THEORETICAL STUDIES ON SOMMERFELD SURFACE WAVE RESONATOR

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ABSTRACT

The present report consists of a brief résumé of the properties of microwave resonators, such as, mode degeneracy, coupling of companion modes etc , and the derivation of the equivalent circuit by using Lagrangian method. After making a comparative study of the Sommerfeld and Harms--Goubau surface wave lines, the report deals with the theory of surface wave resenator excited in E_0 *and* EH *and* HE *modes. As each of the latter two modes are coupled modes it is expected that the* Q *factor will be very low, so emphasis is laid on the* E_0 *–mode resonator, which may be called the Sommerfeld surface wave resonator. Numerical Computations for* Q (E_0) *and guide wavelength* λ_g (E_0) *as function of the length* l *of the resonator and frequency of excitation for the Sommerfeld resonator show that* Q (E_0) *increases linearly with increasing length of the resonator for different frequencies of excitation, whereas,* λ_g *decreases almost exponentially with increase in frequency.*

1. INTRODUCTION

The study of electromagnetic oscillations in resonators is inherently associated with Maxwell's equations and the concept of standing waves. The study of standing waves in resonant cavities first made by Lord Rayleigh remained for many years a subject of theoretical speculation. Almost half a century elapsed before the practical importance of standing waves could be realised and resonators became very useful practical tools for microwave work. The obvious answer as to why standing waves were for such a long period of only academic interest is that the technique of generation of microwave power was not sufficiently advanced so as to make microwave work possible ; and yet this is hardly an adequate answer as the original experiments of Hertz were done with millimeter waves. The practical

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application of resonators to microwave work was made possible due mainly to the work of Southworth and Schelkunoff at the Bell Telephone Laboratories, and Barrow, C hu and Stratton at the Massachusetts Institute of Technology in the middle of 1930's.

The resonance phenomena in microwave resonators of simple and some complicated shapes have been studied by several authors[1-35]. The concept of resonance in enclosed type of microwave cavities has been utilised by several authors[36 41] to study the surface-wave properties of Sommerfeld and Harms-Goubau lines. The investigations on electromagnetic wave propagation[42] initiated by Sommerfied and Zenneck[43] and followed by Wait[44-46], Bowwkamp[47], Barlow and many others[48-66] have led to the modern concept of surface-waves and the evalution of different types of surface-wave structures which can be used as waveguides or antennas depending on the nature of surface-reactance.

The present investigations have been motivated with the object of making a theoretical study of the resonance properties of a surface-wave resonator consisting of Sommerfeld surface-wave line of radius terminated by identical plane metalic circular plates of each of radius a ($a >> d$) at both ends such that the surface-wave line forms the axis of the resonator (Fig. 1). The resonator has been developed with a view to make a systematic experimental study of the surface-wave properties such as field distribution, attenuation constant, etc. of a corrugated cylindrical metallic structure. The present study is the first step towards undertaking the more involved problem of surface wave modulated structures. It is thought worth while to give a brief résumé of some of the fundamental properties of a microwave cavity resouator[21] which will have some bearing on the study of the resonance properties of the Sommerfeld surface wave resonator.

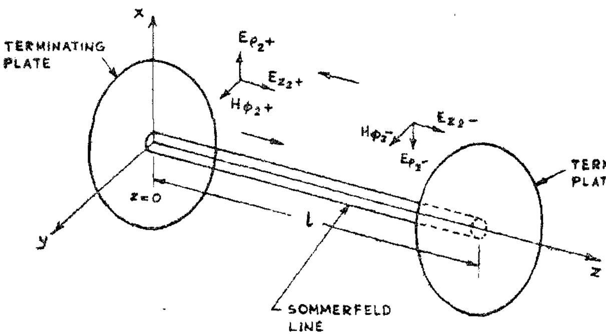

FIG. 1

Field components and coordinate system in surface wave resonator.

2. MICROWAVE RESONATOR—BRIEF RÉSUMÉ

2.1 *Equivalent Circuit*

A microwave cavity resonator, like the conventional resonant circuit, can be described as composed of an inductance—capacitance network with the help of the Lagrangian equation, which for a holonomic system is expressed as follows, in terms of the generalised co-ordinates $q_1, q_2, q_3 \ldots q_n$ and the corresponding velocities $\dot{q}_1, \dot{q}_2, \dot{q}_3 \ldots \dot{q}_n$

$$p\,(\partial L/\partial \dot{q}_k) - (\partial L/\partial q_k) = F_k\,;\; k = 1,\ 2,\ 3 \ldots n \qquad [1]$$

where, $p = d/dt$. F_k represents the dissipative forces and any external applied forces present in the system. The symbol L representing the Lagrangian is a function of q and $\dot{q}$ and is expressed in terms of the kinetic energy T and the potential energy V of the system as $L = T - V$. The charges $Q_1, Q_2, Q_3 \ldots Q_n$ and the currents $\dot{Q}_1, \dot{Q}_2, \dot{Q}_3 \ldots \dot{Q}_n$ in an electrical network can be considered as equivalent[66] to $q_1, q_2, q_3, \ldots q_n$ and $\dot{q}_1, \dot{q}_2, \dot{q}_3 \ldots \dot{q}_n$ respectively. So, the Lagrange equation for a single lossless cavity can be written as

$$p\,(\partial T/\partial \dot{Q}) + (\partial V/\partial Q) = 0 \qquad [2]$$

The kinetic and the potential energies of the cavity of volume **V** can be written as

$$T = 1/2\ \mu \sum_a k_a^2\, v,\ \dot{Q}_a^2 = 1/2 \sum_a \mathcal{L}_a\, \dot{Q}_a^2,\ V = 1/2\ \epsilon \sum_a k_a^4\, v,\ Q_a^2 = 1/2 \sum_a (Q_a^2/C_a) \qquad [3]$$

where k_a represents the wave number for the a^{th} mode of oscillation. The constants of the medium inside the cavity are represented by μ and ϵ. The equivalent lumped inductance and capacitance of the cavity are represented by L_a and C_a respectively.

From equations [2] and [3], the differential equation for a lossless cavity is obtained as follows:

$$\mathcal{L}_a \ddot{Q} + Q/C = 0 \qquad [4]$$

which represents a parallel inductance and capacitance network having resonant frequency.

$$\omega_a = \sqrt{(1/\mathcal{L}_a C_a)} \qquad [5]$$

A microwave cavity is usually coupled to external circuits by means of loops or coupling holes. The equivalent circuit of a single or a double loop coupled cavity can be similarly found with the help of Lagrange's equation.

2.2 *Resonant Frequencies of aCavity*

Let us consider the case of an ideal right circular cylindrical cavity having infinitely conducting walls and end-plates and enclosing completely a lossless dielectric. Natural electromagnetic oscillations once started in such a cavity will persist indefinitely and would be subject to Maxwell's equations expressed as follows in m.k.s. rationalised units.

$$\nabla \times \vec{E} = -\mu\,(\partial \vec{H}/\partial t)\,; \quad \nabla \times \vec{H} = \epsilon\,(\partial \vec{E}/\partial t)\,; \quad \nabla \cdot \vec{H} = 0\,; \quad \nabla \cdot \vec{E} = 0 \qquad [6]$$

inside the volume of the cavity. The following boundary conditions should be satisfied.

$$\vec{n} \,.\, \vec{H} = 0\,; \quad n \times E = 0. \qquad [7]$$

at the inside surface of the cavity. The symbols have their usual significance.

Let us assume that an oscilating electromagnetic field whose $\vec{E}$ and $\vec{H}$ components are given by the following equations has been set up inside the cavity

$$\vec{E} = \frac{1}{\sqrt{\epsilon}}\,\vec{e}\sin\left(\frac{k}{\sqrt{\epsilon\mu}}\,t + \phi\right); \quad \vec{H} = \frac{1}{\sqrt{\mu}}\,\vec{h}\cos\left(\frac{k}{\sqrt{\epsilon\mu}}\,t + \phi\right) \qquad [8]$$

where, the electric and the magnetic field configurations are given by the mode vectors $\vec{e}$ and $\vec{h}$ which are vector functions of position only. k and ϕ are constants.

The field satisfies Maxwell's equations

if

$$\nabla \times \vec{h} = k\,\vec{e}\,; \quad \nabla \times \vec{e} = k\,\vec{h} \qquad [9]$$

within the volume of the cavity and

$$n.\vec{h} = n \times \vec{e} = 0 \qquad [10]$$

at the boundary wall of the cavity.

These equations when solved show that any given cavity can sustain an infinite number of modes of oscillation having eigen frequencies $k_1 | 2\pi\sqrt{\epsilon\mu}$, $k_2 | 2\pi\sqrt{\epsilon\mu}$. . , , , . , $k_n |\ 2\pi\sqrt{\epsilon\mu}$ with eigeuvalues k_1, k_2, k_3 . . . k_n.

The resonant frequencies of a cavity depend on the manner in which the cavity is excited. Broadly, two general classifications are made, namely transverse electric (H), having the electric field transverse to the axis, and transverse magnetic (E) having the magnetic field transverse to the axis. The resonant frequencies of a cavity whether it is excited in the H or E mode is given by

$$f_{l,m,n} = \sqrt{[(cn/2L)^2 + (f_o)^2_{lm}]} \qquad [11]$$

where,

L = Length of the cavity

l = number of full period variation of E_r along the angular θ cordinate.

m = number of half period variations of E_θ along the radial r cordinate.

n = number of half period variations of E_r along the axial or z cordinate.

c = velocity of electromagnetic waves in free space.

The cut-off frequencies $(f_o)_{l,m}$ are given by

$$(f_o)_{l,m} = (c\,k'_{l,m}/2\pi a) \text{ for } H_{l,m} \text{ mode}$$

and

$$(f_0)_{l,m} = (ck_{l,m}/2\pi a) \text{ for } E_{l,m} \text{ mode} \quad [12]$$

Where, a represents the radius of the cavity. The quantities $k'_{l,m}$ and $k_{l,m}$ are the roots of the following equations :

$$J'_l(k'_{l,m}) = 0; \quad \text{for } H_{l,m} \text{ modes}; \quad J_l(k_{l,m}) = 0; \quad \text{for } E_{l,m} \text{ modes} \quad [13]$$

There will be a distinct resonance for each combination of l, m, n, which is referred to as a resonant mode of the system. Theoretically, a triple infinity of modes for each class is possible, but only the several lowest modes are of practical interest.

2.3 *Mode Degeneracy*

In experimental work on cavity resonators, generally the H_{01} mode is used, whereas, for surface-wave work, the mode of primary interest is E_0, since all the other modes have very high attenuation. From eqn. [13], $k'_{01} = k_{11} = 3.832$ as $J'_0(x) = -J_1(x)$. So, the resonant frequencies eqn. [11] f_{01n} and f_{11n} for the H_{01n} and E_{11n} modes respectively are identically the same. This is an important case of double degeneracy. When a cavity is excited in the desired mode H_{01}, the other mode E_{11} which is the companion mode invariably appears.

2.4. *Field Components*

The Field components for these two modes are given by the following expression.

H_{011} mode :

$$E_\rho = E_z = H_\theta = 0; \qquad E_\theta = J_0'(k_1\rho)\sin k_3 z; \qquad H_\rho = (k_3/k)\,J_0'(k_1\rho)\cos k_3 z;$$

$$H_z = (k_1/k)\,J_0(k_1\rho)\sin k_3 z \quad [14]$$

E_{111} mode :

$$E_\rho = -(k_3/k)\ J_1'\ (k_1\rho)\ \cos\theta \sin k_3 z$$
$$E_\theta = (k_3/k)\ [J_1\ (k_1\rho)/k_1\rho] \sin\theta \sin k_3 z$$
$$E_z = (k_1/k)\ J_1\ (k_1\rho) \cos\theta \cos k_3 z$$
$$H_\rho = -[J_1'\ (k_1\rho)/k_1\rho] \sin\theta \cos k_3 z$$
$$H_\theta = -J_1'\ (k_1\rho) \cos\theta \cos k_3 z$$
$$H_z = 0 \qquad [15]$$

2.5 *H_{01} mode*

In spite of the double degeneracy, the H_{01} mode is invariably used for cavity excitation for the following reasons :

(i) The field distribution of the H_{01} mode shows that the wall currents flow in circles perpendicular to the axis of the cylinder and hence no current can cross the contact surface of an adjustable plunger used to resonate the cavity to the excitation frequency. So, a non-contact type of plunger can be used for turning a cavity, This avoids any fluctuating loss taking place at the surface of contact with the walls of the cavity.

(ii) When a cavity is excited in any desired mode, a number of crossing and interfering modes appear depending on the volume of the cavity and the wavelength of excitation. For a cavity of of volume V and wavelength λ of excitation, the number of modes N that can appear is given approximately by the following relation[35]

$$N \simeq (8\pi/3)\ (v/\lambda^3) \qquad [16]$$

But the Q of the cavity is given by the following relation :

$$Q = 2\pi f (\bar{W}/P) \qquad [17]$$

As the mean energy $\bar{W}$ stored in the resonator is given by a volume integral, whereas, the rate of dissipation of energy P is given by a surface integral, it is evident from eqn. [17] that, in order to obtain high Q, the volume of the cavity must be large. This is undesirable as it will make a larger number of spurious modes appear in a cavity. It can also be shown that H_{01} is the mode which gives the highest possible Q with minimum volume of the cavity.

2.6 *Interaction of H_{01} and E_{11} modes*

In the absence of perturbation, it can be shown[67], that there is very little interaction between the two modes H_{01} and E_{11} so that the two modes can co-exist without interacting with each other. It has been shown by Wien[34] that the interaction between the free vibrations of two resonators depend on the coupling coefficient k' and the ratio of the resonance frequencies of the two resonators. We shall calculate the coupling coefficient k' of the two companion modes under normal condition with the help of the field theory.

The coupling coefficient between the two modes is defined broadly as follows :

$$k' = (W_{1,2})/\sqrt{(W_1 W_2)} \qquad [18]$$

Where, W_1 and W_2 represent the energies stored in the H_{01} and E_{11} modes respectively and $W_{1,2} = W_{2,1}$ represents the mutual energy, or the energy interchanged between the two modes. The total energy of the two modes in the resonator is given by the following relations[68] in m.k.s. rationalised units.

$$W = \frac{1}{2\mu} \int_v (\vec{B}_1 + \vec{B}_2) \cdot (\vec{B}_1 + \vec{B}_2)\, dv$$

$$= \frac{\mu}{2} \left[\int_v H_1^2\, dv + 2 \int_v \vec{H}_1 \cdot \vec{H}_2\, dv + \int_a H_2^2\, dv \right] \qquad [19]$$

The first and the last terms of the right hand side in eqn. [19] give the energy stored in the desired and the companion modes respectively. The second term gives the energy used in bringing the two modes into interaction, or, in other words, the mutual energy between the two coupled modes.

Hence

$$W_{1,2} = \mu \int_v H_1 \cdot H_2\, dv \qquad [20]$$

For a cylindrical cavity resonator having radius a and length L, the expression equation [20] for the mutual energy becomes

$$W_{1,2} = \mu \int_0^a \int_0^{2\pi} \int_0^L \vec{H}_1 \cdot \vec{H}_2\, \rho\, d\rho\, d\theta\, dz$$

The expressions for the components H_1 and H_2 for the two modes H_{01} and E_{11} are

$$|\vec{H}_1| = \{(k_3^2/k^2)\,[J_0'(k_1\rho)]^2 \cos^2 k_3 z + (k_1^2/k^2)\, J_0^2(k_1\rho) \sin^2 k_3 z\}^{1/2} \qquad [21]$$

$$|\vec{H}_2| = \{(J_1^2(k_1\rho)/k_1^2\rho^2) \sin^2\theta \cos^2 k_3 z + [J_1'(k_1\rho)]^2 \cos^2\theta \cos^2 k_3 z\}^{1/2} \qquad [22]$$

Substituting eqns. [21] and [22] in [20] and integrating and making some approximations, the following expression for the mutual energy $W_{1,2}$ is obtained[21]

$$W_{1,2} \simeq -\frac{2\pi \mu k_1^3}{4\,k\,k_3}\left[\frac{L}{2}-\frac{\sin 2k_3 L}{4\,k_3}\right] \times \frac{128\,k_1}{9\pi}\left[\frac{a^4}{8}-\frac{a^3}{4k_1}\cos 2k_1 a + \frac{3}{8k_1^2}a^2 \sin 2k_1 a + \frac{3}{8k_1^3}a \cos 2k_1 a - \frac{3}{16k_1^4}\sin 2k_1 a\right] \quad [23]$$

The maximum energies stored in the electric field of the resonator operating in the H_{01} and E_{11} modes respectively are given by the following expressions[21]

$$W_1 = \frac{\epsilon}{2}\int_0^a\int_0^{2\pi}\int_0^L \rho\,[J_0'(k_1\rho)]^2 \sin^2 k_3 z\; d\rho\, d\theta\, dz$$

$$= +\frac{\epsilon\,\pi\,a^2\,L}{4} J_0^2(k_{01}) \quad [24]$$

$$W_2 = \frac{\epsilon}{2}\left[\frac{k_3^2}{k^2}\int_0^a\int_0^{2\pi}\int_0^L \rho\,[J_1'(k_1\rho)]^2 \cos^2\theta \sin^2 k_3 z\; d\rho\, d\theta\, dz\right.$$

$$+\frac{k_3^2}{k^4}\int_0^a\int_0^{2\pi}\int_0^L \rho\,\frac{J_1^2(k_1\rho)}{k_1^2\rho^2}\sin^2\theta \sin^2 k_3 z\; d\rho\, d\theta\, dz$$

$$\left.+\frac{k_1^2}{k^2}\int_0^a\int_0^{2\pi}\int_0^L \rho\,J_1^2(k_1\rho)\cos^2\theta\cos^2 k_3 z\; d\rho\, d\theta\, dz\right]$$

$$= \frac{\epsilon L}{4}\left[\frac{\pi\,a^2}{2}-\frac{\pi\,k_3^2}{k^2\,k_1^2}+\frac{\pi k_3^2}{k^4 k_1}\right] J_0^2(k_{11}) \quad [25]$$

The coupling coefficient k' between the two modes is found from equations [18], [23-25] as follows :

$$k' = -256\,\mu\,k_1^4\left[\frac{L}{2}-\frac{\sin 2k_3 L}{4k_3}\right]$$

$$\times\left\{\frac{[a^4/8-(1/4k_1)a^3\cos 2k_1 a+(3/8k_1^2)a^2\sin 2k_1 a+(3/8k_1^3)a\cos 2k_1 a-(3/16k_1^4)\sin 2k_1 a]}{9\,a\,\epsilon\,L\,k\,k_3\,J_0(k_{01})\,J_0(k_{11})\sqrt{\{\pi(\pi a^2/2-\pi k_3^2/k^4k_1^2+\pi k_3^2/k^4k_1)\}}}\right\} \quad [26]$$

2.7. *Coupled Frequencies :*

When the two modes H_{01n} and E_{11n} coexist, the cavity will oscillate in two different resonant frequencies, one slightly above and the other slightly below the uncoupled resonant frequencies $f_{01n}=f_{11n}=f_0$ of the two individual modes. These coupled frequencies depend on the coupling coefficient k' and are given by

$$f_{c1}=f_0/\sqrt{(1+k')}\ ; \quad f_{c2}=f_0/\sqrt{(1-k')} \qquad [27]$$

If the coupling is loose these two coupled resonance frequencies may be quite close together and the effect is that the the cavity will oscillate over a band of frequencies Δf given by the difference of the two frequencies $f_{c2}-f_{c1}$. This can be reduced to $\Delta f=k' f_0/\sqrt{[1-(k')^2]}$, provided k' is small.

3. Sommerfeld Surface Waveguide

Sommerfeld surface waveguide (see Fig. 2) consists of an infinitely long straight metallic wire of circular cross-section having finite conductivity imbedded in a dielectric of infinite extent and excited by E_0 wave. Treating this as a boundary value problem and matching the fields at the surface $(\rho=d)$, the following characteristic equation for the E_0 wave is obtained.

$$\frac{k_1^2}{\mu_1 u}\,\frac{J_1(u)}{J_0(u)}=\frac{k_2^2}{\mu_2 v}\,\frac{H_1^{(1)}(v)}{H_0^{(1)}(v)} \qquad [28]$$

where

$$u=\gamma_1 d\,; \quad v=\gamma_2 d\,; \quad \gamma_1^2=k_1^2-h^2\,; \quad \gamma_2^2=k_2^2-h^2\,;$$

$$k_1^2=\omega^2\mu_1\epsilon_1+i\,\omega\,\sigma_1\,\mu_1\,; \quad k_2^2=\omega^2\mu_2\epsilon_2.$$

h = axial propagation constant.

The following cases are of interest :

For $\sigma_1=\infty$ eqn. [28] reduces to

$$\frac{H_0^{(1)}(v)}{H_1^{(1)}(v)}=0 \qquad [29]$$

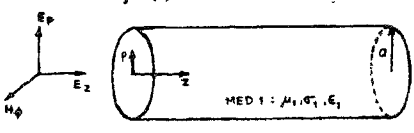

Fig. 2

Sommerfeld surfaoe wave guide: Coordinats system

If ν is small *i.e.* for a very thin conductor

$$H_0^{(1)}(v) \cong \frac{2i}{\pi} \ln \frac{\gamma v}{2i}; \qquad H_1^{(1)}(v) \cong \frac{2i}{\pi v} \tag{30}$$

The principal branch of $H_n^{(1)}(v)$ vanishes at all infinite points of the positive imaginary half-plane. The roots of only the principal branch of the multi-valued Hankel function are of interest, but the principal branches of $H_0^{(1)}(v)$ and $H_1^{(1)}(v)$ have no roots so,

$$\frac{H_0^{(1)}(v)}{H_1^{(1)}(v)} = -v \ln \frac{\gamma v}{2i} = 0 \tag{31}$$

where, $\gamma = 1.781$

possesses the only solution $v = 0$ *i.e.* $h = k_2$.

This means that when a cylindrical conductor of infinite conductivity imbedded in a dielectric medium is excited by the fundamental E wave, the field is propagated in the axial direction with a velocity which is solely determined by the characteristics of the external medium. If the conductor is imbedded in free space, the field is propagated along the cylinder without attenuation and with phase velocity equal to the free space velocity c.

If σ is large but not equal to infinity, $h \neq k_2$ but the difference $h \sim k_2$ for E_0 wave is very small. So, v is small but $|k_1| >> k_2$ since $k_2 \cong |h|$ and $u \simeq k_1 d >> 1$. Therefore, representing $J_0(u)$ and $J_1(u)$ by asymptotic expansions and $H_0^{(1)}(v)$ and $H_1^{(1)}(v)$ by small argument approximations, the following equation is obtained from equation [28].

$$v^2 \ln \frac{\gamma v}{2i} = i \frac{\mu_1}{\mu_2} \frac{k_2^2 d}{k_1} \tag{32}$$

$$\text{which reduces to } \xi \ln \xi = \eta \tag{33}$$

where,

$$\xi = \left(\frac{\gamma v}{2i}\right)^2$$

and

$$\eta = -\frac{i\gamma^2}{2} \frac{\mu_1}{\mu_2} \frac{k_2^2 d}{k_1}$$

Equation [33] is the basic Sommerfeld equation which when solved gives the characteristics, such as the propagation factor in the axial direction, attenuation constant, phase constant and phase velocity of the Sommerfeld surface wave

Sommerfeld's analysis leads to the following conclusions.

(i) A solid cylindrical conductor of circular cross-section can support an infinite number of propagating modes. The amplitudes of these modes are coefficients involved, in the field components. These coefficients are determined by the nature of the source,

(ii) of all the modes, only the Eo mode possesess relatively low attenuation. All the other symmetric E and symmetric H and all the asymmetric modes suffer rapid attenuation within a very short distance from the source, even at very low frequencies, and as such are of no practical interest.

(iii) In order that the Eo mode may be bound to the surface of the conductor. the conductivity of the conductor can be high but must be finite.

(iv) The electric lines of forces outside the conductor are almost perpendicular but not exactly to the surface of the conductor. The wave front in the external medium is slightly tilted forward in the direction of propagation. The Poynting vector being directed towards the conductor, the energy flow into the conductor accounts for the Joule heat losses.

(v) The phase velocity of the wave is slightly less than the free space velocity for conductors having high conductivity and radius of curvature greater than the skin depth.

(vi) The radial field decay in the region outside the conductor is governed by the Hankel function $H_1^{(1)}(\gamma_2\rho)$. The field extension in the radial direction is large and can be reduced by decreasing the conductivity and radius of the conductor and by increasing the frequency of excitation.

(vii) Since the wave is guided along the conductor, its attenuation in the x-direction is produced solely due to definite conductivity of the wire. In the conceptual limit of infinite conductivity, the E wave passes to a T-wave and decreases in amplitude in the radial direction as $1/\rho$

(viii) An ohmic loss on the surface of the guide is essential for the Sommerfeld surface wave to be supported by the conductor.

Sommerfeld surface waveguide is not of much use in practice, as the *Eo* wave supported by the structure is not tightly bound to the surface *i.e.* the extent of the field in the radial direction is inconveniently large. As such any discontinuity in the path of the wave such as a bend or kink in the wire produces considerable loss of power by radiation. The inherent short-coming of the Sommerfeld guide is that its ohmic loss is essential to its operation in contrast with the conventional waveguide for which the ohmic loss is only incidental. This difficulty has been obviated by Goubau[56] by coating the wire with a thin layer of dielectric. The dielectric coating loads the surface in such a way that the *Eo* wave is guided by the structure and the extent of the field spread in the radial direction is comparatively much smaller even in the case of the conductor having infinite conductivity. In a surface wave structure of this type, the ohmic loss is only incidental and the extent of the radial field spread is controlled solely by the thickness and dielectric constant of the dielectric film. The characteristics of the dielectric coated structure was first studied by Harms[69] and then more exhaustively by Goubau[56].

4. HARMS-GOUBAU SURFACE WAVEGUIDE

Harms[69] made a theoretical study of the problem of wave propagation along a cylindrical wire of radius d coated with a dielectric of thickness $(b-d)$ and dielectric constant E_1 (see Fig. 3). Goubau[56-59] made a detailed theoretical and experimental study of the problem and evaluated its practical usefulness as a transmission line for microwaves. Treating this as a boundary-value problem and using the impedance matching technique at $\rho=b$ for *Eo* wave, the following characteristic equation isobtained.

$$\frac{1}{\epsilon_1}\frac{\gamma_1 J_0(\gamma_1 b)\, Y_0(\gamma_1 d)-J_0(\gamma_1 d)\, J_0(\gamma_1 b)}{J_1(\gamma_1 b)\, Y_0(\gamma_1 d)-J_0(\gamma_1 d)\, Y_1(\gamma_1 b)} = -\frac{\gamma_2}{\epsilon_2}\frac{H_0^{(1)}(i\,\gamma_2 b)}{H_1^{(1)}(i\,\gamma_2 b)} \qquad [34]$$

which yields the following equation[56] after some simplication.

$$(\mu_2/\epsilon_2)^{1/2}\,(\gamma_2^2/k_2)\; b\; ln\, 0{,}89\,\gamma_2 b = -(\mu_1/\epsilon_1)^{1/2}\,(\gamma_1/k_1)\; b\; ln\,(b/a) \qquad [35]$$

where $k_1^2=\omega^2\mu_1\epsilon_0$ and $k_2^2=\omega^2\mu_0\epsilon_2$.

In order that the radial impedances at $\rho=b$ be continuous, it is necessary that the axial propagation constant in the two media be the same, *i.e.*

$$\sqrt{(k_1^2-\gamma_1^2)}=\sqrt{(k_2^2+\gamma_2^2)} \qquad [36]$$

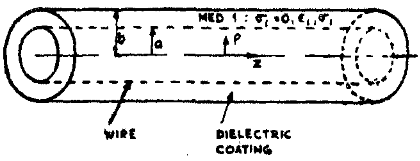

FIG. 3

Harms—Goubau surface wave guide

The radial constant γ_2 and hence the phase velocity $v_p = \omega\sqrt{(k_2^2 + \gamma_1^2)}$ of the wave can be determined from equations [35] and [36]. Since, γ_2 is positive and real $v_p < \omega/k_2$ which is the free space velocity in the case of the medium 2 being air.

The division of power between the two media 1 and 2 is calculated from the Goubau relation

$$\frac{P_1}{P_2} = -\frac{\epsilon_2}{\epsilon_1} \frac{ln\,(b/d)}{ln\,0.89\,\gamma_2\,b + 0.5} \qquad [37]$$

where, P_2 represents the power of the surface wave which is contained in the external medium. This equation is used to determine the thickness of the dielectric coating $(b-d)$ required for a dielectric material to constrain a certain percentage of power of the surface wave within a specified distance from the surface of the structure. The above results are derived on the basis of no loss. The effect of dissipation is determined by using the perturbation method *i.e.* by assuming that the field distribution in an equiphase plane is the same as that in the case when there is no loss. The conclusions drawn from the foregoing analysis are:

(i) The field structure of Harms-Goubau guide is the same as that of Sommerfeld guide.

(ii) The extent of the field spread in the radial direction decreases with increasing dielectric constant and thickness of coating *i.e.* the radial extension of the field can be controlled by modifying the surface of Sommerfeld guide.

(iii) In the case of Sommerfeld guide, if the couductivity is increased indefinitely, the radial extension of the field would increase in such a way that the power carried by a wave of finite amplitude would become infinite which is physically inadmissible. But in the case of Harms-Goubau guide, the wave will still remain a guided wave with a limited radial extension of the field, even in the case when the conductivity of the wave is increased indefinitely, The field is only slightly affected,

(iv) Compared to the Sommerfeld guide the Harms-Goubau guide possesses higher loss. Losses of this type of guide consists of (*a*) the ohmic loss in the conductor which is also present in the Sommerfeld guide (*b*) the loss in the dielectric film which is not present in the Sommerfeld guide (c) the loss due to the finite size of the launching device. As the radial field spread is more in Sommerfeld guide, it requires a much longer dimension of the launching device than Goubau guide. For the same dimension of the launching device, Sommerfeld line will have more loss.

(v) The phase velocity of the wave guided by the dielectric coated structure is less than the free space velocity.

(vi) As $H_0^{(1)}(i\,\gamma_2 b)$ is negative imaginary and $H_1^{(1)}(i\,\gamma_2 b)$ is negative real for positive imaginary argument, it follows that the surface impedance is negative imaginary, *i. e.* the surface impedance of this guide is purely inductive. Or, in other words, coating the wire with a dielectric amounts to loading inductively the surface of the conductor.

(vii) The axial component of the Poynting vector integrated over a plane perpendicular to the axial direction yields a finite value which leads to the physical realisability of Goubau wave.

5. Attenuation Constants

5.1 *Sommerfeld Line:*

From the relations

$$h=\alpha+j\beta\,;\quad \gamma_2=a_2-jb_2\,;\quad k_2^2=\omega_2\,\mu_0\,\epsilon_0=-(h_2+\gamma_2^2) \qquad [38]$$

and assuming that at microwave frequencies

$$k_2^2>>(a_2^2-b_2^2)\,;\quad a_2>>b_2 \qquad [39]$$

the attenuation constant of the Sommerfeld line is[53]

$$a(\text{sommerfeld})=(c/\omega)\,(a_2 b_2) \qquad [40]$$

5.2 *Harms-Goubau Line*

Assuming that there is no loss due to radiation, the total loss in Harms-Goubau line is due to the ohmic loss in the wire (α_c) and dielectric loss in the coating (α_d). The attenuation constants α_c and α_d are[56]

$$\alpha_c\simeq\frac{1}{2d}\sqrt{\frac{\epsilon_0\,\omega\,\mu_w}{2\mu_0\sigma_w}}\;\frac{1}{ln\,\gamma_2 b+0.38}\,\text{Nepers}/m \qquad [41]$$

$$\alpha_d\simeq\frac{\gamma_2^2}{2k}\;\frac{\epsilon_0}{\epsilon_d-\epsilon_0}\left(1-\frac{0.5}{ln\,\gamma_2 b+0.38}\right)\tan\delta\,\text{Neper}/m \qquad [42]$$

The radial propagation factor is obtained from

$$G(\gamma_2 b)=-(\gamma_2 b/2\,\pi)^2\,ln\,(0.89\,\gamma_2 b) \qquad [43]$$

where,

$$G(\gamma_2 b)=\frac{ln\,(b/d)}{(\epsilon_d/\epsilon_d-\epsilon_0)\,(\lambda/b)^2}$$

The attenuation constant α (Harms-Goubau line) $= \alpha_d + \alpha_c$

where,

k = free space wave propagation

constant $= \omega\,(\mu_0\,\epsilon_0)^{1/2}$

ϵ_d = dielectric constant of the dielectric coating

6. Comparative Study of the Sommerfeld and Harms Goubau Lines

A comparative study of the characteristics such as radial decay factor (γ) as a function of the radius of the Sommerfeld line and as a function of the dielectric constant for different coating thickness in the case of the Harms-Goubau line for different wavelength of excitation, ratio of the radii of the constant percentage power contour as a function of coating thickness $b-d=a$ for different wavelength of excitation and percentage reduction in phase velocity as a function of wire radius are presented in figures (4-7) respectively. Fig. 8 shows the percentage power flow for the Harms Goubau and Sommerfeld line as a function of the radial distance from the line. Fig. 9 shows a comparative study of the conduction and dielectric loss in the case of the Harms-Goubau line as a function of dielectric coating thickness in the X and K band.

7. Theory of the Surface Wave Resonator

The resonator (see Fig. 1) consists of a metallic wire of radius d terminated at both ends by large circular metallic plates each of radius $a >> d$. The length l of the wire is adjusted such that it is an integral multiple of half the guide wavelength λ_g corresponding to the mode of excitation. The resonator is open on all sides except at the two ends.

7.1 *Field Components of Resonant Waves*

The components of resonant waves, when the resonator oscillates in pure E or H modes are respectively[42].

E mode :

$$E_{zr}(\rho) = 2\,\chi \cos\theta \cos(m_z\pi/l)\, z\, J_1(\gamma_e\rho)$$

$$E_{\rho r}(\rho) = 2j\,(h\,\gamma_e/\omega^2\,\mu_0\epsilon_0)\,\chi \cos\theta \sin(m_z\pi/l)\, z\, J_1'(\gamma_e\rho)$$

$$E_{\theta r}(\rho) = -2j\,\chi\,(h/\omega^2\,\mu_0\epsilon_0)\; 1/\rho \sin\theta \sin(m_z\pi/l)\, z\, J_1(\gamma_e\rho)$$

$$H_{\rho r}(\rho) = -2j\,\chi\,(1/\omega\mu_0)\,(1/\rho) \sin\theta \cos(m_z\pi/l)\, z\, J_1(\gamma_e\rho)$$

$$H_{\theta r}(\rho) = -2j\,\chi\,(\gamma_e/\omega\mu_0) \cos\theta \cos[(m_z\,\pi)/l]\, z\, J_1'(\gamma_e\rho) \qquad [44]$$

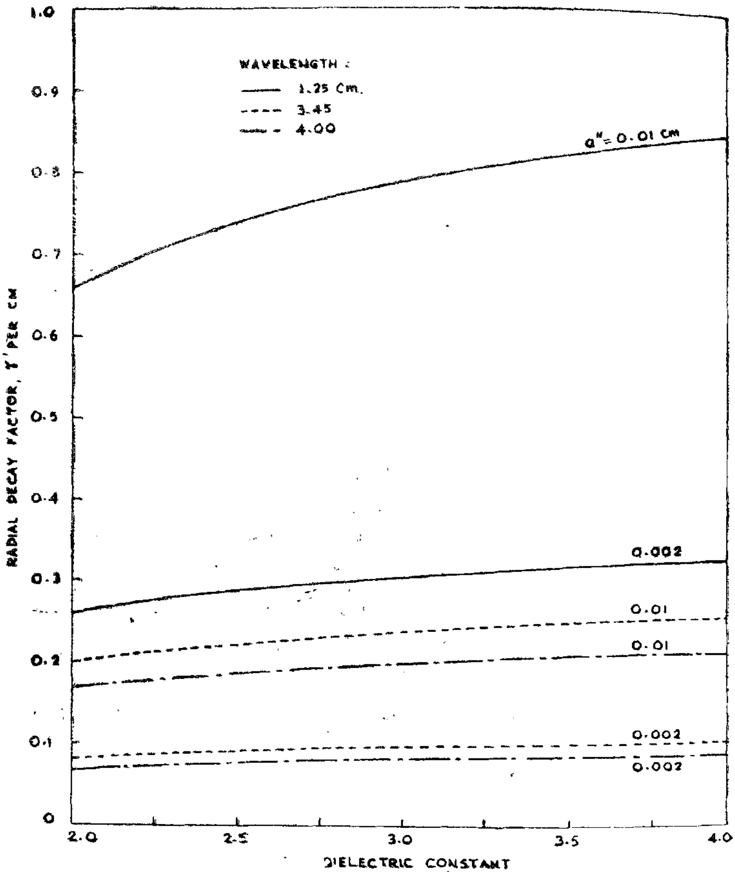

FIG. 4

Radial decay factor as a function of dielectric constant of the coating for different wavelengths Wire radius, $a=0.10$ cm.

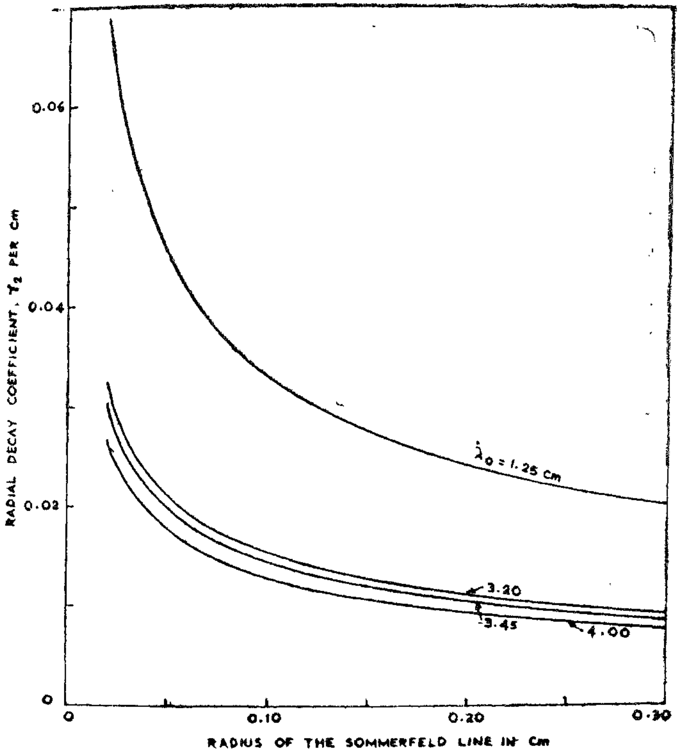

FIG. 5

Radial decay factor as a function of wire radius for different wavelengths.

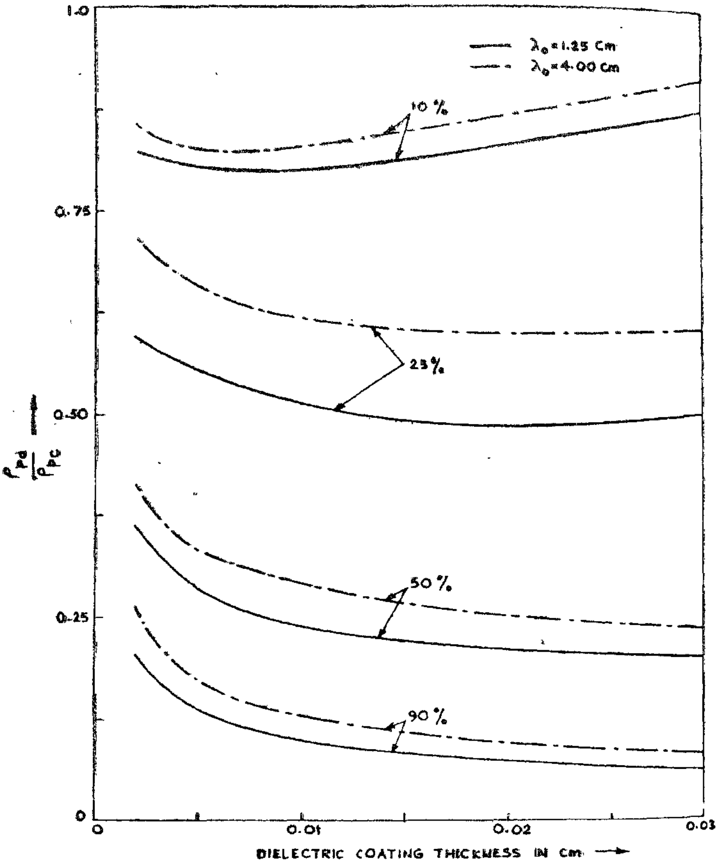

FIG. 6

Ratio of radii of the constant percentages power contour as a function of the dielectric coating thickness for different wavelengths. a=0.10 cm, ϵ=2.4.

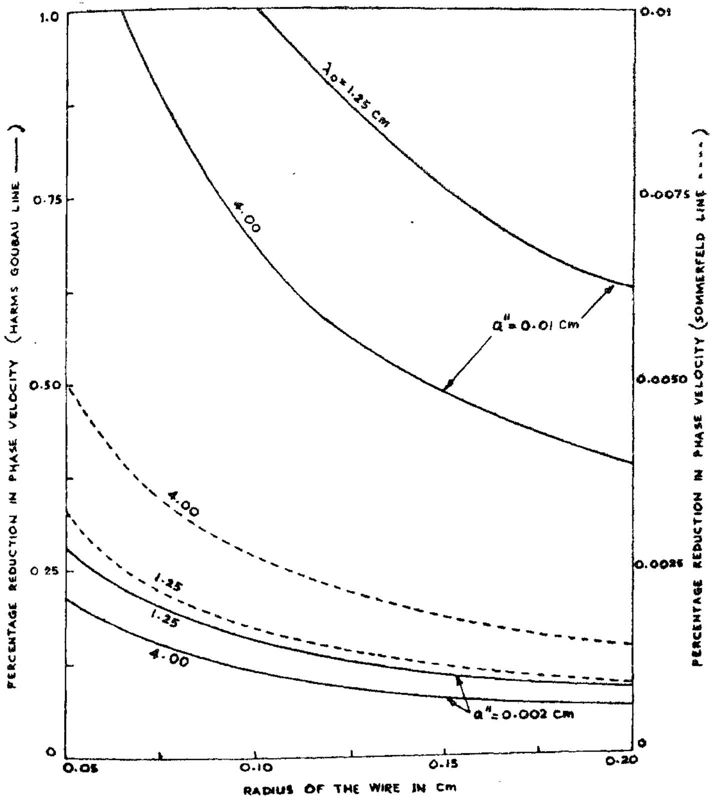

FIG. 7

Percentage reduction in phase velocity as a function of wire radius for different wavelengths. $\epsilon=2\ 4$.

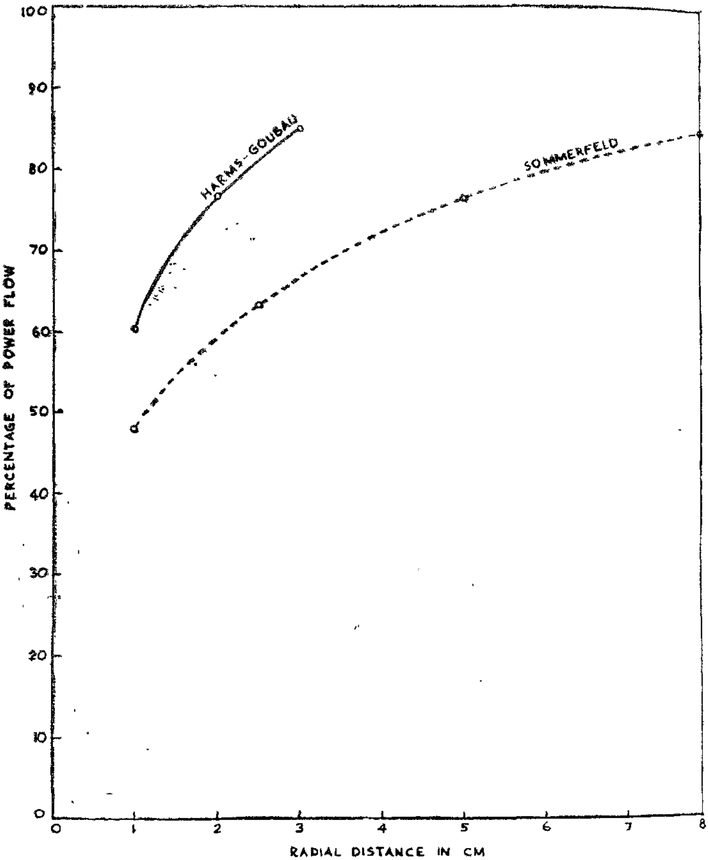

FIG. 8

Power distribufion curves for a wire of radius $a=0.08$ cm, coating thickness $a''=0.005$ cm. $\lambda_0=3.57$ cm.

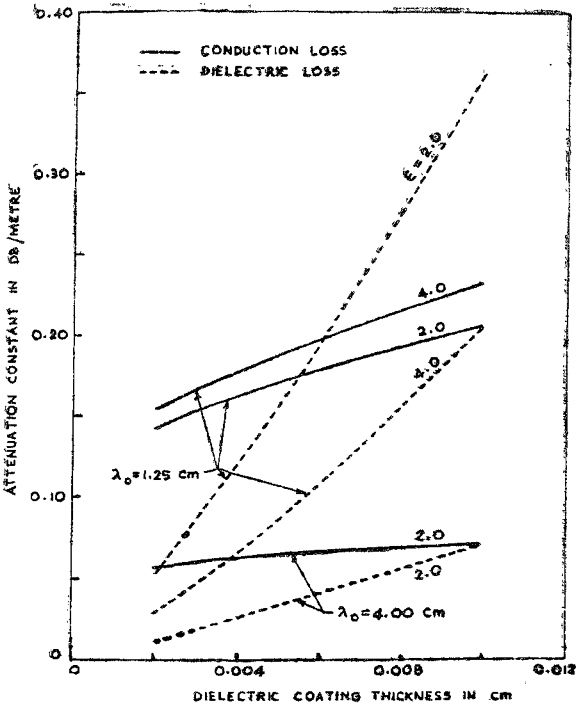

FIG. 9

Attenuation constant as a function of dielectric coating thickness for different dielectric constants and wavelengths. Wire radius $a = 0.10$ cm.

H mode :

$$H_{zr}(\rho) = -2j\psi \sin\theta \sin(m_z \pi/l)\, z\, J_1(\gamma_h \rho)$$
$$H_{\rho r}(\rho) = 2\psi\, (h\,\gamma_h/\omega^2\mu_0\epsilon_0) \sin\theta \cos(m_z \pi/l)\, z\, J_1'(\gamma_h \rho)$$
$$H_{\theta r}(\rho) = 2\psi\, (h/\omega^2\mu_0\epsilon_0)\,(1/\rho) \cos\theta \cos(m_z \pi/l)\, z\, J_1(\gamma_h \rho)$$
$$E_{\rho r}(\rho) = -2\psi\, (1/\omega\epsilon_0)\,(1/\rho) \cos\theta \sin(m_z \pi/l)\, z\, J_1(\gamma_h \rho)$$
$$E_{\theta r}(\rho) = 2\psi\, (\gamma_h/\omega\epsilon_0) \sin\theta \sin(m_z \pi/l)\, z\, J_1'(\gamma_h \rho)$$
$$E_{zr}(\rho) = 0$$

The above field components are derived on the assumption that the electromagnetic energy is contained wholly within the volume $\pi a^2 l$ of the resonator and that there is no loss of energy by radiation.

7.2 *Conditions of Resonance*

The conditions of no radiation leads to

$$H_1^{(2)}(\gamma_e d) = -H_1^{(1)}(\gamma_e d)$$

which yields

$$J_1(\gamma_e d) = 0 \qquad [46]$$

as the condition of resonance when the resonator is oscillating in a pure E-mode. The condition of resonance when the resonator is oscillating in a pure H-mode and the energy is completely enclosed within the volume of the resonator is

$$H_1^{(2)\prime}(\gamma_h d) = -H_1^{(1)\prime}(\gamma_h d)$$

which yields

$$J_1'(\gamma_h d) = 0 \qquad [47]$$

The eigen values $\gamma_h d$ which satisfy the above equation is obtained when $J_1(\gamma_h d)$ is maximum *i.e.* $\gamma_h d = 1.84,\ 8.54,\ 14.86$, etc.

7.3 *Coupled E and H modes*

In the case of a conventional type of cavity resonator enclosed on all sides by highly conducting metallic walls, no loss of energy occurs by radiation and E or H modes can exist independently, whereas, in the case of an open type resonator, due to the discontinuity which is invariably present at the edge ($\rho = a$) of the end plates, some energy will be lost by radiation. As the radiated wave in free space is a T-wave, E_ρ^e, H_ρ^e and E_ρ^h, H_ρ^h of the E and H modes respectively, must vanish inside the resonator or approach zero value at $\rho = a$. But the radial components of E and H modes of the non-radiating standing wave part of the total field within the resonator cannot independently become zero. So, it may be said that

$$E^e_\rho + E^h_\rho = 0 \quad [48]$$

or

$$H^e_\rho + H^h_\rho = 0 \quad [49]$$

at $\rho = a$, which signifies that the E and H modes are coupled.

The characteristic equation for the coupled mode obtained by imposing proper bourdary conditions on appropriate field components and utilising the no-radiation condition is

$$-\frac{j\omega\mu_0}{h\gamma a}\,\frac{y\,H_1^{(1)}(\gamma a) - H_1^{(2)}(\gamma a)}{x\,H_1^{(1)}(\gamma a) - H_1^{(2)}(\gamma a)} = -j\frac{\gamma d\,\omega\mu_0}{h}\,\frac{y\,H_1^{(1)\prime}(\gamma d) - H_1^{(2)}(\gamma d)}{x\,H_1^{(1)}(\gamma d) - H_1^{(2)}(\gamma d)} \quad [50]$$

where,

$$x = \frac{H_1^{(2)}(\gamma_e d)}{H_1^{(1)}(\gamma_e d)} \quad [51]$$

and

$$y = \frac{H_1^{(2)\prime}(\gamma_h d)}{H_1^{(1)\prime}(\gamma_h d)}$$

and

$$\gamma = \gamma_{eh}$$

It can be shown that for the resonance conditions $J_1(\gamma_e d) = 0$, $x = 1$ and similarly $y = -1$ from the definition of x and y. By using appropriate recurrence relations and $x = 1$, $y = -1$ eqn. [50] reduces to

$$\frac{J_1(\gamma a)}{\gamma a\,J_0(\gamma a) - J_1(\gamma a)} = \frac{\gamma d\,J_0(\gamma d) - J_1(\gamma d)}{J_1(\gamma d)} \quad [52]$$

which yields on differentiation with respect to γa

$$\gamma a\,J_0^2(\gamma a) - 2\,J_1(\gamma a)\,J_0(\gamma a) + \gamma a\,J_1^2(\gamma a) = 0 \quad [53]$$

since

$$\frac{J_1(\gamma a)}{\gamma a\,J_0(\gamma a) - J_1(\gamma a)} = \text{constants.}$$

Let the roots of eqn. [53] be δ_n $(n = 1, 2, 3, \ldots)$. For mth mode, the eigen values $\gamma_{eh} = \gamma$ is given by $\gamma_{eh} = \delta_m/a$ and the condition of resonance for the coupled mode is

$$\left[\frac{m_z\pi}{l} = \frac{4\pi^2}{\lambda_0^2} - \frac{\delta_m^2}{a^2}\right]^{1/2} \quad [54]$$

since, $m_2\pi/l$ is positive and real $(\delta_m/a) < (2\pi/\lambda_0)$ and $(\delta_m^2/a^2) << (4\pi^2/\lambda_0^2)$. Hence,

$$l = m_2\lambda_0/2 \quad [55]$$

which states that the resonance condition is established, when the distance between the two terminating end plates is an integral multiple of half wavelength.

In practice $d << a$ and if $\gamma d << 1$, then by making small argument approximation of $J_0(\gamma d) \cong 1$ and $J_1(\gamma d) \cong \gamma d/2$, eqn. [52] reduces to

$$J_0(\gamma_{eh} a) = 0 \qquad [36]$$

which gives the successive eigen values when resonator is oscillating under the condition that the modes are coupled.

7.4. *Q of the Resonator*

The Q of the resonator is defined as

$$Q = \omega \frac{W_E}{P} \text{ or } \omega \frac{W_M}{P} \qquad [37]$$

where, ω is the angular frequency at resonance, W_E and W_M represent the maximum energy stored in the electric and magnetic fields respectively inside the resonator and P is the total power loss inside the resonator.

The total power lost is equal to the sum of the power lost in the end plates (P_e) and power lost in the wire (P_w) and the power lost by radiation. Assuming that there is no loss of power by radiation and that the resonator is oscillating in pure E or H modes and calculating

$$W_E = \frac{\epsilon_0}{2} \int_{\theta=0}^{2\pi} \int_{\rho=d}^{a} \int_{z=0}^{l} |E|^2 \rho \, d\rho \, d\theta \, dz$$

$$W_M = \frac{\mu_0}{2} \int_{\theta}^{2\pi} \int_{\rho=}^{a} \int_{z=0}^{l} |H|^2 \rho \, d\rho \, d\theta \, dz$$

$$P_e = 2 \times 1/2 \sqrt{\frac{\pi f \mu_0}{\sigma_e}} \int_{\theta=0}^{2\pi} \int_{\rho=d}^{a} |H_{tan}|^2 \rho \, d\theta \, d\rho \qquad [58]$$

for both the end plates and

$$P_w = \sqrt{\frac{\pi f \mu_0}{\sigma_w}} \int_{\theta=0}^{2\pi} \int_{z=0}^{l} |H_{tan}|^2 \, d \, d\theta \, dz$$

the Q factor for the $E\,(Q^E)$ and H (Q^H) modes are

$$Q^E = \langle [\{2-\gamma_e^2 a^2\}\, J_1^2(\gamma_e a) - \{\gamma_e^2 a^2 - 2\gamma_e - 2\}\, J_0^2(\gamma_e a) + \{\gamma_e^2 d^2 + 2\gamma_e - 2\}\, J_0^2(\gamma_e d)]\, f^{-3/2} h^2 l \times 81.64 \times 10^{29} + [a^2 J_1^2(\gamma_e a) - a^2 J_0(\gamma_e a) J_2(\gamma_e a) - d^2 J_0^2(\gamma_e d)] f^{5/2} \times 1.57 \rangle \div \{-120.31 \times 10^{-39} [-\tfrac{1}{2}\gamma_e^2 d^2 J_0^2(\gamma_e d) - \tfrac{1}{2}\gamma_e^2 a^2 J_0^2(\gamma_e a) - (1-\tfrac{1}{2}\gamma_e^2 a^2) J_1^2(\gamma_e a)] - 52.2 \times 10^{-39}\gamma_e^2\, d\, l\, J_0^2(\gamma_e d)\} \quad [59]$$

$$Q_H = 179 \times 10^8 f^{-1} l\, [\tfrac{1}{2}\gamma_h^2 a^2 - 1)\, J_1^2(\gamma_h a) + \tfrac{1}{2}\gamma_h^2 a^2 J_0^2(\gamma_h a) - (\tfrac{1}{2}\gamma_h^2 d^2 - 1)\, J_1^2(\gamma_h d) - \tfrac{1}{2}\gamma_h^2 d^2 J_0^2(\gamma_h d)] \div \{694.22 \times 10^{22} h^2 f^{-7/2} [(\tfrac{1}{2}\gamma_h^2 a^2 - 1)\, J_1^2(\gamma_h a) + \tfrac{1}{2}\gamma_h^2 a^2 J_0^2(\gamma_h a) - (\tfrac{1}{2}\gamma_h^2 d^2 - 1)\, J_1^2(\gamma_h d) - \tfrac{1}{2}\gamma_h^2 d^2 J_0^2(\gamma_h d)] + 2\,61 \times 10^{-7} d\, l f^{1/2} [52 \times 10^{29} (h^2/d) f^{-4} - 1]\, J_1^2(\gamma_h d)\} \quad [60]$$

The Q factor for the coupled EH and HE modes are respectively

$$Q_{EH} = 81.64 \times 10^{29} f^{-3/2} h^2 l\, [\{2-\gamma^2 a^2\}\, J_1^2(\gamma a) - \{2-\gamma^2 d^2\}\, J_1^2(\gamma d) + \{\gamma^2 d^2 + 2\gamma - 2\}\, J_0^2(\gamma d) + 1.57 f^{5/2} l\, [a^2 J_1^2(\gamma a) - d^2 J_1^2(\gamma d) + d^2 J_0(\gamma d)\, J_2(\gamma d)] \div \{-120.31 \times 10^{-39} \times [-\tfrac{1}{2}\gamma^2 d^2 J_0^2(\gamma d) + (1 - \langle \gamma^2 d^2/2 \rangle)\, J_1^2(\gamma d) - (1-\tfrac{1}{2}\gamma^2 a^2)\, J_1^2(\gamma a)] - 52.2\,\gamma^2 d\, l \times 10^{-39} [J_0^2(\gamma d) + \langle J_1^2(\gamma d)/\gamma^2 d^2 \rangle - \langle 2 J_0(\gamma d)\, J_1(\gamma d)/\gamma d \rangle]\} \quad [61]$$

where, $\gamma = \gamma_{eh.}$

$$Q^{HE} = 179 \times 10^8 l f^{-1} [(\tfrac{1}{2}\gamma^2 a^2 - 1)\, J_1^2(\gamma a) - (\tfrac{1}{2}\gamma^2 d^2 - 1)\, J_1^2(\gamma d) - \tfrac{1}{2}\gamma^2 d^2 J_0^2(\gamma d)] \div 694.22 \times 10^{22} f^{-7/2} h^2 [(\tfrac{1}{2}\gamma^2 a^2 - 1)\, J_1^2(\gamma a) - (\tfrac{1}{2}\gamma^2 d^2 - 1)\, J_1^2(\gamma d) - \tfrac{1}{2}\gamma^2 d^2 J_0^2(\gamma d)] + 2.61 \times 10^{-7} l d f^{1/2} [52 \times 10^{29} f^{-4} h^2 d^{-1} - 1]\, J_1^2(\gamma d)\} \quad [62]$$

where, $\gamma = \gamma_{he}$

7.5 *Guide Wavelength*

The values of the axial propagation constant h in the case of the resonator oscillating in pure E or H mode are obtained from the condition $W_E = W_M$ at resonance from the following equations.

E mode :

$$26\times10^{29} f^{-4} h^2 [-\gamma_e^2 a^2 \{J_1^2(\gamma_e a)+J_0^2(\gamma_e a)\}$$
$$+\gamma_e^2 d^2 J_0^2(\gamma_e d)+2J_1^2(\gamma_e a)+2\gamma_e \{J_0^2(\gamma_e a)$$
$$+J_0^2(\gamma_e d)\}+2\{J_0^2(\gamma_e a)-J_0^2(\gamma_e d)\}]$$
$$=22\times10^{14} f^{-2}[J_1^2(\gamma_e a)+\tfrac{1}{2}\gamma_e^2 d^2 J_0^2(\gamma_e d)-[(\gamma_e^2 a^2)/2]$$
$$\times\{J_0^2(\gamma_e a)+J_1^2(\gamma_e a)\}]$$
$$-\tfrac{1}{2}a^2\{J_1^2(\gamma_e a)-J_0(\gamma_e a)J_2(\gamma_e a)\}-\tfrac{1}{2}d^2 J_0^2(\gamma_e d) \quad [63]$$

H *mode* :

$$26\times10^{29} f^{-4} h^2 [-\gamma_h^2 d^2 J_1^2(\gamma_h d)+2\{J_1^2(\gamma_h d)$$
$$+J_0^2(\gamma_h d)\}+\gamma_h^2 a^2\{J_1^2(\gamma_h a)+J_0^2(\gamma_h a)\}-2\{J_1^2(\gamma_h a)+J_0^2(\gamma_h a)\}]$$
$$=22\times10^{14} f^{-2}[\gamma_h^2 a^2/2)\{J_1^2(\gamma_h a)+J_0^2(\gamma_h a)\}$$
$$-(\gamma_h^2 d^2/2)\{J_1^2(\gamma_h d)+J_0^2\ \gamma_h d)\}-\{J_1^2(\gamma_h a)$$
$$-J_1^2(\gamma_h d)\}]-\tfrac{1}{2}d^2\{J_1^2(\gamma_h d)-J_0^2(\gamma_h d)$$
$$-J_0(\gamma_h d)J_2(\gamma_h d)\}+\tfrac{1}{2}a^2\{J_1^2(\gamma_h a)-J_0(\gamma_h a)$$
$$J_2(\gamma_h a)\}-(\gamma_h^2/2)\{J_1^2(\gamma_h d)-J_1^2(\gamma_h a)-J_0^2(\gamma_h a)$$
$$+J_0^2(\gamma_h d)\}-(1/\gamma_h^2\{J_0^2(\gamma_h a)-J_0^2(\gamma_h d)\} \quad [64]$$

The total propagation constants h for the coupled EH and HE modes is obtained from eqn. [55] and [56] respectively by replacing γ_e by γ_{eh} and γ_h by γ_{he} in equations [53] and [54] respectively and using the resonance condition $J_0(\gamma_{eh} a)=0$.

EH Mode :

$$26\times10^{19} f^{-4} h^2 [-\gamma^2 a^2 J_1^2(\gamma a)+\gamma^2 d^2\{J_1^2(\gamma d)+J_0^2(\gamma d)\}$$
$$+2\{J_1^2(\gamma a)-J_1^2(\gamma d)\}+2\gamma J_0^2(\gamma d)-2J_0^2(\gamma d)]$$
$$=-22\times10^{11} f^{-2}[\{J_1^2(\gamma d)-J_1^2(\gamma a)\}-\gamma^2 d^2\{J_1^2(\gamma d)+\tfrac{1}{2}J_0^2(\gamma d)\}$$
$$+(\gamma^2 a^2/2)J_1^2(\gamma a)]-\tfrac{1}{2}a^2 J_1^2(\gamma a)+\tfrac{1}{2}d^2\{J_1^2(\gamma d)$$
$$-J_0(\gamma d)J_2(\gamma d)\} \quad [65]$$

where, $\gamma=\gamma_{eh}$

HE Mode :

$$26 \times 10^{29} f^{-4} h^2 [- \gamma^2 d^2 J_1^2(\gamma d) + 2 \{J_2^2(\gamma d) + J_0^2(\gamma d)\} + \gamma^2 a^2 J_1^2(\gamma a) - 2 J_1^2(\gamma a)]$$
$$= 22 \times 10^{14} f^{-2} [(\gamma^2 a^2/2) J_1^2(\gamma a) - (\gamma^2 d^2/2) \{J_1^2(\gamma d) + J_0^2(\gamma d)\} - \{J_1^2(\gamma a) - J_1^2(\gamma d)\}] - \tfrac{1}{2} d^2 \{J_1^2(\gamma d) - J_0^2(\gamma d) - J_0(\gamma d) J_2(\gamma d)\} + \tfrac{1}{2} a^2 J_1^2(\gamma a) - (\gamma^2/2) \{J_1^2(\gamma d) - J_1^2(\gamma a) + J_0^2(\gamma d)\} + (1/\gamma^2) J_0^2(\gamma d) \quad [66]$$

where, $\gamma = \gamma_{he}$

It is evident that the guide wave length λ_g determined from h in each case is a function of d, a, and f_0.

When γ and the argument of the Bessel functions are large equations [59] to [66] reduce respectively to

$$Q^E = \{[-(2\gamma a/\pi) \langle \cos^2(\gamma a - 3\pi/4) + \cos^2(\gamma a - \pi/4)\rangle + 2\gamma J_0^2(\gamma d)] f^{-3/2} h^2 l \times 81.64 \times 10^{29} + [(2a/\pi\gamma) \langle \cos^2(\gamma a - 3\pi/4) + \cos^2(\gamma a - \gamma/4)\rangle - (4/\pi\gamma^2) \cos(\gamma a - 3\pi/4) \cos(\gamma a - \pi/4) - d^2 J_0^2(\gamma d)] f^{5/2} l \times 1.57\} \div \{120.31 \times 10^{-39} [\tfrac{1}{2} \gamma^2 d^2 J_0^2(\gamma d) + (\gamma a/\pi) \cos 2\gamma a] - 52.2 \times 10^{-39} \gamma^2 d l J_0^2(\gamma d)\} \quad [67]$$

where, $\gamma = \gamma_e$

$$Q^H = 179 \times 10^8 f^{-1} l [(\gamma a/\pi) \{\cos^2 \langle \gamma a - (3\pi/4)\rangle + \cos^2 \langle \gamma a - (\pi/4)\rangle\} - (\tfrac{1}{2} \gamma^2 d^2 - 1) J_1^2(\gamma d) - \tfrac{1}{2} \gamma^2 d^2 J_0^2(\gamma d)] \div \{694.22 \times 10^{22} h^2 f^{-7/2} [(\gamma a/\pi) \langle \cos^2(\gamma a - 3\pi/4) + \cos^2(\gamma a - \pi/4)\rangle - (\tfrac{1}{2} \gamma^2 d^2 - 1) J_1^2(\gamma d) - \tfrac{1}{2} \gamma^2 d^2 J_0^2(\gamma d)] + 2.61 \times 10^{-7} l d \sqrt{(f)} [52 \times 10^{19} (h^2/d) f^{-4} - 1] \times J_1^2(\gamma d)\} \quad [68]$$

where $\gamma = \gamma_h$

$$Q^{EH} = 81.64 \times 10^{29} f^{-3/2} h^2 l [\{2 - \gamma^2 a^2\} (2/\pi\gamma a) \cos^2(\gamma a - 3\pi/4) - 2 J_1^2(\gamma d) + (2\gamma - 2) J_0^2(\gamma d)] + 1.57 f^{5/2} l \langle (2a/\pi\gamma) \cos^2(\gamma a - 3\pi/4) - d^2 \{J_1^2(\gamma d) - [2 J_1(\gamma d) J_0(\gamma d)/(\gamma d)] + J_0^2(\gamma d)\}\rangle \div \{120.31 \times 10^{-39} [J_1^2(\gamma d) - (2/\pi\gamma a)(1 - \tfrac{1}{2}\gamma^2 a^2) \times \cos^2(\gamma a - 3\pi/4)] - 52.2 \gamma^2 d l \times 10^{-39} \langle J_0^2(\gamma d) + J_1^2(\gamma d)/(\gamma^2 d^2) - 2 J_0(\gamma d) J_1(\gamma d)/(\gamma d)\rangle\} \quad [69]$$

where, $\gamma = \gamma_{eh}$

$$Q^{HE} = 179 \times 10^8 \, l \, f^{-1} \left[\left(\tfrac{1}{2} \gamma^2 a^2 - 1\right) (2/\pi \gamma a) \cos^2 \left\{ \gamma a - (3\pi/4) + J_1^2 (\gamma d) \right] \right.$$
$$\div \left\{ 694.22 \times 10^{22} h^2 f^{-7/2} \left[\left(\tfrac{1}{2} \gamma^2 a^2 - 1\right) (2/\pi \gamma a) \times \cos^2 (\gamma a - 3\pi/4) \right.\right.$$
$$\left. + J_1^2 (\gamma d) \right] + 2.61 \times 10^{-7} \, l \, d \, f^{1/2}$$
$$\left. \times [52 \times 10^{29} h^2 f^{-4} d^{-1} - 1] \, J_1^2 (\gamma d) \right\} \qquad [70]$$

where, $\gamma = \gamma_{he}$

The variation of λ_g and Q^E with respect to the frequency of excitation and the variation of Q^E with respect to the length of the resonator are shown in figures 10 and 11 respectively. The following values for the constants have been used in computing the results

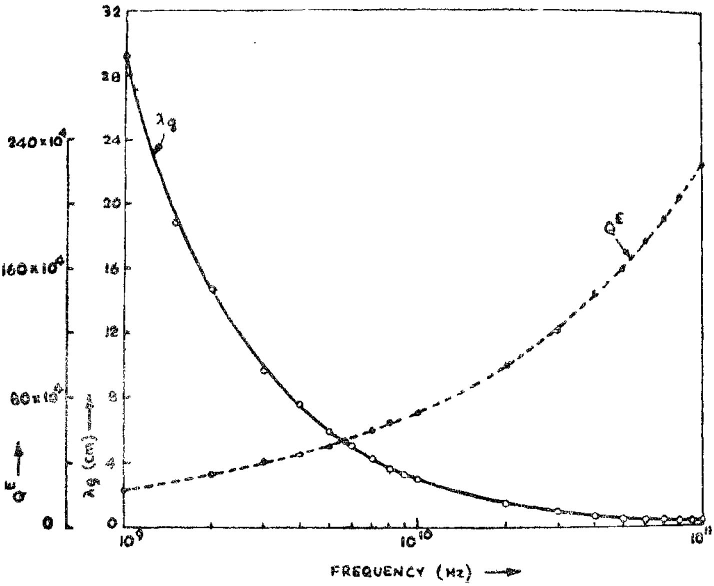

FIG. 10

Variation of guide wavelength and Q factor of E mode with frequency of excitation of the resonator

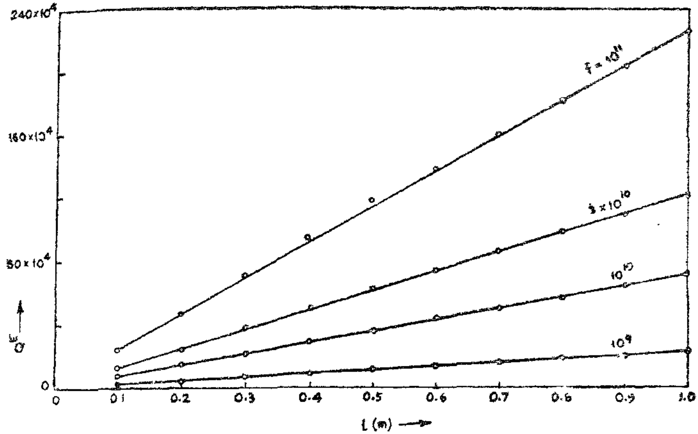

FIG. 11

Variation of Q (E-mode) with respect to the length of the resonator at different frequencies of excitation.

$a = 1$ metre ; $d = 10^{-3}$ metre ; $\sigma_e\,(Al) = 3.54 \times 10^7\ v/m$;

$\sigma_w\,(Cu) = 5.8 \times 10^7\ v/m$; $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H}/m$; $\epsilon_0 = 8.85 \times 10^{-12}\ F/m$.

Q of Sommerfeld Surface Wave Resonator

8.1. *Field Components :*

The mode of practical intest in cylindrical surface wave transmission is rhe E_0 mode, since all other modes have very high attenuation. For Sommerfeld surface wave line having radius d and immersed in air the field components for the resonant waves E_0 are[40]

$$E_{zr} = 2\,B\,\mathrm{H}_0^{(1)}\,(j\,\gamma_2\,\rho)\cos(n\,\pi\,z/l)$$

$$E_{\rho r} = -j\,2\,B\,(n\pi/\gamma_2\,l)\,\mathrm{H}_1^{(1)}\,(j\,\gamma_2\,\rho)\sin(n\,\pi\,z/l)$$

$$\mathrm{H}_{\theta r} = 2\,B\,(\omega\,\epsilon_0/\gamma_2)\,\mathrm{H}_1^{(1)}\,(j\,\gamma_2\,\rho)\cos(n\,\pi\,z/l) \qquad [71]$$

where $\gamma_2 = a_2 - j\,b_2$

8 2. *Maximum Energy Stored* ;

The maximum energy W_M stored inside the resonator is

$$W_M = -\frac{2\pi \mu_0 B^2 \omega^2 \epsilon_0^2 l}{\gamma_2 \gamma_2^* (\gamma_2^{*2} - \gamma_2^2)} [-j \gamma_2^* d H_1^{(1)} (j \gamma_2 d) H_0^{(2)} (-j \gamma_2^* d) -j \gamma_2 d H_0^{(1)} (j \gamma_2 d) H_1^{(2)} (-j \gamma_2^* d)] \quad [72]$$

8.3. *Power Lost* :

$$P_e = -\sqrt{\left(\frac{\omega \mu_0}{2 \sigma_e}\right)} \frac{8 \pi B^2 \omega^2 \epsilon_0^2}{\gamma_2 \gamma_2^* (\gamma_2^{*2} - \gamma_2^2)} [-j \gamma_2^* d H_1^{(1)} (j \gamma_2 d) H_0^{(2)} (-j\gamma_2^* d) -j \gamma_2 d H_0^{(1)} (j \gamma_2 d) H_1^{(2)} (-j \gamma_2^* d)] \quad [73]$$

$$P_w = \sqrt{\left(\frac{\omega \mu_0}{2 \sigma_w}\right)} \frac{2 \pi B^2 \omega^2 \epsilon_0^2 l d}{\gamma_2 \gamma_2^*} H_1^{(1)} (j \gamma_2 d) H_1^{(2)} (-j \gamma_2^* d) \quad [74]$$

8.4. Q *Factor* :

The Q factor for the Sommerfeld surface wave resonator is

$$\begin{aligned} Q (E_0) &= \omega (W_M/[P_E + P_W]) \\ &= \sqrt{\omega \mu_0} [-j \gamma_2^* d H_1^{(1)} (j \gamma_2 d) H_0^2 (-j \gamma_2^* d) \\ &\quad -j \gamma_2 d H_0^{(1)} (j \gamma_2 d) H_1^{(2)} (-j \gamma_2^* d)] \\ &\quad - [4/l \sqrt{(2\sigma_e)}] [-j \gamma_2^* d H_1^{(1)} (j \gamma_2 d) H_0^{(2)} (-j \gamma_2^* d) \\ &\quad -j \gamma_2 d H_0^{(1)} (j \gamma_2 d) H_1^{(2)} (-j \gamma_2^* d)] \\ &\quad -[d (\gamma_2^{*2} - \gamma_2^2)/\sqrt{(2 \sigma_w)} [H_1^{(1)} (j \gamma_2 d) H_1^{(2)} (-j \gamma_2^* d)] \end{aligned} \quad [75]$$

The magnitude of the arguments of the Hankel functions in most of the practical cases is less than 0.05. Therefore, using the following small argument approximations

$$\begin{aligned} H_1^{(1)} (j \gamma_2 d) &= -(2/\pi \gamma_2 d) \\ H_1^{(2)} (-j \gamma_2^* d) &= -(2/\pi \gamma_2^* a) \\ H_0^{(1)} (j \gamma_2 d) &= j (2/\pi) (m + j n) \\ H_0^{(2)} (-j \gamma_2^* d) &= -j (2/\pi) (m + j n) \end{aligned} \quad [76]$$

where,

$$m = \tfrac{1}{2} \ln [(0\ 89\ d)^2 (a_2^2 + b_2^2)]$$

$$n = \arctan (b_1/a_2)$$

the expression for Q (E_0) reduces to

$$Q(E_0) \cong \frac{\sqrt{(\omega\ \mu_0\ \sigma_w\ \sigma_e)}}{\{2\sqrt{2}\ \sigma_w / l\}\ \{a_2\ b_2 \sqrt{(2\ \sigma_e / d[n\ (a_2^2-b_2^2)+2\ a_2\ b_2\ m]\}}} \qquad [77]$$

where, the values of a_2 b_2 are determined from the solution of the following characteristic equation given by Barlow and Brown[53]

$$\frac{\gamma_2\, H_0^{(1)}(j\,\gamma_2 d)}{\omega\,\epsilon_0\, H_1^{(1)}(j\,\gamma_2 d)} = \frac{j\,\gamma_1}{\sigma_w + j\,\omega\epsilon_1}\ \frac{J_0(j\,\gamma_1 d)}{J_1(j\,\gamma_1 d)} \qquad [78]$$

where, γ_1 is the radial propagation constant for the region inside the conductor and ϵ_1 represents the dielectric constant of the conducting medium. By using large argument approximation $|j\,\gamma_1 d| >> 1$ for the Bessel functions and small argument approximation $|j\,\gamma_2\, d| << 1$ for the Hankel functions, eqn. [78] is solved to yield

$$a_2 = [(1.123\,|\xi|^{1/2})/d]\cos\delta/2, \quad b_2 = [(1.123\,|\xi|^{1/2})/d]\sin\delta/2 \qquad [79]$$

where,

$$\delta = -\frac{\pi}{4}\left(1 - \frac{1}{ln\,|\xi|+1}\right)$$

$$-|\eta| = |\xi|\ ln\,|\xi|$$

and $$|\eta| = \frac{24\ 6\times 10^{-14}}{\sqrt{\sigma_w}}\, f^{3/2}\, d \qquad [80]$$

The Q (E_0) of the Sommerfeld resonator at $f = 9500$ *M*Hz and with

$$\sigma_e = 3.54\times 10^7\ \upsilon/m, \quad \sigma_w = 5.8\times 10^7\ \upsilon/m$$

is

$$Q(E_0) = 10^{-6}\left[\frac{1.74}{l} - \frac{0\ 678\ a_2\ b_2}{d\,[n\,a_2^2 - b_2^2) + 2\,a_2 b_2\, m]}\right] \qquad [81]$$

which for $l = 0.75\ m$ and $d = 1.1\times 10^{-3}\ m$ yields

Q (E_0) = 18.830 and for $l = 0\ 1\ m$ Q (E_0) = 14660.

9. Power Flow

In deriving the expression for Q (E_0), it has been assumed that the only losses in the resonator occurs due to ohmic dissipation in the end plates and the wire surface and the loss due to the radiation has been ignored. An idea of the radiation loss can be gained from the power flowing outside a radius ρ_e corresponding to the radius of the terminating end plates.

The total power flow P_t outside the Sommerfelc line is

$$P_t=\frac{1}{2} Re \int_{\theta=0}^{2\pi} \int_{\rho=d}^{\infty} E_{\rho 2} H_{\theta 2}^{*} \rho \, d\rho \, d\theta$$

$$= Re \left[\frac{\pi \epsilon_0 \omega h B^2}{j \gamma_2 \gamma_2^{\times} (\gamma_2^{\times 2} - \gamma_2^2)} \left\{ j \gamma_2^{\times} d H_1^{(1)} (j \gamma_2 d) \right.\right.$$

$$\left.\left. H_0^{(2)} (-j \gamma_2^{*} d) + j \gamma_2 d H_0^{(1)} (j \gamma_2 d) H_1^{(2)} (-j \gamma_2^{*} d) \right\} \right]$$

$$\cong \frac{2 \epsilon_0 \omega B^2 \beta}{\pi a_2 b_2 (a_2^2 + b_2^2)^2} \left[(b_2^2 - a_2^2) n - 2 a_2 b_2 m \right] \qquad [82]$$

where, $h=\alpha+j\beta \cong j\beta$ and small argument approximations for the Hankel functions have been used.

The energy flow outside a radius ρ_e is

$$P_{\rho e}=\frac{1}{2} Re \int_{\rho=\rho_e}^{\infty} \int_{\theta=0}^{2\pi} E_{\rho 2} H_{\theta 2}^{*} \rho \, d\rho \, d\theta$$

$$= Re \left[\frac{\pi \omega \epsilon_0 h B^2}{j \gamma_2 \gamma_2^{*} (\gamma_2^{*2} - \gamma_2^2)} \left\{ j \gamma_2^{*} \rho_e H_1^{(1)} (j \gamma_2 \rho_e) \right.\right.$$

$$H_0^{(2)} (-j \gamma_2^{*} \rho_e) + j \gamma_2 \rho_e H_0^{(1)} (j \gamma_2 \rho_e)$$

$$H_1^{(2)} (-j \gamma_2^{*} \rho_e) \qquad [83]$$

Therefore, the percentage of power flow outside a radius

ρ_e is

$$(P_{\rho e}/P_t) \times 100$$

$$= 100 (\pi^2 (a_2^2 + b_2^2) \rho_e / 8 \beta)$$

$$\frac{Re [h \{ j\gamma_2^{*} H_1^{(1)} (j \gamma_2 \rho_e) H_0^{(2)} (-j \gamma_2^{*} \rho_e) + j \gamma_2 H_0^{(1)} (j \gamma_2 \rho_e) H_1^{(2)} (-j \gamma_2^{*} \rho_e) \}]}{[n (a_2^2 - b_2^2) + 2 a_2 b_2 m]} \qquad [84]$$

The variation of the percentage of power flow outside a radins $\rho_e = 1$ m and $\rho_e = 0.45$ m as function of the radius of the Sommerfield line having σ_w and σ_e values same as stated previously is shown in fig. 12. It is found that for 25 s.w.g. wire and end plate radius of 1 metre only about .03% power flows outside the resonator, whereas, for end plate of radius 0.45 metre, the percentage of power flow outside the resonator, is about 1%.

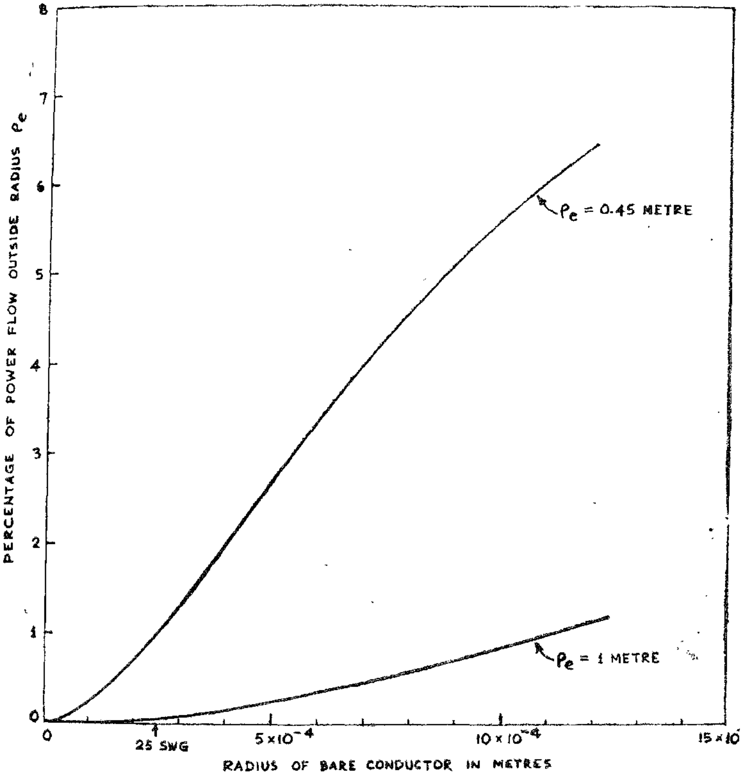

FIG. 12

Power flow outside a radius ρ_e=1m and ρ_e=0.45 *m* as a function of the radius of Sommerfeld line.

10. Further Scope of Work

The following work in connection with the surface wave resonator which has been developed (see fig. 13) is under progress.

(i) The effect of the tilt of one of the terminating end plates on the Q of the resonator.

(ii) The problem of excitation of a metallic corrugated surface wave structure.

(iii) Experimental study of the field decay guide wavelength, attenuation of surface wave lines with metal disc loading.

(iv) Extension of the surface wave resonator technique to the study of corrugated dielectric rod characteristic.

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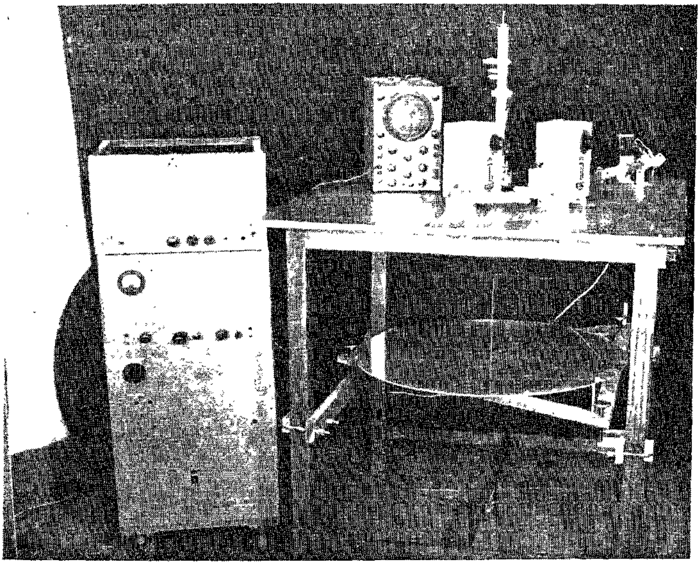

FIG. 13

Photograph of the experimental setup of Sommerfeld surface wave resonator

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