FLUTTER OF CLAMPED SKEW PANELS WITH MID-PLANE FORCES IN SUPERSONIC FLOW*

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ABSTRACT

The flutter problem of uniform, thin, flat, isotropic, skew panels clamped on all the edges and under the action of mid-plane forces is formulated on the basis of the classical small deflection thin plate theory. For the aerodynamic loading, the two-dimensional static approximation is used.

Approximate flutter analysis is made by using the Galerkin method employing a double series of beam characteristic functions to represent the deflection surface. Results of numerical calculation for the critical dynamic pressure for a few configurations of rhombic panel under direct stress in the stream-wise direction are presented.

1. INTRODUCTION

Panel flutter is a self-excited oscillation of the thin skin forming the external surface of high speed aircraft and missiles. This is a dynamic instability brought about by the interaction of aerodynamic, inertia and elastic forces on the panel. It is a local instability of the panels of the exposed surface of the vehicle distinct from the flutter of a lifting surface or vehicle as a whole.

The failure of early V-2 rockets was supposed to have been due to panel flutter[1]. Panel flutter came to be reckoned as a serious design problem indeed since the time that wind-tunnel tests indicated the susceptibility of some parts of X-15 surface to this instability, further confirmed by flight tests on other aircraft (see, for example, References 2 and 3). It is significant to note that a survey of the U.S. Aircraft Industry in 1962 apparently revealed a total of 82 incidences of panel flutter in flight of the attack, fighter and experimental aircraft of that time, the bulk of which were for flat, uniform panels[4].

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There have been many theoretical investigations involving different panel configurations, support conditions and the use of different aerodynamic theories. Similarly, there have been several experimental investigations on flutter of panels of different configurations under a variety of conditions. All the pre-1960 investigations have been quite comprehensively reviewed in the literature[5,6]. In Reference 7, Fung reports some of the subsequent contributions. Johns[8] discussed the status of panel flutter in a comprehensive review of the entire bibliography on panel flutter upto that date and followed it by a survey[9] which also outlined the efforts of different groups around the world. It is interesting to note, however, that among all these investigations, the flutter problem of skew panels has not received much attention. The few papers published so far appear to be by Kornecki[10] and the author[11-14].

It has been shown earlier for two-dimensional panels and rectangular panels that a panel with mid-plane compressive force is more susceptible to flutter. Hedgepeth[15] has shown, for rectangular simply supported panels, that compressive mid-plane force in the chordwise (stream-wise) direction lowers the critical dynamic pressure while the spanwise (cross-stream) mid-plane force does not have any influence. Eisley and Luessen[16], in a detailed study of the effect of in-plane loads including edge shear loading on the flutter of rectangular panels, have shown that the edge shear loads have a pronounced effect on the critical dynamic pressure and are indeed detrimental when combined with either spanwise or chordwise normal edge loadings. These and other investigations by Kobayashi[17], Movchan[18], and Frahlich[19] on simply supported rectangular panels indicate convincingly that a panel on the verge of buckling is very much prone to flutter. Experiments[20,21] in which the boundary condition is more nearly that of clamping have also amply corroborated this. Analytical studies on the influence of in-plane forces on flutter of clamped panels do not seem to have been made in sufficient detail. Dixon[22], however, used a simple 4-term approximation in treating the "Transtability Flutter" of rectangular clamped panels. No results are available at all in published literature with regard to flutter of stressed clamped skew panels.

In this paper, therefore, the influence of in-plane forces on the critical dynamic pressure of clamped skew panels is investigated using the conventional small deflection, thin plate theory and the two-dimensional static approximation for the aerodynamic loading. The present formulation may also be used to obtain the results for stressed rectangular clamped panels by setting the skew angle to zero. Results of numerical calculations made, for a rhombic panel under the mid-plane force N_x are presented in this paper.

2. Mathematical Analysis

In Reference 12, the flutter problem of a clamped skew panel with mid-plane forces is formulated in detail and the results for panels without mid-plane forces have been presented. Consequently, in this paper, the essential equations only are put down for the sake of brevity.

The skew panel considered is flat and is clamped all round. It is acted upon by uniform in-plane loads. A sketch of the panel, the coordinate system and the orthogonal mid-plane force system on an element of the panel together with the assumed positive sign convention is shown in Fig. 1. The panel is assumed to be uniform, thin and isotropic. It is exposed to supersonic flow on one side and to still air on the other. The damping in the structure is neglected in the present analysis. Under the above mentioned assumptions and using the classical, small deflection theory, the governing differential equation is

$$D \nabla^4 w + N_x w_{,xx} + 2 N_{xy} w_{,xy} + N_y w_{,yy} + \rho h w_{,tt} = l(x, y, t) \qquad [1]$$

where the subscripts after comma denote differentiation. The boundaries of the panel, in oblique coordinates, are

$$x_1 = 0, \ x_1 = a; \quad y_1 = 0, \ y_1 = b \qquad [2]$$

The rectangular and oblique coordinates are related by the equations.

$$x_1 = x - y \tan \psi; \quad y_1 = y \sec \psi \qquad [3]$$

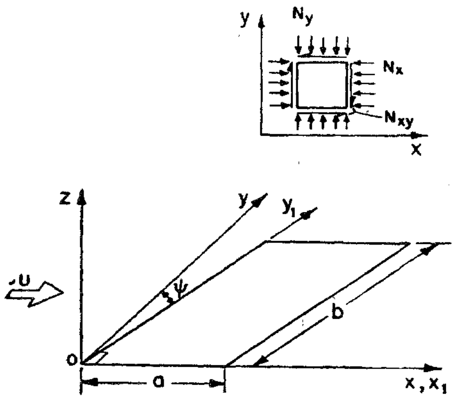


FIG. 1

Sketch of Skew Panel and the Mid-Plane Force System

For the aerodynamic loading, the two-dimensional static approximation is used in the first instance as it has been shown to be reasonable for flat panels at sufficiently high Mach numbers[15, 23]. The effect of yaw of the skew panel is included. The system of axes and the skew panel in yaw are shown in Fig. 2. The aerodynamic loading is then written as

$$l(x, y, t) = -(2q/\beta)\, \partial w/\partial \bar{x} \qquad [4)$$

The non-dimensional coordinates

$$\xi = x_1/a \quad \text{and} \quad \eta = y_1/b \qquad [5]$$

are introduced. Using Eqs. [3], [4] and [5], Eq. [1] becomes

$$\begin{aligned} & w,_{\xi\xi\xi\xi} + (a/b)^4\, w,_{\eta\eta\eta\eta} + 2\,(1 + 2\sin^2\psi)\,(a/b)^2\, w,_{\xi\xi\eta\eta} - 4\sin\psi\,(a/b)\,(w,_{\xi\xi\xi\eta} \\ & \quad + (a/b)^2\, w,_{\xi\eta\eta\eta}) + (a/b)^2\, w,_{\xi\xi}\,(R_x^* - 2R_{xy}^*\sin\psi + R_y^*\sin^2\psi) \\ & \quad + w,_{\eta\eta}\, R_y^*\,(a/b)^4 + 2w,_{\xi\eta}\,(a/b)^3\,(R_{xy}^* - R_y^*\sin\psi) + (\rho h a^4\cos^4\psi/D)\, w,_{tt} \\ & = -[(2q\,a^3\cos^4\psi)/\beta D]\,[(\cos\Lambda - \sin\Lambda\tan\psi)\, w,_{\xi} + (a/b)\sin\Lambda\sec\psi\, w,_{\eta}] \qquad [6] \end{aligned}$$

For a plate which is clamped all round, the boundary conditions are

$$w = \partial w/\partial n = 0 \text{ on all the edges} \qquad [7]$$

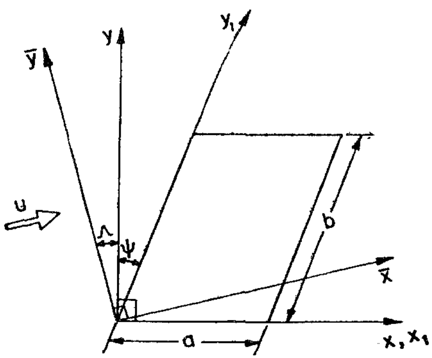


FIG. 2

Co-ordinate system of Panel in Yaw.

where n denotes the direction of the outward normal to the edge. The boundary conditions of the present problem are entirely of the "geometric" or "kinematic" type. It can be shown, by the use of Eqs. [3] and [5], that the boundary conditions, Eq. [7] reduce to the form

$$w = \partial w/\partial \xi = 0 \text{ on } \xi = 0 \text{ and } 1 \qquad [8a]$$

$$w = \partial w/\partial \eta = 0 \text{ on } \eta = 0 \text{ and } 1 \qquad [8b]$$

At the critical flutter condition, as the motion is simple harmonic, one can write

$$w(\xi, \eta, t) = Re\, W(\xi, \eta)\, e^{i\omega t} \qquad [9]$$

Substituting this in Eq. [6] results in

$$\begin{aligned} &W,_{\xi\xi\xi\xi} + (a/b)^4\, W,_{\eta\eta\eta\eta} + 2(1 + 2\sin^2\psi)(a/b)^2\, W,_{\xi\xi\eta\eta} - 4\sin\psi\,(a/b)\,(W,_{\xi\xi\xi\eta} \\ &\quad + (a/b)^2\, W,_{\xi\eta\eta\eta}) + (a/b)^2\, W,_{\xi\xi}\,(R_x^* - 2R_{xy}^*\sin\psi + R_y^*\sin^2\psi) \\ &\quad + W,_{\eta\eta}\, R_y^*\,(a/b)^4 + 2\,W,_{\xi\eta}\,(a/b)^3\,(R_{xy}^* - R_y^*\sin\psi) - \bar{k}^{*2}\, W \\ &\quad + \lambda^*\,[(\cos\Lambda - \sin\Lambda\tan\psi)\, W,_{\xi} + (a/b)\sin\Lambda\sec\psi\, W,_{\eta}] = 0. \end{aligned} \qquad [10]$$

The solution of the problem now consists of finding the critical value of λ^* at which a non-zero $W(\xi, \eta)$ satisfying Eq. [10] and the boundary conditions, given by Eqs. [8] is possible.

An approximate solution of this problem is attempted by using the Galerkin method[24]. The deflection of the panel is expressed in terms of a double series of beam characteristic functions representing the normal modes of a uniform clamped-clamped beam[25]. That is, it is written as

$$W(\xi, \eta) = \sum_{r=1}^{M} \sum_{s=1}^{N} C_{rs}\, X_r(\xi)\, Y_s(\eta) \qquad [11]$$

where

$$X_r(\xi) = (\cosh \epsilon_r \xi - \cos \epsilon_r \xi) - \alpha_r (\sinh \epsilon_r \xi - \sin \epsilon_r \xi) \qquad [12a]$$

$$Y_s(\eta) = (\cosh \epsilon_s \eta - \cos \epsilon_s \eta) - \alpha_s (\sinh \epsilon_s \eta - \sin \epsilon_s \eta) \qquad [12b]$$

Each term in the series, Eq. [11] satisfies the boundary conditions, Eq. [12]. These functions are tabulated by Young and Felgar[25] and the integrals involving these functions and there derivatives are given by Felgar[26]. We define the ingrals

$$I_{m,r}^{(1)} = \int_0^1 X_m X_r\, d\xi\,;\quad I_{m,r}^{(2)} = \int_0^1 X_m X_r'\, d\xi\,;\quad I_{m,r}^{(3)} = \int_0^1 X_m X_r''\, d\xi\,;\quad I_{m,r}^{(4)} = \int_0^1 X_m X_r'''\, d\xi \qquad [13]$$

where primes denote differentiation with respect to the appropriate non-dimensional independent variable. Similar integrals involving $Y_n(\eta)$, $Y_s(\eta)$ are labelled as *J*-integrals. Since the fuuctions $X_m(\xi)$, $Y_n(\eta)$ are the same in the present problem in view of the fact that the boundary conditions on both the pairs of opposite edges are identical, the *J*-integrals have the same values as the corresponding *I*-integrats. It is easily seen that

$$I_{rm}^{(1)} = I_{mr}^{(1)}\ ;\ I_{rm}^{(2)} = -I_{mr}^{(2)}\ ;\ I_{rm}^{(3)} = I_{mr}^{(3)}\ ;\ I_{rm}^{(4)} = -I_{mr}^{(4)} \qquad [14]$$

Corresponding relationships exist among the *J*-integrals. In Ref. [26] these integrals are given as functions of ϵ_m, ϵ_r, α_m, α_r from which the numerical values may be calculated. These values, along with the values for a few other combinations of commonly occurring boundary conditions, are tabulated in Ref. [27]

Applying the Galerkin method, by substituting Eq. [11] in Eq. [10] and orthogonalising the resulting error in the differential equation with respect to each of the product functions $X_m(\xi)\, Y_n(\eta)$ of Eq. [11], we get a set of homogeneous, linear, simultaneous, algebraic equations which can be written in the matrix notation as follows :

$$[E_{mnrs}]\{C_{rs}\} = \bar{k}^{*2}\ \{C_{rs}\} \qquad [15]$$

where

$$E_{mnrs} = (1/\pi^4)\,[\{\epsilon_r^4 + (a/b)^4\,\epsilon_s^4\}\ \delta_{mnrs} + 2\,(1+2\sin^2\psi)\,(a/b)^2\,I_{m,r}^{(3)}\,J_{n,s}^{(3)}$$
$$-4\sin\psi\,(a/b)\,\{I_{m,r}^{(4)}\,J_{n,s}^{(2)} + (a/b)^2\,I_{m,r}^{(2)}\,J_{n,s}^{(4)}\}] - \bar{R}_x^*\ H_{mnrs}^{(1)} - \bar{R}_{xy}^*\,H_{mnrs}^{(2)}$$
$$-\bar{R}_y^*\ H_{mnrs}^{(3)} - Q^*\,L_{mnrs} \qquad [16]$$

with

$$H_{mnrs}^{(1)} = -(1/\pi^2)\,I_{m,r}^{(3)}\,J_{n,s}^{(1)} \qquad [17]$$

$$H_{mnrs}^{(2)} = (2/\pi^2)\,[\sin\psi\ I_{m,r}^{(3)}\,J_{n,s}^{(1)} - (a/b)\,I_{m,r}^{(2)}\,J_{n,s}^{(2)}] \qquad [18]$$

$$H_{mnrs}^{(3)} = -(1/\pi^2)\,[\sin^2\psi\ I_{m,r}^{(3)}\,J_{n,s}^{(1)} + (a/b)^2\,I_{m,r}^{(1)}\,J_{n,s}^{(3)} - 2\,(a/b)\,\sin\psi\ I_{m,r}^{(2)}\,J_{n,s}^{(2)}] \qquad [19]$$

and

$$L_{mnrs} = -[(\cos\Lambda - \sin\Lambda\ \tan\psi)\ I_{m,r}^{(2)}\,J_{n,s}^{(1)} + (a/b)\,\sin\Lambda\ \sec\psi\ I_{m,r}^{(1)}\,J_{n,s}^{(2)}] \qquad [20]$$

Eq. [15] reprents the algebraic eigenvalue problem corresponding to the general problem of panel flutter of a clamped skew panel acted upon by mid-plane forces N_x, N_{xy} and N_y. It is clear that the problems of free vibration, buckling under the action of N_x, N_{xy} and N_y individually or in combination and panel flutter of unstressed panels (*i.e.* with $\bar{R}_x^* = \bar{R}_{xy}^* = \bar{R}_y^* = 0$) are special cases of the preceding general equation by dropping the appropriate terms. The final results of the free vibration and buckling calculations are reported in Refs. 28, 29 and the flutter of unstressed panels in Ref. 12.

The eigenvalues $\bar{k}^{*2}$ of the matrix $[E]$ represent squares of the frequencies of vibration of the panel. For the static aerodynamic theory that is used, all eigenvalues of Eq. [15] are real for sufficiently small values of Q^*. In fact, for $Q^*=0$, the problem posed by Eq. [15] is really a free vibration problem and the resulting eigenvalues correspond to the in-vacuo frequencies of the panel. As Q^* is gradually increased, some of the eigenvalues approach each other and for a certain value of Q^* two of them coalesce forming an eigenvalue loop. If Q^* is increased further, these two would become complex. When $\bar{k}^{*2}$ becomes complex, the corresponding motion clearly is a divergent oscillation. Thus the value of Q^* at which two eigenvalues coalesce defines the critical value Q^*_{cr} for flutter. This criterion for critcal flutter is well-known and is commonly adopted in theoretical panel flutter analyses using static approximation[15, 23, 30]. Further, it is generally the case that the eigenvalues at the lower end of the spectrum are the ones which tend to coalesce, leading to instability. In the present problem the eigenvalues and eigenvectors of the matrix $[E]$ were determined using a library routine based on the QR-transformation method[31].

3. Numerical Calculations

Numerical calculations have been performed for rhombic panels only, *i.e.*, $a/b=1$. Only $\bar{R}^*_x$ was considered and the value has been varied upto or slightly below the buckling value. A sixteen-term series ($M=4$, $N=4$) has been used. From earlier investigations[12, 14], it is concluded that a 16-term series would be quite adequate in the present problem of clamped skew panel with $\bar{R}^*_x$ alone from considerations of convergence.

4. Results and Discussion

Results have been obtained for $a/b=1$ with only N_x present. The variation of the frequencies with the dynamic pressure parameter Q^* for representative cases is shown in Figures 3 to 7. The eigenvalue loops for different values of $\bar{R}^*_x$ for a given rhombic panel are plotted on the same graph. The coalescence is mostly between the first two eigenvalues only; there are a few cases in which it is between the third and the fourth (see, for example, Figures 5 and 6). In these cases the third and the fourth had to be plotted separately with magnification since the eigenvalues for $Q^*=0$, corresponding to the natural frequencies in-vacuo, are themselves close to each other. We notice that the Q^* at which the two pairs 1,2 and 3,4 coalesce are quite close indeed requiring extra care in establishing the more critical (*i.e.* lower) one of the two coalescences.

The results are presented in the Table. They have also been presented in Fig. 8 for different skew angles showing the variation of Q_{cr} with $\bar{R}_x^*$. The figure shows the usual trend of lowering of the Q_{cr} with increase in the mid-plane compressive force, the decrease being more rapid for higher skew angles.

The effect of the other two components of midplane force namely, N_{xy} (both positive and negative values) and N_y when each one is acting individually and the effect of all the three components acting simultaneously can also be investigated using the computer programme developed.

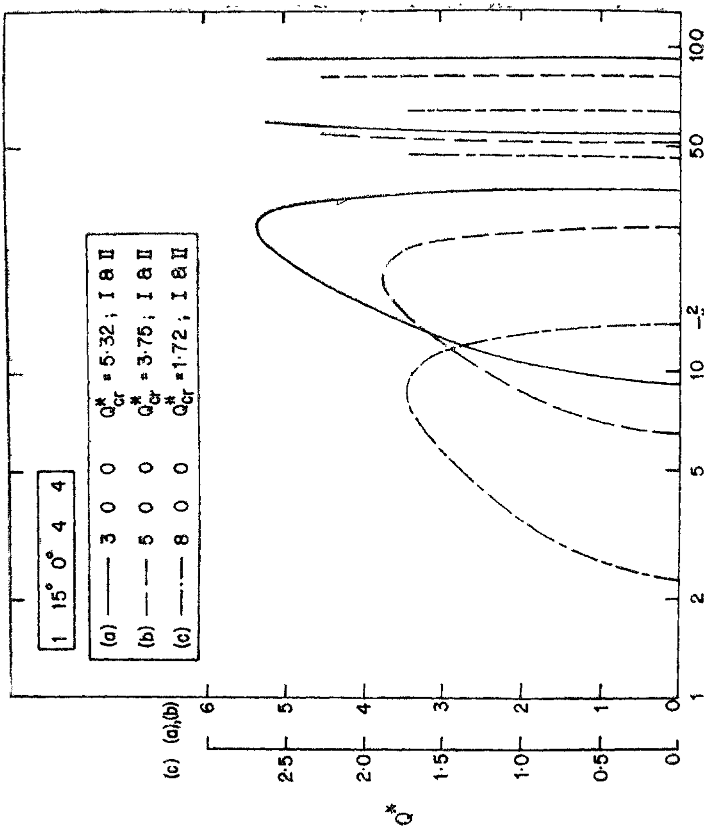
5. Conclusions

The supersonic flutter problem of isotropic, flat, clamped, skew panels subjected to mid-plane forces is considered using the two-dimensional static approximation for the aerodynamic loading. Results have been obtained in the present paper for $a/b = 1$ and $\Lambda = 0$ with only N_x present using a 16-term series. The critical dynamic pressure is lowered by the mid-plane compressive force and the decrease is more rapid for higher skew angles.

Table

Critical Dynamic Pressure for Flutter of Clamped Skew Panels with Mid-Plane Forces

a/b	ψ	M	N	$\bar{R}_x^*$	$\bar{R}_{xy}^*$	$\bar{R}_y^*$	Q_{cr}^*	Coalescence	Q_{cr}
1	15°	4	4	0	0	0	7.92	I & II	9.10
				3	0	0	5.32	,,	6.11
				5	0	0	3.75	,,	4.31
				8	0	0	1.72	,,	1.98
				9.46	0	0	0.916	,,	1.05
	30°	4	4	0	0	0	6.35	,,	11.29
				2	0	0	4.47	,,	7.95
				4	0	0	2.26	III & IV	4.01
				6	0	0	1.26	,,	2.24
				7.636	0	0	0.657	I & II	1.17
	45°	4	4	0	0	0	4.16	,,	16.6
				1.5	0	0	2.53	,,	10.1
				3	0	0	1.46	,,	5.84
				4.5	0	0	0.715	,,	2.86
				5.41	0	0	0.396	,,	1.58


1 15° 0° 4 4
(a) —— 3 0 0 Q*cr = 5·32 ; I & II
(b) — — 5 0 0 Q*cr = 3·75 ; I & II
(c) —·— 8 0 0 Q*cr = 1·72 ; I & II
(c) (a),(b)
Q*
L*²
1 2 5 10 50 100

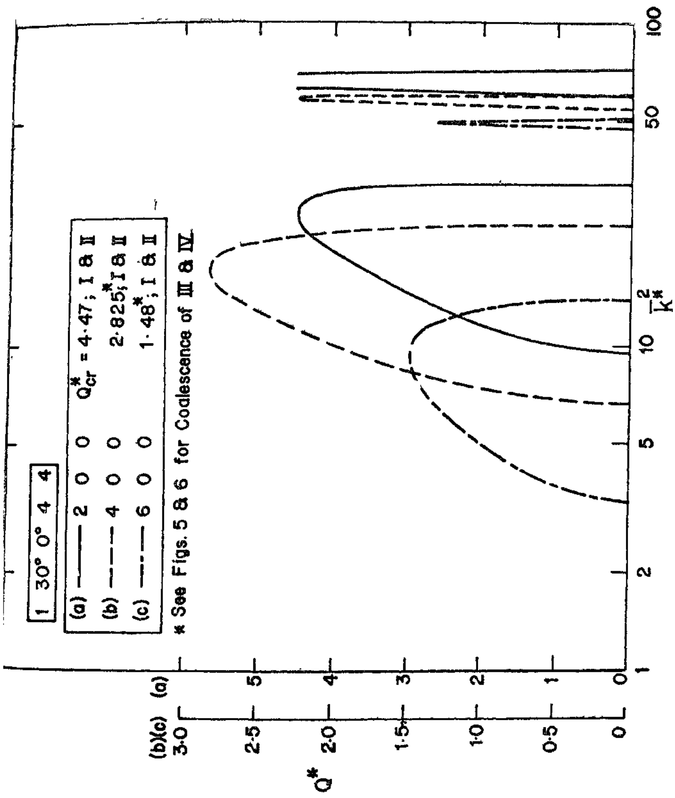


FIG. 4
Variation of frequencies with dynamic pressure

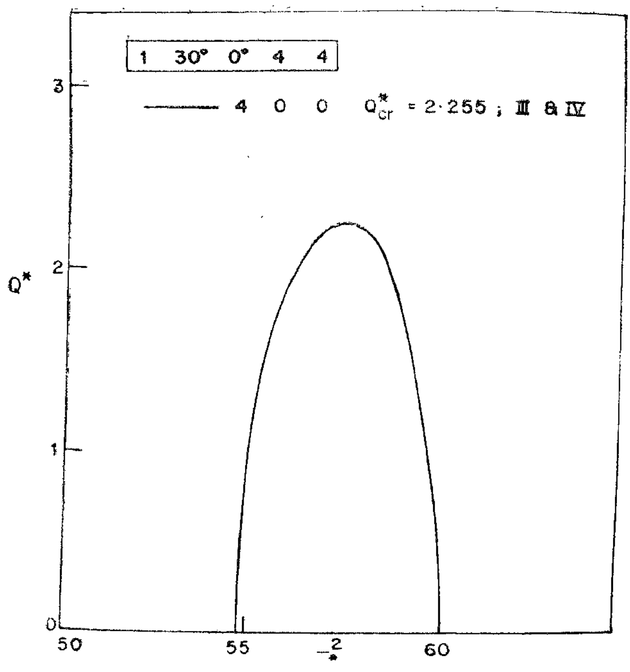


FIG. 5

Variation of frequencies with dynamic pressure

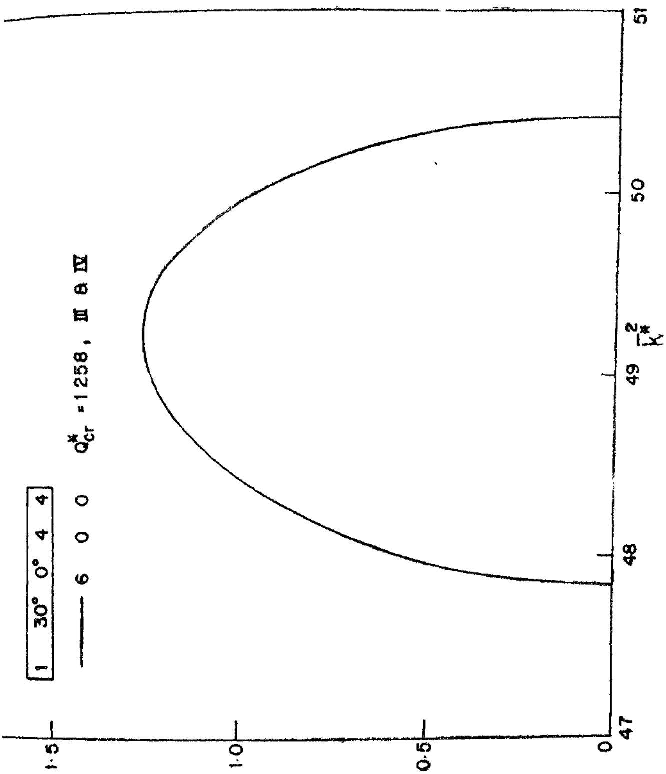


FIG. 6

Variation of frequencies with dynamic pressure

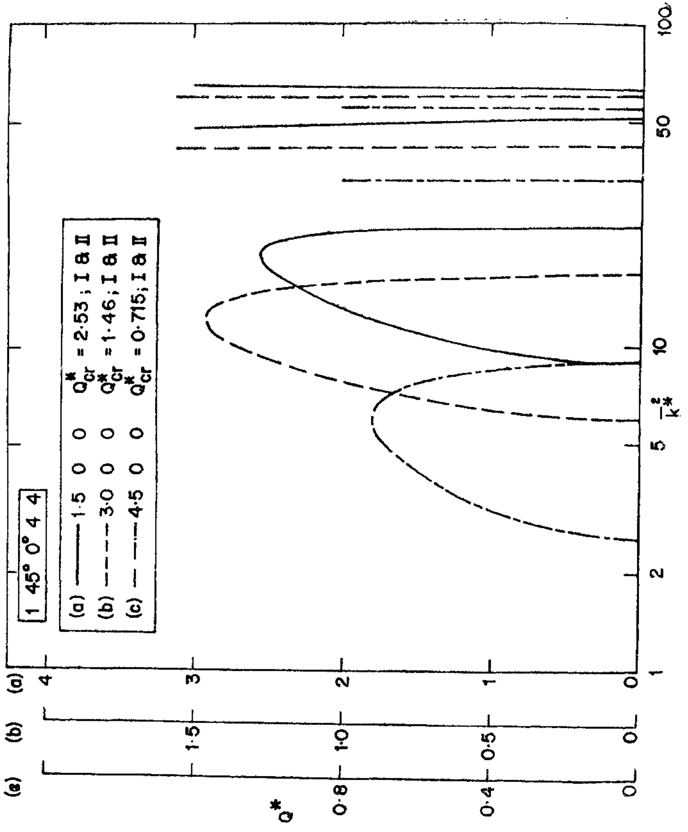


FIG. 7

Variation of frequencies with dynamic pressure

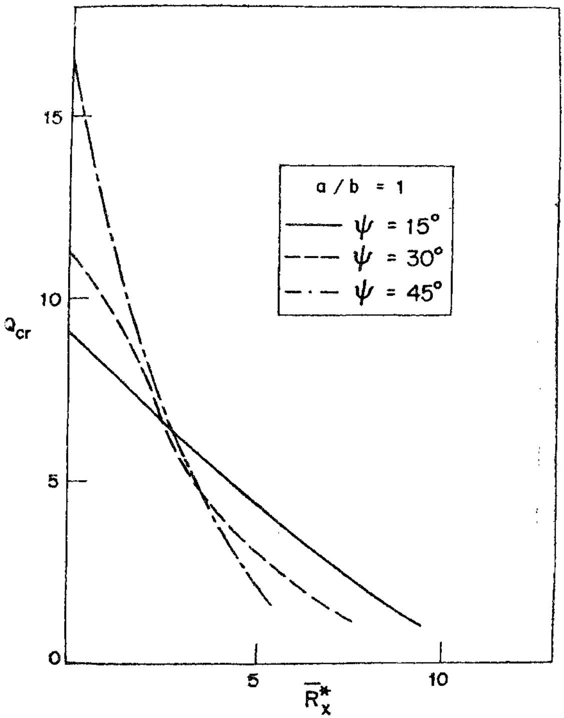


FIG. 8

Variation of critical dynamic pressure with mid-plane force

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NOTATION

a, b	..	panel dimensions along the x_1 and y_1 axes respectively.
C_{rs}	..	coefficient in the series expansion of deflection.
D	..	plate rigidly, $Eh^3/12\,(1-\nu^2)$
E	..	Young's modulus of the material of the panel.
E_{mnrs}	..	element of the matrix $[E_{mnrs}]$.
h	..	plate thickness
$I_{mr}^{(1)}$, $I_{mr}^{(2)}$, $I_{mr}^{(3)}$, $I_{mr}^{(4)}$	..	integrals defined in Eq. [13].
$\bar{k}^{*2}$	..	frequency parameter, $\rho h\,\omega^2 a^4 \cos^4\psi/D\,\pi^4$.
$l\,(x,y,t)$	..	aerodynamic loading per unit area.
L_{mnrs}	..	elements of aerodynamic matrix, Eq. [20].
$m,n,r,s,$	..	indices in the deflection series.
M	..	Mach number, also maximum value of indices m, r.
N	..	maximum value of indices n, s.
N_x, N_{xy}, N_y	..	mid-plane forces per unit length.
q	..	dynamic pressure, $\frac{1}{2}\,\rho_a u^2$.
Q, Q^*	..	dynamic pressure parameter, $2\,qa^3/\beta\,D\,\pi^4$ and $2qa^3 \cos^4\psi\,/\beta D\,\pi^4$ respectively.
$\bar{R}_x^*$, $\bar{R}_{xy}^*$, $\bar{R}_y^*$	..	non-dimensional mid-plane force parameters $N_x a^2 \times \cos^4\psi/D\,\pi^2$; $N_{xy}\,a^2 \cos^3\psi/D\,\pi^2$; $N_y a^2 \cos^2\psi/D\,\pi^2$ respectively.
t	..	time.
T	..	kinetic energy.
u	..	velocity of free stream.
U	..	potential energy.
$w\,(x, y, t)$	..	time dependent deflection of panel.
$W(x, y)$	..	deflection surface of the panel.
x, y, z	..	rectangular coordinate system, defined in Figs. 1 and 2.
x_1, y_1	..	oblique coordinates, defined in Fig. 1 and 2.
$\bar{x}$, $\bar{y}$	..	rectangular coordinates, defined in Fig. 2.

$X_r(\xi)$	.. r^{th} beam characteristic function in the ξ-direction, see Eq. [12*a*].
$Y_s(\eta)$	.. s^{th} beam characteristic function in the η-direction, see Eq, [12*b*].
α_r, α_s	.. parameters in beam characteristic functions, Eq. [12].
β	.. $(M^2-1)^{1/2}$.
ϵ_r, ϵ_s	.. beam eigenvalues, Eq. [12].
ρ	.. mass density of panel material.
ρ_a	.. mass density of free stream air.
ψ	.. angle of skew, defined in Figs. 1 and 2.
Λ	.. angle of yaw, defined in Fig. 2.
ξ, η	.. non-dimensional coordinates, x_1/a and y_1/b respectively.
ν	.. Poissson's ratio.
ω	.. frequency of oscillation, radians per second.
δ_{mnrs}	.. Kronecker delta, $=1$, for $m=r$ and $n=s$ $=0$, for $m\neq r$ or $n\neq s$
∇^4	.. biharmonic operator in rectangular coordinates, $\partial^4/\partial x^4+2\,\partial^4/\partial x^2\,\partial y^2+\partial^4/\partial y^4$.

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