THEORY OF OPEN MICROWAVE RESONATOR WITH AN AXIAL CORRUGATED METAL ROD*

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ABSTRACT

The characteristics of a microwave resonator consisting of circular cylindrical corrugated metal rod terminated at both ends by large circular metallic plates and excited in E_0*-mode are studied theoretically. Expressions for the* Q*-factor and resonance frequency of the resonator as functions of groove spacing, depth and other parameters of the axial rod are derived.*

1. INTRODUCTION

The open-type of microwave resonator utilises the surface wave characteristics[1,2,3], of the corrugated metallic rod. Wait[4-6] has made significant contributions to different aspects of surface wave phenomena. The characteristics of different types of microwave resonators have been studied extensively by Chatterjee, *et al*[7-17]. The object of the present report is to make a theoretical study of the resonance properties of a surface-wave resonator consisting of a circular cylindrical corrugated metal rod excited in E_0- surface wave mode and terminated at the two ends by two identical circular metallic plates of radius much larger than the radius of the structure. The large size of the end-plates and excitation of the structure by surface wave mode ensures that the loss of electromagnetic energy by radiation, if there is any, through the open side of the resonator is negligibly small.

2. FIELD COMPONENTS

The corrugated structure consists of three media (Fig. 1). The first medium consists of the uniform portion of the rod ($0 \leq \rho \leq a$), The second

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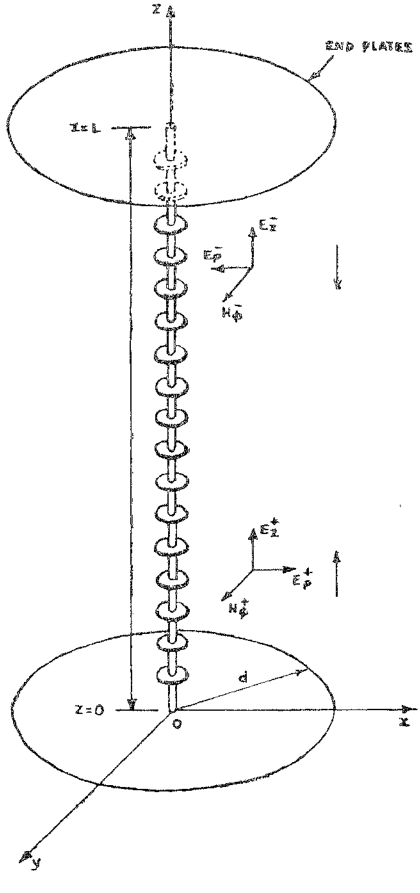

Fig. 1 (a)

Corrugated Metal open type Resonator with end Plates.

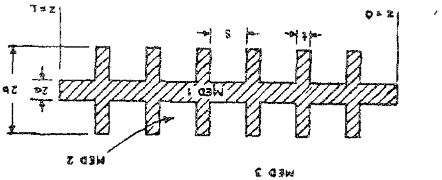

FIG. 1 (*b*)

Corrugated Metallic Structure.

medium consists of the grooved region ($a \leqslant \rho \leqslant b$) and the third medium ($\rho \geqslant b$) is air. The grooved region is considered as a homogeneous dielectric medium of dielectric constant.

$$\epsilon_2 = \frac{s+t}{s} \qquad [1]$$

which has been derived from the consideration of equality of optical path.

The field components in the three different media are ;

Medium 1 : $0 \leqslant \rho \leqslant a$

$$E_{z1} = A\ J_0\ (ju_1\ \rho)\ \exp\ (-\gamma\ z)$$

$$E_1 = A\ \frac{\gamma}{ju_1} J_1\ (ju_1\ \rho)\ \exp\ (-\gamma\ z)$$

$$H_{\phi 1} = A\ \frac{\sigma_1 + j\ \omega\ \epsilon_1}{j\ u_1}\ J_1\ (ju_1\ \rho)\ \exp\ (-\gamma\ z) \qquad [2]$$

Medium 2 : $a \leqslant \rho \leqslant b$

$$E_{z2} = [B\ J_0\ (ju_2\ \rho) + C\ Y_0\ (ju_2\ \rho)]\ \exp\ (-\gamma\ z)$$

$$E_2 = \frac{\gamma}{ju_2}\ [B\ J_1\ (ju_2\ \rho) + C\ Y_1\ (ju_2\ \rho)]\ \exp\ (-\gamma\ z)$$

$$H_{\phi 2} = \frac{\sigma_2 + j\ \omega\ \epsilon_2}{ju_2}\ [B\ J_1\ (ju_2\ \rho) + C\ Y_1\ (ju_2\ \rho)]\ \exp\ (-\gamma\ z) \qquad [3]$$

Medium 3 : $b \leqslant \rho \leqslant \infty$

$$E_{z3} = D\ H_0^{(1)}\ (ju_3\ \rho)\ \exp\ (-\gamma\ z)$$

$$E_3 = D\ \frac{\gamma}{ju_3}\ H_1^{(1)}\ (ju_3\ \rho)\ \exp\ (-\gamma\ z)$$

$$H_{\phi 3} = D\ \frac{\omega\ \epsilon_0}{u_3}\ H_1^{(1)}\ (ju_3\ \rho)\ \exp\ (-\gamma\ z) \qquad [4]$$

where, $\quad u_1 = a_1 + j\ b_1,\quad u_2 = a_2 - j\ b_2,\quad u_3 = a_3 - j\ b_3$, and $\gamma = \alpha + j\ \beta$ [5]

2.1 *Standing Waves.*

The standing waves are represented on a vector basis as follows.

$$E_{z2s} = E_{z2} + E_{z2-}$$

$$E_{\rho 2s} = E_{\rho 2} + E_{\rho 2-}$$

$$H_{\phi 2s} = H_{\phi 2} + H_{\phi 2-} \quad [6a]$$

$$E_{z3s} = E_{z3} + E_{z3-}$$

$$E_{\rho 3s} = E_{\rho 3} + E_{\rho 3-}$$

$$H_{\phi 3s} = H_{\phi 3} + H_{\phi 3-} \quad [6b]$$

for the E_0 wave. The subscripts + and − indicate components of the wave travelling in the $+z$ and $-z$ directions respectively. Due to reflections taking place at the two ends ($z=0$ and $z=L$) of the structures the following relations hold good.

$$E_{z2-} = +E_{z2+} \qquad E_{z3-} = +E_{z3+}$$

$$E_{\rho 2-} = -E_{\rho 2+} \qquad E_{\rho 3-} = -E_{\rho 3+}$$

$$H_{\phi 2-} = +H_{\phi 2+} \qquad H_{\phi 3-} = +H_{\phi 3+} \quad [6c]$$

Using the above relations, the field components for the standing waves in media 2 and 3 respectively are

Medium 2 :

$$E_{z2s} = 2\,[\,B\,J_0\,(ju_2\,\rho) + C\,Y_0\,(ju_2\,\rho)\,]\cos\beta z$$

$$E_{2s} = -\frac{2j\beta}{u_2}\left[\,B\,J_1\,(ju_2\,\rho) + C\,Y_1\,(ju_2\,\rho)\right]\sin\beta z$$

$$H_{\phi 2s} = \frac{(2\sigma_2 + j\omega\,\epsilon_2)}{ju_2}\left[\,BJ_1\,(ju_2\,\rho) + C\,Y_1\,(ju_2\,\rho)\right]\cos\beta z \quad [7]$$

Medium 3 :

$$E_{z3s} = 2\,DH_0^{(1)}\,(ju_3\,\rho)\cos\beta z$$

$$E_{\rho 3s} = -j\,\frac{2\,D\,\beta}{u_3}\,H_1^{(1)}\,(ju_3\,\rho)\sin\beta z$$

$$H_{\phi 3s} = \frac{2\,D\omega\epsilon_0}{u_3}\,H_1^{(1)}\,(ju_3\,\rho)\cos\beta z \quad [8]$$

2.2 *Resonant Waves.*

The field components of the standing wave satisfy the boundary conditions at the end plates *i.e.*, $E_{\rho 3s}=0$ at $z=0$ and $z=L$ (*i e.*) $\sin \beta L=0$, which requires that $\beta=n\pi/L$, where n is a positive integer and indicates the number of half cycles variations of the field components in the z-direction.

The field components of the resonant waves are

Medium 2:

$$E_{z2r}=2\left[B J_0(ju_2\rho)+C Y_0(ju_2\rho)\right]\cos\left(\frac{n\pi}{L}\right)z$$

$$E_{\rho 2r}=-\frac{2j\beta}{u_2}\left[B J_1(ju_2\rho)+C Y_1(ju_2\rho)\right]\sin\left(\frac{n\pi}{L}\right)z$$

$$H_{\phi 2r}=\frac{2(\sigma_2+j\omega\epsilon_2)}{ju_2}\left[B J_1(ju_2\rho)+C Y_1(ju_2\rho)\right]\cos\left(\frac{n\pi}{L}\right)z \qquad [9]$$

Medium 3:

$$E_{z3r}=2DH_0^{(1)}(ju_3\rho)\cos\left(\frac{n\pi}{L}\right)z$$

$$E_{\rho 3r}=-2j\frac{D}{u_3}\left(\frac{n\pi}{L}\right)H_1^{(1)}(ju_3\rho)\sin\left(\frac{n\pi}{L}\right)z$$

$$H_{\phi 3r}=2D\frac{\omega\epsilon_0}{u_3}H_1^{(1)}(ju_3\rho)\cos\left(\frac{n\pi}{L}\right)z \qquad [10]$$

3. Energy Stored in Magnetic Field

The total energy stored in magnetic fields of the two media (2 and 3) is

$$W_M=W_{M2}+W_{M3} \qquad [11]$$

where,

$$W_{M2}=\frac{\mu_2}{2}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\int_{z=0}^{L}\left|H_{\phi 2r}\right|^2\rho\, d\rho\, d\phi\, dz$$

$$= \frac{\mu_2}{2} \int_{0=\phi}^{2\pi} \int_{\rho=a}^{b} \int_{z=0}^{L} \frac{4(\sigma_2^2+\omega^2 \epsilon_2^2)}{u_2 u_2^*} \left\{ B J_1 (ju_1 \rho) + C Y_1 (ju_2 \rho) \right\} \times$$

$$\left\{ B^* J_1 (-ju_2^* \rho) + C^* Y_1 (-ju_2^* \rho) \right\} \cos^2 \left(\frac{n\pi}{L} \right) z \rho \, d\rho \, d\phi \, dz$$

$$= \frac{2\pi L \mu_2 (\sigma_2^2 + \omega^2 \epsilon_2^2)}{u_2 u_2^*} \left[I_1 + I_2 + I_3 + I_4 \right] \quad [12]$$

where,

$$I_1 = \int_{\rho=a}^{b} B^2 J_1 (ju_2 \rho) J_1 (-ju_2^* \rho) \rho \, d\rho$$

$$= \frac{B^2}{(u_2^{*2} - u_2^2)} \left[-j u_2^* \rho J_1 j(u_2 \rho) J_0 (-ju_2^* \rho) - ju_2 \rho J_1 (-ju_2^* \rho) J_0 (ju_2 \rho) \right]_{\rho=a}^{b} \quad [12a]$$

$$I_2 = \int_{\rho=a}^{b} C^2 Y_1 (ju_2 \rho) Y_1 (-ju_2^* \rho) \rho \, d\rho$$

$$= \frac{C^2}{(u_2^{*2} - u_2^2)} \Big[- ju_2^* \rho Y_1 (-ju_2^* \rho) Y_0 (-ju_2^* \rho) - ju_2 \rho Y_1 (-ju_2^* \rho) Y_0 (ju_2 \rho) \Big]_{\rho=a}^{b} \quad [12b]$$

$$I_3 = \int_{\rho=a}^{b} B C J_1 (ju_2 \rho) Y_1 (-ju_2^* \rho) \rho \, d\rho$$

$$= \frac{BC}{(u_2^{*2} - u_2^2)} [-j u_2^* \rho Y_0 (-ju_2^* \rho) J_1 (ju_2 \rho) - ju_2 \rho J_0 (ju_2 \rho) Y_1 (-ju_2^* \rho)]_{\rho=a}^{b} \quad [12c]$$

$$I_4 = \int_{\rho=a}^{b} BC J_1 (-ju_2^* \rho) Y_1 (ju_2 \rho) \rho \, d\rho$$

$$= \frac{BC}{(u_2^{*2} - u_2^2)} [- j u_2^* \rho J_0 (-ju_2^* \rho) Y_1 (ju_2 \rho) - j u_2 \rho J_1 (-ju_2^* \rho) Y_0 (ju_2 \rho)]_{\rho=a}^{b} \quad [12d]$$

$$W_{M3} = \frac{\mu_0}{2} \int_{\phi=0}^{2\pi} \int_{\rho=b}^{d} \int_{z=0}^{L} |H_{\phi 3r}|^2 \rho \, d\rho \, d\phi \, dz$$

$$= \frac{\mu_0}{2} \int_{\phi=0}^{2\pi} \int_{\rho=b}^{d} \int_{z=0}^{L} \frac{4 D^2 \omega^2 \epsilon_1^2}{u_3 u_3^{*}} H_1^{(1)} (ju_3 \rho) H_1^{(2)} (-ju_3^{*} \rho) \times$$

$$\cos^2 \left(\frac{n\pi}{L} \right) z \, \rho \, d\rho \, d\phi \, dz$$

$$= \frac{2 \pi L \mu_0 \omega^2 \epsilon_0^2 D^2}{u_3 u_3^{*} (u_3^{*2} - u_3^2)} \Big[-j u_3^{*} \rho \, H_1^{(1)} (ju_3 \rho) H_0^{(2)} (-ju_3^{*} \rho)$$

$$- ju_3 \rho \, H_0^{(1)} (ju_3 \rho) H_1^{(2)} (-ju_3^{*} \rho) \Big]_{\rho=b}^{d} \qquad [13]$$

4. Energy Stored in Electric Field

The energy stored in electric field in medium 3 is

$$W_{E3} = \frac{\epsilon_0}{2} \int_{\phi=0}^{2\pi} \int_{\rho=b}^{d} \int_{z=0}^{L} |E|^2 \rho \, d\rho \, d\phi \, dz$$

$$= \frac{\pi \epsilon_0 L}{2} \Bigg[\frac{4 D^2}{u_3^{*2} - u_3^2} \Big\{ -ju_3 \rho \, H_0^{(1)} (ju_3 \rho) H_{-1}^{(2)} (-ju_3 \rho)$$

$$- ju_3 \rho \, H_{-1}^{(1)} (ju_3 \rho) H_0^{(2)} (-ju_3 \rho) \Big\}_{\rho=d}^{\rho=x}$$

$$+ \frac{4D^2 n^2 \pi^2}{(u_3^{*2} - u_3^2) L^2 u_3 u_3^{*}} \Big\{ -ju_3^{*} \rho \, H_1^{(1)} (ju_3 \rho) H_0^{(2)} (-ju_3^{*} \rho)$$

$$- ju_3 \rho \, H_0^{(1)} (ju_3 \rho) H_1^{(2)} \rho (-ju_3^{*} \rho) \Big\}_{\rho=b}^{\rho=d} \qquad [14]$$

Where,

$$|E|^2 = |E_{z3}|^2 + |E_{\rho 3}|^2 \qquad [14a]$$

Using the following relations of Hankel functions

$$H_{-1}^{(1)} (ju_3 \rho) = e^{+\pi i} H_1^{(1)} (ju_3 \rho) = -H_1^{(1)} (ju_3 \rho)$$

$$H_{-1}^{(2)} (ju_3 \rho) = e^{-\pi i} H_1^{(2)} (ju_3 \rho) = -H_1^{(2)} (ju_3 \rho) \qquad [14b]$$

equation [14] reduces to

$$W_{E2}=\frac{\pi\epsilon_0 L}{2}\left[\frac{4\,D^2}{u_3^{*2}-u_3^2}\ \{+\mathrm{j}u_3^*\ \rho\ H_0^{(1)}(\mathrm{j}u_3\,\rho)\ H_1^{(2)}(-\mathrm{j}u_3^*\,\rho)\right.$$

$$+\mathrm{j}u_3\,\rho\ H_1^{(1)}(\mathrm{j}u_3\,\rho)\ H_0^{(2)}(-\mathrm{j}u_3^*\,\rho)\ \}_{\rho=b}^{\rho=d}$$

$$+\frac{n^2\pi^2}{L^2\,u_3\,u_3^*}\ \{-\mathrm{j}u_3^*\ \rho\ H_1^{(1)}(\mathrm{j}u_3^*\,\rho)\ H_0^{(2)}(-\mathrm{j}u_3^*\,\rho)$$

$$\left.-\mathrm{j}u_3\,\rho\ H_0^{(1)}(\mathrm{j}u_3\,\rho)\ H_1^{(1)}(-\mathrm{j}u_3^*\,\rho)\ \}_{\rho=b}^{\rho=d}\ \right] \qquad [14c]$$

Similarly,

$$W_{E2}=\frac{\epsilon_2}{2}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\int_{z=0}^{L} 4\,|\,(BJ_0(\mathrm{j}u_2\,\rho)+CY_0(\mathrm{j}u_2\,\rho)\,|^2\cos^2\left(\frac{n\pi}{L}\right)z\rho\,d\rho\,d\phi\,dz$$

$$+\frac{4\pi^2 n^2}{L^2\,u_2\,u_2^*}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\int_{z=0}^{L}|\,BJ_1(\mathrm{j}u_2\,\rho)+CY_1(\mathrm{j}u_2\,\rho)\,|^2\times$$

$$\sin^2\left(\frac{n\pi}{n}\right)z\ \rho\ d\ \rho\ d\,\phi\ dz \qquad [15]$$

$$=2\,\pi\ L\,\epsilon_2\,R_1+2\,\pi\,\epsilon_2\,\frac{n^2\pi^2}{L\,u_2\,u_2^*}\ R_2$$

where,

$$R_1=R_1^{(1)}+R_1^{(2)}+R_1^{(3)}+R_1^{(4)}$$

$$R_2=R_2^{(1)}+R_2^{(2)}+R_2^{(3)}+R_2^{(4)}=I_1+I_2+I_3+I_4 \qquad [15a]$$

$$R_1^{(1)}=\frac{B\,B^*}{(u_2^{*2}-u_2^2)}[-\mathrm{j}\,u_2^*\ \rho\ J_0(\mathrm{j}u_2\,\rho)\ J_{-1}(-\mathrm{j}u_2^*\,\rho)$$

$$-\mathrm{j}u_2\,\rho\ J_0(-\mathrm{j}u_2^*\,\rho)\ J_{-1}(\mathrm{j}u_2\,\rho)]_{\rho=a}^{b}$$

$$R_1^{(2)}=\frac{C\ C^*}{u_2^{*2}-u_2^2}\ [-ju_2^*\ \rho\ Y_0(j\ u_2\ \rho)\ Y_{-1}(-ju_2^*\ \rho)$$

$$-ju_2\ \rho\ Y_0(-ju_2^*\ \rho)\ Y_{-1}(ju_2\ \rho)]_{\rho=a}^{b}$$

$$R_1^{(3)}=\frac{B\ C^*}{u_2^{*2}-u_2^2}\ [-ju_2^*\ \rho\ J_0(Ju_2\ \rho)\ Y_{-1}(-ju_2^*\ \rho)$$

$$-ju_2\ \rho\ J_{-1}(ju_2\ \rho)\ Y_0(-ju_2\ \rho)]_{\rho=a}^{b}$$

$$R_1^{(4)}=\frac{B^*\ C}{u_2^{*2}-u_2^2}\ [-ju_2^*\ \rho\ J_{-1}(-ju_2^*\ \rho)\ Y_0(ju_2\ \rho)$$

$$-ju_2\ \rho\ J_0(-ju_2^*\ \rho)\ Y_{-1}(ju_2\ \rho)]_{\rho=a}^{\rho=b} \qquad [15b]$$

6. Power Lost

The power lost (P) in the resonator is

$$P=P_1+P_2+P_3+P_e+P_r \qquad [16]$$

where,

P_1 = Power lost along the rim of the disc at $\rho=b$

P_2 = Power lost along the inner rod at $\rho=a$

P_3 = Power lost along the surface of disc at $a\leq\rho\leq b$

P_e = Power lost at the end plates

P_r = Power lost by radiation from the structure and flowing radially out of the resonator.

As the end plates diameter ($2d$) is much larger compared to the diameter of the surface wave structure and the structure is excited by E_0 – surface wave made, the radiation loss out of the resonator may be considered to be insignificantly small and therefore P_r may be ignored in calculating P.

$$P_1=\tfrac{1}{2}\left(\frac{\omega\ \mu_0}{2\sigma_2}\right)^{\frac{1}{2}}\left[\int_{\phi=0}^{2\pi}(H_{\phi3}\ H_{\phi3}^*)_{\rho=b}\ b\ d\phi\right]t$$

$$=\left(\frac{\omega\ \mu_0}{2\sigma_2}\right)^{\frac{1}{2}}\ \frac{\pi\ D^2\ \omega^2\ \epsilon_0^2\ b\ t}{u_3\ u_3^*}\ H_1^{(1)}(ju_3\ b)\ H_1^{(2)}(-ju_3^*\ b) \qquad [16a]$$

$$P_2 = \frac{1}{2}\left(\frac{\omega\,\mu_0}{2\sigma_{cu}}\right)^{\frac{1}{2}}\left[\int_{\phi=0}^{2\pi}(H_{\phi 2}\,H^*_{\phi 2})\,a\,d\phi\right]s$$

$$=\left(\frac{\omega\,\mu_0}{2\sigma_{cu}}\right)^{\frac{1}{2}}\frac{\pi\,a\,s\,(\sigma_2^2+\omega^2\,\epsilon_2^2)}{u_2\,u_2^*}\,|\,B\,J_1\,(\,ju_2\,a)+C\,Y_1\,(\,ju_2\,a)\,|^2 \qquad [16b]$$

$$P_3=\frac{1}{2}\left(\frac{\omega\,\mu_0}{2\sigma_{cu}}\right)^{\frac{1}{2}}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}(H_{\phi 2}\,H^*_{\phi 2})\,\rho\,d\rho\,d\phi$$

$$=\left(\frac{\omega\,\mu_0}{2\sigma_{cu}}\right)^{\frac{1}{2}}\frac{(\sigma_2^2+\omega^2\,\epsilon_2^2)}{u_2\,u_2^*}\,[I_1+I_2+I_3+I_4] \qquad [16c]$$

$$P_e=2\cdot\frac{1}{2}\left(\frac{\omega\,\mu_e}{2\,\sigma_e}\right)^{\frac{1}{2}}\int_{\rho=b}^{d}\int_{\phi=0}^{2\pi}|\,H_{\phi 3}\,|^2\,\rho\,d\rho\,d\phi$$

$$=\left(\frac{2\,\omega\,\mu_e}{\sigma_e}\right)^{\frac{1}{2}}\frac{\pi\,D^2\,\omega^2\,\epsilon_0^2}{u_3\,u_3^*\,(u_3^{*2}-u_3^2)}\,[\,-ju_3^*\,\rho\,H_1^{(1)}\,(\,ju_3\,\rho)\,H_0^{(2)}\,(\,-ju_3^*\,\rho)$$

$$-ju_3\,\rho\,H_0^{(1)}\,(\,ju_3\,\rho)\,H_1^{(2)}\,(\,-ju_3^*\,\rho)]\,{}_{\rho=b}^{\rho=d} \qquad [16d]$$

Total power lost in the resonator is

$$P_{\text{Total}}=\frac{(P_1+P_2+P_3)\,L}{s+t}+P_e \qquad [17]$$

where, s = spacing of discs *i.e.*, width of groove

t = thickness of discs

and L = length of the structure.

7. QUALITY FACTOR OF RESONATOR

The quality factor Q of the resonator is defined in terms of the energy stored and power lost as

$$Q=\frac{W_{M2}+W_{M3}}{P_{\text{Total}}} \qquad [18]$$

which yields

$$Q/\omega = \frac{\dfrac{2\pi\, L\, \mu_2\, (\sigma_2^2+\omega^2\,\epsilon_2^2)}{u_2\, u_2^*}\,[I_1+I_2+I_3+I_4] + \dfrac{2\pi L\,\mu_0\,\omega^2\,\epsilon_0^2\, D^2}{u_3\, u_3^*\,(u_3^* \; - u_3^2)}\; Y}{\left[\left\{\left(\dfrac{\omega\;\mu_0}{2\;\sigma_2}\right)^{1/2}\dfrac{\pi\; D^2\,\omega^2\,\epsilon_0^2\, b\; t}{u_3\; u_2^*}\; H_1^{(1)}(\,ju_3\, b)\; H_1^{(2)}(-ju_3^*\; b) + \left(\dfrac{\omega\;\mu_0}{2\;\sigma_{cu}}\right)^{1/2}\dfrac{\pi\; a\; s\;(\sigma_2^2+\omega^2\,\epsilon_2^2)}{u_2\; u_2^*}\;\left|\, B\, J_1\,(\,ju_2\; a) + C\; Y_1\,(\,ju_2\; a)\,\right|^2 + \left(\dfrac{\omega\;\mu_0}{2\;\sigma_{cu}}\right)^{1/2}\dfrac{(\sigma_2^2+\omega^2\,\epsilon_2^2)}{u_2\; u_2^*}\;(I_1+I_2+I_3+I_4)\right\}\dfrac{L}{s+t} + P_r\right]} \quad [19]$$

where,

$$P_e = \left(\frac{2\;\omega\;\mu_e}{\sigma_e}\right)^{1/2}\frac{\pi\; D^2\,\omega^2\,\epsilon_0^2}{u_3\; u_3^*\;(u_3^{*2}-u_3^2)}\; Y$$

$$Y = [\,-ju_3^*\;\rho\;H_1^{(1)}(\,ju_3\,\rho)\;H_0^{(2)}(-ju_3^*\;\rho) - ju_3\,\rho\;H_0^{(1)}(\,ju_3\;\rho)\;H_1^{(2)}(-ju_3^*\;\rho)]_{\rho=b}^{d} \quad [19a]$$

8. Resonance Frequency

At resonance $\quad W_E = W_{M2}$ [20]

where, $\quad W_E = W_{E2} + W_{E3}$

$$W_M = W_{M2} + W_{M3} \quad [20a]$$

Using (20) and equations (12-15) the following expression is obtained.

$$\frac{2\;L\;\mu_0\;\omega^2\,\epsilon_0^2\, D^2}{u_3\, u^{3*}\;(u_3^{*\;2}-u_3^2)}\; Y + \frac{2\;\pi\;L\;\mu_2\,(\sigma_2^2+\omega^2\;\epsilon_2^2)}{u_2\; u_2^*}\;[I_1+I_2+I_3+I_4]$$

$$= 2\;\pi\;L\;\;\mu_2\left(R_1^{(1)}+R_1^{(2)}+R_1^{(3)}+R_1^{(4)}\right)+2\pi\;\epsilon_2\;\frac{n^2\;\pi^2}{L\,u_2\,u_2^*}(I_1+I_2+I_3+I_4) + \frac{2\pi\;L\,\epsilon_0\,D^2}{(u_3^{*2}-u_3^2)}\left[\;X+\frac{n^2\;\pi^2}{L^2\;u_3\;u_3^*}\;Y\right] \quad [21]$$

Since

$$J_{-1}\,(\,ju_2^*\;\;\rho) = \frac{-H_1{}^{(1)}\,(-ju_2^*\;\rho) - H_1{}^{(2)}\,(-ju_2^*\;\rho)}{2}$$

$$= -J_1\,(-ju_2^*\;\rho)$$

$$Y_{-1}(ju_2\rho) = -Y_1(ju_2\rho)$$

$$R_1^{(1)} = \frac{BB^*}{u_2^{*\,2} - u_2^2}\Big[ju_2^*\rho\, J_0(ju_2\rho)\, J_1(-ju_2^*\rho) + (ju_2\rho)\, J_0(-ju_2^*\rho)\, J_1(ju_2^*\rho)\Big]_{\rho=a}^{b}$$

Similarly for $R_1^{(2)}$, $R_1^{(3)}$ and $R_1^{(4)}$

$$X = \Big[ju_3^*\rho\, H_0^{(1)}(ju_3\rho)\, H_1^{(2)}(-ju_3^*\rho) + ju_3\rho\, H_1^{(1)}(ju_3\rho)\, H_0^{(2)}(-ju_3^*\rho)\Big]_{\rho=b}^{\rho=d}$$

$$Y = [-ju_3^*\rho\, H_1^{(1)}(ju_3\rho)\, H_0^{(2)}(-ju_3^*\rho) - ju_3\rho\, H_0^{(1)}(ju_3\rho)\, H_1^{(2)}(-ju_3\rho)]_{\rho=b}^{d}$$

If however, the structure parameters are such that $W_{M2} << W_{M3}$ and $W_{E2} << W_{E3}$, then the resonance frequency is derived from the equation

$$W_{M3} = W_{E3}$$

which leads to

$$X + \frac{n^2\pi^2}{L^2 u_3 u_3^*}\, Y = \frac{\omega^2}{C^2 (u_3 u_3^*)}\, Y$$

which yields

$$f_{\text{resonance}} = \left[\frac{XC^2 u_3 u_3^*}{4\pi^2 Y} + \frac{C^2 n^2}{4L^2}\right]^{1/2} \tag{22}$$

9. EVALUATION OF B AND C IN TERMS OF D

The excitation constants B and C are evaluated in terms of the constant D by using proper field components and appropriate boundary conditions.

$$C = \frac{D}{X'}\left[H_0^{(1)}(ju_3 b)\, J_1(ju_2 b) - \frac{H_1^{(1)}(ju_3 b)\, J_0(ju_2 b)}{Y'}\right]$$

and

$$B = D\left[\frac{H_1^{(1)}(ju_3 b)}{J_1(ju_2 b)}\frac{1}{Y'} - \frac{1}{X}\left\{H_0^{(1)}(ju_3 b)\, Y_1(b\, ju_2 b) - \frac{Y_1(ju_2 b)\, H_1^{(1)}(ju_3 b)\, J_0(ju_2 b)}{Y'}\right\}\right] \tag{23}$$

where,

$$X' = \frac{2}{\pi j\, u_2 b}$$

$$Y' = \frac{(\sigma_2 + j\omega\epsilon_2)u_3}{j\omega\epsilon_0 u_2} \tag{23a}$$

10. Argument Approximation

In order to simplify the relations for Q and resonance frequency the following two sets of approximations may be made.

Case 1 : Assuming $\sigma_2 \simeq 0$ in medium 2, the following approximations are made.

$$ju_3 b << 1, \quad ju_2 b << 1, \quad ju_2 a << 1, \quad ju_1 a >> 1 \tag{24}$$

which leades to the following approximations forms for the functions

$$J_0\,(ju_2\rho) = 1$$

$$J_1\,(ju_2\rho) = \frac{ju_2\,\rho}{2}$$

$$Y_0\,(ju_2\,\rho) = \frac{j}{2\,3} + \frac{2}{\pi}\,\log\,u_2\rho + 0.18$$

$$Y_1\,(ju_2\rho) = -\,\frac{2}{j\,\pi\,u_2\rho}$$

$$H_0^{(1)}\,(ju_3\,\rho) = j\,\frac{2}{\pi}(m - jn')$$

$$H_0^{(2)}(ju_3^*\,\rho) = -j\,\frac{2}{\pi}(m + j\,n')$$

$$H_1^{(1)}(ju_3\rho) = -\,\frac{2}{\pi u_3\rho}$$

$$H_1^{(2)}\,(-\,ju_3^*\rho) = -\,\frac{2}{\pi u_3^*\,\rho} \tag{25}$$

Case 2: The conductivity $\sigma_2 \neq 0$ in medium 2, but is given as a function of spacing $\sigma_2 = \sigma_{cu}\,\frac{t}{s+t}$.

In this case the following assumptions are justified.

$$ju_3 b << 1, \quad ju_2 b >> 1, \quad ju_2 a >> 1, \quad ju_1 a >> 1 \tag{26}$$

which lead to

$$J_0(ju_2\rho) = \left(\frac{2}{\pi\, ju_2\rho}\right)^{1/2}\cos\left(ju_2\rho - \frac{\pi}{4}\right)$$

$$J_1(ju_2\rho) = \left(\frac{2}{\pi\, ju_2\rho}\right)^{1/2}\cos\left(ju_2\rho - \frac{3\pi}{4}\right)$$

$$Y_0(ju_2\rho) = \left(\frac{2}{\pi\, ju_2\rho}\right)^{1/2}\sin\left(ju_2\rho - \frac{\pi}{4}\right)$$

$$Y_1(ju_2\rho) = \left(\frac{2}{\pi\, ju_2\rho}\right)^{1/2}\sin\left(ju_2\rho - \frac{3\pi}{4}\right) \tag{27}$$

11. APPROXIMATE RELATIONS OF POWER AND ENERGY

The power and energy relations under approximations made above reduce to,

Case 1:

$$P_1 = \frac{16 D^2 \omega^2 \epsilon_0^2\, t}{b\,(a_3^2 + b_3^2)^2}\left(\frac{\mu_0}{\epsilon_2}\right)^{1/2} \tag{28a}$$

$$P_2 = \frac{\pi a \rho \omega^2 \epsilon_2^2}{(a_2^2 + b_2^2)}\left(\frac{\omega\,\mu_0}{2\sigma_{cu}}\right)^{1/2}\left|\frac{Bu_2 a}{2} + \frac{2C}{u_2 a}\right|^2 \tag{28b}$$

$$P_3 = \frac{\pi\,\omega^2\,\epsilon_2^2}{(a_2^2 + b_2^2)}\,[I_1' + I_2' + I_3' + I_4']\left(\frac{\omega\,\mu_0}{2\sigma_{cu}}\right)^{1/2} \tag{28c}$$

$$P_c = \frac{-\pi\, D^2 \omega^2 \epsilon_0^2 (m' - m)}{2\,(a_3^2 + b_3^2)^2}\left(\frac{2\,\omega\,\mu_c}{\sigma_c}\right)^{1/2} \tag{28d}$$

$$W_{M2} = \frac{2\pi L\,\mu_2 \omega^2\,\epsilon_2^2}{(a_2^2 + b_2^2)}\,[I_1' + I_2' + I_3' + I_4'] \tag{28e}$$

$$W_{M3} = \frac{-\,8\,L\,\mu_0\,D^2\,\omega^2\,\epsilon_0^2}{(a_3^2 + b_3^2)^2}\,(m' - m) \tag{28f}$$

where,

$$I_1' = 0$$

$$I_2' = \frac{C^2}{(a_2^2+b_2^2)\,a_2\,b_2}\left[-\frac{(a_2^2-b_2^2)}{2\cdot 3} + 2a_2\,b_2\,(0\cdot 18) + \frac{4}{\pi}\,a_2\,b_2 \log b/a\right]$$

$$I_3' + I_4' = \frac{B\,C\,(a_2^2+b_2^2)}{4\,a_2\,b_2}\left[\frac{8\,b_2\,a_2}{(a_2^2+b_2^2)^2} - \frac{(b^2-a^2)}{2\,3} - j\,\frac{(b^2-a^2)}{\pi}\log\frac{(a_2-j\,b_2)^2}{(a_2^2+b_2^2)}\right]$$

$$m' = \tfrac{1}{2}\ \ln\,(0.89\,b)^2 \quad (a_3^2+b_3^2)$$

$$m = \tfrac{1}{2}\ \ln\,(0.89\,d)^2 \quad (a_3^2+b_3^2)$$

$$n = \tan^{-1} b_3/a_3 \qquad (28g)$$

Case 2 :

$$P_1 = \frac{4\,D^2\,\omega^2\,\epsilon_0^2\,l}{(a_3^2+b_3^2)_3^2}\left(\frac{\omega\,\mu_0}{2\,\sigma 2)}\right)^{1/2} \qquad [29a]$$

$$P_2 = \left(\frac{\omega\,\mu_2}{2\,\sigma_{cu}}\right)^{1/2}\frac{2\,s\,(b_2-j\,a_2)}{(a_2^2+b_2^2)^2}\,(\sigma_2^2+\omega^2\,\epsilon_2^2)$$
$$\times\ |\,B\,\cos[(b_2+ja_2)-(3\pi/4)]+C\,\sin[(b_2+ja_2)-(3\pi/4]\,|^2 \qquad [29b]$$

$$P_3 = \left(\frac{\omega\,\mu 2}{2\,\sigma_{cu}}\right)^{1/2}\frac{(\sigma_2^2+\omega^2\,\epsilon_2^2)}{(a_2^2+b_2^2)}\,[I_1''+I_2''+I_3''+I_4''] \qquad [29c]$$

$$Pe = -\left(\frac{2\,\omega\,\mu_c}{\sigma_c}\right)^{1/2}\frac{\pi\,D^2\,\omega\,\epsilon_0^2}{2\,(a_3^2+b_3^2)^2}\,(m'-m)$$

$$W_{M2} = \frac{2\pi\,L\,\mu_2\,(\sigma_2^2+\omega^2\,\epsilon_2^2)}{a_2^2+b_2^2}\,[I_2''+I_2''+I_3''+I_4'']$$

$$W_{M3} = \frac{-8L\,\mu_0\,D^2\omega^2\epsilon_0^2\,(m'-m)}{(a_3^2+b_3^2)^2}$$

where,

$$I_1'' = \frac{BB^*}{2\pi\,(a_2^2+b_2^2)^{1/2}\,a_2b_2}\,[a_2\,(\cos 2\,b_2b-\cos 2ab_2)\ -b_2\,(\sinh 2a_2b-\sinh 2a_2a)]$$

$$I_2'' = \frac{-CC^*}{2\pi\,a_2\,b_2\,(a_2^2+b_2^2)^{1/2}}\,[a_2\,(\cos 2\,b\,b_2-\cos 2a\,b_2)-b_2\,(\sinh 2\,a_2\,b-\sinh 2a_2\,a)]$$

$$I_3'' = \frac{BC^*}{2\pi\,a_2\,b_2\,(a_{22}^{2})^{1/2}+b}\,[a_2\,(\sin 2\,b\,b_2-\sin 2a\,b_2)-\ jb_2\,(\cosh b_2\,b-\cosh b_2\,a)\,]$$

$$I_4'' = \frac{B^*\,C}{2\,\pi\,a_2\,b_2\,(a_2^2+b_2^2)^{1/2}}[a_2\,(\sin 2bb_2-\sin 2\,ab_2)+jb_2\,(\cosh 2\,bb_2-\cosh 2aa_2)]$$

The radical propagation constants u_i $(i=1, 2, 3)$ are related to the axial propagation constant γ as follows :

$$-(\gamma^2+u_1^2)=-j\,\omega\mu_0\,(\sigma_1+j\omega\epsilon_1)$$

$$-(\gamma_2+u_2^2)=-j\,\omega\mu_0\,(\sigma_2+j\omega\epsilon_2)$$

$$-(\gamma^2+u_3^2)=\omega\mu_0\epsilon_0$$

12. Approximate Expressions for Quality Factor

The expression for Q (equation 19) reduces to the following expressions in the two cases of approximations.

Case 1 :

$$\frac{Q}{\omega}=\frac{\dfrac{-8L\,\mu_0\,D^2\omega^2\epsilon_0^2}{\pi\,(a_3^2+b_3^2)^2}(m'-m)+\dfrac{2\pi\,L\,\mu_2\omega^2\epsilon_2^2}{(a_2^2+b_2^2)}(I_1'+I_2'+I_3'+I_4')}{\left[\dfrac{L}{s+t}\left\{\dfrac{16\,D^2\,\omega^2\,\epsilon_0^2\,t}{b\,(a_3^2+b_3^2)^2}\left(\dfrac{\mu_0}{\epsilon_2}\right)^{1/2}+\dfrac{\pi\,a\,s\,\omega^2\epsilon_2^2}{a_2^2+b_2^2}\left(\dfrac{\omega\mu_0}{2\sigma_{cu}}\right)^{1/2}\left|\dfrac{Bu_2a}{2}+\dfrac{2\,C_c}{u_2a}\right|^2+\dfrac{\pi\,\omega^2\epsilon_2^2}{a_2^2+b_2^2}\left(\dfrac{\omega\mu_0}{2\sigma_{cu}}\right)^{1/2}(I_1'+I_2'+I_3'+I_4')\right\}-\dfrac{\pi D^2\omega^2\epsilon_0^2\,(m'-m)}{2\,(a_3^2+b_3^2)^2}\left(\dfrac{2\omega\mu_e}{\sigma_e}\right)^{1/2}\right]}$$

Case 2 :

$$\frac{Q}{\omega}=\frac{\dfrac{-8L\mu_0D^2\,\omega^2\epsilon_0^2\,(m'-m)}{\pi\,(a_3^2+b_3^2)^2}+\dfrac{2\pi\,L\,\mu_2\,(\sigma_2^2+\omega^2\epsilon_2^2)}{a_2^2+b_2^2}(I_1''+I_2''+I_3''+I_4'')}{\left[\dfrac{L}{s+t}\left\{\left(\dfrac{\omega\mu_2}{2\sigma_2}\right)^{1/2}\dfrac{16D^2\omega^2\epsilon_0^2t}{\pi b\,(a_3^2+b_3^2)^2}+\left(\dfrac{\omega\mu_0}{2\sigma_{cu}}\right)^{1/2}\dfrac{\pi\,s\,(\sigma_2^2+\omega^2\,\epsilon_2^2)\,(b_2-ja_2)}{(a_2^2+b_2^2)^2}\left|B\cos\left(ju_2a-\dfrac{3\pi}{4}\right)+C\sin\left(ju_2b-\dfrac{3\pi}{4}\right)\right|^2+\left(\dfrac{\omega\mu_0}{2\sigma_{cu}}\right)^{1/2}\dfrac{(\sigma_2^2-\omega^2\epsilon_2^2)}{(a_2^2+b_2^2)}(I_1''+{}_2''I+I_3''+I_4'')\right\}-\left(\dfrac{2\omega\mu_e}{\sigma_e}\right)^{1/2}\dfrac{\pi\,D^2\,\omega^2\epsilon_0^2}{2(a_3^2+b_3^2)}(m'-m)\right]}$$

13. Approximate Expression for Resonance Frequency

Equation [21] can be simplified by using appropriate argument approximation.

Case 1 :

Equation [21] in this case is reduced to

$$\omega^2 \left[\frac{\mu_0^2 \epsilon_0^2 D^2}{u_3 u_3^* (u_3^{*2} - u_3^2)} Y + \frac{\mu_2 \epsilon_2^2}{u_2 u_2^*} (I_1' + I_2' + I_3' + I_4')\right.$$

$$= \epsilon_2 (R_1^{(1)\prime} + R_1^{(2)\prime} + R_1^{(3)\prime} + R_1^{(4)\prime}) + \frac{\epsilon_2 n^2 \pi^2}{L^2 u_2 u_2^*} (I_1' + I_2' + I_3' + I_4')$$

$$+ \frac{\epsilon_0 D^2}{u_3^{*2} - u_3^2} \left(X + \frac{n^2 \pi^2}{L^2 u_3 u_3^*} Y\right)$$

which gives the resonance frequency

$$f_{res} = \frac{1}{2\pi} \left\{ \frac{\epsilon_2 (R_1^{(1)\prime} + R_1^{(2)\prime} + R_1^{(3)\prime} + R_1^{(4)\prime}) + \epsilon_2 \dfrac{n^2 \pi^2}{L^2 u_2 u_2^*} (I_1' + I_2' + I_3' + I_4') + \dfrac{\epsilon_0 D^2}{u_3^{*2} - u_3^2} \left(X + \dfrac{n^2 \pi^2}{L^2 u_3 u_3} Y \right)}{\dfrac{\mu_0 \epsilon_0^2 D^2 Y}{u_3 u_3^* (u_3^{*2} - u_3^2)} + \dfrac{\mu_2 \epsilon_2^2}{u_2 u_2^*} (I_1' + I_2' + I_3' + I_4')} \right\}^{\frac{1}{2}}$$

Case 2 :

$$f_{res} = \frac{1}{2\pi} \left\{ \frac{\epsilon_2 (R_1^{(1)\prime\prime} + R_1^{(2)\prime\prime} + R_1^{(3)\prime\prime} + R_4^{(4)\prime\prime} = \left(\dfrac{\epsilon_2 n^2 \pi^2}{L^2} - \mu_2 \sigma_2^2\right) \dfrac{(I_1'' + I_2'' + I_3'' + I_4'')}{u_2 u_2^*} + \dfrac{2\epsilon_0 D^2}{u_3^{*2} - u_3^2} \left(X + \dfrac{n^2 \pi^2}{L^2 u_3 u_3^*} Y \right)}{\dfrac{\mu_0 \epsilon_0^2 D^2}{u_3 u_3^* (u_3^{*2} - u_3^2)} Y + \dfrac{\mu_2 \epsilon_2^2}{u_2 u_2^*} (I_1'' + I_2'' + I_3'' + I_4'')} \right\}^{1/2}$$

where,

$$R_1^{(1)}=\frac{B\,B^*}{u_2^{*2}-u_2^2}\Big[\,ju_2^*\,\rho\,J_0\,(ju_2\rho)\,J_1\,(-ju_2^*\,\rho)+ju_2\rho\,J_0\,(-ju_2^*\,\rho)\,J_1\,(ju_2\rho)\Big]_{\rho=a}^{b}$$

$$R_1^{(2)}=\frac{C\,C^*}{u_2^{*\,2}-u_2^2}\Big[\,ju_2^*\,\rho\;Y_0(ju_2\,\rho)\,Y_1\,(-ju_2^*\,\rho)+j\,u_2\,\rho\;Y_0(-j\,u_2^*\,\rho)\,Y_1\,(j\,u_2\,\rho)\Big]_{\rho=a}^{\rho=b}$$

$$R_1^{(3)}=\frac{B\,C^*}{u_2^{*2}-u_2^2}\Big[\,j\,u_2^*\,\rho\;J_0(j\,u_2\,\rho)\;Y_1\,(-j\,u_2^*\,\rho)+j\,u_2\,\rho\;J_1\,(j\,u_2\,\rho)\;Y_0(-j\,u_2^*\,\rho)\Big]_{\rho=a}^{b}$$

$$R_1^{(4)}=\frac{B\,C^*}{u_2^{*2}-u_2^2}\Big[\,j\,u_2^*\,\rho\;J_1\,(-j\,u_2^*\,\rho)\;Y_0(j\,u_2\,\rho)+j\,u_2\,\rho\;J_0(-j\,u_2^*\,\rho)\;Y_1\,(j\,u_2\,\rho)\Big]_{\rho=a}^{b}$$

When $W_{M2}<<W_{M3}$ and $W_{E2}<<W_{E3}$, equation (22) changes under argument approximation to

$$f_{\text{resonance}}=\left[\frac{C^2}{8\,\pi^2}\,\frac{(a_3^2+b_3^2)^2\,n'}{(m'-m)\;a_3b_3}+\frac{C^2n^2}{4\,L^2}\right]^{1/2}$$

since X and Y reduces to

$$Y=\frac{+8}{\pi^2\,(a_3^2+b_3^2)}\,[2\,j\;a_3\,b_3\;(m-m')]$$

$$X=\frac{-8\;j\,n'}{\pi^2}$$

where,

$n'=$ arc tan (b_3/a_3)

$m=\frac{1}{2}\,ln\;(0{\cdot}89\,d)^2\;(a_3^2+b_3^2)$

$m'=\frac{1}{2}\,ln\;(0{\cdot}89\,b)^2\;(a_3^2+b_3^2)$

$C=$ free space velocity

$L=$ length of the resonator

$n=$ no of half cycle variations in the z-direction.

Applying argument approximation to $R_1^{(1)}$, $R_1^{(2)}$, $R_1^{(3)}$, $R_1^{(4)}$, these expressions will reduce to :

Case 1 :—

$$R_1^{(1)} = 0$$

$$R_1^{(2)'} = \frac{C^2}{\pi a_2 b_2} \left[\frac{1}{2 \cdot 3} - 2 \tan^{-1} (b_2/a_2) \right]$$

$$R_1^{(3)'} + R_1^{(4)'} = \frac{B C (b^2 - a^2)}{2 a_2 b_2} \left[\frac{2 (a_2^2 - b_2^2)}{2 \cdot 3} + a_2 b_2 (0 \cdot 18) \right.$$

$$\left. + \frac{a_2 b_2}{\pi} \log (a_2^2 + b_2^2) (b/a) - \frac{a_2^2 - b_2^2}{b^2 - a^2} \tan^{-1} (b_2/a_2) \right]$$

Case 2 :—

$$R_1^{(1)''} = \frac{B^2}{2 \pi a_2 b_2 (a_2^2 + b_2^2)^{1/2}} [b_2 (\sin h\ 2 a_2 b - \sin h\ 2 a_2 a$$

$$- a_2 (\cos 2 b_2 b - \cos b_2 a)]$$

$$R_1^{(2)''} = \frac{C^2}{2 \pi a_2 b_2 (a_2^2 + b_2^2)^{1/2}} [a_2 (\cos (2 b_2 b) - \cos (2 b_2 a)$$

$$+ b_2 (\sin h\ 2 a_2 b - \sin h\ 2 a_2 a)]$$

$$R_1^{(3)''} + R_1^{(4)} = \frac{B C}{\pi b_2 (a_2^2 + b_2^2)^{1/2}} [\sin 2 b_2 a - \sin 2 b_2 b]$$

The Q-factor, etc have been computed as functions of disc radius b and disc spacing s with the help of IBM 360 computer. The results are given in Figure 2.

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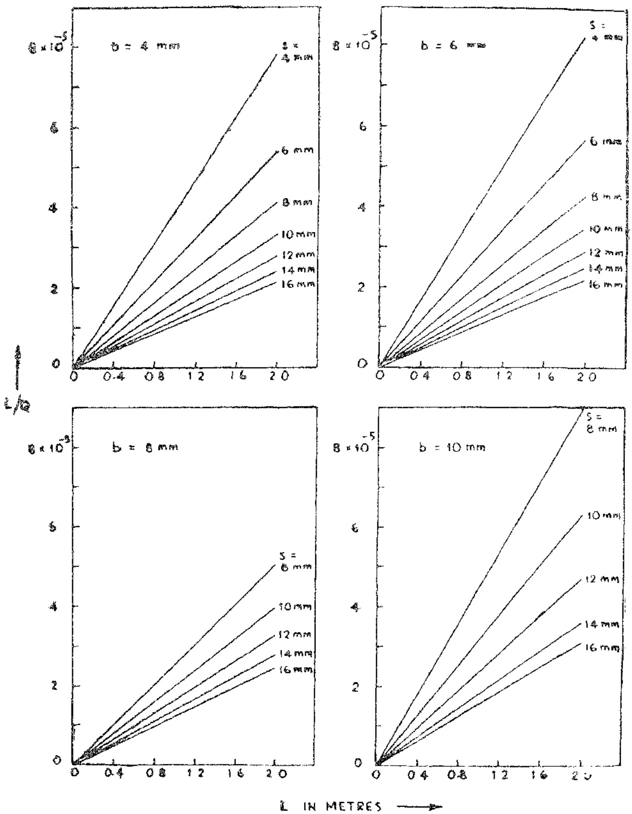
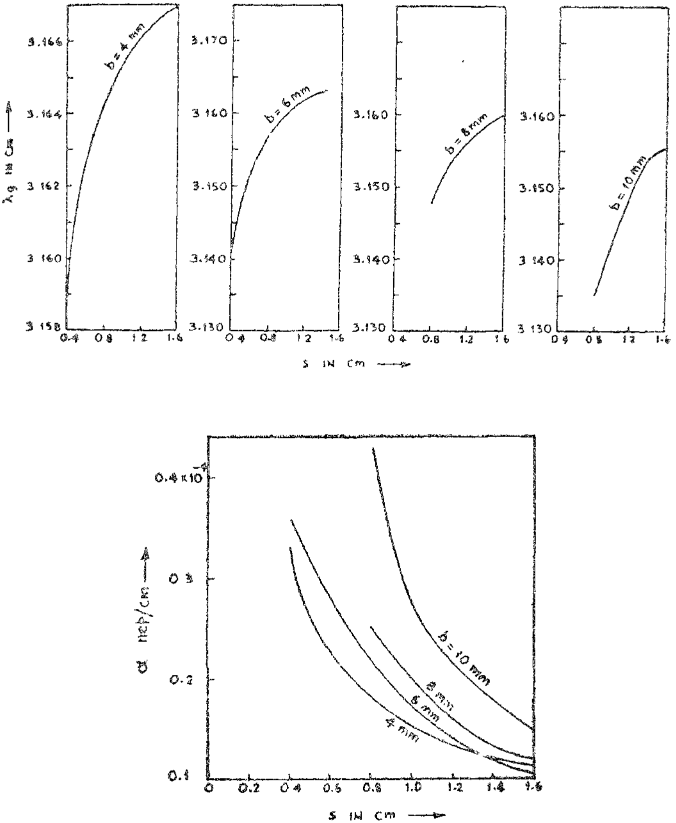


L = Length of the Resonator in Metres, S = Disc spacing, b = Disc Radius, f = 9460 Mc/s.

Fig. 2 (*a*)

Plots of L/Q *Vs* L for A Corrugated Metallic Cylindrical Structure



λ_g = Guide Wavelength, s = Disc Spacing. b = Disc Radius α = Attenuation Coastant

FIG 2 (b)

Plots of λ_g Vs S and α Vs S

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