

# BUCKLING COEFFICIENTS OF VARIOUSLY SUPPORTED SKEW PLATES\*

By M. S. S. PRABHU AND S. DURVASULA

(Department of Aeronautical Engineering, Indian Institute of Science, Bangalore-12, India)

[Received : October 5, 1970]

## ABSTRACT

*The buckling problems of skew plates with different edge support conditions involving simple support and clamping are considered. The in-plane stresses are represented in terms of oblique components. Rayleigh-Ritz method is used employing a double series of functions appropriate to the combination of the edge conditions. Numerical results are presented for several combinations of side ratio, skew angle and different loadings.*

## 1. INTRODUCTION

Skew plates have their application in construction of modern swept wing aircraft. The buckling problems of plates of such shape are of interest to the designer. The boundary conditions obtaining on individual panels are more nearly in the nature of elastic restraint against rotation. Analytical treatment of this boundary condition, however, is somewhat tedious and it is even more so for skew geometry. Consequently, the ideal boundary conditions of simple support or clamping are usually analysed.

While considerable literature is available on buckling of rectangular plates under different loadings (Refs. 1,2,3) yet buckling coefficients for the many different combinations of edge conditions involving simple support and clamping are not fully available.

The problem of buckling of clamped skew plate under uniform compression was studied by Guest<sup>4</sup>. He applied the Lagrangian Multiplier method to get upper bounds and rather doubtful lower bounds (see Ref. 5). In another paper<sup>6</sup> he considered the buckling of clamped rhombic plate under bending and compression. Using Rayleigh-Ritz method, Wittrick studied the buckling problem of clamped skew plates under uniform compression<sup>7</sup>

---

\* Paper presented at the 22nd Annual General Meeting of the Aeronautical Society of India held at Hyderabad on the 20th and 21st March 1970.

and pure shear<sup>8</sup>. He used Iguchi functions and found that the convergence was slow particularly in the case of positive shear. Hasegawa<sup>9</sup> calculated the buckling coefficients of clamped rhombic plate under the action of pure shear by the Rayleigh-Ritz method using polynomials. Hamada<sup>10</sup> used Lagrangian multiplier method to study the problem of buckling of clamped skew plates under the action of uniform compression and oblique shear. Matrix methods have also been applied<sup>11</sup> to find the buckling coefficients of the parallelogramic plates under the action of shear and compression. Durvasula<sup>5</sup> investigated the above problem using Galerkin Method and expressing the deflection as a series of beam characteristic functions. The buckling coefficients have been calculated when direct and shear forces are acting either individually or in combination. Ashton<sup>12</sup> also investigated the problem using beam characteristic functions and Rayleigh-Ritz method. Mansfield<sup>13</sup> obtained a rough estimate for the buckling coefficient under uniform compression.

Yoshimura and Iwata<sup>14</sup> obtained the buckling coefficients for the simply supported skew plates under oblique shear and compression. Durvasula<sup>15</sup> solved the problem using double Fourier sine series and Rayleigh-Ritz method with in-plane stresses expressed in terms of orthogonal components. Durvasula and Nair<sup>16</sup> have also considered the buckling problem of simply supported skew plates with in-plane stresses expressed in terms of oblique components. Extensive numerical results were presented for various combinations of skew angles and side ratios. Interaction curves have also been given.

In this paper, the buckling problems of skew plates supported differently on different edges are considered. The support conditions considered are confined to different combinations of simple support and clamping on the four edges. The in-plane stresses are represented in terms of oblique components. Rayleigh-Ritz method has been used with the buckling mode expressed as a double series of beam characteristic functions appropriate to the combinations of the edge conditions in each case. Numerical calculations have been made to obtain the buckling coefficients mainly when each of the stress components is present individually for different combinations of side ratio, skew angle and boundary condition and for a few combined loadings. Convergence has been examined in a few typical cases and is found to be satisfactory.

## Notation:

$a, b$	dimensions of the plate
$C_{rs}$	Coefficient in the series expansion of deflection
$D$	flexural rigidity of the plate. $[Eh^3/12(1-\nu^2)]$
$E$	Young's Modulus of the material of the plate
$G, H^{(1)}, H^{(2)}, H^{(3)}$	Matrices defined in Equation [15]
$G_1$	matrix defined in Equation [18]
$h$	plate thickness
$J_{mn}^{pq}, J_{ns}^{rq}$	integrals defined in Equation [14]
$M$	maximum value of indices $m, r$
$N$	maximum value of indices $n, s$
$N_x, N_y, N_{xy}$	midplane forces (oblique components), $h \sigma_x, h \sigma_y, h \sigma_{xy}$ respectively
$m, n, r, s$	integers
$M_n$	normal bending moment
$\bar{R}_x, \bar{R}_y, \bar{R}_{xy}$	non-dimensional midplane force parameters $\sigma_x b^2 h / \pi^2 D, \sigma_y b^2 h / \pi^2 D, \sigma_{xy} b^2 h / \pi^2 D$ respectively.
$\bar{R}_x^*, \bar{R}_y^*, \bar{R}_{xy}^*$	non-dimensional midplane force parameters $(\sigma_x a^2 h \cos^3 \psi) / D, (\sigma_y a^2 h \cos^3 \psi) / D, (\sigma_{xy} a^2 h \cos^3 \psi) / D$ , respectively.
$U$	strain energy of the plate
$V$	potential energy of the middle surface forces.
$W(\xi, \eta)$	deflection of the plate
$X_m(\xi), Y_n(\eta)$	beam characteristic functions
$x, y, z$	oblique coordinate system defined in Fig. 1
$\xi, \eta$	non-dimensional coordinates, $x/a$ and $y/b$ respectively
$\nu$	Poisson's ratio
$\sigma_x, \sigma_y, \sigma_{xy}$	oblique stress components defined in Fig. 1
$\nabla^2$	Skew differential operator $= \text{Sec}^2 \psi [\partial^2 / \partial x^2 - 2 \text{Sin} \psi (\partial^2 / \partial x \partial y) + (\partial^2 / \partial y^2)]$
$\nabla_1^2$	Non-dimensional skew differential operator $= \text{Sec}^2 \psi [\partial^2 / \partial \xi^2 - 2\lambda \text{Sin} \psi (\partial^2 / \partial \xi \partial \eta) + \lambda^2 (\partial^2 / \partial \eta^2)]$
$\psi$	Skew angle as defined in Fig. 1
$\lambda$	$a/b$ . side ratio

## 2. FORMULATION OF THE PROBLEM

A sketch of the skew plate is shown in Fig. 1 along with the in-plane stresses represented in terms of oblique components. Since the geometry of the plate is oblique in nature, the use of oblique stress components instead of usual orthogonal components is preferable. In terms of oblique components, expressions for the strain energy of the plate and the potential energy of the middle surface forces are simpler and the structure of these expressions is similar to those in the case of rectangular plates with orthogonal stress components. The plate is assumed to be thin, uniform and isotropic.

Using the classical, small deflection thin plate theory, the differential equation for the deflection of a plate of constant thickness under the action of middle surface forces is given by<sup>17</sup>,

$$D \cos \psi \nabla^4 W = -h [\sigma_x (\partial^2 W / \partial x^2) + 2\sigma_{xy} (\partial^2 W / \partial x \partial y) + \sigma_y (\partial^2 W / \partial y^2)] \quad [1]$$

In terms of oblique coordinates, the boundaries of the skew plate are given by,

$$x=0, x=a; y=0, y=b. \quad [2]$$

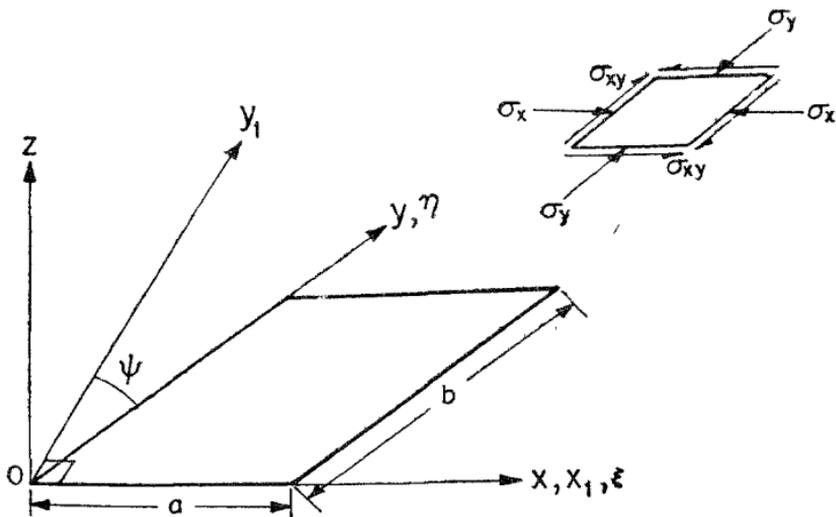


FIG. 1

Sketch of the Skew Plate and the in-plane Stress System (Oblique Components)

The boundary conditions considered are confined to combinations of simple support and clamping. These conditions are stated as follows :

$$\text{Simple support : } W = M_n = 0 \quad [3a]$$

or alternatively for a polygonal plate<sup>18</sup>

$$W = \nabla^2 W = 0 \quad [3b]$$

$$\text{Clamping : } W = [(\partial W / \partial n)] = 0 \quad [3c]$$

If the edge  $x = a$ , for example, is simply supported, then boundary condition, Eq (3b) takes the form,

$$W = [(\partial^2 / \partial x^2) - 2 \sin \psi (\partial^2 / \partial x \partial y)] W = 0 \quad [4]$$

If the edge  $y = b$  is clamped, then the boundary condition Eq. (3c) takes the form<sup>4</sup>.

$$W = [(\partial W / \partial y)] = 0 \quad [5]$$

In this paper, the buckling problems with different edges supported differently are considered. An approximate solution of the buckling problem stated by Equation [1], together with boundary conditions such as given by Equations [3b, 3c] appropriate to each edge is solved using the Rayleigh-Ritz method.

Non-dimensional coordinates  $\xi$  and  $\eta$  are defined as follows :

$$\xi = x/a ; \eta = y/b \quad [6]$$

For the stress system shown in Fig. 1. the expressions for the strain energy of the plate and the potential energy of the middle surface forces are given respectively by<sup>17</sup>,

$$U = \frac{D \cos \psi b}{2a^3} \int_0^1 \int_0^1 [(\nabla_1^2 W)^2 - 2\lambda^2(1-\nu) \sec^2 \psi \{W_{,\xi\xi} W_{,\eta\eta} - W_{,\xi\eta}^2\}] d\xi d\eta \quad [7]$$

$$V = \frac{hb}{2a} \int_0^1 \int_0^1 (\sigma_x W_{,\xi}^2 + 2\lambda \sigma_{xy} W_{,\xi} W_{,\eta} + \lambda^2 \sigma_y W_{,\eta}^2) d\xi d\eta \quad [8]$$

For polygonal boundaries with  $W = 0$  along the edges the expression for  $U$  reduces to<sup>17,4</sup>

$$U = \frac{D \cos \psi b}{2a^3} \int_0^1 \int_0^1 (\nabla_1^2 W)^2 d\xi d\eta \quad [9]$$

The deflection  $W$  is expressed as a double series in terms of "admissible functions", i.e., functions which satisfy the geometric boundary conditions. Beam characteristic functions which have been widely used in the literature have been made use of in the present analysis. The series is written as,

$$W(\xi, \eta) = \sum_{m=1}^M \sum_{n=1}^N C_{mn} X_m(\xi) Y_n(\eta) \quad [10]$$

where  $X_m(\xi)$ ,  $Y_n(\eta)$  are the beam characteristic functions which are appropriate to the particular boundary conditions specified. For example, if the edges  $\xi=0$  and  $\xi=1$  are both clamped, the clamped-clamped beam functions are taken for  $X_m(\xi)$ . Similarly, if the edge  $\eta=0$  is clamped and edge  $\eta=1$  is simply supported, the clamped-simply supported beam functions are taken for  $Y_n(\eta)$ .

Substituting the expressions for  $W$  in Eqs. [8] and [9], we get,

$$U = \frac{D \sec^3 \psi b}{2a3} \int_0^1 \int_0^1 \sum_{m=1}^M \sum_{n=1}^N \sum_{r=1}^M \sum_{s=1}^N C_{mn} C_{rs} \left[ X_m'' Y_n - 2 \lambda \sin \psi X_m' Y_n' + \lambda^2 X_m Y_n'' \right] \left[ X_r'' Y_s - 2 \lambda \sin \psi X_r' Y_s' + \lambda^2 X_r Y_s'' \right] d\xi d\eta \quad [11]$$

$$V = - \frac{hb}{2a} \int_0^1 \int_0^1 \sum_{m=1}^M \sum_{n=1}^N \sum_{r=1}^M \sum_{s=1}^N C_{mn} C_{rs} [\sigma_x X_m' X_r' Y_n Y_s + 2 \lambda \sigma_{xy} X_m' X_r Y_n Y_s' + \lambda^2 \sigma_y X_m Y_n' X_r Y_s'] d\xi d\eta \quad [12]$$

The coefficient  $C_{mn}$  are determined from the condition<sup>19</sup>

$$[(\partial/\partial C_{mn}) (U+V)] = 0 \text{ for } m=1, 2, \dots, M, \text{ for } n=1, 2, \dots, N \quad [13]$$

The integrals involving the functions  $X_m(\xi)$ ,  $Y_n(\eta)$  and their derivatives are defined as follows:

$$I_{mr}^{pq} = \int_0^1 X_m^p(\xi) X_r^q(\xi) d\xi; \quad J_{ns}^{pq} = \int_0^1 Y_n^p(\eta) Y_s^q(\eta) d\eta \quad [14]$$

where  $p$  and  $q$  represent the order of the derivative. The formulae for such integrals were given by Felgar<sup>20</sup> and the numerical values of some of these integrals are given in Ref. 21. Using the expressions for  $U$  and  $V$  from Eqs. [11] and [12] in Eq. [13], and using the relationships between the integrals, we get, finally, a set of linear simultaneous algebraic equations in  $C_{rs}$  which can be expressed in the form of a matrix equation as follows:

$$[G] \{C_{rs}\} = \bar{R}_x^* [H^{(1)}] \{C_{rs}\} + \bar{R}_y^* [H^{(2)}] \{C_{rs}\} + \bar{R}_{xy}^* [H^{(3)}] \{C_{rs}\} \quad [15]$$

where

$$G_{mnr s} = \{ I_{mr}^{22} J_{ns}^{00} + \lambda^4 (I_{mr}^{00} J_{ns}^{22}) - 2\lambda \sin \psi [I_{mr}^{21} J_{ns}^{01} + I_{mr}^{12} J_{ns}^{10} + \lambda^2 (I_{mr}^{10} J_{ns}^{12} + I_{mr}^{01} J_{ns}^{21})] + 2\lambda^2 (1 + 2 \sin^2 \psi) I_{mr}^{11} J_{ns}^{11} \} \quad [16]$$

$$H_{mnr s}^{(1)} = I_{mr}^{11} J_{ns}^{00}; \quad H_{mnr s}^{(2)} = \lambda^2 I_{mr}^{00} J_{ns}^{11}; \quad H_{mnr s}^{(3)} = 2\lambda I_{mr}^{01} J_{ns}^{01} \quad [17]$$

This is an algebraic eigenvalue problem. To get the buckling loads when  $N_x$ ,  $N_y$ ,  $N_{xy}$  are present individually or in combination, numerical values are given to two of the three parameters  $\bar{R}_x^*$ ,  $\bar{R}_y^*$ ,  $\bar{R}_{xy}^*$  and the third is treated as the eigenvalue. For example, if we wish to determine the buckling parameter  $\bar{R}_x^*$ , when both  $N_{xy}$  and  $N_y$  are acting, we assign appropriate numerical values to  $\bar{R}_y^*$  and  $\bar{R}_{xy}^*$  and obtain the  $G_1$  matrix as,

$$[G_1] = [G] - \bar{R}_x^* [H^{(2)}] - \bar{R}_{xy}^* [H^{(3)}] \quad [18]$$

Equation [15] then reduces to

$$[G_1] \{C_{rs}\} = \bar{R}_x^* [H^{(1)}] \{C_{rs}\} \quad [19]$$

which can be written as,

$$[G_1^{-1}] [H^{(1)}] \{C_{rs}\} = (1/\bar{R}_x^*) \{C_{rs}\} \quad [20]$$

For combinations of boundary conditions symmetric about the diagonals, the Equation [15] splits into two cases:  $(m+n)$  Even and  $(r+s)$  Even;  $(m+n)$  Odd and  $(r+s)$  Odd. The Even case corresponds to skew symmetric case consisting of modes which are doubly symmetric and doubly antisymmetric. The Odd case corresponds to skew antisymmetric case consisting of modes which are symmetric-antisymmetric and antisymmetric-symmetric. This splitting reduces the order of the matrix to be considered for finding out the eigenvalues and eigenvectors. If  $K(=M \times N)$  is the order of the original matrix, then the size of matrix for the even case will be  $(K+1)/2$  if  $K$  is Odd and  $K/2$  if  $K$  is Even; and the matrix size for the Odd case will be  $(K-1)/2$  if  $K$  is Odd and  $K/2$  if  $K$  is Even.

The eigenvalue  $\bar{R}_x^*$  can now be determined by using any of the standard methods. The two groups give two eigenvalues; the lower of the two is the desired critical buckling load. Similar procedure can be adopted to determine the eigenvalues for other combinations of loads.

For cases where such symmetry of boundary conditions about the diagonals is not present, this splitting is not possible and the full matrix of order  $K$  will have to be handled.

## 3. NUMERICAL CALCULATIONS

Numerical calculations have been made for different combinations of side ratio  $a/b$  and skew angle  $\psi$  for different edge conditions. Since the accuracy of the eigenvalues decreases with increasing value of  $\psi$ , more terms have been considered for higher skew angles. For skew  $\psi \leq 30^\circ$ , the number of terms considered is upto  $M=N=6$  except in the case of  $N_x$  acting alone in which case the number of terms considered is upto  $M=N=5$  only. For  $\psi = 45^\circ$ , terms upto  $M=N=8$  have been taken. The calculations made are mainly for  $N_x$ ,  $N_y$  or  $N_{xy}$  acting alone, though the combined action of  $N_x$ ,  $N_y$  and  $N_{xy}$  has also been studied in a typical case. Convergence study has been made for one representative boundary condition when  $N_x$  and  $N_{xy}$  are each acting alone. The numerical values are presented in Tables 1 to 5.

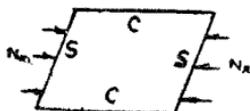
## 4. RESULTS AND DISCUSSION

Results of the convergence study for one typical boundary condition in the case of a rhombic plate with  $\psi = 30^\circ$  are given in Tables 1 and 2. Table 1

TABLE 1  
Convergence Study:  $N_x$  Acting Alone

$$\lambda = a/b = 1, \psi = 30^\circ$$

Boundary conditions	M	N	Matrix size	Eigen value* $\bar{R}_x$
	2	2	2	12.89
	3	3	4	10.41
	4	4	8	9.663
	5	5	12	9.510
	6	6	18	9.391
	7	7	24	9.352
	8	8	32	9.302
	9	9	41	9.282



\*These values are all from (M+N) ODD case; (M+N) EVEN case gives higher values.

TABLE 2  
Convergence Study:  $N_{xy}$  Acting Alone

$$\lambda = a/b = 1; \psi = 30^\circ$$

Positive Shear



Negative Shear



Boundary conditions	M	N	Matrix size	Eigenvalue* $\bar{R}_{xy}$	
				Positive	Negative
	2	2	2	18.69	-37.78
	4	4	8	8.550	-34.84
	6	6	18	8.314	-34.79
	8	8	32	8.233	-34.75

\* These values are all from (M+N) EVEN case. (M+N) ODD case gives higher values.

gives the eigenvalues when  $N_x$  alone is acting. Table 2 gives the eigenvalues when  $N_{xy}$  alone is acting. It can be seen from Table 1 that the convergence of the eigenvalues is satisfactory. When  $N_{xy}$  is acting the convergence is equally good for positive as well as negative shears (Table 2).

The buckling coefficient  $\bar{R}_x$  has been obtained for seven different combinations of boundary conditions for  $a/b$  equal to 0.5 and 1 and  $\psi$  equal to  $0^\circ$ ,  $15^\circ$ ,  $30^\circ$  and  $45^\circ$ . These are given in Table 3 along with results, where available, for comparison. Similarly for the same combinations of  $a/b$  and  $\psi$  and different boundary conditions, the buckling coefficients  $\bar{R}_{xy}$  and  $\bar{R}_y$  are given in Tables 4 and 5 respectively. In Table 6, the buckling coefficient  $\bar{R}_x$  in the presence of inplane forces  $N_{xy}$  and  $N_y$  is given for a rhombic plate with skew angle  $\psi = 30^\circ$  for a typical boundary condition. From Table 3 it may be seen that even for rectangular plates complete results are not available. For skew plates with different combinations of boundary conditions no results could be found in the literature for comparison. The results of the present paper are in good agreement with the available results.

The buckling coefficient  $\bar{R}_x = N_x b^2 / \pi^2 D$  increases with the skew angle, as may be expected, and decreases with  $a/b$ . Also the values in Table 3 are indicative of the relative stiffnesses of the plates with different combinations of boundary conditions for a given combination of  $a/b$  and  $\psi$ . One can expect that for a given  $a/b$  and  $\psi$  the buckling coefficient for a plate with combination of clamped edge conditions (C) and simply supported edge conditions (S) should be in between the values for a plate of the same geometry with all edges clamped and all edges simply supported; this is borne out by the present results except for  $a/b=0.5$ , for the obvious reason that in this case the order of approximation is lower ( $M=N=4$ ).

In Table 4, the buckling coefficients under positive and negative shears are given along with some available results for rectangular plates. The agreement between the present results and available results is quite good. As in the case of  $\bar{R}_x$ , the buckling coefficient  $\bar{R}_{xy}$  decreases with  $a/b$  and increases with  $\psi$ . The buckling coefficient  $\bar{R}_{xy}$  for positive shear is less than that for negative shear irrespective of  $a/b$ ,  $\psi$  and boundary condition. This is in conformity with the observation made previously<sup>16</sup>.

In Table 5, the critical buckling coefficient  $\bar{R}_y$  is given for different combination of  $a/b$ , and boundary condition. The buckling coefficient  $\bar{R}_y$ , for a certain  $a/b$  and  $\psi$  and boundary condition, can be related to  $\bar{R}_x$  for corresponding  $b/a$  and  $\psi$ , for appropriate boundary conditions. For example for  $\psi=0^\circ$ ,  $a/b=0.5$  for boundary conditions (Case 6)  $\bar{R}_y$  can be interpreted as the value of  $\bar{R}_x$  for  $\psi=0$  and  $a/b=2$  for boundary conditions (Case 5). For this to be valid, the corresponding orders of approximations have to be necessarily equal; the slight difference that is seen in the case of  $\bar{R}_x$  for  $a/b=1$  (Table 3) and  $\bar{R}_y$  for  $a/b=1$  (Table 5) is because the corresponding orders are not the same.

In Table 6, the critical buckling coefficient  $\tilde{R}_x$  in the presence of different combinations of inplane forces  $N_{xy}$  and  $N_y$  is given for a typical combination of  $a/b$ ,  $\psi$  and boundary condition. The computer programme, however, can generate data for other combinations of  $a/b$ ,  $\psi$  and any combined loading and is thus capable of generating interaction surfaces which should prove useful in design.

TABLE 3  
Buckling Coefficient  $\bar{R}_s$  For Different Edge Conditions

Case	Boundary Condition	$\psi=0^\circ$		$\psi=15^\circ$		$\psi=30^\circ$		$\psi=45^\circ$	
		$a/b=0.5$	1	0.5	1	0.5	1	0.5	1
1*		6.25	4.00	6.84	4.32	9.17	5.55	15.7	8.40
2		10.4	4.85	11.3	5.21	14.9	6.51	24.1	9.22
3		6.86 (6.85) <sup>a</sup>	5.74 (5.74) <sup>a</sup>	7.45	6.16	9.68	7.62	15.2	10.2
4		10.9	6.22	11.8	6.55	15.3	7.87	24.3	10.4
5		18.2	6.75 (6.74) <sup>a</sup>	19.8	7.14	25.6	8.51	38.0	11.3
6		7.70 (7.69) <sup>a</sup>	7.70 (7.69) <sup>a</sup>	8.30	8.10	10.5	9.51	15.9	12.0
7		18.7	8.07	20.2	8.47	25.9	9.83	38.3	12.5
8		11.6	8.09	12.6	8.58	16.0	10.2	24.8	13.1
9		19.4	10.1	20.8	10.5	26.3	11.8	38.8	14.3

(a) Ref. 18 (Levy's Method)

(c) Ref. 2 (Taken from the graph)

<sup>a</sup>The eigenvalues corresponding to this case are taken from Ref. 16 (note that for  $a/b=0.5$ ,  $M=N=4$  and for  $a/b=1$ ,  $M=N=6$ .)

TABLE 4



Positive Shear



Negative Shear

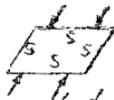
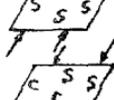
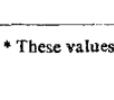
Buckling Coefficient  $\bar{R}_{xy}$  For Different Edge conditions

Case	Boundary Conditions	$\psi=0^\circ$		$\psi=15^\circ$		$\psi=30^\circ$		$\psi=45^\circ$	
		$a/b=0.5$	1	0.5	1	0.5	1	0.5	1
1*			$\pm 9.40$		7.15 -14.71		6.52 -27.4		
2		$\pm 26.5$ (26.88) <sup>a</sup>	$\pm 10.7$ (10.98) <sup>a</sup>	20.3 -40.3	8.05 -16.7	17.8 -72.5	7.09 -30.6	18.0 -163	7.26 -70.0
3		$\pm 32.4$ (33.72) <sup>a</sup>	$\pm 10.7$ (10.98) <sup>a</sup>	24.6 -49.6	8.05 -16.7	21.7 -89.5	7.09 -30.6	22.2 -199	7.76 -70.0
4		33.1 -33.2	11.7 -11.9	24.9 -50.9	8.79 -18.4	21.9 -90.7	7.63 -33.6	22.4 -201	7.66 -76.2
5		$\pm 40.1$ (40.16) <sup>a</sup>	$\pm 12.6$ (12.6) <sup>a,b</sup>	30.8 -60.3	9.52 -19.3	26.7 -107	8.31 -34.8	26.8 -238	8.37 -78.5
6		$\pm 26.9$ (2.688) <sup>a</sup>	$\pm 12.6$ (12.6) <sup>a,b</sup>	20.5 -41.0	9.52 -19.3	18.2 -73.9	8.31 -34.8	18.3 -166	8.37 -78.5
7		$\pm 40.5$	$\pm 13.4$	30.9 -61.1	10.1 -20.5	27.0 -109	8.75 -37.0	26.9 -242	8.71 -83.1
8		$\pm 34.2$	$\pm 13.4$	25.7 -51.8	10.1 -20.5	22.3 -91.7	8.75 -37.0	22.5 -203	8.71 -83.1
9		$\pm 41.0$	$\pm 14.7$	31.2 -62.2	11.1 -22.3	27.2 -111	9.50 -40.0	27.1 -246	9.35 -89.3

\* Eigenvalues for this case are taken from Ref. 16; a) Ref. 23, b) Ref. 3.

TABLE 5

Buckling Coefficient  $\bar{K}_y$  For Different Edge Condition

Case	Boundary Conditions	$\psi=0^\circ$		$\psi=15^\circ$		$\psi=30^\circ$		$\psi=45^\circ$	
		$a/b=0.5$	1	0.5	1	0.5	1	0.5	1
1*			4.00	4.32		5.55		8.40	
2		22.4	5.74	23.5	6.15	27.3	7.56	34.5	10.2
3		17.0	4.85	18.0	5.19	21.5	6.46	28.0	9.22
4		22.7	6.22	26.1	6.53	27.8	7.80	34.8	10.4
5		27.9	7.70	29.4	8.08	34.2	9.39	42.3	12.0
6		19.4	6.75	20.2	7.13	23.0	8.47	28.5	11.3
7		28.9	8.09	30.3	8.56	34.6	10.1	42.8	13.1
8		24.9	8.07	25.9	8.46	29.3	9.78	36.0	12.5
9		31.6	10.1	32.5	10.5	35.8	11.8	43.1	14.3

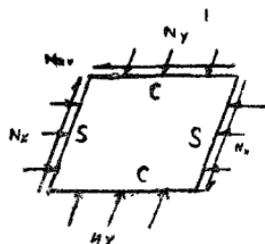
\* These values are taken from Ref. 16.

TABLE 6

Buckling Coefficient  $\bar{R}_x$  under combined Loading for SCSC case

$$a/h=1; \psi=30^\circ; N_y = \alpha N_x; N_{xy} = \beta N_x$$

Boundary conditions	$\alpha$	$\beta$	$\bar{R}$ critical
	0	0	9.51
	0	0.5	6.66
	0	1.0	4.93
	0.5	0	6.77
	0.5	0.5	5.08
	0.5	1.0	3.96
	1.0	0	4.87
	1.0	0.5	3.96
	1.0	1.0	3.27



## 5. CONCLUSIONS

The buckling problems of skew plates with different edge conditions involving simple support and clamping are considered with the in-plane stresses represented in terms of oblique components. Rayleigh-Ritz method is used expressing the deflection in terms of beam characteristic functions in oblique coordinates. Buckling coefficients have been obtained mainly when the in-plane forces  $N_x$ ,  $N_{xy}$ ,  $N_y$  are acting individually for different combinations of  $a/b$ ,  $\psi$  and boundary condition and for a few combined loadings. For buckling under shear loading (oblique components) two critical values exist; the positive shear (acting in a way so as to reduce the skew angle) is found to be less than the negative shear in magnitude for all the plate configurations and boundary conditions considered. The computer programme developed can be used for generating extensive design data in the form of buckling charts and interaction surfaces for buckling under combined loading.

## 6. ACKNOWLEDGEMENTS

The authors express their sincere thanks to Prof. C. V. Joga Rao for his keen interest and helpful criticism. One of us (MSSP) is thankful to Mr. P. S. Nair for many useful discussions.

## 7. REFERENCES

1. Anonymus . . . . . Data Sheets, Stressed Skin Structures, R. Aero. Soc., 2.
2. Gerard, G. and Becker, H. . . . . Handbook of Structural Stability, 1957, NACA TN 3781.
3. Leggett, D. M. A. . . . . *A. R. C. Tech. Rep.* 1941. R & M No. 1991.
4. Guest J. . . . . Rep. SM. 172, Aero. Res. Lab., Melbourne, 1951.
5. Durvasula, S. . . . . Rep. No. AE 2458, Dept. Aero. Engg., I.I.Sc., 1959.
6. Guest, J. . . . . Rep. SM. 227, Aero. Res. Lab., Melbourne, 1955.
7. Wittrick, W. H. . . . . *Aero. Quart.*, 1953, 4, 151.
8. ————— . . . . . *Ibid*, 1954, 5, 39.
9. Hasegawa, M. . . . . *J. Aero. Soc.*, 1954, 21, 720.
10. Hamada, M. . . . . *Bull. Jap. Soc. Mech. Engrs.*, 1959, 2, 520.
11. Argyris, J. H. . . . . Matrix Methods in Structural Mechanics, (Proceedings of the Conference held at Wright-Patterson Air Force Base), 1966, 11.
12. Ashton, J. E. . . . . *J. Appl. Mech., Trans. ASME*, 1969, E 91, 139.
13. Mansfield, E. H. . . . . *Aircraft Eng.*, 1952, 24, 48.
14. Yoshimura, Y. and Iwata . . . . . *J. Appl. Mech.*, 1963, 30, 363.
15. Durvasula, S. . . . . Paper presented at the 17th Annual General Meeting of the Aero. Soc. India, held at Bangalore, 1965.
16. Durvasula, S. and Nair, E. S. . . . . *Israel J. Tech.*, 1969, 7, 303.
17. Morley, L. S. D. . . . . Skew Plates and Structures, Pergamon Press, 1963.
18. Timoshenko, S. and Gere, J. M. . . . . Theory of Elastic Stability, McGraw Hill Book Co., 1952.
19. Bleich, F. . . . . Buckling Strength of Metal Structures, McGraw Hill Book Co., 1952.
20. Felgar, R. P. . . . . Univ. Texas, Bur. Eng. Res. Circular No. 14, 1950.
21. Durvasula, S. Bhatia, P. and Nair, P. S. . . . . Rep. No. AE 220, S, Dept. Aero. Engg., I.I.Sc., 1969.
22. Flugge, W. (Ed.) . . . . . Handbook of Engineering Mechanics, McGraw Hill Book Co., 1962, 44-35.
23. Chilver, A. H. (Ed.) . . . . . Thin Walled Structures, Chatto and Windus, 1967, 251.