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Simple method of tuning PI controllers for stable inverse response systems

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Abstract

A simple method is proposed to design PI controllers for stable inverse response systems with and without delay. The method is based on (i) matching the corresponding coefficient of s in the numerator and in the denominator of the closed loop transfer function for a servo problem, and (ii) by specifying the initial (inverse) jump. This method gives simple equations for controller settings in terms of model parameters. PI controllers are also developed by IMC principles. A numerical optimization method is proposed to obtain the tuning parameters for the simple method. Simulation results are given for robust performance of the controllers for uncertainty in process gain, time constant and the location of zero. The performance of the proposed controllers is evaluated by simulation on nonlinear isothermal CSTR carrying out Van de Vusse reaction.

Keywords: PID controller, stable system, inverse response system, IMC.

1. Introduction

Inverse response is the dynamic behaviour of the system with a positive zero showing step response in the opposite direction initially to that of the steady-state direction. Such a behaviour is exhibited by processes such as (i) the level of a drum boiler to variations in the heating medium flow rate, (ii) exit temperature of a tubular exothermic reactor to changes in the inlet reactant temperature, (iii) the tray composition of a distillation column to variations in vapor flow rate, (iv) the temperature of the municipal waste incinerator to variations in inlet load rate [1], and (v) recycling of energy and material to a reactor makes poles and zeros move to the right half plane [2]. Methods of designing PI/ PID controllers for stable inverse response systems are Ziegler–Nichols method [3], IMC method [4]–[6], Modified phase margin and Gain margin method [7], Extension of Haalman method [8], Optimization method using artificial neural networks [9], Gain–phase margin tester method [10], and extension of smith predictor using modern H_∞ control theory [11]. In all the above methods, the design procedures are somewhat complicated. Scali and Rachid [4] have considered the design of PI/PID controller for the second-order inverse response systems without delay. Luyben [7] proposed PI controller for the second-order inverse response system with delay. PI tuning parameters are functions of the positive zero and dead time and are given for critically damped open loop system. The servo responses for various combinations of dead time and positive zeros are compared with that of the Ziegler–Nichols method. Jyothi *et al*. [8] also extended the method proposed by Haalman [12] to design PID controller to second -order systems with a positive zero and with delay. The performance of their method is almost same as that of Ziegler–Nichols method.

In the present work, a simple method is proposed for the design of PI/PID controller for first-order inverse response systems with and without delay. Chemical reactor examples exibiting such transfer function models have been studied [8], [13]–[17].

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Recently, a simple method has been proposed by Chidambaram *et al*. [18] to design PID controller for stable first-order plus time delay (FOPTD) system by equating the coefficients of the corresponding powers of s in the numerator and in the denominator of the closed loop transfer function for a servo problem. Performance of the controller designed by this method is shown to be similar to that of the controller designed by Ziegler–Nichols method. Since the performance specifications for stable systems cannot be met for the unstable systems, Chidambaram *et al*. [18] have used one tuning parameter, α (i.e. each term in the numerator is equal to α times the corresponding term in the denominator). The performance of the controller designed by the method is significantly better than that of the controller designed by pole placement method. Later, Chidambaram and Padma Sree [19] have extended the method to an integrating system with dead time and the performance of the controller designed is significantly better than that of the controller designed by optimization method proposed by Visioli [20].

In the present work, the simple method [18] is extended to design PI controllers for stable first-order inverse response systems. Since the system with a zero shows an initial inverse response, it is proposed in the present controller design method to use this value and also to match coefficient of power of s in numerator with α times that of denominator of closed loop transfer function for a servo problem. A method of designing PI controllers by IMC principles is also proposed. The performance of the controllers designed by both the methods is compared.

2. The proposed method-1

2.1. *First-order inverse-response system without delay*

Let us consider a stable first-order system with a positive zero $[k_pG_p = k_p(1-ps)/(\tau s+1)]$. Let us design a PI controller. The closed loop transfer function relating the output variable (y) to the set point (y_r) is given by

$$y/y_r = k_c k_p(1-ps)(\tau_I s+1)/[\tau_I s(\tau s+1) + k_c k_p(1-ps)(\tau_I s+1)]. \tag{1}$$

Equation (1) can be written as

$$y/y_r = (1 + a_1 s + a_2 s^2)/(1 + b_1 s + b_2 s^2). \tag{2}$$

$$\text{Let } \alpha = a_1/b_1 \text{ and } \beta = a_2/b_2 \tag{3}$$

where

$$a_1 = (\tau_I - p) \tag{4a}$$

$$b_1 = [\tau_I + k_c k_p(\tau_I - p)]/\{k_c k_p\} \tag{4b}$$

$$a_2 = -p\tau_I \tag{4c}$$

$$b_2 = [\tau_I(\tau - k_c k_p p)]/\{k_c k_p\} \tag{4d}$$

and α, β are the ratios of the corresponding coefficients of s and s^2 in the numerator with that of the denominator.

The response of the under-damped closed loop system is given by

$$y(t) = 1 - k[\cos(qt) + \{(x_1 - \zeta)/q\}\sin(qt)] \tag{5}$$

where, $$k = 1-\beta\ ;\ q = (1-\zeta^2)^{0.5};\ x_1 = b_1\,(\alpha-\beta)/(1-\beta). \tag{6}$$

Therefore, the servo response of the system is a function of the ratio of the corresponding coefficients of s and s^2 in the numerator and denominator, respectively. By selecting appropriate values of α and β (and hence the values for the controller parameters), we can shape the response.

$y\,(t)$ at $t = 0$ gives the value of the inverse jump. From the initial value theorem, we know that $y(t)$ at $t = 0$ can be obtained from the limiting value of value of [s y(s)] as s tends to infinity.

$$\lim_{s \to \infty} [s\ y(s)] = -\,[\,k_c\, k_p\, p/(\tau - k_c\, k_p\, p)] = -\,\phi. \tag{7}$$

Here ϕ $(= -\beta)$ is initial jump of the closed loop system.

From eqn (7), we get

$$k_c = \phi\ \tau/[(1 + \phi)\ k_p\, p]. \tag{8}$$

The overshoot and settling time for systems with positive zero is large. The numerator term of the coefficient of s is made equal to α times that of the corresponding denominator term. By doing so, we get

$$k_c\, k_p\, (\tau_I - p) = \alpha\,[\tau_I + k_c\, k_p\, (\tau_I - p)]. \tag{9}$$

From eqn (9), we get

$$\tau_I = k_c\, k_p\, p\ (1-\alpha)/\ [\ k_c\, k_p\, (1-\alpha) - \alpha\ ]. \tag{10}$$

For a stable system we can specify the value for ϕ. The limits of ϕ and α can be obtained by using Routh array stability criteria for the characteristic equation of the system as

$$\phi > 0$$
$$\alpha < [\phi\ \tau/(\phi\ \tau + \phi\, p + p)]. \tag{11}$$

For stability ϕ should be greater than 0. But if the initial jump is allowed to be small, the overshoot will be large. Basically we have to compromise between initial jump and overshoot. We can get the starting value of ϕ (closed loop inverse jump) from the knowledge of open loop jump, which is equal to ($k_p\, p/\tau$). If open loop jump is less than 1, then $\phi = 0.3$ is suggested. On the other hand if open loop jump is greater than 1, then $\phi = 0.25$ is recommended. α value is tuned less than the value obtained from the RHS of eqn (11). From simulation studies it is observed that if the ratio of (p/τ) is less (i.e. of the order of 0.1–0.3), α value is recommended as 0.96 to 0.98 times the value of α obtained by eqn (11). On the other hand, if the ratio of (p/τ) is large (i.e. of the order of 1 or more), α value is recommended as 0.1 to 0.3 times the value of α obtained by eqn (11). The value of ϕ dictates the value of k_c and the value of τ_I depends both on ϕ and α.

2.2. *Stable FOPTD system with a positive zero*

Let us consider stable first-order plus time delay system with a positive zero. The transfer function of the process is given by $k_p\, G_p = k_p\,(1-p\,s)\, e^{-Ls}/(\tau s + 1)$. Let us use a PI controller. The closed loop transfer function relating the output variable (y) and set point (y_r) is given by

$$y/y_r = k_c k_p (1-p\mathrm{s})\,(\tau_I \mathrm{s}+1)\,e^{-Ls}/[\tau_I \mathrm{s}(\tau\,\mathrm{s}+1)+k_c\,k_p(1-p\mathrm{s})\,(\tau_I\,\mathrm{s}+1)\,e^{-Ls}]\,. \quad (12)$$

In the above equation we shall not consider e^{-Ls} term in the numerator for further analysis, since this will only shift the corresponding time axis. Using Pade's approximation for e^{-Ls} in the denominator, the order of the numerator is made same as that of the denominator.

$y\,(t)$ at $t = L$ gives the value of the inverse jump. From the initial value theorem, we know that $y\,(t)$ at $t = L$ can be obtained from the limiting value of $[\mathrm{s}y(\mathrm{s})]$ as s tends to infinity.

$$\lim_{\mathrm{s}\to\infty} [\mathrm{s}\;y(\mathrm{s})] = -[\,k_c\,k_p\,p/\tau] = -\,\phi\,. \quad (13)$$

From eqn (13), we get

$$k_c = \phi\,\tau\,/[k_p\,p]. \quad (14)$$

By equating the coefficient of s in the numerator with α times that in the denominator we get the following equation:

$$k_c k_p\,(\tau_I - p + 0.5L\,) = \alpha\,[\tau_I + k_c\,k_p\,(\tau_I - p - 0.5L)\,]. \quad (15)$$

From eqn (15), we get

$$\tau_I = k_c\,k_p\,[p\,(1-\alpha)-0.5L\,(1+\alpha)]/[k_c\,k_p\,(1-\alpha) - \alpha]. \quad (16)$$

The limits of ϕ and α can be obtained by using Routh array stability criteria for the characteristic equation of the system as

$$\phi > 0$$

$$\alpha < [\phi\,\tau\,/\,(\phi\,\tau\, + p)]. \quad (17)$$

From the analysis of the closed loop system it can be shown that the initial jump is less for system with delay than system without delay. From the stability analysis ϕ should be greater than zero. We can get the starting value of ϕ (closed loop inverse jump) from the open loop jump, which is equal to $(k_p\,p/\tau)$. Therefore, the starting value of ϕ should be less than $(k_p\,p/\tau)$. α is tuned less than the value α obtained from the RHS of eqn (17). From simulation studies it is observed that if the ratio of (p/τ) is less (i.e. of the order of 0.1–0.3), α value is recommended as 0.95 to 0.98 times the value of α obtained by eqn (17). On the other hand, if the ratio of (p/τ) is large (i.e. of the order of 1 or more), α value is recommended as 0.1 to 0.3 times the value of α obtained by eqn (17).

3. The proposed method-2

3.1. *Modified IMC method for first-order system with a zero*

The process transfer function is $k_p\,(1-p\mathrm{s})/(\tau\mathrm{s}+1)$.

The process transfer function is factored as

$$P = P_A\,P_M = [(1-p\mathrm{s})\,/(1+p\mathrm{s})]\;k_p\;[(1+p\mathrm{s})/(\tau\,\mathrm{s}+1)]. \quad (18)$$

The IMC controller is

$$Q = P_M^{-1} f = [(\tau\, s + 1)/\{k_p(1{+}p s)\}]\ (1/[\lambda s + 1]). \tag{19}$$

The equivalent feedback controller G_c can be written as

$$G_c = Q/(1 - PQ). \tag{20}$$

Substituting eqn (18) and eqn (19) in eqn (20), we get

$$G_c = (1/k_p)\ (\tau\, s + 1)/[(2p + \lambda)\, s + \lambda p\, s^2] = k_c(1 + \{1/\tau_I\, s\})\ (1/[\tau_f\, s + 1]) \tag{21}$$

with

$$k_c = \tau/[k_p(2p + \lambda)]; \tag{22}$$

$$\tau_I = \tau \tag{23}$$

$$\tau_f = \lambda p/(2p + \lambda). \tag{24}$$

We get a PI controller with first-order filter and the value of λ filter time constant is selected by simulation.

3.2. *FOPTD system with a positive zero*

The process transfer function: $k_p\ (1{-}p s)\ e^{-Ls}/(\tau\, s + 1)$. Here e^{-Ls} is approximated as (1–Ls).

The process transfer function is factored as

$$P = P_A\, P_M = [(1 - p s)(1 - L s)/(1 + p s)]\ k_p[(1 + p s)/(\tau\, s + 1)]. \tag{25}$$

The IMC controller is

$$Q = P_M^{-1} f = [(\tau\, s + 1)/\{k_p\ (1 + p s)\}]\ (1/[\lambda s + 1]). \tag{26}$$

The equivalent feedback controller G_c can be written as [eqn (20)]

$$G_c = (1/k_p)\ (\tau\, s + 1)/[(2p + \lambda + L)s + (\lambda p - pL)\, s^2] = k_c(1 + \{1/\tau_I\, s\})\ (1/[\tau_f s + 1]) \tag{27}$$

with

$$k_c = \tau/[k_p(2p + \lambda + L)] \tag{28}$$

$$\tau_I = \tau \tag{29}$$

$$\tau_f = (\lambda p - pL)/(2p + \lambda + L). \tag{30}$$

In this method, filter constant (λ) is a tuning parameter and is selected by simulation. For large value of λ, the response of the system is sluggish and the controller is robust. On the other hand, for small λ values the response is fast at the expense of robustness. Hence there is a trade off between these two values.

4. Simulation results

4.1. *Case Study* 1

Let us consider a first-order stable system with a positive zero where $k_p = 1$, $\tau = 1$ and $p = 1$. The open loop jump for the system is –1. Therefore, closed loop jump is taken as 30% of the open

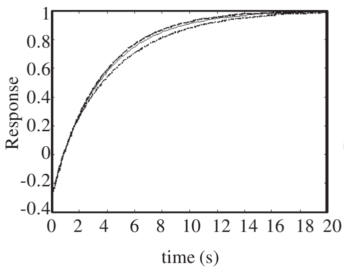

FIG. 1. Servo response of the system: $(1-s)/(s+1)$ $k_c = 0.2537$. Solid: τ_I = 1.0886, chain: τ_I = 1.1994, dash: τ_I = 1.3418.

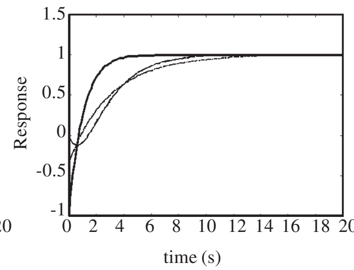

FIG. 2. Servo response of the system: $(1-s)/(s+1)$. Solid: optimization method, chain: present method, dash: IMC method.

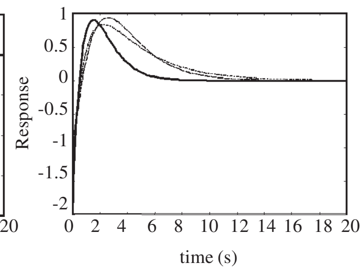

FIG. 3. Regulatory response of the system: $(1-s)/(s+1)$. Solid: optimization method, chain: present method, dash: IMC method.

loop jump. For a value of $\phi = 0.3$ and the value of α from the RHS of eqn (11) is 0.188. α value is varied as 0.1, 0.2 and 0.3 times the α value obtained from eqn (11). The corresponding value of k_c [from eqn (8)] is 0.2537 and τ_I values [from eqn (10)] are 1.0428, 1.0903 and 1.2, respectively. Servo response of the system is shown in Fig. 1. Since the larger value of τ_I gives sluggish response, the lower value of τ_I is selected. In the IMC method, λ is selected by simulation by giving different values for λ. Among these values, the value of λ that gives the best performance is selected. For this case study, $\lambda = 1$, 1.5, 2 and 2.5 are tried. Among these values, the value of $\lambda = 2$ gives the best performance. However, it is observed that suitable value of λ cannot be selected which will give a similar performance as that of the simple method as the latter method has two tuning parameters, whereas IMC method has one. The performance of the controller designed by the present method is compared with that of the controller designed by modified IMC method {eqn (22)–eqn (24)} [For $\lambda = 2$, $k_c = 0.25$ and $\tau_I = 1, \tau_f = 0.5$]. Comparison of servo responses by the two methods is shown in Fig. 2. IMC method gives less undershoot. Table I shows that the performance of the controller designed by the present method is comparable with that of the controller designed by the modified IMC method. The present method gives lesser ISE value compared to IMC method. The regulatory response of the system is shown in Fig. 3. Simple method does not have any filter; however, if we use the same filter as the IMC method, the servo and regulatory responses are similar. The PI controller is designed for nominal value of p, whereas we use +20% or –20% perturbation in p while simulating. Table II shows the robust performance of the present method in terms of ISE values for uncertainty in p. Similar response is obtained for uncertainty in model parameters τ and separately in k_p also (refer to Tables III and IV).

Table I
Comparison of the performance (ISE) of the present method with IMC method (for case studies 1, 2 and 4)

Case study	Perfect parameter			
	Present method		Modified IMC method	
	Servo	Regulatory	Servo	Regulatory
1	2.964	2.964	3.005	3.005
2	3.133	0.915	3.505	0.973
4	0.0206	3.73×10^{-4}	0.0223	4.08×10^{-4}

Table II
Robustness comparison in terms of ISE values for uncertainty in *p*

Case study	ISE values for uncertainty in p							
	Servo problem				Regulatory problem			
	+20%		–20%		+20%		–20%	
	Present	MIMC	Present	MIMC	Present	MIMC	Present	MIMC
1	2.951	3.2192	2.6059	2.8175	3.28	3.5412	2.3913	2.5943
2	3.239	3.6892	2.913	3.338	0.858	1.0030	0.811	0.9462

MIMC: Modified IMC.

A numerical optimization (*leastsq* of Matlab) method is used to minimize ISE value to obtain optimal values for ϕ and α. This method uses the initial guesses for the values of ϕ and α from the above simple method. For the case study 1, the optimal values obtained are $\phi = 0.9621$ and $\alpha = -0.0097$ and hence $k_c = 0.49$ and $\tau_I = 0.98$. Using the optimized values, the responses are evaluated and shown in Fig. 2. The results show that though the initial jump is large compared to the previous methods, the settling time is significantly improved and hence gives a lesser ISE value (= 2.0198). Figure 3 shows the regulatory response of the system (for case study 1). Here also the optimal values by optimization method give the lower ISE (=2.0197).

In the present work, the tuning parameters are selected based on performance for servo problem. The settings may not be optimal for regulatory problem. To get a better performance for regulatory problem also, the controller has to be detuned.

4.2. *Case Study* 2

Let us consider a first-order stable system with a positive zero. $k_p=1$, $\tau=10$, $p=1$. The open loop jump is 0.1. If the value of ϕ is taken equal to open loop jump sluggish response is observed. Therefore in the present work a value of 0.25 is suggested. The value of α obtained from the RHS of eqn (11) is 0.67. α value is varied as 0.94, 0.96 and 0.98 times the value obtained from eqn (11). The corresponding value of k_c [from eqn (8)] is 2 and τ_I values [from eqn (10)] are 6.22, 9.0 and 17.3, respectively. Since larger values of τ_I give sluggish response and smaller values

Table III
Robustness comparison in terms of ISE values for uncertainty in τ

Case study	ISE values for uncertainty in τ							
	Servo problem				Regulatory problem			
	+20%		–20%		+20%		–20%	
	Present	MIMC	Present	MIMC	Present	MIMC	Present	MIMC
1	2.86	3.117	2.69	2.894	2.625	2.872	3.016	3.2052
2	3.409	3.86	2.74	3.167	0.828	0.9649	0.84	0.9863

MIMC: Modified IMC.

lead to large overshoot, the value of $\tau_I = 9$ is selected. Servo response of the system for different values of τ_I is shown in Fig. 4. The performance of the proposed controller is compared with that of the controller designed by modified IMC method {eqn (22)–eqn (24)} [For $\lambda = 3$, $k_c = 2$, $\tau_I = 10$ and $\tau_f = 0.6$]. Figure 5 shows the servo responses of the system by both the methods. IMC method gives lesser undershoot. Figure 6 shows the regulatory response of the system. Performance comparison of the two methods in terms of ISE values is shown in Table I. The proposed method gives lesser ISE values compared to IMC method. Robustness of the controller under parameter uncertainty in p is shown in Table II. Similar results are observed for uncertainty in parameters τ and k_p separately (refer to Tables III and IV).

The optimal values obtained by using numerical optimization method are $\phi = 0.9777$ and $\alpha = 0.8162$ and hence $k_c = 4.9436$ and $\tau_I = 9.8349$. Using the optimized values, the responses are evaluated (Fig. 5). The results show that though the initial jump is large compared to the previous method the settling time is significantly improved and hence gives a lesser ISE value (= 2.0199). Figure 6 shows the regulatory response of the system. The controller designed by optimization method gives much improved performance (ISE value is 0.2012).

4.3. *Case study* 3

Let us consider a second-order stable system with a positive zero and with a delay. $k_p = 1$, $\tau_1 = 1, \tau_2 = 1$, $p = 1.6$ and $L = 0.2$. Using model reduction method proposed by Sundaresan and Krishnaswamy [21] to stable FOPTD system with a positive zero ($k_p = 1$, $\tau = 1.4439$, $p = 1.6$ and $L = 0.8327$). The value of the open loop jump is 1.107. Closed loop jump is taken as 20–30% of the open loop jump. For a value of $\phi = 0.3$, the value of α obtained from the RHS of eqn (17) is 0.213. α value is varied as 0.1, 0.2 and 0.3 times the value obtained from eqn (17). The corresponding value of k_c [from eqn (14)] is 0.2707 and τ_I values [from eqn (16)] are 1.27, 1.37 and 1.51, respectively. The value of $\tau_I = 1.37$ is considered. The performance of the proposed controller is compared with that of the PI controller designed by the method of Luyben [7] [$k_c = 0.3812$ and $\tau_I = 1.5858$] and PI controller with a first-order filter designed by IMC method [$k_c = 0.261$, $\tau_I = 1.4439$, $\tau_f = 0.193$] using eqn (28) – eqn (30). Figures 7 and 8 show, respectively, the servo and regulatory responses of the system. ISE values of the present method, Luyben method and IMC method for servo problem are 4.25, 4.09 and 4.36, respectively, and 4.59, 3.95 and 4.69, respectively, for regulatory problem. Luyben method gives the best performance. Robustness of the controller under parameter uncertainty in p is shown in Table V. Similar results are observed for uncertainty in parameters L and k_p separately (refer to Table V).

Table IV
Robustness comparison in terms of ISE values for uncertainty in k_p

Case study	ISE values for uncertainty in k_p							
	Servo problem				Regulatory problem			
	+20%		–20%		+20%		–20%	
	Present	MIMC	Present	MIMC	Present	MIMC	Present	MIMC
1	2.4564	2.743	3.262	3.4424	3.5319	3.95	2.0878	2.203
2	2.7067	3.1409	3.6256	4.082	0.892	1.051	0.759	0.8798

MIMC: Modified IMC

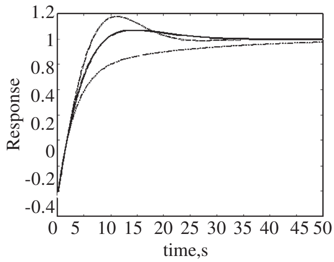

FIG. 4. Servo response of the system: $(1-s)/(10s+1)$ $k_c = 2$. Dash: $\tau_I = 6.22$, solid: $\tau_I = 9.0$, chain: $\tau_I = 17.3$.

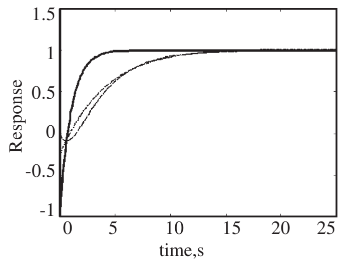

FIG. 5. Servo response of the system: $(1-s)/(1+10s)$. Solid: optimization method, chain: present method, dash: IMC method.

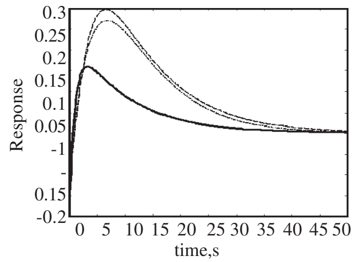

FIG. 6. Regulatory response of the system: $(1-s)/(1+10s)$. Solid: optimization method, chain: present method, dash: IMC method.

The optimal values obtained (using *leastsq* of Matlab to minimize ISE) are $\phi = 0.4999$ and $\alpha = 0.19$ and hence $k_c = 0.4508$ and $\tau_I = 2.0606$. Using the optimized values, the responses are evaluated (Fig. 7). The results show that the inverse peak and the settling time are almost the same as the other methods. ISE value for the servo problem is 3.9974. Figure 8 shows the regulatory response of the system. The controller designed by optimization method gives improved performance (ISE value is 3.8214).

4.4. *Case Study* 4: *Application to a nonlinear isothermal CSTR*

Let us consider an isothermal CSTR wherein the following isothermal series-parallel reactions are occuring. Here product B is the desired one.

$$\mathrm{A} \xrightarrow{k_1} \mathrm{B} \xrightarrow{k_2} \mathrm{C}$$
$$2\mathrm{A} \xrightarrow{k_3} \mathrm{D}$$

The mass balance equations for the species A and B are given by

$$dx_1/dt = -k_1\, x_1 - k_3\, x_1^2 + (C_{Ao} - x_1)u \tag{31}$$

$$dx_2/dt = k_1\, x_1 - k_2\, x_2 - x_2\, u \tag{32}$$

Here, $u = F/V$, where F is the flow rate (l/min), V, the volume of the reactor (l), x_1 and x_2, the concentration, respectively, of A and B in the reactor (mol/l) and C_{Ao} is the feed concentration of A (mol/l). The parameter values considered in the present work are given by $k_1 = 0.8333$ (l/min), $k_2 = 1.6667$(l/min), $k_3 = 0.16667$ (l/mol-min), $C_{Ao} = 10$ mol/l. The plot of x_{2s} (steady-state value of x_2) versus u, shows steady state input multiplicities in u on the product concentration (x_{2s}). That is the two values of u_s give the same value of x_{2s}. For example $x_{2s} = 1.117$ can be obtained at $u_s = 0.5714$ and also at $u_s = 2.8746$. The steady-state gain is 0.5848 at $u_s = 0.5714$ and gain is -0.1208 at $u_s = 2.8746$. The transfer function relating the deviation variable (at each value of u_s) is obtained from the linearized version of eqn (31) and Eqn (32)[8].

$$\Delta x_2/\Delta u = -0.1208\ (0.3546\,s + 1)/[(0.1742\ s + 1)(0.2202\ s + 1)] \text{ at } u_s = 2.8746 \tag{33}$$

$$\Delta x_2/\Delta u = 0.5848\ (-0.3546\ s + 1)/[(0.4149\ s + 1)(0.4464\ s + 1)] \text{ at } u_s = 0.5714 \tag{34}$$

Combining the two terms in the denominator as a single term and considering a measurement delay of 0.1min, the system in eqn (34) becomes

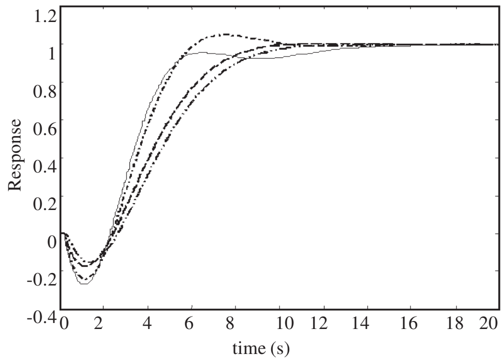

FIG. 7. Servo response of the system: $(1–1.6s)\, e^{-0.2s}/(s+1)^2$. Solid: Optimization method, dash: Present method, chain: Luyben method, long dash dot: IMC method.

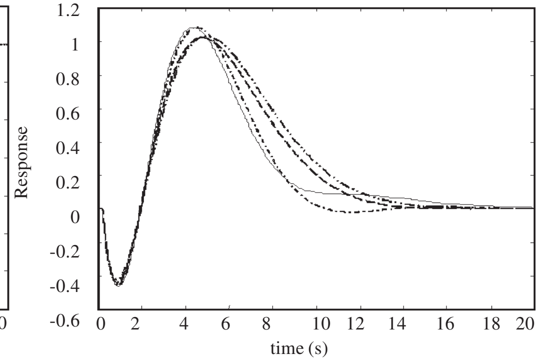

FIG. 8. Regulatory response of the system: $(1–1.6s)\, e^{-0.2s}/(s+1)^2$. Solid: Optimization method, dash: Present method, chain: Luyben method, long dash dot: IMC method

$$\Delta x_2/\Delta u = 0.5848\,(-0.3546\,s+1)\,e^{-0.1s}/(0.8613\,s+1)\ \text{at}\ u_s = 0.5714. \tag{35}$$

PI controller settings calculated by the present method for the system in eqn (35) are $k_c = 0.415$ and $\tau_I = 0.6429$ and controller settings by modified IMC method are $k_c = 0.5243$, $\tau_I = 0.8163$ and $\tau_f = 0.2398$ for a λ value of 2.

Using model reduction method proposed by Sundaresan and Krishnaswamy[21] for the system in eqn (34) with a measurement delay of 0.1 min, we get

$$\Delta x_2/\Delta u = 0.5848\,(-0.3546\,s+1)\,e^{-0.3567s}/(0.6302\,s+1)\ \text{at}\ u_s = 0.5714. \tag{36}$$

PI controller settings calculated by the present method for the system in eqn (36) are $k_c = 0.3647$ and $\tau_I = 0.5065$ and controller settings by modified IMC method are $k_c = 0.42$, $\tau_I = 0.6302$ and $\tau_f = 0.158$ for a λ value of 1.5.

The servo response of the above two settings [one based on eqn (35) and the other based on eqn (36)] on the actual nonlinear system is almost the same. Therefore, combination of both the terms in the denominator is as good as model reduction method. The servo response of the system with PI settings based on eqn (35) is shown in Fig. 9. However, in general, for any system the second model reduction method [21] is recommended. ISE value comparison is shown in Table I. Regulatory response is shown in Fig. 10. The proposed method shows the superior performance over the IMC method both for regulatory as well as servo response.

Table V
Robustness comparison in terms of ISE values for case study 3

Uncertainty	ISE values											
	Servo problem						Regulatory problem					
	+20%			–20%			+20%			–20%		
	P	L	I	P	L	I	P	L	I	P	L	I
Process gain	4.25	4.16	4.46	4.73	4.33	5.20	5.96	5.77	6.28	2.95	2.68	3.26
Location of zero	4.90	4.77	5.14	3.94	3.59	4.32	5.36	5.3	5.56	3.47	3.08	3.88
Time delay	4.73	4.15	4.73	4.31	4.04	4.65	4.30	4.02	4.63	4.19	3.89	4.54
Time constant	4.51	4.22	4.83	4.21	3.97	4.56	4.17	3.83	4.52	4.34	4.1	4.68

P–Simple method, L–Luyben method, I–MIMC method

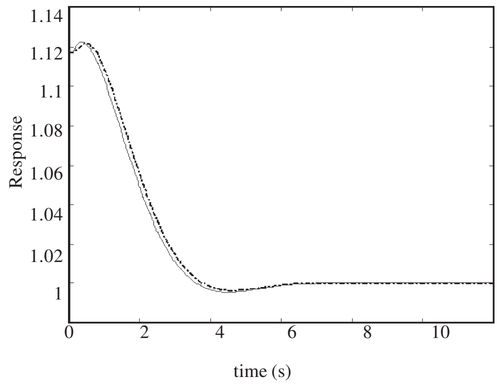

FIG. 9. Servo response of the CSTR (nonlinear model equations). Solid: Present method, chain: IMC method.

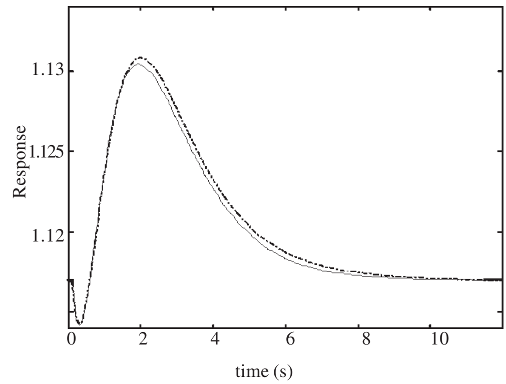

FIG. 10. Regulatory response of the CSTR (nonlinear model equations). Solid: Present method, chain: IMC method.

5. Conclusions

Two methods are proposed to design controllers for inverse response first-order systems with and without delay. The first method has two tuning parameters (ϕ and α). The tuning parameters obtained by numerical optimization method (using the values of ϕ and α obtained by simple method as initial guesses) gives an improved performance in terms of ISE. In the second method, λ is a single tuning parameter. The tuning parameters are selected by simulation. Simulation studies on two transfer function models for the first-order inverse response system, one transfer function model for the second-order inverse response system and one on nonlinear CSTR, show that first method is better than that of the second method in terms of ISE values. The proposed methods are also comparable with that of the Luyben method [7].

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Nomenclature and Units:

C_{Ao} Feed concentration, mole/l

k_1 Reaction rate constant, l/min

k_2 Reaction rate constant, l/min

k_3 Reaction rate constant, l/mole-min

k_c Controller gain

k_p Process gain

L Time delay, s

p Inverse of process zero, s

t Time, s

u Ratio of feed flow rate to the volume of the reactor (manipulated variable)

u_s Steady-state value of u

x_1 Concentration of A in the reactor, mole/l

x_2 Concentration of B in the reactor, mole/l

x_{1s} Steady-state product concentration of A, mole/l

x_{2s} Steady-state product concentration of B, mole/l

y Output

y_r	Set point
α	Ratio of coefficient of s in the numerator to that of the denominator of closed loop transfer function
β	Ratio of coefficient of s^2 in the numerator to that of the denominator of closed loop transfer function
τ	Process time constant, s
τ_I	Integral time, s
τ_f	Filter time constant, s