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Elastic-plastic analysis of R/C coupled shear walls: The equivalent stiffness ratio of the tie elements

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Abstract

Bar frame modelling is a popular method in coupled shear wall systems in structural design. In this process, determining the stiffness of the tie beams is important. In this study, results obtained by finite-element analysis of R/C coupled shear wall systems having several geometries in elastic-plastic space are considered. Using SPSS (Ver.5.0) statistical package program, an equivalent tie beam stiffness modification parameter is provided. The formula which defines the ratio between the plastic and elastic equivalent stiffness modification parameters is also given.

Keywords: Coupled shear walls, elastic-plastic analysis, stiffness modification parameter.

1. Introduction

Shear walls are generally used in the design of multistoried buildings because of good performance under lateral loads like lateral pressure and/or earthquake inertia forces. Coupled shear walls, which are special cases of shear wall systems, comprise an effective earthquake-resisting structure of high rigidity and reasonable ductility due to their short span tie beams (Fig. 1). In this study, a proposal for the estimation of stiffness of tie members of coupled shear walls is examined in a plastic space. The parameters of the problem are defined and the necessary explanations dealing with the subject are discussed. The stiffness of the coupling members, the geometrical and material parameters that affect the stiffness directly are defined in plastic space.

2. Analysis of coupled shear walls using elastic and plastic relations

It is well known that in mid-sized and multistoried buildings shear walls and coupled shear wall systems are usually used to provide the necessary stiffness, strength and ductility.

Two types of modeling are considered in this study for the lateral load analysis of coupled shear wall systems: (1) finite element, and (2) equivalent bar frame. The use of equivalent bar frame modeling for the analysis of coupled wall system is one of the popular methods in design [1]–[5]. The structural behavior of coupled shear walls is greatly influenced by the behavior of their coupling beams; therefore, the analysis and design of these elements are of importance.

The geometrical parameters d, b, h, ℓ, L, t of the problem are shown in Fig. 2; d and ℓ are the gross height and the clear length of tie elements, respectively, h, the height of the flat, b, the width of the wall and L, the length measured center to center. The mechanical parameters of the problem are modulus of elasticity (E), shear modulus (G) and the Poisson's ratio (μ) [4].

The behavior of tie elements, which connect two shear walls, depends on the geometry of the tie elements and the mechanical characteristics of the concrete and reinforcement [2], [4]. To estimate the real behavior of the coupled shear wall systems, it is necessary to consider the system in plastic space.

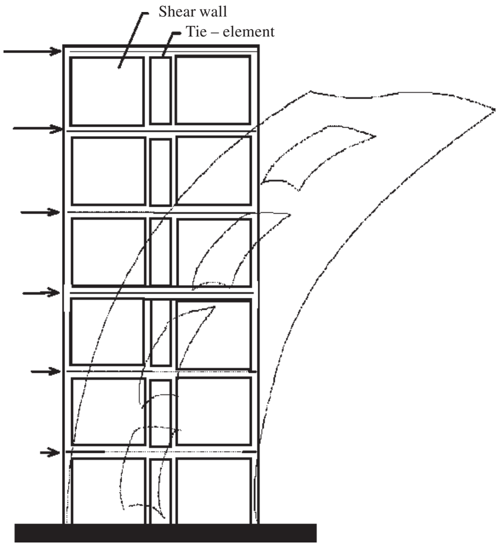


FIG.1. A typical coupled shear wall.

3. Numerical applications

Numerical applications are made with LUSAS (Powerful FE technology for specialist applications) program which has a wide range of finite-element types and a sophisticated material models. Choice and concepts are discussed here.

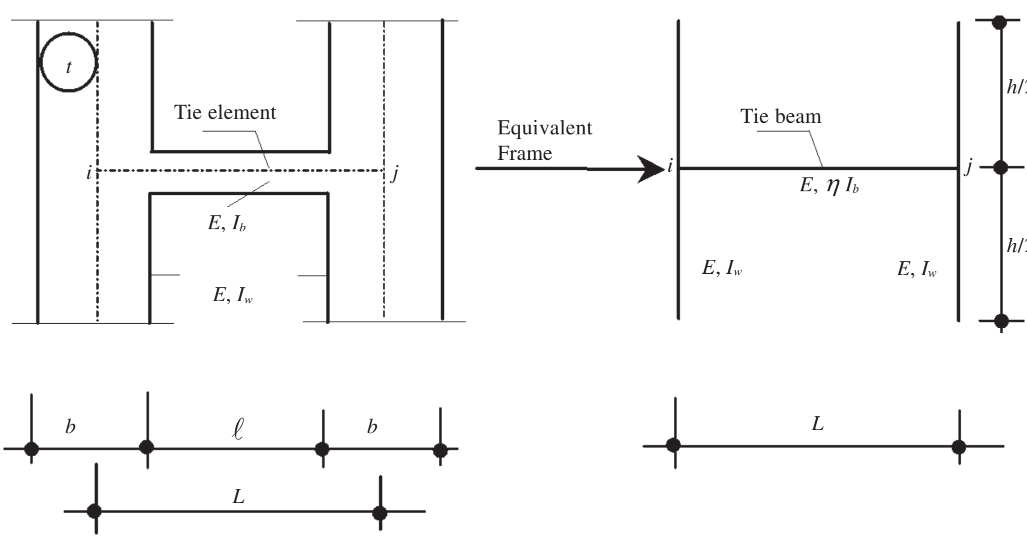
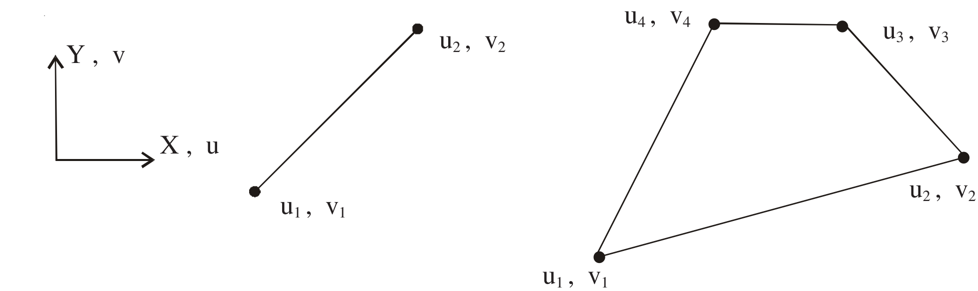


FIG. 2. Coupled shear wall and equivalent frame [4].



(a) For reinforcement. (b) For concrete.

FIG. 3. Finite element types.

3.1. *Finite element types for concrete and reinforcement*

In this study, QPM4 for concrete and BAR2 for the reinforcements are chosen (Fig. 3).

3.2. *Material models for concrete and reinforcement*

To produce sample data of statistical assessment, coupled wall systems are analyzed using classical plasticity concepts of the LUSAS program employing NLFEA (nonlinear finite element analysis) techniques. Since concrete is a quasi-brittle material, its nonlinear behavior can be modeled using the concepts of classical plasticity theory. It is essential to choose a suitable yield criterion in order to analyze any member using classical plasticity concepts [6], [7]. The analytical model of Drucker–Prager yield criterion which is a smooth approximation of the Mohr–Coulomb theory is used to model the nonlinear behavior of concrete [4]. A smooth approximation of the Mohr–Coulomb surface was expressed by Drucker and Prager in the following form [6]:

$$f(I_1, J_2) = \alpha\, I_1 + \sqrt{J_2} - k = 0, \tag{1}$$

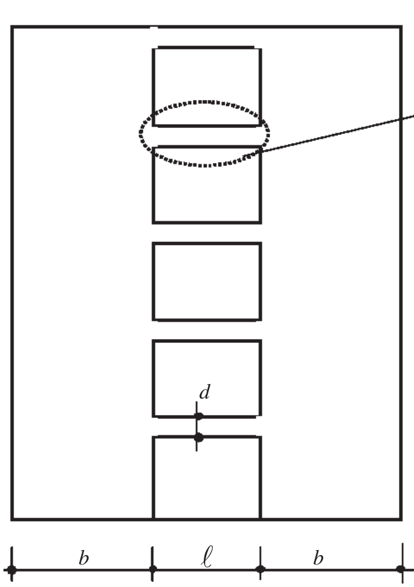
where I_1, J_2 are the first invariant of the stress tensor and second invariant of the stress deviator tensor, respectively, α and k are positive constants pertaining to the material. α and k are related to Mohr–Coulomb constants c (cohesion) and ϕ (friction) by

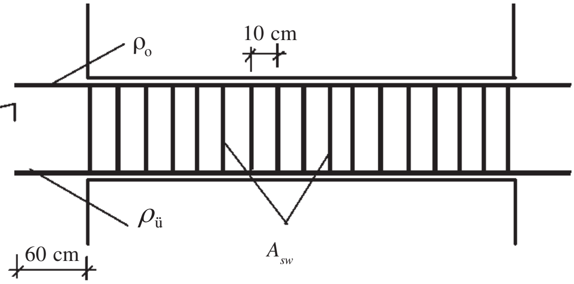
$$\alpha = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)}, \quad k = \frac{6c \cos \phi}{\sqrt{3}(3 - \sin \phi)}. \tag{2}$$

These two parameters which define the strength of the material are used by the LUSAS program in plastic analysis. The most suitable values for the parameters, $c = 2.80$–3.70 MPa and $\phi = 25°$–$35°$, are selected [4], [7]–[9].

The nonlinear behavior of the reinforcement can also be modeled using the concepts of classical plasticity theory. The analytical model of von-Mises yield criterion which states that yielding begins when the octahedral shearing stress reaches a critical value k is used to model the nonlinear behavior of the reinforcement [7], [8]. The yield surface was expressed by von- Mises in the following form [6]:

$$f(J_2) = J_2 - k^2 = 0, \tag{3}$$





$$\rho_{ü} = \rho_{\alpha,}\rho_{min} = \frac{f_{ctd}}{f_{yd}} \geq 0.003,\ \rho_{min} = 0.002$$

FIG. 4. Single span coupled shear wall and the configuration of the reinforcement [4].

where k is the yield stress in pure shear.

In this study, the effect of geometrical configuration of reinforcement of tie beams is the only factor taken into account in the frame analyses (Fig. 4). The effect of the reinforcement of the shear wall on the structural behaviour is neglected [4]. This approach is valid for the coupled shear wall systems of 4–10 stories.

To represent the yield surface in Drucker–Prager and von-Mises criteria, some parameters (Figs 5, 6) must be introduced first for the analysis using LUSAS program.

In this program, L_l and C (slope of the σ–ε curve) are the input values,

$$L_1 = \varepsilon_1 - \frac{\sigma_1}{E}; \tag{4}$$

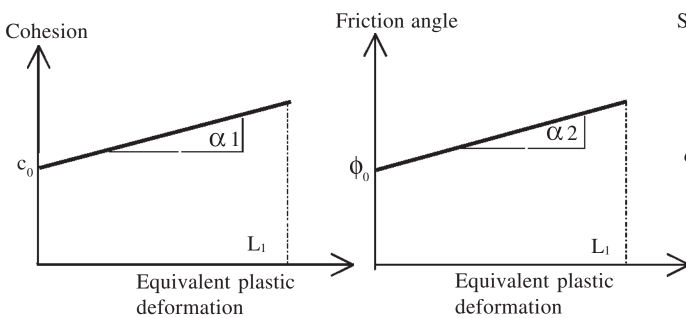


FIG. 5. Drucker–Prager yield criterion.

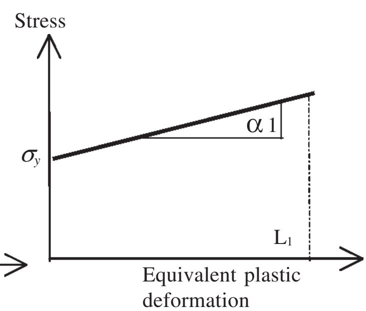


FIG. 6. von-Mises yield criterion.

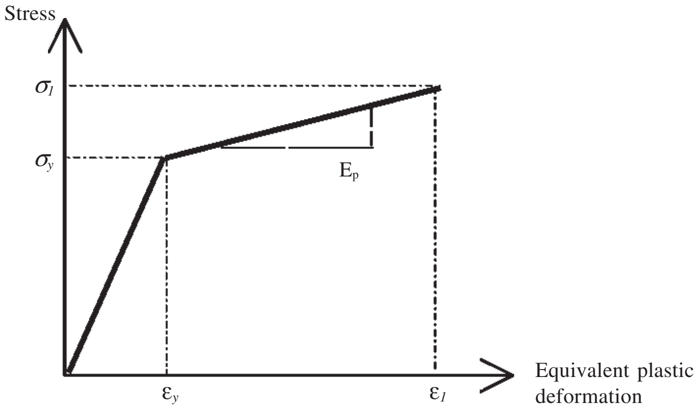


FIG. 7. $\sigma-\varepsilon$ curve.

$$C = \frac{E_p}{(1-\frac{E_p}{E})}. \tag{5}$$

Here, E_p can be defined as (Fig. 7),

$$E_p = \frac{\sigma_1 - \sigma_y}{\varepsilon_1 - \varepsilon_y}. \tag{6}$$

4. Method of analysis

The coupled shear wall systems are analyzed using 2D elements for the geometrical parameters $h = 3$ m, $L = 6$ m, $b = 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2, 4.4, 4.6$ (m), $d = 0.20, 0.40, 0.60, 0.80, 1.00, 1.20$ (m). The stiffness of the tie elements is evaluated as

$$m_{i\theta i} = \frac{M_i}{\theta_i}, \tag{7}$$

where M_i and θ_i are the bending moment and the rotation for section i (Fig. 2), respectively. On the other hand, the stiffness which takes the shear effect of tie element of the equivalent frame into account, is given as

$$\overline{m}_{i\theta i} = \frac{6EI}{L}\,\frac{L^2}{(L^2 + 3.9\,d^2)}, \tag{8}$$

where EI is the bending stiffness and L and d are the geometrical parameters (Fig. 2). Thus, the stiffness of the coupling members (ij member), the geometrical parameters and the material parameters that affect the stiffness directly are defined in plastic space.

$$m_{i\theta i} = \eta^p \overline{m}_{i\theta i}, \tag{9}$$

where η^p is the stiffness modification parameter. By the way, modification parameter can also be provided as a function [4]:

$$\eta^p = \alpha_o \left(\frac{h}{\ell}\right)^{a1} \left(\frac{b}{\ell}\right)^{a2} \left(\frac{d}{\ell}\right)^{a3} \left(\frac{\alpha_c}{f_c}\right)^{a4}. \tag{10}$$

All the constants in eqn (10) can be evaluated by using SPSS program. The results obtained by finite-element analysis in plastic space are considered adequate as a statistical sample data [10]–[12]. Using SPSS package program, the constants, a_0, a_1, a_2, a_3, a_4 in eqn (10), can be obtained [4]:

$$\eta^p = 1.507 \left(\frac{h}{\ell}\right)^{0.0281} \left(\frac{b}{\ell}\right)^{1.6896} \left(\frac{d}{\ell}\right)^{-0.5124} \left(\frac{\sigma_c}{f_c}\right)^{-0.345} \tag{11}$$

or by rounding the powers,

$$\eta^p = 1.5 \left(\frac{h}{\ell}\right)^{0.03} \left(\frac{b}{\ell}\right)^{1.70} \left(\frac{d}{\ell}\right)^{-0.51} \left(\frac{\sigma_c}{f_c}\right)^{-0.35} \tag{12}$$

where σ_c/f_c (stress in exciting zones/characteristic compressive strength of concrete) defines the various stress levels . For example, considering $\sigma_c/f_c \cong 0.40$, the behavior of the system can be idealized as linear [4]. Solutions obtained with the proposed formulae indicate that the recommended procedure can produce sufficiently accurate results for the structural analysis of coupled shear walls. Also, the stiffness of coupling members, and the geometrical and material parameters that affect the stiffness directly are defined in linear elastic space [4], [5]. The stiffness of *ij* member is :

$$m_{i\theta i} = \eta^e \overline{m}_{i\theta i} \tag{13}$$

In a similar way, the stiffness modification parameter in elastic space is provided [4], [5]:

$$\eta^e = 1.9210 \left(\frac{h}{\ell}\right)^{0.0282} \left(\frac{b}{\ell}\right)^{1.6824} \left(\frac{d}{\ell}\right)^{-0.5860} \tag{14}$$

or by rounding the powers

$$\eta^e = 1.9 \left(\frac{h}{\ell}\right)^{0.03} \left(\frac{b}{\ell}\right)^{1.70} \left(\frac{d}{\ell}\right)^{-0.60}. \tag{15}$$

To simplify, dividing eqn (12) by eqn (15);

$$\frac{\eta^p}{\eta^e} = 0.80 \left(\frac{h}{\ell}\right)^{0.09} \left(\frac{\alpha_c}{f_c}\right)^{-0.35}. \tag{16}$$

As shown in Fig. 8, at higher stress levels, values of η^p/η^e decrease rapidly.

And if the numerical data are pointed in Euclid space (E3), it is possible to obtain a linear relation between η^p/η^e and σ_c/f_c (Fig. 8).

As indicated in Fig. 8, the plastic behavior of the tie elements should be taken into account in regions which have $0.40 \leq \sigma_c/f_c \leq 0.80$.

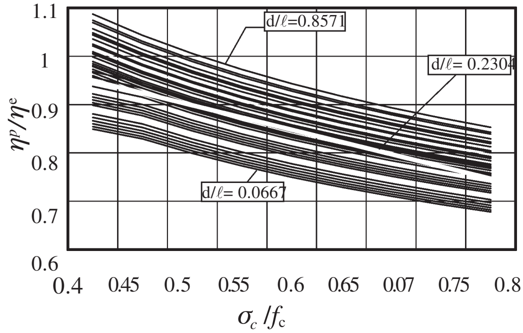


FIG. 8. ($\frac{\eta^p}{\eta^e} - \frac{\sigma_c}{f_c}$) curves for different values of d/ℓ [4].

Table I
Lateral deflections (mm)

Height from the base (m)	FE solution	Present work	Pala and Özmen [13]
3	1.879	2.037	2.015
6	5.612	5.890	5.804
9	10.180	10.594	10.411
12	14.970	15.510	15.208
15	19.680	20.296	19.862

5. Example

A typical coupled shear wall system and equivalent frame are shown in Fig. 9. In this example, geometrical parameters are $d = 0.4$ m, $\ell = 2$ m, b = 4 m and $t = 0.3$ m, mechanical parameters are $\mu = 0.3$ and $E = 3.10^7$ kN/m^2 and applied load is $P = 50$ kN. Using the above algorithm, lateral deflections for the coupled shear wall, the equivalent frame (present work) and the data from other investigators [13] are given in Table I and Fig. 10 for comparison.

6. Conclusions

An equivalent bar frame modeling for the analysis of coupled shear wall structures is presented in this work considering only the stiffness of tie elements as a variable. This approach gives practically more accurate results if the stiffness of tie members has been determined using the generated formulae. In the case of multistoried buildings, the configuration of the reinforcement of the shear wall should be taken into consideration.

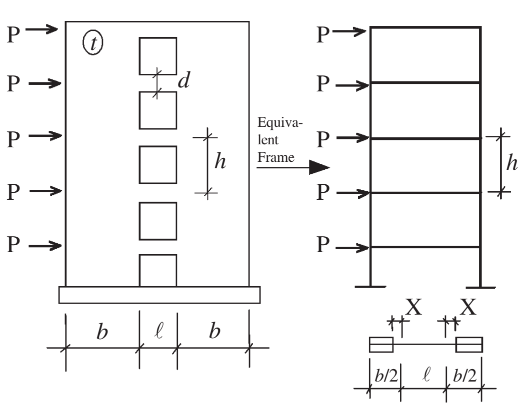


FIG. 9. Coupled shear wall and equivalent frame.

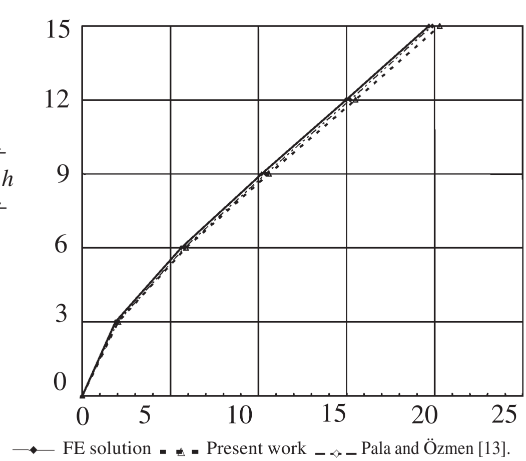


FIG. 10. Lateral deflections.

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