. Linhart(1960) and Fowler *et al.* (1960) have discussed the stability of a radially accelerated cylindrical shell of conducting material in the presence of poloidal or toroidal magnetic field, the field being either inside the shell or outside the shell. Harris (1962) has considered a shell of infinitesimal thickness and discussed its stability against azimuthal and axial disturbances in the presence of axial magnetic fields. Recently Bhatnagar and Bhat (1968) reconsidered the problem considered by Harris (1962) with finite thickness of the shell in the presence of both poloidal and toroidal magnetic fields against axial disturbances. They found that an exploding shell is overstable and the frequency of unbounded oscillations depends on the initial magnetic field, the rate of explosion and on the thickness of the shell; while, an imploding shell is purely unstable and the growth rate of instability depends on the rate of collapse and the thickness of the shell.

In the present paper we have considered the effect of a steady volume current distribution on the stability of a cylindrical shell, which is composed of incompressible, inviscid and ideally conducting fluid of finite thickness against axisymmetric disturbances. A magnetic field having both poloidal and toroidal components prevails inside the shell. In the inner vacuum of the shell there is a co-axial conducting rod of radius 'R', carrying a uniform axial current and the rod is insulated by means of a non-conducting sheet.

2. EQUILIBRIUM STATE

At time $t=0$, let the inner and outer radii of the shell be R_1 and R_2 and let 'R' be the radius of the coaxial conducting rod. The state of magnetic field and plasma pressure in non-dimensional form is as follows:

$$\vec{H}^* = \begin{cases} (0,\ \lambda_1^* r^*,\ 1) & r^* \leq R^*, \\ \left(0,\ \dfrac{\lambda_2^*}{r^*},\ 1\right) & R^* \leq r^* \leq 1, \\ \left(0,\ \lambda_3^* r^* + \dfrac{\lambda_4^*}{r^*},\ H_2^*\right) & 1 \leq r^* \leq R_2^* \\ \left(0,\ \dfrac{\lambda_5^*}{r^*},\ H_3^*\right) & r^* \geq R_2^*, \end{cases} \qquad [2.1]$$

$$P_{\text{plasma}} = -2\lambda_3^* \left(\frac{\lambda_3^* r^{*2}}{2} + \lambda_4^* \ln r^*\right) + A, \qquad [2.2]$$

where 'A' is a constant. Here

$$\left.\begin{aligned} \lambda_1^* &= \frac{I_0 R_1}{2 H_0}, \quad \lambda_2^* = \lambda_1^* R^{*2}, \quad \lambda_3^* = \frac{I_1 R_1}{2 H_0} \\ \lambda_4^* &= (\lambda_1^* R^{*2} - \lambda_3^*), \\ \lambda_5^* &= (\lambda_3^* R_2^* + \lambda_1^{*\,2} R^{*2} - \lambda_3^*). \end{aligned}\right\} \quad [2.3]$$

In non-dimensionalising the physical quantities, we have used the following characteristic quantities :

$$\left.\begin{array}{lll} \text{Characteristic length} & & = R_1, \\ \quad \text{do} & \text{magnetic field} & = H_0, \\ \quad \text{do} & \text{velocity} & = F/R_1, \\ \quad \text{do} & \text{pressure} & \mu = H_0^2, \\ \quad \text{do} & \text{time} & = R_1^2/F, \end{array}\right\} \quad [2.4]$$

In the above I_0, I_1 are axial uniform line currents in the central rod and in the plasma column, respectively and H_0, H_2 and H_3 are axial magnetic fields. We shall drop the stars over the quantities in our further analysis.

Starting from this initial state, we impose a radial velocity $u_r = F/r$ on the system, where F is a pure constant, and calculate the physical and dynamical state at any time $t > 0$. We record the solutions below :

(a) *Exploding case* :

Conducting fluid: $R_i < r < R_0$

$$R_i^2 = 1 + 2t; \quad R_0^2 = R_2^2 + 2t.$$

$$\vec{V} = [(1/r),\ 0,\ 0); \quad \vec{H} = \left(0,\ \lambda_3 r + \frac{\lambda_4 r}{r^2 - 2t},\ H_2\right),$$

$$\vec{E} = \left[0,\ (H_2/r),\ -\left(\lambda_3 + \frac{\lambda_4}{r^2 - 2t}\right)\right],$$

$$\vec{J} = \left[0,\ 0,\ 2\lambda_3 - \frac{4\lambda 4t}{(r^2 - 2t)^2}\right],$$

$$P_{\text{plasma}} = \tfrac{1}{2}(1 - H_2^2) + \tfrac{1}{2}(1/R_i^2 V_A^2) + \lambda_3 R_i^2 + 2\lambda_3 \lambda_4 t + \lambda_4^2 t - (\lambda_4 + \lambda_3)^2 (1 + 2t)$$
$$+ \frac{\lambda_2}{2R_i^2} - \frac{1}{2r^2 V_A^2} - \lambda_3^2 r^2 - \frac{2\lambda_3 \lambda_4 t}{(r^2 - 2t)} - \lambda_3 \lambda_4 \ln(r^2 - 2t) - \frac{\lambda_4^2 t}{(r^2 - 2t)^2}.$$

Outer Vacuum : $r > R_0$.

$$\vec{V} = (0,\ 0,\ 0)\ ;\quad \vec{H} = (0,\ 0,\ H_3)$$

$$\vec{J} = (0,\ 0,\ 0)\ ;\quad \vec{E} = \left[0,\quad \frac{H_3 - H_2}{R_0},\quad \frac{1}{R_0}\left\{\lambda_3 R_0 + \frac{\lambda_4 R_0}{R_2^2} - \frac{\lambda_5}{R_0}\right\}\right]$$

$$P_{\text{mag}} = \tfrac{1}{2}\left(\frac{\lambda_5^2}{r^2} + H_3^2\right).$$

Inner Vacuum : $r < R_i$

$$\vec{V} = (0,\ 0,\ 0)\ ;\quad \vec{H} = [0,\ (\lambda_2/r),\ 1]$$

$$\vec{J} = (0,\ 0,\ 0)\ ;\quad \vec{E} = \left[0,\quad \frac{H_2 - 1}{R_i},\quad -\frac{1}{R_i}\left\{\lambda_3 R_i + \lambda_4 R_i - \frac{\lambda_2}{R_i}\right\}\right]$$

$$P_{\text{mag.}} = \tfrac{1}{2}\,[\lambda_2^2/r^2 + 1].$$

Inner surface : $r = R_i$

$$\vec{J}^* = [0,\ (1 - H_2),\ R_i(\lambda_3 + \lambda_4) - \lambda_2/R_i].$$

Outer surface : $r = R_0$

$$\vec{J}^* = \{0,\ (H_3 - H_2),\ R_0[\lambda_3 + \lambda_4/R_2^4] - \lambda_5/R_0\}.$$

We note that in the above, unsteady state helical surface current exist at the inner and outer surfaces of the shell.

(b) *Imploding case* :

In this case the solutions are obtained by putting $-t$ for t in the above solutions.

3. *Perturbation equations* : In order to study the stability of the system under small axial disturbances, at any time t we impose an axial perturbation on the system of the type $\tilde{X} = \hat{X}e^{ikz}$ and study its growth with time. Here k is the wave number of the perturbation, z is the axial coordinate in dimentionless form. Let $\tilde{\vec{V}}$, $\tilde{P}$, $\tilde{\vec{E}}$, $\tilde{\vec{H}}$ and $\tilde{\vec{J}}$ denote the perturbations in the velocity, pressure, electric field, magnetic field and the current density respectively. We have the following equations determining them in dimensionless form :

Plasma :

$$(\partial\tilde{\vec{V}}/\partial t)+(\tilde{\vec{V}}\cdot\nabla)\,\vec{V}+(\vec{V}\cdot\nabla)\,\tilde{\vec{V}}=V_A^2\,[-\text{grad}\,\tilde{P}+\text{curl}\,\tilde{\vec{H}}\times\vec{H}+\text{curl}\,\vec{H}\times\tilde{\vec{H}}], \qquad [3.1]$$

$$(\partial\tilde{\vec{H}}/\partial t)=\text{curl}\,(\vec{V}\times\tilde{\vec{H}})+\text{curl}\,(\tilde{\vec{V}}\times\vec{H}), \qquad [3.2]$$

$$\text{div}\,\tilde{\vec{V}}=0, \qquad [3.3]$$

$$\text{div}\,\tilde{\vec{H}}=0, \qquad [3.4]$$

$$\tilde{\vec{E}}=-\vec{V}\times\tilde{\vec{H}}-\tilde{\vec{V}}\times\vec{H}. \qquad [3.5]$$

Here V_A^2 is the square of dimensionless Alfvén velocity.

Inner and Outer vacuum :

$$\text{div}\,\tilde{\vec{H}}=0, \qquad [3.6]$$

$$\text{curl}\,\tilde{\vec{H}}=0, \qquad [3.7]$$

$$\text{curl}\,\vec{E}=-(\partial\tilde{\vec{H}}/\partial t), \qquad [3.8]$$

$$\text{div}\,\tilde{\vec{E}}=0, \qquad [3.9]$$

The boundary conditions satisfied by the perturbations are the following :

$$\vec{n}\cdot[\tilde{\vec{H}}]+\tilde{\vec{n}}\cdot[\vec{H}]=0, \qquad [3.10]$$

$$\vec{n}\times[\tilde{\vec{H}}]+\tilde{\vec{n}}\times[\vec{H}]=\tilde{\vec{J}}^*, \qquad [3.11]$$

$$\vec{n}\times[\tilde{\vec{E}}]+\tilde{\vec{n}}\times[\vec{E}]=\tilde{u}\,[\vec{H}]+u\,[\tilde{\vec{H}}], \qquad [3.12]$$

$$\vec{n}\,[\tilde{P}]+\tilde{\vec{n}}\,[P]=\tilde{\vec{J}}^*\times\vec{H}+\vec{J}^*\times\tilde{\vec{H}}+\tilde{q}^*\vec{E}+q^*\tilde{\vec{E}}, \qquad [3.13]$$

$$\vec{n}\cdot[\tilde{\vec{E}}]+\tilde{\vec{n}}\cdot[\vec{E}]=\tilde{q}^*, \qquad [3.14]$$

$$\vec{n}\cdot\tilde{\vec{V}}+\tilde{\vec{n}}\cdot\vec{V}=\tilde{u}. \qquad [3.15]$$

Where u stands for the velocity of the boundary and $\tilde{q}^*$ and $\vec{\tilde{J}}^*$ denote the perturbations in the surface charge density and surface current density respectively. The equation of the boundary after perturbation is

$$r = R_{i,0} + \tilde{\delta}\, r_{i,0} \exp(ikz), \qquad [3.16]$$

Where $\tilde{\delta} r_{i,0}$ is the radial displacement of the boundary and R_i stands for the inner radius of the shell and R_0 for the equation of the outer surface of the shell. The disturbance in the unit normal at the surface of the shell is

$$\vec{\tilde{n}} = (0, \quad 0, \quad -ik\, \tilde{\delta} r_{i,0} \exp(ikz), \qquad [3\ 17]$$

so that

$$\vec{\hat{n}} = (0, \quad 0, \quad -ik\, \hat{\delta} r_{i,0}). \qquad [3.18]$$

4. *Solutions of the problem in the exploding case :* We have solved the above set of equations [3.1]–[3 9] with boundary conditions [3.10]–[3.15], for small wave number k corresponding to the large wave length disturbances, which are of particular interest in such problems. Thus we set

$$\hat{X} = \hat{X}_0 + k\, \hat{X}_1 + 0\,(k^2), \qquad [4.1]$$

and evaluate $\hat{X}_0$, $\hat{X}_1$.

The zeroth order solutions in the three regions of the system are as follows :

Plasma :

$$\vec{\tilde{V}}_0 = (0, \quad 0, \quad 0)\,; \quad \tilde{P}_0 = -H_2\, g\,(r^2 - 2\,t)\ \exp(ikz),$$

$$\vec{\tilde{H}}_0 = [0, \ 0, \ g\,(r^2 - 2\,t) \exp(ikz)]\,; \ \vec{\tilde{E}} = [0, \ (1/r)\, g\,(r^2 - 2\,t) \exp ikz, \ 0] \qquad [4.2]$$

Inner Vacuum :

$$\vec{\tilde{H}}_0 = (0, \quad 0, \quad 0)\,; \quad \vec{\tilde{E}}_0 = (0, \quad 0, \quad 0). \qquad [4.3]$$

Outer Vacuum :

$$\vec{\tilde{H}}_0 = (0, \quad 0, \quad 0)\,; \quad \vec{\tilde{E}}_0 = (0, \quad 0, \quad 0). \qquad [4.4]$$

where $g\,(\xi)$ is the arbitrary function of ξ. In the imploding case the solutions are obtained by changing t to $-t$ in the above.

The first order solutions after feeding the set of zeroth order solutions are :

$$\hat{u}_{r1}=F_1(t)/r, \tag{4.5}$$

$$\hat{u}_{\theta 1}=(1/r)\ g_3\ (r^2-2\ t)+V_A^2\left[c\ \lambda_3 r+\frac{c\ \lambda_4 r\ (r^2-4\ t)}{2\ (r^2-2\ t)^2}\right.$$

$$\left.-(i/4)\ \lambda_3\ r3g(r^2-2t)-\frac{i\ \lambda_4\ g\ (r^2-2t)}{12\ (r^2-2t)^2}\ (r^2-2t)\right], \tag{4.6}$$

$$\hat{u}_{z1}=i\ V_A^2\ H_2\ g\ (r^2-2t)+g_2\ (r^2-2t), \tag{4.7}$$

$$\hat{H}_{r1}=\frac{c}{r}\ -\frac{i}{r}\int^{r} rg\ (r^2-2t)\ dr\ , \tag{4.8}$$

$$\hat{H}_{\theta 1}=\frac{2\ \lambda_4\ r}{(r^2-2t)^2}\int^{t} F_1\ (t)\ dt+rg_1\ (r^2-2t), \tag{4.9}$$

$$\hat{H}_{z1}=g_4\ (r^2-2t), \tag{4.10}$$

$$\hat{E}_{r1}=-\left[\frac{H_2}{r}\ g_3\ (r^2-2t)-\left\{\lambda_3 r+\frac{\lambda_4 r}{(r^2-2t)}\ g_2\ (r^2-2t)\right\}\right] \tag{4.11}$$

$$\hat{E}_{\theta 1}=[(1/r)\ g_4\ (r_2-2t)+(H_2/r)\ F_1(t)], \tag{4.12}$$

$$\hat{E}_{z1}=-\left[\frac{2\ \lambda_4}{(r^2-2t)^2}\int^{t} F_1(t)\ dt+g_1\ (r^2-2t)+\lambda_3 r+\frac{\lambda_4 r}{r^2-2t}\ \frac{F_1(t)}{r}\right], \tag{4.13}$$

$$\hat{\phi}_1=\phi_1(t)-\frac{1}{V_A^2}\left[F_1'(t)\ ln\ r+\frac{F_1(t)}{r^2}\right]-\left\{\frac{4\lambda_4^2\ t}{3\ (r^2-2t)^3}\ -\frac{2\lambda_4\ \lambda_3}{(r^2-2t)}\right\}\int^{t} F_1\ (t)\ dt$$

$$-2\ \lambda_3\int^{r} g_1\ (r^2-2t)\ dr+4\lambda_4\ t\int^{r}\frac{rg\ (r^2-2t)}{(r^2-2t)^3}\ dr$$

$$-4\lambda_4\ \left\{-\frac{\lambda_3^2\ r}{2\ (r^2-2t)^2}\ +\frac{\lambda_4\ (3r^2+2t)}{12\ (r^2-2t)^3}\right\}\ \int^{t} F_1\ (t)\ dt-2\ \lambda_3\int^{r} rg_1\ (r^2-2t)\ dr$$

$$-2\lambda_3\int^r r^3 g_1'(r^2-2t)\,dr - 2\lambda_4\int^r \frac{rg_1(r^2-2t)}{(r^2-2t)}\,dr$$

$$-2\lambda_4\int^r \frac{r^3 g_1'(r^2-2t)}{(r^2-2t)}\,dr - 2H_2\int^r rg_4(r^2-2t)\,dr. \qquad [4\ 14]$$

Here $F_1(t)$ and $\phi_1(t)$ are arbitrary functions of integration and C is a pure constant. And g_1, g_2, g_3 and g_4 are arbitrary functions of the argument.

The first order solutions in inner and outer vacuum respectively are:

$$\hat{\vec{H}}_1=[0,\quad 0,\quad iX(t)], \qquad [4.15]$$

$$\hat{\vec{E}}_1=[0,\quad -iX'(t)\,r/(2),\quad f_4(t)], \qquad [4.16]$$

$$\hat{\vec{H}}_1=[0,\quad 0,\quad 0], \qquad [4.17]$$

$$\hat{\vec{E}}_1=[f_5(t)/(r),\quad f_1(t)/(r),\quad f_7(t)), \qquad [4.18]$$

where $f_4(t)$, $X(t)$, $f_1(t)$, $f_5(t)$ and $f_7(t)$ are arbitrary functions of the argument. A dash denotes differenciation with respect to time t.

After applying the corresponding first order boundary conditions, we obtain the following set of equations which enable us to determine the arbitrary constants and functions in the solutions:

$$c\equiv 0,\quad g(r^2-2t)\equiv 0,\quad g_3(r^2-2t)\equiv 0,\quad g_2(r^2-2t)=0,$$
$$f_5(t)\equiv 0,\quad g_1(r^2-2t)\equiv 0 \qquad [4.19]$$

$$X'(t)+2X(t)/R_i^2=2i\,F_1(t)/R_i^2, \qquad [4.20]$$

$$\frac{F_1(t)}{R_i}-\frac{x}{R_i^2}=\frac{dx}{dt}, \qquad [4.21]$$

$$\frac{F_1(t)}{R_0}-\frac{y}{R_0^2}=\frac{dy}{dt}, \qquad [4.22]$$

$$iX(t)=\phi_1(t)-\frac{1}{V_A^2}\left\{F_1'(t)\ln R_i+\frac{F_1(t)}{R_i^2}\right\}-\left\{\frac{\lambda_4 t}{3}(4\lambda_4+\tfrac{1}{2})\right.$$

$$\left.-2\lambda_4\lambda_3(1+R_i^2)+\tfrac{1}{2}R_i^2\left(\lambda_3+\frac{\lambda_4}{2}\right)-2\lambda_4^2R_i^2\right\}\int^t F_1(t)\,dt+\frac{x}{R_i^3V_A^2}, \qquad [4.23]$$

$$0=\phi_1(t)-\frac{1}{V_A^2}\left\{F_1'(t)\,ln\,R_0+\frac{F_1(t)}{R_0^2}\right\}-\left\{\frac{4\lambda_4^2\,t}{3\,R_2^6}-\frac{2\lambda_4\lambda_3}{R_2^2}\right.$$

$$\left.+\frac{\lambda_4^2\,(3R_0^2+2t)}{3R_2^6}-\frac{2\lambda_4^2\,R_0^2}{R_2^6}\right\}\int^t F_1(t)\,dt+\frac{y}{R_0^3\,V_A^2}\,, \quad [4.24]$$

$$f_4(t)=-F_1(t)\,\lambda_2/R_i^2\,, \quad [4.25]$$

$$f_7(t)=-F_1(t)\,\lambda_5/R_0^2, \quad [4.26]$$

$$f_1(t)=F_1(t)\,H_3, \quad [4.27]$$

where $(\hat{\delta}\,R_i)_1=x$ and $(\hat{\delta}\,R_0)_1=y$ are the first order displacements of external and internal boundaries respectively.

5. *Method of solution:*

From [4.20], we obtain

$$i\,X(t)=-2\,z_1(t)/R_i^2+i\,l_1/R_i^2, \quad [5.1]$$

where l_1 is an arbitrary constant of integration and $z_1=\int^t F_l(t)\,dt$

Again from [4.21] and [4.22], we obtain

$$x=z_1(t)/R_i+l_2/R_i, \quad [5.2]$$

$$y=z_1(t)/R_0+l_3/R_0, \quad [5.3]$$

where l_2 and l_3 are arbitrary constants of integration.

Making use of [5.1]—[5.3] in the result got by subtracting [4.24] from [4.23], we obtain

$$\frac{d^2z_1}{dt^2}-\frac{\bar{a}^2}{(1+2t)\,(R_2^2+2t)\,ln\,[(R_2^2+2t)/(1+2t)]}\cdot\frac{dz_1}{dt}+\frac{M}{ln\,[(R_2^2+2t)/(1+2t)]}\,z_1$$

$$=\left[\frac{l_3}{(R_2^2+2t)^2}-\frac{l_2}{(1+2t)^2}+\frac{i\,V_A^2\,l_1}{(1+2t)}\right]\frac{1}{ln\,[(R_2^2+2t)/(1+2t)]}\,,$$

where $\bar{a}^2=R_2^2-1$ [5.4]

and

$$M=\left[\frac{\bar{a}^2 (R_2^2+4t+1)}{(R_2^2+2t)^2((1+2t)^2}+V_A^2\left\{-\frac{\lambda_4}{R_2^4}(\lambda_4+2\lambda_3)-\frac{4\lambda_4^2 t}{3}\right.\right.$$

$$\left.\left.+\tfrac{10}{3}\lambda_4 t+4\lambda_3\lambda_4(1+t)-\lambda_3(t+\tfrac{1}{2})+\tfrac{7}{4}\lambda_4+\frac{2}{1+2t}\right\}\right].$$

(a) *Exploding case* :

In this case t can take any value > 0, hence we have solved equation [5.4] asymptotically (Ford 1960) when $t\to\infty$ and its general solution is as follows:

$$z_1=D_1 t^{\rho_1}\exp\left[\frac{1}{2}\left\{a_0 \ln t+\frac{a_1}{t}-\frac{a^2}{2t^2}+\ldots\right\}+f_1(t)\right]\times\left[1+\frac{A_{1,1}}{t}+\ldots\right]$$

$$-\left\{t^{\rho_2}\exp\left[\frac{1}{2}\left(a_0 \ln t+\frac{a_1}{t}-\frac{a_2}{2t^2}+\ldots\right)+f_1(t)\right]\left(1+\frac{A_{1,1}}{t}+\ldots\right)\right\}\times$$

$$\times\int\frac{t^{\rho_2}\left(1+\frac{A_{2,1}}{t}+\ldots\right)r(t)\exp\left[-\frac{1}{2}\left(a_0 \ln t+\frac{a_1}{t}-\frac{a_2}{2t^2}+\ldots\right)-f_1(t)\right]}{\Delta}dt$$

$$+D_2 t^{\rho_2}\exp\left[\frac{1}{2}\left\{a_0 \ln t+\frac{a_1}{t}-\frac{a^2}{2t^2}+\ldots\right\}+f_2(t)\right]\times\left[1+\frac{A_{2,1}}{t}+\ldots\right]$$

$$+\left\{t^{\rho_2}\exp\left[\frac{1}{2}\left(c_0 \ln t+\frac{a_1}{t}-\frac{a_2}{2t_2}+\ldots\right)-f_2(t)\right]\left(1+\frac{A_{2,1}}{t}+\ldots\right)\right\}.\times$$

$$\times\int\frac{t^{\rho_1}\left(1+\frac{A_{1,1}}{t}+\ldots\right)r(t)\exp\left[-\frac{1}{2}\left(a_0 \ln t+\frac{a_1}{t}-\frac{a_2}{2t^2}+\ldots\right)-f_2(t)\right]}{\Delta}dt, \quad [5.5]$$

where D_1 and D_2 are constants of integration, and

$$\rho_i=-\frac{m_i+a_{i,-1}^2+b_0}{2m_i^2},\quad i=1,\ 2$$

$$m_1=\frac{i V_A}{\bar{a}}\sqrt{\left[2\left\{-\frac{4\lambda_4^2}{3}+\frac{10}{3}\lambda_4+4\lambda_3\lambda_4-\lambda_3\right\}\right]},$$

$$m_2=-\frac{iV_A}{\bar{a}}\sqrt{\left[2\left\{-\frac{4\lambda_4^2}{3}+\frac{10}{3}\lambda_4+4\lambda_3\lambda_4-\lambda_3\right\}\right]}.$$

$$a_{1,-1}=-\frac{b_1}{2m_1}\,;\quad a_{2,-1}=-\frac{b_1}{2m_2}\,,$$

$$b_0=\frac{2V_A^2}{\bar{u}^2}\left[1+\frac{(R_2^2+1)}{4}\left\{-\frac{\lambda_4}{R_2^4}(\lambda_4+2\lambda_3)+4\lambda_3\lambda_4-\frac{\lambda_3}{2}+\frac{7}{2}\lambda_4\right\}\right.$$

$$\left.-\frac{1}{4}\left\{\frac{(R_2^2+1)^2}{4}-\frac{(R_2^6-1)}{3}\right\}\left\{-\frac{4\lambda_4^2}{3}+\frac{10}{3}\lambda_4+4\lambda_3\lambda_4-\lambda_3\right\}\right],$$

. . . .

$$a_0=\frac{1}{2}\,;\quad a_1=\frac{R_2^2+1}{8}\,;\quad a_2=\frac{R_2^4+R_2^2+1}{2}\,,$$

. . . .

$$f_1(t)=\left\{\frac{m_1t^2}{2}+(a_{1,-1})\,t\right\},\;;\quad f_2(t)=\left\{\frac{m_2t^2}{2}+(a_{2,-1})\,t\right\},$$

$$\Delta=t^{\rho_1}\left(1+\frac{A_{1,1}}{t}+\ \ldots\right)\left[\rho_2\,t^{\rho_2-1}\left(1+\frac{A_{2,1}}{t}+\ \ldots\right)\right.$$

$$\left.+\,t^{\rho_2}\left\{\frac{1}{2}\left(\frac{a_0}{t}-\frac{a_1}{t^2}+\ \ldots\right)+m_2+a_{2,-1}\right\}\left(1+\frac{A_{2,1}}{t}+\ \ldots\right)-t^{\rho_2-2}A_{2,-1}\right]$$

$$-\,t^{\rho_2}\left(1+\frac{A_{2,1}}{t}+\ \ldots\right)\left[\rho_1\,t^{\rho_1-1}\left(1+\frac{A_{1,1}}{t}+\ \ldots\right)\right.$$

$$+\,t^{\rho_1}\left\{\frac{1}{2}\left(\frac{a_0}{t}-\frac{a_1}{t^2}+\ \ldots\right)+m_1+a_{1,-1}\right\}\times\left(1+\frac{A_{1,1}}{t}+\ \ldots\right)$$

$$\left.-\,t^{\rho_1-2}A_{1,-1}\right]\,;\quad A_{i,1}=-\frac{A_{i,0}\{2\rho_i\,a_{i,-1}+b_3\}}{2(\rho_i-1)m_i+K_i}\,;\quad i=1,2.$$

CONCLUSION

We note that all the zeroth order solution is identically zero. But the disturbances in the first order plasma pressure and that of first order plasma velocity grow with time. In fact they manifest in the form of unbounded oscillations, the frequencies proportional to

$$\left[\frac{V_A^2}{2\,\bar{a}^2}\left\{-\frac{4\lambda_4^2}{3}+\frac{10}{3}\lambda_4+4\lambda_3\lambda_4-\lambda_3\right\}\right]^{1/2}$$

Hence we conclude that the exploding shell is overstable and the growth rate of these unbounded oscillations depends on the thickness of the shell $\bar{a}^2$, the rate of explosion V_A^2, and the initial magnetic field configuration. Further, we find that the effect of volume current distribution which is characterised by 'λ_3' and the presence of ring surface current which exists in the configuration as a result of postulated magnetic field reduces the frequency of unbounded oscillations. This is due to the surface forces coming into picture on account of surface currents. The general feature of the volume current distribution is therefore to add to the stability of the system[6].

(*b*) *Collapsing Case* :

In this case $R_i \to R$ when the inner vacuum of the shell is extinct. This means that $t \to (1-R^2)/2$. Therefore effecting the transformation

$$t = \frac{1-R^2}{2} - \xi, \quad \text{where} \quad t \to \xi \to 0 \text{ as } \frac{1-R^2}{2},$$

and setting $\xi = (\bar{a}^2/\nu)$ where as $\nu \to \infty$, $\xi \to 0$ the transformed equation is

$$\frac{d^2 z_1}{d\nu^2} + \left[\frac{2}{\nu} + \frac{\bar{a}^4}{(R^2\nu + 2\bar{a}^2)(\nu\bar{a}^2 + R^2\nu + 2\bar{a}^2)\, ln\{[\nu(\bar{a}^2 + R^2) + 2\bar{a}^2]/(R^2\nu + 2\bar{a}^2)\}}\right]\frac{dz_1}{d\nu}$$

$$+ \frac{M_1}{ln\{[\nu(\bar{a}^2 + R^2) + 2\bar{a}^2]/(R^2\nu + 2\bar{a}^2)\}}\, z_1$$

$$= \left[\frac{l_3}{\nu^2(\bar{a}^2\nu + R^2\nu + 2\bar{a}^2)^2} - \frac{l_2}{\nu^2(\nu R^2 + 2\bar{a}^2)} + \frac{i V_A^2 l_1}{\nu^3(R^2\nu + 2\bar{a}^2)}\right]$$

$$\times \frac{\bar{a}^4}{ln\{[\nu(R^2 + \bar{a}^2) + 2\bar{a}^2]/(R^2\nu + 2\bar{a}^2)\}}, \qquad [5.6]$$

where

$$M_1 = \left[\frac{\bar{a}^6\{(R_2^4 + 2R^2 - 1)\nu + 4\bar{a}^2\}}{\nu\{(R_2^2 + R^2 - 1)\nu + 2\bar{a}^2\}^2(\nu R^2 + 2\bar{a}^2)^2} - \frac{\bar{a}^4 V_A^2}{\nu^4}\cdot\frac{\lambda_4}{R_2^4}(\lambda_4 + 2\lambda_3)\right.$$

$$\left. + \ldots + \frac{7\bar{a}^4 V_A^4}{4\nu^4} + \frac{\bar{a}^4}{\nu^4}\cdot\frac{2V_A^2}{R^2}\left\{1 - \frac{2\bar{a}^2}{\nu R^2} + \frac{4\bar{a}^4}{\nu^2 R^4}\right\}\right]$$

The two linearly independent solutions of [5.6] are

$$z_1 = D_3 u_1(\nu) - u_1(\nu)\int\frac{u_2(\nu)\, r(\nu)}{\Delta}\, d\nu + D_4 u_2(\nu) + u_2(\nu)\int\frac{u_1(\nu)\, r(\nu)}{\Delta}\, d\nu. \qquad [5.7]$$

Here D_3 and D_4 are constants of integration, and

$$u_1(\nu) = \left[\frac{1}{\nu} - \frac{b_3 - a_1}{2\nu^2} + \ldots\right],$$

$$u_2(\nu) = \left[\ln \nu \left\{\frac{b_3}{\nu} + \frac{b_3(b_3 - a_1)}{2\nu^2} + \ldots\right\} + \left\{1 + \frac{a_1}{\nu} + \frac{b_3}{\nu} + \ldots\right\}\right],$$

$$b_3 = -\frac{5\bar{a}^4 V_A^2 \lambda_4 (1 - R^2)}{3},$$

$$a_1 = \frac{\bar{a}^4}{R^2 \ln[\bar{a}^2 + R^2)/R^2]\,(\bar{a}^2 + R^2)};$$

$$a_2 = \frac{2\bar{a}^6}{R^4 \ln[(R^2 + \bar{a}^2)/R^2]\,(\bar{a}^2 + R^2)^2}\left\{\frac{\bar{a}^2}{\ln[(R^2 + a^2)/R^2]} - (\bar{a}^2 + 2R^2)^2\right\},$$

$$r(\nu) = \left[\frac{l_3}{(\bar{a}^2 + R^2)^2} - \frac{l_2}{R^4} + \frac{i V_A^2 l_1}{R^2}\right] 1/\nu^4) + 0(1/\nu^5),$$

Δ = Wronskian of (u_1, u_2).

5. Conclusions

In this case the disturbances in the plasma pressure become large as the shell collapses and at the same time the disturbances in the velocity of the shell also grows. The growth rate of this instability is proportional to

$$\ln\left[\frac{2\bar{a}^2}{(1 - R^2) - 2t}\right]$$

Hence the imploding shell is purely unstable and the growth rate of the disturbance depends on the rate of collapse t, the thicknets of the shell $\bar{a}^2$ and on the radius of the conducting rod. However, there is no contribution from the volume current distribution to this growth rate which is on account of the fact that their contribution is not felt in the dominating terms of the solution.

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REFERENCES

1. Linhart, J. G. .. .. Il Nuovo Cimento, 1960, **17**, 850.
2. Fowler, C. M., Gram, W B. and Caird, R. S. .. *J Appl Phys*. 1960, **31**, 588.
3. Harris, E. G. .. . *Phys. Fluids*, 1962, **5**, 1057.
4. Bhatnagar, P. L. and Bhat, P. K. .. *Proc Nat. Inst., India*, 1968, 34, A. No. 3, 115.
5. Ford, W. B. .. .. "Studies on Divergent series and Asymptotic Series" *Chlsea Publication Company*, 1960.
6. Tayler, R. J. .. .. *Proc. Phys., Soc.*, 1957, **70 B**, 48.