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BOOK REVIEWS

The laws of the web: Patterns in the ecology of information by Bernardo A Huberman. The MIT Press, 5, Cambridge Center, Cambridge, MA 02142, USA, 2001, pp.105, $ 24.95.

In this slim volume of less than 100 pages, the author discusses some interesting phenomenological observations on the web and how they can be used in better designs, for example, search engines. The book is aimed at the 'layman' though many of the intuitions discussed in the book have been inspired by models in physics (statistical mechanics, etc.) and many active workers in this field are also physicists.

The first empirical observation, after large statistical studies on the web, is that of the power law governing many relations such as the number of pages per site and the number of outgoing or incoming links to a page. This property is often termed as the self-similarity, i.e. the existence of the same distribution at all length scales. Because of this law, the average behaviour of the system is not that of the mode. A lognormal distribution results if we assume that changes in the sizes of sites over time are proportional to their sizes with random multiplicative factors. For power law (instead of the lognormal), two more factors are needed: sites appear at different times and that sites grow at different rates. The power law is similar to Zipf 's law in linguistics where the frequency of the k th most occurring word in a language is inversely proportional to k.

Another interesting phenomenon is that of the 'small worlds': the existence of a chain of about 6–7 acquaintances before closure. On the web, it has been shown that four clicks are enough to navigate from a given random web site to another. This can be used to design better search engines: for any search, one can get the page rank (the frequency with which a page is referred to) and the closeness of the match for all the candidate pages.

Some editing errors: it would have been better to illustrate the lognormal distr with a figure when it is discussed in p. 28 than later in Ch. 7 (Fig. 7.1). The third sentence of the last para on p. 63 has some words missing. The word page rank is used without explanation in p. 38: it is unlikely to be known to a layman. In Table 8.1, the captions "% sites" and "%volume by user" need to be interchanged to be consistent with the text.

Overall, the book gives an useful insight into the dynamics of the web.

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Applied computational economics and finance by M. J. Miranda and P. L. Fackler. The MIT Press, Cambridge, MA 02142, USA, 2002, pp. 510, $70.

Many interesting economic and finance models cannot be solved analytically using mathematical techniques of algebra and calculus. This is often true of applied economic models that attempt to capture the complexities inherent in real-world economic behavior. For example, every day the financial markets bravely price trillions of dollars in such risky securities as stocks, bonds, options, futures, and derivatives. The systematic determination of their values gives rise to functional equations in which the unknown is not simply a vector in Euclidean space, but rather an entire function defined on a continuum of points. Except in a very limited number of special cases, the functional equation lacks a known closed-form solution, even though the solution can be shown theoretically to exist and to be unique. Since the introduction of the digital computer, scientists have turned increasingly to computer methods to solve such models. In many cases where analytical approaches fail, numerical methods are often used to successfully compute highly accurate approximate solutions. Over the past several decades there have been a lot of exciting developments in this area due to enormous increase in processor speed and affordability.

The growing popularity of applied computational economics and finance has been impeded by the absence of adequate textbooks and computer software. The methods of numerical analysis and much of the available computer software have been largely developed for non-economic disciplines, most notably the physical, mathematical, and computer sciences. Mathematical prerequisites and unfamiliar domain knowledge (most solutions are for physical, and biological systems) have made the work inaccessible to economists and financial analysts (more so traditional MBAs) to completely appreciate the numerical approaches. Many available software packages, moreover, are designed to solve problems that are specific to the physical sciences.

Miranda and Fackler's book for advanced undergraduate students, graduate students, and practicing economists, fills this void. The book discusses methods for applying numerical methods and solving dynamic stochastic models in economics and finance. The book is divided into two major sections. The first six chapters cover basic numerical methods, error analysis, interpolation and approximation techniques, linear and nonlinear equation methods, complementary methods, finite-dimensional optimization, numerical integration and differentiation, and function approximation. This will be familiar material to readers from the physical sciences and engineering, and is useful for those coming from other backgrounds wishing for a concise overview. References are given for those wanting to go deeper. The last five chapters deal with methods to solve dynamic programming problems. The book also looks at methods that do not rely on dynamic programming techniques. These include almost-linear methods and several nonlinear methods, such as parameterized expectations and weighted residual methods. Examples are used to illustrate the methods where appropriate. The real strength of the book lies in its presentation of how to evaluate the accuracy of different methods. As there is no single 'right' way to solve most computational problems, the authors share their experiences and the rationale for the choices they made along the way. This is a major learning for doctoral students and researchers attempting their initial modeling experiences.

This book blends the three key ingredients for successful technical subject to be of interest to both undergraduate and advanced requirements. The first is its focus on the mingling of intuition and rigor that is required to analyze economic and financial problems such as stochastic growth models, asset pricing models with symmetric and asymmetric information, life-cycle optimization problems, overlapping generations models, and static and dynamic games. This is accomplished not only in the discussion and in examples, but also especially in the proofs. Second, a number of practical applications are presented to illustrate the capacity of the theoretical techniques to contribute insights in a variety of areas; such presentations greatly increase the reader's motivation to grasp the theoretical material. The emphasis on learning-by-doing is the final major element in the learning process, and to this end an extensive collection of problems and codes for problems widely differing difficulty is incorporated. The book is replete with precise examples used to explain every theory in the book. Hundreds of graphs and diagrams, all developed in MATLAB, illustrate the subtleties of the subject. These utilities and demonstration programs provide interested researchers with a starting point for their own computer models.

Although the authors have attempted to keep the mathematical prerequisites for this book to a minimum, some mathematical training and insight is necessary to work with computational economic models and numerical techniques. A reader with familiarity of ideas and methods of linear algebra and calculus would appreciate the book. Overall, the book is a stimulating introduction that would induce the reader to further development and application of the numerical methods in the fields of econometrics and finance.

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Counting, sampling and integrating: Algorithms and complexity by Mark Jerrum. Birkhauser Verlag AG, Klösterberg 23, CH 4010, Basel, Switzerland, 2003, pp. 124, chF 38.

It is well known that exact counting of combinatorial structures is rarely possible in polynomial time. Attempts have been made by mathematicians and computer scientists to get an approximate counting, within arbitrarily small specified relative error. This book mainly deals with such approximate counting, employing a novel approach. The author suggests a method on the basis of sampling of combinatorial structures at random from an almost uniform distribution or from the precisely uniform distribution in a suitably defined model of computation. To achieve such sampling, Markov Chain (MC) simulation has been used. Such simulation has been extensively used in literature to derive statistical behavior/properties of a system, but the author uses such simulation to solve the enumeration problems–indeed an altogether new approach for solving such difficult problems.

The book starts with two counting problems which are solved in polynomial time: # of spanning trees of a graph and # of perfect matchings in a planar graph. In usual decision problems we deal with a predicate $\phi : \Sigma^* \to \{0,1\}$ where Σ is same finite alphabet in which problem instances are encoded, Σ^* denotes the set of all finite sequences of symbols in Σ. In counting problems we deal with functions of the form $f : \Sigma^* \to N$. Thus the complexity class P and NP have to be modified for the fact that we are dealing with functions with codomain N rather than predicates. The modified classes have been termed as class FP and class #P and also defined appropriately. Computation of the permanent of a binary matrix is shown to be #P-complete.

By virtue of proving Proposition 3.4, the author has reduced the problem of approximate counting to almost uniform sampling. This is one of the very strong and important results in the book. Such sampling is realized by simulating appropriately defined MC. However, it is important that the MC in question is 'rapidly mixing', i.e. it converges to near-equilibrium state in time polynomial in the size of the problem instance. It is shown that the MC, in the context of colourings of a low degree graph, does achieve this desired property (Proposition 4.5). Natural MC on matchings in a graph G is also shown to be rapidly mixing. Sophisticated probability theory has been employed all through.

MC methods are also used to estimate the volume of convex body; in this situation, MC with continuous state space is used. Also used is the famous Poincare inequality (Theorem 6.7). A nice and detailed proof of this theorem is given.

It is observed that not all enumeration problems are efficiently approximable. Examples are: number of Hamiltonian cycles in a graph, number of independent sets in a graph, etc. However, it is shown that the second problem is approximable if we place a bound on the maximum degree of the graph.

Throughout the book, a number of open problems are mentioned, which I am sure will draw the attention of researchers. The bibliography is excellently done. Both the content and style of presentation are very rich.

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Subgroup growth by Alexander Lubotzky and Dan Segal, Progress in Mathematics, Vol. 212, Birkhauser Verlag, AG, Klösterberg 23, CH 4010, Basel, Switzerland, 2003, pp. 476, chF 148.

If G is a finitely generated group, then for every integer $n \geq 1$ there are only a finite number $a_n(G)$ of subgroups H having index n in G. This book is devoted to the study of the sequences $(a_n(G))$. The two main themes of study are: (i) the relationship of the asymptotic behaviour of the sequence $(a_n(G))$, i.e. the subgroup growth of G, and the algebraic properties of the group G; (ii) explicit formulas for the numbers $a_n(G)$ and their asymptotic and arithmetical properties. The major part of the book, Chapters 1–13, is devoted to the first theme; the last there chapters, Chapters 14–16, are devoted to the second theme. The wide mathematical spread of the book is best expressed in the authors' own words. "Attempts to answer the simple question: 'how many subgroups of index n does a group possess?' have thus encompassed a surprising broad sweep of mathematics. The full range will become apparent on looking through this book; it includes methods and results from the theories of finite simple groups, permutation groups, linear groups, algebraic and arithmetic groups, p-adic Lie groups, analytic and algebraic number theory, algebraic grometry, probability and logic."

We mention here just a sample of the topics considered in the book in order to give the reader a flavour of the subject.

Let $s_n(G) = \sum_{j=1}^{n} a_j(G)$. A group G is said to have *polynomial subgroup growth* if $s_n(G) \leq n^c$ for all n, where c is some constant. One of the major results in the book is the PSG Theorem which states that a finitely generated residually finite group G has polynomial subgroup growth if and only if it is virtually soluble of finite rank. An interesting class of finitely generated groups is that of *arithmetic groups*. It is of interest to study the congruence subgroup growth in arithmetic groups. The book provides definitive results on this topic. A group G is said to have *growth type f*, for some function f, if there exist positive constants a, b such that $s_n(G) \geq f(n)^a$ for all large n and $s_n(G) \geq f(n)^b$ for infinitely many n. Clearly the infinite cyclic group $\mathbb{Z}$ is of growth type n. Every finitely generated non-Abelian free group has subgroup growth of (strict) type n^n, and this is the upper limit for the growth type of finitely generated groups. The spectrum of admissible subgroup growth types has been explicitly studied.

A useful tool for the study of the arithmetical properties of the sequence $(a_n(G))$ is the Dirichlet series $\zeta_G(s) := \sum a_n(G)$, called the *zeta function of the group* G. The object here is to relate the analytic properties of the function $\zeta_G(s)$ to the structure of G. This study can be viewed as *non-commutative analytic number theory*. For $G = \mathbb{Z}$, $\zeta_G(s)$ is the usual Riemann zeta function $\zeta(s) = \sum n^{-s}$. Already for $G = \mathbb{Z}^r$, the free Abelian group of rank r, there occurs an interesting result: $\zeta_G(s) = (s)\,\zeta(s-1) \ldots \zeta(s-r+1)$; the authors give five distinct proofs of this result. The two concluding chapters study the zeta functions of nilpotent and p-adic analytic groups.

A helpful feature of the book is the set of twelve *Windows* provided separately after the conclusion of main text. The material made available in these sections will prove very convenient for the readers in working their way through the text.

Subgroup growth is an extremely well-written book and is a delight to read. It has a wealth of information making a rich and timely contribution to an emerging area in the theory of groups which has come to be known as *Asymptotic Group Theory*. This monograph and the challenging open problems with which it concludes are bound to play a fundamental role in the development of the subject for many years to come.

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Fractals in Graz 2001 edited by Peter Grabner and Wolfgang Woess. Birkhauser Verlag AG, Klösterberg 23, CH 4010, Basel, Switzerland, 2003, pp. 292, chF. 148.

This book contains the proceedings of the conference 'Fractals in Graz 2001–Analysis, Dynamics, Geometry, Stochastics' that was held in June 2001 at Graz University of Technology, Austria. The proceedings contain 12 invited and refereed papers, which cover different directions of active current research work in the modern theory of fractal structures, such as potential theory, random walks, spectral theory, fractal groups, dynamical systems and fractal geometry. The book is addressed to mathematicians and scientists interested in fractal dimension, fractal energies, fractal groups, stochastic processes on fractals, self-similarity, spectra of random walks, tilings, analysis on fractals, and dynamical systems.

Although these papers are essentially written by mathematicians for mathematicians, the results and open problems discussed in them would interest not only mathematicians working in the field of fractal structures but also others working in related fields, particularly physics and computer science.

I can review this book only from the perspective of a physicist interested in applications of fractals. During the last two decades, ideas of fractal geometry have become very popular, mainly due to several textbooks on the subject. But by and large these textbooks are confined to fractal geometry, which deals with the properties of fractal sets and measures on them. As a result, to a large extent, applications of fractals have so far been limited to static description of objects by quantities such as fractal dimension.

Questions concerned with dynamical aspects of fractals are not as well known. For example, how would a Sierpinsky gasket, having point masses at its vertices and springs along its edges, vibrate. Or how would heat flow through a Sierpinsky gasket. Such questions motivated the search for generalization of the Laplacian operator for the case of fractals, because the important basic equations of physics, such as the Poisson equation, wave equation, diffusion equation and Schrödinger equation, contain the Laplacian operator. This is a formidable challenge because the Laplacian operator is a differential operator, whereas fractals do not have smooth structures to permit differentiation.

Two approaches have been suggested to meet this challenge. Both approaches make use of pre-fractals F_n that converge to the fractal F as $n \to \infty$. The 'probabilistic approach' defines random walks W_n on F_n that converge to a diffusion process on F as $n \to \infty$. The 'analytical approach' on the other hand defines 'discrete Laplacians' L_n on F_n, which converge to an operator L on F that can be considered a suitable generalization of the Laplacian operator.

The background given above is enough to show the relevance of the papers presented in this book for physicists interested in applying modern developments in the field of fractals.

A brief review of the papers in the book is given below:

1. 'The spectrum of the Laplacian on the pentagasket' by Bryant Adams, S. Alex Smith, Robert S. Strichartz and Alexander Teplyaev follows the analytic approach to define a fully symmetric Laplacian on the fractal pentagasket. Properties of the eigenvalues and eigenfunctions of this Laplacian for both Dirichlet and Neumann boundary conditions are investigated theoretically as well as numerically using the finite element method.

2. 'From fractal groups to fractal sets' by Laurent Bartholdi, Rostislav Grigorchuk, and Volodymyr Nekrashevych presents a survey of ideas and results connected with self-similarity of groups, semigroups and their actions and exhibits new connections between them and fractal objects such as the Julia sets.

3. 'Pointwise estimates for transition probabilities of random walks on infinite graphs' by Thierry Couthon and Alexander Grigor'yan presents several results concerned with estimates of the heat kernel $p_n(x,y)$ for random walks on infinite graphs. To each edge (x,y) which connects the vertices x and y of a graph Γ, one can assign a symmetric positive weight $\mu_{xy} = \mu_{yx}$. An example is the standard weight in which each edge of the graph is assigned unit weight. The weight $\mu(x)$ of a vertex x is defined as the sum of the weights of all the edges of the graph that connect x to some vertex. A graph equipped with a weight μ is called a weighted graph. A weighted graph admits a random walk defined by a one-step transition probability $P(x,y)$, which is zero if y is not connected to x by an edge and which otherwise is equal to the ratio of the weight μ_{xy} of the edge (x,y) and the weight $\mu(x)$ of the vertex x. If $P_n(x,y)$ denotes the transition probability from x to y after exactly n steps of the random walk, the heat kernel $p_n(x,y)$ is defined as the ratio of $P_n(x,y)$ to the weight of vertex y.

4. 'Piecewise isometries–An emerging area of dynamical systems' by Arek Goetz describes dynamical systems generated by piecewise isometries. A piecewise isometry is a pair (T, P) where $P = \{P_0, P_1, \ldots.., P_{r-1}\}$ is a partition of a subset X of n-dimensional Euclidean space and $T: X \to X$ is a map such that its restriction on each P_i (called atom) is an Euclidean isometry. The orbit of a point $x_0 \in X$ is given by the set of points $x_k = T^k(x_0)$. Each point x_k of the orbit

belongs to a unique atom P_{wk}. The orbit of x_0 can be encoded into a sequence via the itinerary map defined by $I(x_0) = w_0 w_1 \ldots w_k$. A cell is defined as the set of all points having the same itineraries. Cells whose itineraries are eventually periodic are called rational cells. The set of points with irrational itineraries is called the exceptional set E. The central unsolved problem in the area of piecewise isometries is to determine the size of the set E. In all cases, for which the Lebesgue measure of E has been computed, it is zero, but no general proof is available. This paper also discusses several other open problems in the area. It also gives references to literature on potential applications to dual billiards, Hamiltonian systems and digital filters.

5. 'Random walks on Sierpinski graphs: Hyperbolicity and stochastic homogenization' by Vadim A. Kaimanovich introduces two new techniques for analysis on fractals. The first is based on the presentation of a fractal as the boundary of an associated graph. The second emerged from the need to understand the highly symmetric appearance of the Sierpinsky gasket despite the absence of a sufficiently big symmetry group.

6. 'Some remarks for stable-like jump processes on fractals' by Takashi Kumagai summarizes recent work on non-local Dirichlet forms on fractals whose corresponding processes are stable-like jump processes. A symmetric bilinear closed form E on the space of functions is called a Dirichlet form if it is Markovian, i.e. for each u in the domain of E, $v = \min\{\max\{0, u), 1\}$ is in the domain of E and $E(v,v) \leq E(u,u)$. A Dirichlet form is local if $E(u,v) = 0$ for each u,v whose supports are disjoint compact sets. The paper discusses the connection between functional spaces and stochastic processes. Three natural non-local Dirichlet forms on sets of Hausdorff dimension d are also introduced and their equivalence proved.

7. 'Fractals, multifunctions and Markov operators' by Andrzej Lasota and Jozef Myjak explores the relationship between the dynamics of sets and dynamics of measures. Thanks to Barnsley's popular book *Fractals everywhere* it is well known that fractals can be obtained as an attractor $A*$ of an iterated function system consisting of a finite family of transformations w_i and more easily by chaos game using the same transformations w_i with probabilities p_i. The IFS with probabilities allows one to construct a Markov operator P acting on the space of Borel measures. Let the attractor of this operator be the invariant measure $\mu*$. If the w_is are Lipschitz maps with Lipschitz constants L_i and sup $\{L_i\} < 1$, then $A* = \text{supp}\ m*$. However, the invariant measure $\mu*$ exists even if sup $\{L_i\}$ is not less than 1 but $\sum p_i L_i < 1$. For this case the usual IFS (without probabilities) does not have a unique attractor. One of the interesting questions explored in this paper is: How to describe the geometric properties of the support of $\mu*$ using the transformations w_i without any probabilistic tools.

8. 'Infinite chains of springs and masses' by Michel Mendes France and Ahmed Sebbar investigates the vibrations of sequences of masses m_n, each mass m_n connected to its neighbors m_{n-1} and m_{n+1} by springs of equal spring constant k and equal unstretched length. The problem to be solved is: In a vibration mode ω determine the mass distribution m_n so that each mass vibrates with the same amplitude, i.e. the displacement of m_n is equal to A_n sin ω twhere $A_n = 1$ or -1. Interesting relations between the sequences A_n and m_n are derived. Little that is known about the generalization of this problem in which A_n is allowed to take values in the interval $[\alpha, \beta]$ where $\alpha < 1$ and $\beta > 1$ is also discussed briefly.

9. 'Self-similar fractals and self-similar energies' by Volker Metz discusses the existence and uniqueness of diffusions adapted to self-similar finitely ramified fractal, i.e. fractals which can be disconnected by removal of finite number of points. The problem is reducible to a finite-dimensional nonlinear eigenvalue problem for the renormalization map acting on all possible energies of the fractal graph.

10. 'Neighbours of self-affine tiles in lattice tilings' by Klaus Scheicher and Jorg M. Thuswaldner gives an algorithm that allows one to determine all neighbors of a tile in the tiling and characterize the sets of points where a tile meets all other tiles. The paper also sheds light on relations between different kinds of characterizations of the boundary of a tile.

11. 'On the Hausdorff dimension of the Sierpinski gasket with respect to the harmonic metric' by Elmar Teufl disproves Kigami's conjecture that Hausdorff dimension with respect to the harmonic metric is equal to the spectral dimension. Vector of hitting probabilities $p_n(x)$ of random walks W_n on the ηth stage pre-Sierpenski gasket G_n is defined with the help of three matrices so that the restriction of p_n on G_m is equal to p_m for $m < n$. It converges to a vector function $p(x)$ defined on the Sierpinski gasket G. This function is harmonic on the set $G - G_0$, and the harmonic metric on G is defined by $h(x, y) = |p(x) - p(y)|$. Consider the Laplacian operator L associated with the Brownian motion to which the random walks W_n converge. The spectral dimension d_s is defined by $|\{\lambda \in$ the set of eigenvalues of $L: \lambda < x\}| \sim x^{ds/2}$. It is shown that the Hausdorff dimension d_h with respect to the harmonic metric h on G is strictly less that the spectral dimension d_s.

12. 'Riesz potential and Besov spaces on fractals' by M. Zahle introduces fractal Liouville operators D_μ^s as inverses of fractal Riesz potentials I_μ^s of order s with respect to a given d-measure μ on a fractal set of Hausdorff dimension d embedded in Euclidean space R^n. These operators play a similar role as the standard Liouville operators D^s and Riesz potential I^s and reduce them for $d = n$.

Readers interested in fractals will definitely find this book very useful.

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