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Short Communication

Cascade controller tuning by relay auto tune method

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Abstract

The present work extends the method of Srinivasan and Chidambaram (An improved auto tune identification method, *Proc. Int. Conf. on Digital Modeling & Simulation* (DAMS-2003), Jan 6–8, Coimbatore, India (2003)) to analyze conventional on–off relay oscillations for a single loop feedback controller to relay tuning of cascade controllers. Using the ultimate gain and ultimate crossover frequency of the two loops, the inner loop (PI) and outer loop (PID) controllers are designed by Ziegler–Nichols tuning method. The performance of the controllers is compared with the results based on conventional relay analysis. The improved method of analyzing biased auto tune method proposed for single feedback controller by Srinivasan and Chidambaram (Modified relay feedback metho for improved system identification, *Comp. Chem. Engng*, **27**, 727–732 (2003)) is also extended to relay auto tune of cascade controllers. The proposed methods give an improved performance over that of the conventional on–off relay tune method.

Keywords: Cascade, symmetric relay, PI controllers, asymmetric relay.

1. Introduction

Åström and Hägglund [1] have suggested the use of an ideal (on–off) relay to generate sustained closed-loop oscillations. The ultimate gain and ultimate frequency can be found [using $k_u = 4h/(\pi a_0)$, where h is the relay height and a_0, the amplitude of the closed loop oscillation]. PID controllers can be designed by using the Ziegler–Nichols method. Luyben [2] has employed the relay feedback method to identify an FOPTD transfer function model. Once, k_u and w_u are known, then the amplitude criterion and phase angle condition can be written down. To get the three parameters of an FOPTD model, the knowledge on the process gain or delay should be known. Luyben has noted the delay from the initial portion of the relay oscillation. In deriving the relation $k_u = 4h/(\pi a_0)$, an assumption made in the conventional relay analysis is that all higher-order harmonics (of the relay output) are filtered by the system. Li *et al*. [3] have pointed out that an error of –18% to +27% is obtained in the calculation of k_u by this method. Yu [4] has given an excellent review of relay feedback method. Srinivasan and Chidambaram [5] have improved the conventional relay auto tune method by proposing a method to calculate the value of k_u by using appropriate value of

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number of harmonics coming out of the system output. This method gives an accurate value of k_u.

Shen *et al*. [6] have used a biased relay for getting the model parameters of an FOPTD model. In this method, the process gain is calculated from

$$k_p = \int_0^{2p} e(t)d(wt) / \int_0^{2p} y(t)d(wt). \tag{1}$$

Since the value of k_u is calculated from $k_u = 4\,h/(pa_0)$, the method does not give good results. Recently, Srinivasan and Chidambaram [7] have proposed an improved analysis of the biased relay auto tune method in which the gain of the process can be easily calculated compared to symmetric relay method.

Cascade control scheme utilizes two control loops: secondary or inner loop embedded within a primary or outer loop. The disturbances entering the inner loop are reduced or eliminated before their effect is felt on the outer loop output variable (y_1). As two control loops present, two controllers need to be tuned. Hang *et al*. [8] have proposed a relay auto tuning of cascade control loops. They have used the conventional on–off relay testing and using the value of k_u from $4\,h/(pa)$ and Ziegler–Nichols tuning formulae the controllers are tuned. With the inner loop under PI control action, the relay test is repeated for the outer loop (Fig. 1). In the present work, the methods of Srinivasan and Chidambaram [5, 7] are applied to tune cascade controllers. The improved performance of the proposed controller is compared with that of the method Hang *et al*. [8]. Luyben and Luyben [9] have recommended the use of PI controller for inner loop and PID controller for the outer loop.

2. Proposed method-1

The cascade control scheme is considered here. First, the conventional on–off relay is considered. The relay is used in the inner loop and the outer loop is kept under manual mode. The relay oscillations are noted. For simulation study, the process models assumed are $(k_pG_p)_2 = \exp(-0.1\,s)/(0.025\,s + 1)$ and $(k_pG_p)_1 = \exp(-0.1\,s)/(0.025\,s + 1)$. For single-loop systems, Li *et al*. [3] have reported that only for large (t_d/t) ratio, the conventional relay

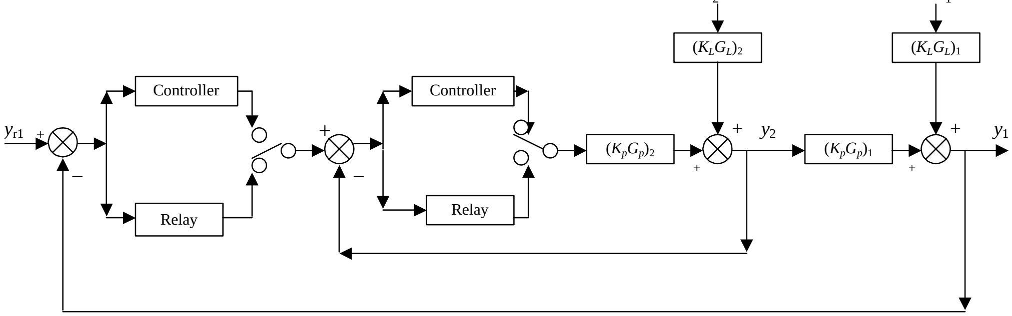

FIG. 1. Relay feedback tuning of cascade control system.

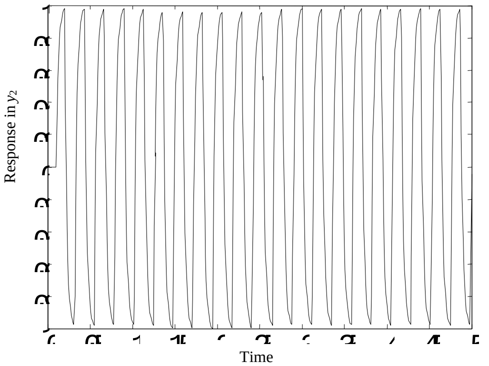

FIG. 2. Response in y_2 using symmetric relay (with relay height ±1) in inner loop.

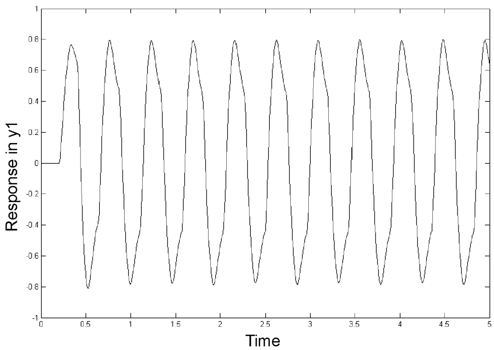

FIG. 3. Response in y_1 for symmetric relay in outer loop (with inner loop on PI settings).

analysis gives a significant error in calculating the value of k_u. Using a symmetric relay height of ±1, the oscillation in the inner loop output variable y_2 is noted (Fig. 2). The amplitude and frequency of oscillations are noted as 0.9814 and 27.0128, respectively. Using the relation $k_u = 4\,h/(pa_0)$, the value of k_u is obtained as 1.2974. Based on the transfer function model, the exact value of k_u is calculated as 1.18. Thus significant error is obtained in k_u by the conventional relay analysis. Using the results of relay testing, the PI settings are calculated by using the Ziegler–Nichols tuning method as $k_c = 0.5838$ and $t_I = 0.1938$.

Using this PI settings in the inner loop instead of the relay and introducing a relay in the outer loop, the resulting oscillation in the outer loop gives an amplitude of 0.695 and frequency of 13.3685 (Fig. 3). Hence the value of $k_u = 1.832$ is obtained. Based on the relay test results, the outer loop PID controller is designed as $k_c = 1.0992$, $t_I = 0.235$ and $t_D = 0.0587$. The closed loop servo response is evaluated for a unit step change in the set

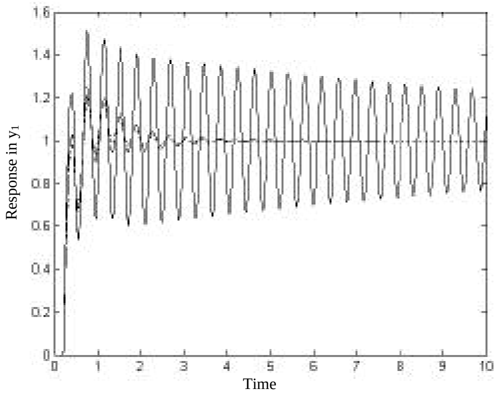

FIG. 4. Servo response in y_1 using PID controller for outer loop and PI for inner loop for the first example. Outer oscillatory response – Hang *et al*. method; Inner solid–proposed method-1; Inner dash–proposed method-2.

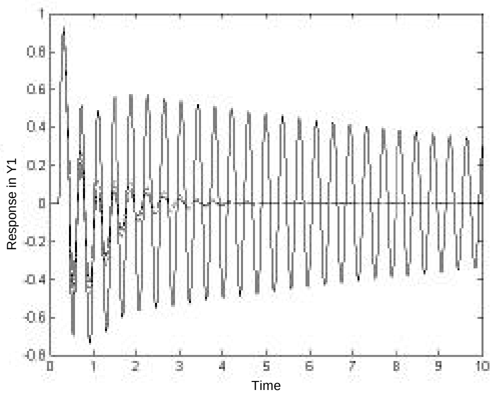

FIG. 5. Regulatory response in y_1 for a disturbance in inner loop. PID controller for outer loop and PI for inner loop, for the first example. Inner solid–proposed method-1; Inner dash–proposed method-2.

Table I
Controller setting comparisons for $(t_d/t)_{\text{inner–loop}} = 4.0$, $(t_d/t)_{\text{outer–loop}} = 4.0$

Loop	Controller settings	Symmetric relay			Asymmetric relay	Asymmetric relay with noise (s = 0.5%)
		$N = 1$	$N = 5$	$N = 5$ (with noise, s = 0.5%)		
Inner	k_c	0.5838	0.5307	0.5282	0.5306	0.5309
	t_I	0.1938	0.1938	0.1950	0.1917	0.2
Outer	k_c	1.0992	0.9382	0.9549	0.9780	1.07
	t_I	0.2350	0.2350	0.2360	0.24	0.2324
	t_D	0.0587	0.0587	0.059	0.06	0.0581

point and the response is shown in Fig. 4. A highly oscillatory response is obtained. Similar response is obtained for a regulatory problem also [with $(k_L G_L)_2 = 1$] as shown in Fig. 5.

The method of Srinivasan and Chidambaram [5] is applied now. As stated earlier, in deriving the relation $k_u = 4\,h/(pa_0)$, Aström and Hägglund [1] made an assumption that all higher-order harmonics (of the relay output) are filtered by the system. Srinivasan and Chidambaram [5] have given a method to find out the value of k_u by considering the higher-order harmonics (refer to appendix A for the summary of the method). The initial portion of the relay output gives an indication of how many higher-order harmonics are present in the relay output. Five higher-order harmonics ($N = 5$) are recommended. Let us use their method for analyzing the cascade auto tuning. For the system under study, the value of k_u for the inner loop is obtained as 1.1794 and the frequency of oscillations as 27.0128. Once PI controller is designed based on this value and w_u, the inner loop is kept under PI mode and then the relay is kept in the outer loop. From the relay oscillation, the value of $k_u = 1.832$ is obtained by the conventional method. The method of Srinivasan and Chidambaram [5] gives k_u as 1.5637. A PID controller is designed based on this value (Table I). The servo response is evaluated for a step change in the set point. The response in y_1 is shown in Fig. 4. The response by the conventional analysis using single harmonics gives a highly oscillatory response, whereas the method of Srinivasan and Chidambaram [5] gives an excellent response. Similar performance is obtained for a regulatory problem $[(k_L G_L)_2 = 1]$ as shown in Fig. 5. The ISE values are given in Table II.

The effect of measurement noise is studied by adding a random noise (Gaussian distribution with zero mean and standard deviation of 0.5%) in inner loop and corrupted signal is used for the feedback. The present method gives $k_u = 1.1737$, and the frequency of oscilla-

Table II
Performance comparison of proposed methods and Hang *et al*. [8] method for $(t_d/t)_{\text{inner–loop}} = 4.0$, $(t_d/t)_{\text{outer–loop}} = 4.0$

Comparison parameters	Symmetric relay		Asymmetric relay
	$N = 1$	$N = 5$	
ISE	0.7989	0.2765	0.2808
Overshoot	0.5135	0.2022	0.2494
Settling time	55	7	7

Table III
Effect of change in relay height (asymmetric relay testing) on PI settings

Asymmetric relay		PI controller		
H	g	K_u	K_c	t_I
0.5	4.0	1.1859	0.5336	0.2050
1.0	2.0	1.1972	0.5387	0.1975
1.0	2.5	1.1902	0.5356	0.2000
1.0	3.0	1.1837	0.5327	0.2008

tions (w) is 26.8512. A PI controller is designed by the Ziegler–Nichols method. The inner loop is kept under PI controller and the symmetric relay is used in the outer loop. Based on the oscillation obtained in y_1, the value of k_u is obtained for the outer loop as 1.5915. A PID controller is designed for the outer loop based on the value of k_u. The results for symmetric relay with the noise are given in Table I and are very close to those obtained by without noise.

3. Proposed method-2

In this section, the biased relay auto tune method is applied with a relay height of +2 and –1. The method proposed by Srinivasan and Chidambaram [7] [refer to Appendix B for a summary of this method] is extended to series cascade systems. The value of k_u for the inner loop based on the assumed FOPTD model ($k_P = 0.8722$, $t = 0.0088$, $t_d = 0.1073$), obtained by the relay method is 1.1792. A PI controller is designed by the Ziegler–Nichols method. The inner loop is kept under a PI controller and the asymmetric relay is then used in the outer loop. Based on the oscillation obtained in y_1 and hence based on the identified FOPTD model ($k_P = 1.0684$, $t = 0.11$, $t_d = 0.162$), the value of k_u is obtained for the outer loop as 1.63. PID controller is designed for the outer loop based on the value of k_u obtained. The results of the relay test are given in Table I. The servo response in y_1 for a unit step change in the set point is shown in Fig. 4. The performance is as good as the performance of the proposed symmetric relay method.

The effect of measurement noise is studied by adding a random noise (Gaussian distribution with zero mean and standard deviation of 0.5%) in inner loop and corrupted signal is used for feedback. The results for the asymmetric relay with the noise are given in Table I. The results are close to results obtained by without any noise.

In the modified symmetrical relay method proposed by Srinivasan and Chidambaram [5], the value of order (N) of higher-order harmonics is to be selected, whereas in the asymmetric method of Srinivasan and Chidambaram [7], no such value of N is required. In the asymmetric method, the model is to be identified and then the controller settings are calculated, whereas in the symmetrical method, the controller settings are calculated based on the ultimate values obtained from relay test. Figure 5 shows the regulatory response for a step change in the inner loop disturbance. The proposed two methods give an improved performance than that by the conventional relay analysis (Table II).

In the literature for single-loop asymmetric relay tuning method, the value of $g = 2$ is used. Therefore, the relay height of +2 and –1 is used in the present study also. However, simulations studies are carried out with different relay heights of $g = 2$, 2.5, 3 and 4 (Table III). We have observed that the resulting PI controller settings have not changed significantly.

4. Second example

A second example is also considered. The model assumed for the inner loop is $(k_pG_p)_2 = \exp(-0.1\,s)/(0.01\,s + 1)$ and for the outer loop is $(k_pG_p)_1 = \exp(-0.1\,s)/(0.1\,s + 1)$. This example considers a lower value of (t_d/t) in the outer loop. Using a symmetric relay with a

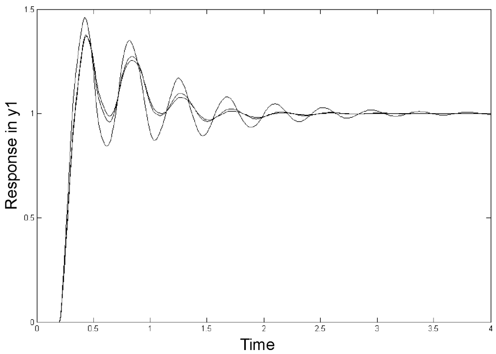

FIG. 6. Servo response in y_1 using PID controller for outer loop and PI for inner loop for the second example. Inner solid–proposed method-1; Inner dash–proposed method-2.

Table IV
Controller setting comparisons for the second example

Loop	Controller settings	Symmetric relay $N = 1$	Symmetric relay $N = 9$	Asymmetric relay
Inner	k_c	0.5730	0.4681	0.4694
	t_I	0.1783	0.1783	0.1828
Outer	k_c	1.74	1.82	1.86
	t_I	0.269	0.2750	0.2806
	t_D	0.0673	0.0688	0.0702
IAE		0.5009	0.411	0.419

height of ±1, the oscillation in the inner loop output variable y_2, is recorded. The amplitude and frequency of oscillations are noted. By the conventional method of analyzing the relay testing, the PI settings are calculated by Ziegler–Nichols tuning method. Using this PI setting for the inner loop, and symmetric relay in the outer loop, the oscillation in the outer loop are noted with an amplitude of 0.44 and frequency of oscillation (w_u) 11.6788. Hence by the conventional method of relay analysis, k_u is calculated as 2.8937. Based on k_u and w_u, the outer loop PID controller is designed and the values of controller settings are given in Table IV.

The proposed methods-1 and 2 are also applied. For method-2, the value of $g = 2$ is used (Table IV). The closed-loop servo response is evaluated for a unit step change in the set point. The response in y_1 is shown in Fig. 6. Since the error is small, integral of the absolute value of the error is calculated for performance comparisons and the results are shown in Table IV. The proposed methods-1 and 2 show good performance.

5. Conclusions

The modified analysis of relay auto tuning proposed for single feedback system by Srinivasan and Chidambaram [5] and modified analysis of asymmetric auto tuning by Srinivasan and Chidambaram [7] are extended to tune cascade controllers. Both the methods take care of higher-order harmonics effectively. The performances of the PI-PID controllers are compared with the conventional relay analysis (principle harmonic analysis). The present methods give a superior performance over that of the conventional analysis.

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Appendix A: Srinivasan and Chidambaram method [5]

Srinivasan and Chidambaram [5] have recently reported the derivations for analyzing symmetric relay tuning. Here, final equations are given to present application to cascade control systems. With a symmetric relay, the output oscillations are recorded. Let us consider the time (t^*) at which

$$t^* = 0.5p/w_u, \tag{A.1}$$

where w_u is the frequency of observed output oscillations. From the output oscillations, it is possible to calculate $y(t^*)$ at time t^*. If observed output oscillations are close to the rectangular waveform then amplitude (a) is calculated [5] as

$$y(t^*) = a[1–(1/3) + (1/5)–(1/7) + (1/9)–.......]. \tag{A.2}$$

Let the number of terms to be considered in the above equation be denoted as N. If the oscillation deviates from pure sine wave and depending on the extent of deviation, $N = 3$ or 5 or 7 can be considered. Using the limiting value for the summation term ($0.25p$), we get from eqn (A.2):

$$a = 1.273y(t^*). \tag{A.3}$$

Similarly, if the observed output oscillations are close to the triangular waveform then the amplitude is calculated [5] as

$$y(t^*) = a[1 + (1/9) + (1/25) + (1/49) + (1/81) +.......]. \tag{A.4}$$

Using the limiting value for the summation term ($0.125p^2$), we get from eqn (A.4):

$$a = 0.810y(t^*). \tag{A.5}$$

After getting the above-corrected amplitude, the value of k_u is given by

$$k_u = 4\,h/(ap). \tag{A.6}$$

Appendix B: Method of Srinivasan and Chidambaram [7] for asymmetric oscillation

Srinivasan and Chidambaram [7] have recently reported the derivations for analyzing asymmetric relay tuning. Here final equations are given for application to a cascade control system.

Let us denote $G(s)$ as the FOPTD model to be identified. With asymmetric relay, the responses in output (y) and input (u) are recorded. The process gain k_p is calculated as

$$k_p = \int_0^{2p} e(t)d(wt) / \int_0^{2p} u(t)d(wt). \tag{B.1}$$

$G(jw)$ can be written by substituting $s = jw$ in the above equation

$$G(jw) = a + jb = (c_1 - j\,f_1)/(c_2 - j\,f_2), \tag{B.2}$$

where a is real part and b is imaginary, and

$$c_1 = \int_0^{p} u(t)\cos(wt)dt; \quad f_1 = \int_0^{p} u(t)\sin(wt)dt, \tag{B.3a}$$

$$c_2 = \int_0^{p} y(t)\cos(wt)dt; \quad f_2 = \int_0^{p} y(t)\sin(wt)dt, \tag{B.3b}$$

where $P = 2p/w$ and w is the frequency of oscillation observed in the output response. (Equations (B.3a) and (B.3b) can be evaluated numerically)

$$a = (c_1c_2 + f_1\,f_2)/(c_2^2 + f_2^2), \tag{B.4a}$$

$$b = (\,f_2c_1 - f_1c_2)/(c_2^2 + f_2^2). \tag{B.4b}$$

We can write $G(jw)$ as

$$a + jb = k_p[\cos(t_d w) - j\,\sin(tw)]/(1 + jtw). \tag{B.5}$$

On cross multiplying and equating the resulting real and imaginary part to zero we get

$$a - btw - k_p\cos(t_d w) = 0 \tag{B.6}$$

$$atw + b + k_p\sin(t_d w) = 0 \tag{B.7}$$

substituting w_u for w, an analytical solution of eqns (B.6), (B.7) gives the values for t and t_d.

Nomenclature

a amplitude of oscillation corresponds to the principle harmonics calculated from eqn (A.2)

a_0 amplitude of oscillation observed from the process output

c_1, f_1	as defined by eqn (B.3a)
c_2, f_2	as defined by eqn (B.3b)
d_1	disturbance entering outer loop
d_2	disturbance entering inner loop
G	process transfer function
h	relay height
k_c	controller gain
$k_{c,\max}$, k_u	controller ultimate gain
k_p	process gain
$(k_pG_p)_1$	transfer function of the outer loop process
$(k_pG_p)_2$	transfer function of the inner loop process
$(k_LG_L)_1$	transfer function for load disturbance in the outer loop
$(k_LG_L)_2$	transfer function for load disturbance in the inner loop
N	number of terms considered in eqns (A.2) or (A.4)
P_u	period of output oscillation
s	Laplace variable
$s_1 = 8/t_s$	
t_s	time taken to reach three invariant cycles of oscillations in the output
t	time
$t^* = 0.5p/w_u$	
u	input variable
y_1	outer loop output variable
y_2	inner loop output variable
t	process time constant
t_d	process time delay
t_I	integral time
t_D	derivative time
w	frequency of oscillation
w_u	ultimate frequency of oscillation

Subscript

0	outer loop
1	inner loop

Symbols

 Manual switch

$\otimes$ Comparator