DUAL PHASE DAMPING IN SIMPLE SHOCK MOUNTS

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ABSTRACT

Response of a simple mounted system to time varying transients like rounded step and pulse displacements applied at the foundation is investigated. The system employs a dual-phase damping characteristic defined by a set of four parameters. Variations of the peak acceleration and displacement with these four parameters are computed. Optimum values of the parameters at which the minimum peak acceleration or minimum peak displacement occurs are then sought. The main advantage in using a dual-phase damping over single phase constant damping is the facility of minimising the peak acceleration or displacement.

Key words: Shock isolation; Damping.

1. INTRODUCTION

This paper presents an investigation of the response of a simple mounted system (Fig. 1. 1 *a* and *b*) having nonlinear dual-phase damping to a transient time varying input impressed on the foundation. The peak acceleration and displacement of the mass M are particularly considered here in bringing out the variations of their respective magnitudes with different damping parameters of the dual-phase damping. Optimum values of the parameters are found in order to achieve either minimum acceleration or displacement, one subject to the constraint of the other.

The purpose of the shock absorber is to reduce the acceleration and displacement of the mass M. The components of M are less affected if they experience less acceleration and displacement. This paper considers the effect of the dual-phase damping instead of constant damping on the peak acceleration and displacement, as the effect of the former is found to be better than that of constant damping.

The damping system may be represented by a linear spring and a non-linear dashpot (Fig. 1. 1 *a* and *b*) along with the dual-phase damping

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characteristic defined by the four parameters ζ_1, ζ_2, A and B. Realisation of the dual-phase damping characteristics is evident in the sketches of Fig. 1.2 where a spring-loaded ball or a return valve adapt to the changes in damping force offering the desired characteristics [1]. The four parameters could be changed by changing the initial and final area of the orifice (for ζ_1 and ζ_2) and by changing the spring constant (for A and B). Some aircraft landing gear systems may be said to display somewhat similar behaviour.

Snowdon [1] has worked out the transient response of such a system for a set of parameters and indicated that the peak acceleration and displacement could be minimised employing the dual-phase damping characteristics. The excursions of the peak values of acceleration and displacement transmitted to the mass M for inputs like rounded step and rounded pulse displacements are sought by the authors for different combinations of values of A, B, ζ_1, ζ_2 (Fig. 1. 1 b) which information, it is felt, would be useful in the design of shock mounts.

2. System Equations

The equation of motion for the system shown in Fig. 1. 1 a may be written as

$$M\ddot{Z}_1 + C\dot{Z}_1 + KZ_1 = C\dot{Z}_3 + KZ_3$$

with the initial conditions

$$\dot{Z}_1(o) = Z_1(o) = 0$$

It is convenient to rewrite the equation as

$$\ddot{Z}_1 + 2\zeta w_0\dot{Z}_1 + w_0^2 Z_1 = 2\zeta w_0\dot{Z}_3 + w_0^2 Z_3$$

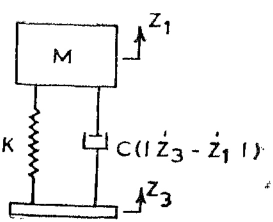

Fig. 1 a

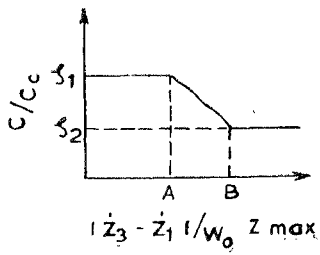

Fig. 1 b

where,

$$w_0^2 = \frac{K}{M}; \quad 2\zeta w_0 = \frac{C}{M}; \quad \zeta = \frac{C}{C_c} = \frac{C}{2\sqrt{KM}}$$

Dividing throughout by $w_0^2 Z_{max}$ results in

$$\frac{\ddot{Z}_1}{w_0^2 Z_{max}} + 2\zeta \frac{\dot{Z}_1}{w_0 Z_{max}} + \frac{Z_1}{Z_{max}} = 2\zeta \frac{\dot{Z}_3}{w_0 Z_{max}} + \frac{Z_3}{Z_{max}}$$

In the non-dimensional form, the above equation becomes

$$\ddot{Y} + 2\zeta \dot{Y} + Y = 2\zeta \dot{Z}_3' + Z_3'$$

where,

$\dot{Y}(o) = 0 = Y(o)$ and Z_3', $\dot{Z}_3'$ are respectively the non-dimensional displacement and velocity.

According to the characteristic shown in Fig. 1. 1 *b*, ζ takes values as under

$$\zeta = \zeta_1 \quad \text{where} \quad |\dot{Z}_3' - \dot{Y}| \leqslant A$$

$$= \frac{\zeta_1 - \zeta_2}{A - B}(X - A) + \zeta_1; \quad |\dot{Z}_3' - \dot{Y}| \text{ lies between } A \text{ and } B$$

$$X = |\dot{Z}_3' - \dot{Y}|$$

$$= \zeta_2; \quad |\dot{Z}_3' - \dot{Y}| \geqslant B.$$

Optimisation for the peak acceleration or displacement though could be done by gradient method, the authors prefer to use direct search because the latter furnishes information about the variations of the peak acceleration and displacement with the four parameters along with the optimum value.

The non-linear differential equation is solved by Runge-Kutta method using a digital computer IBM 360/44. The inputs considered are the rounded step and rounded pulse displacements (Fig. 2 *a*, *b*) for these possess finite first and second derivatives for all values of t including $t = 0$; application of these transients to the foundation of the simple system provides physically realistic values for the velocity and acceleration of both the foundation and the mounted mass M. The reference inputs are defined by

$$Z_3 = 0, \quad t < 0$$

$$= Z_{max}[1 - e^{-\gamma w_0 t}(1 + \gamma w_0 t)], \quad t \geqslant 0 \text{ (Rounded step)}$$

$$= Z_{max}\frac{e^2}{4}(\gamma w_0 t)^2 e^{-\gamma w_0 t}, \qquad t \geqslant 0 \text{ (Rounded pulse)}$$

where,

$$\gamma = \frac{T}{2\tau} = \frac{\pi}{w_0 \tau}; \quad w_0 = \sqrt{\frac{K}{M}}$$

The rise time τ of the rounded step displacement is defined as the time required for the displacement to reach 82% of its final value. The duration τ of the displacement pulse is defined as the length of an equivalent rectangular pulse that has the same area ($Z_{max}\, e^2/2\gamma w_0$) as that of the rounded pulse, but which is higher by 17·6% than Z_{max}.

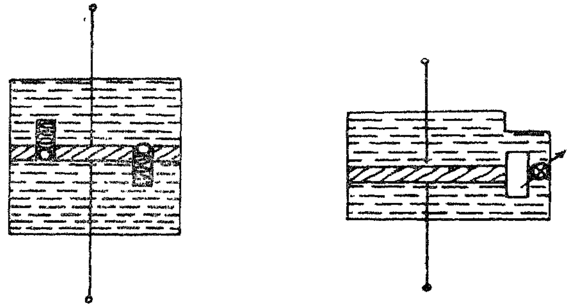

FIG. 1-2

Range of parameters considered in the study for getting data are :

$A \;=\; 0\,(0{\cdot}2)\,1$

$B \;=\; (A + 0{\cdot}2)\;(0{\cdot}2)\;(A + 1)$

$\zeta_1 \;=\; 0{\cdot}1\;(0{\cdot}1)\;0{\cdot}9$

$\zeta_2 \;=\; 0{\cdot}1,\; 0{\cdot}2,\; 0{\cdot}0$

$\gamma \;=\; 10,\; 50$

$\gamma = 10$ defines mild shock and $\gamma = 50$ defines severe shock. These two values are chosen for analysis.

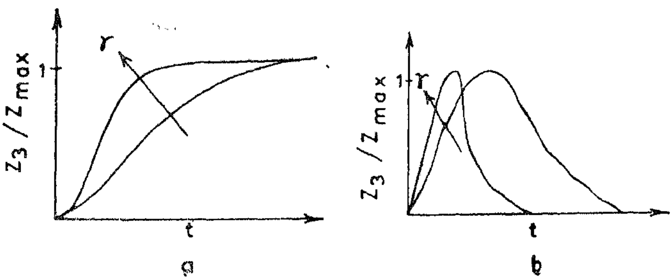

FIG. 2

In the following section, peak acceleration refers to max $|\ddot{Y}|$, peak displacement to max $|Y|$.

3. Results and Discussion

From the data computed, it is found that the peak acceleration and displacement show a variation with A, B, ζ_1, ζ_2 as typified in the accompanying graphs, Fig. 3. 1 through 3.32.

3.1 *Input : Rounded Step*

3.1.1 : $\gamma = 10$ and $\zeta_2 = 0 \cdot 0$

The peak acceleration (i) increases with increase in ζ_1 for the values of A, B, ξ_1, less than or equal to 0·4, 0·6, 0·6 respectively, (ii) decreases and then increases with increase in ζ_1 in the range where A lies between 0·2 and 1·0 and B between 1·0 and 1·4, (iii) increases with increase in ζ_1 for values of B and A greater than 1·4 and 0·6 respectively. These variations are

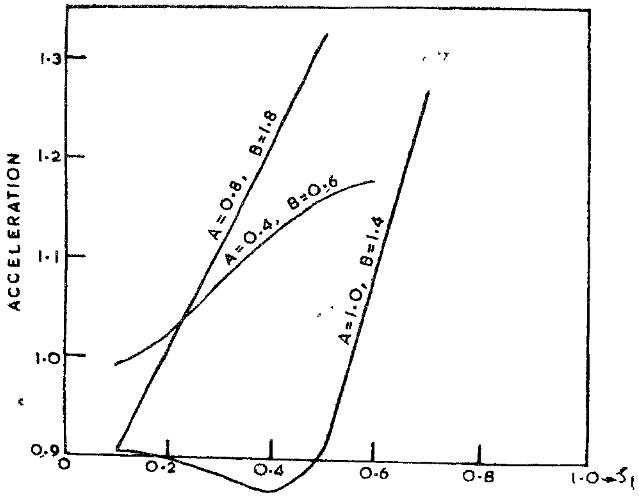

FIG. 3.1 ROUNDED STEP $\gamma = 10$, $\zeta_2 = 0{\cdot}0$

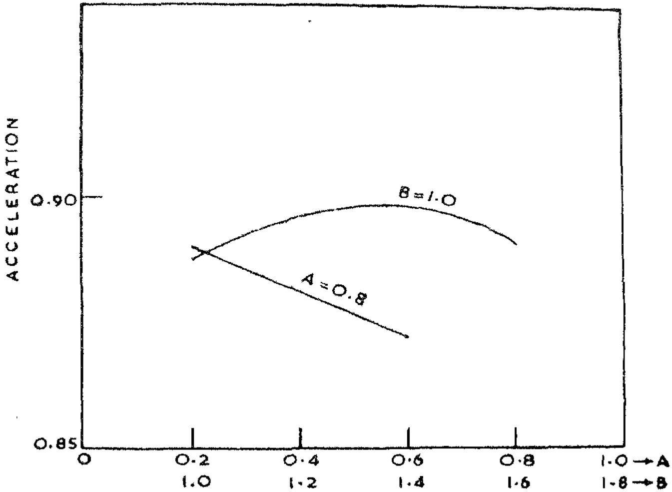

FIG. 3.2 ROUNDED STEP $\gamma = 10$, $\zeta_2 = 0.0$

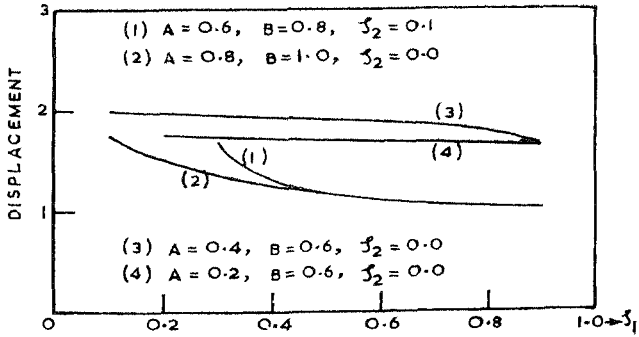

FIG. 3.3 ROUNDED STEP $\gamma = 10$

shown in Fig. 3.1, for certain typical sets of parameters. The minimum value of the peak acceleration found in case (ii) is noted to decrease with increase of B and increase and then decrease with increase of A (Fig. 3.2).

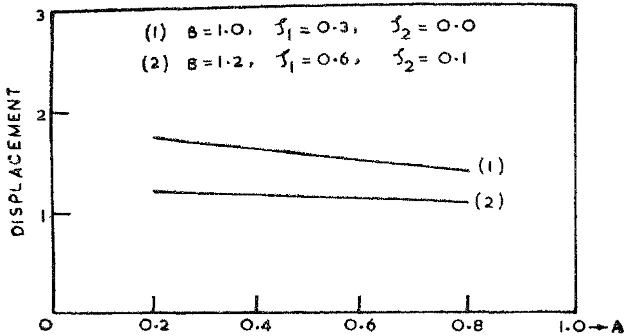

FIG. 3.4 ROUNDED STEP $\gamma = 10$

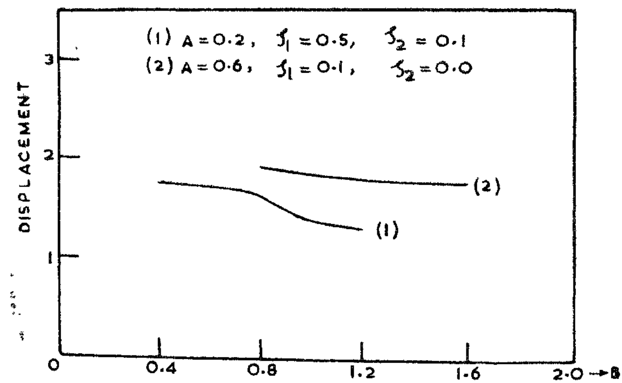

FIG. 3.5 ROUNDED STEP $\gamma = 10$

The peak displacement (i) decreases with increase in ζ_1. This decrement is found to be small for B less than 0·8 (Fig. 3.3); (ii) decreases with increase in A and B (Fig. 3.4, 3.5). Case (ii) is not true always. In many cases

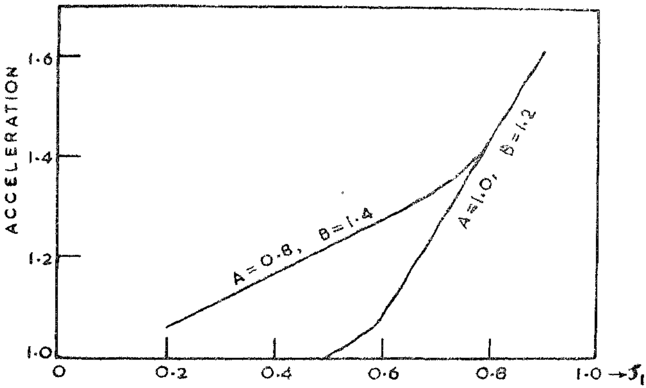

FIG. 3.6 ROUNDED STEP $\gamma = 10$, $\zeta_2 = 0.1$

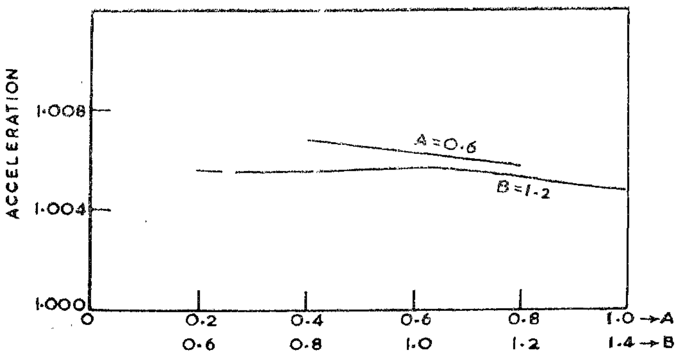

FIG. 3.7 ROUNDED STEP $\gamma = 10$, $\zeta_2 = 0.1$

it is found that the peak displacement increases with increase in A and B (for A greater than 0·6) for $\zeta_1 \geqslant \zeta_c$ where ζ_c itself varies from 0·2 to 0·5. But this increment is very small of the order of 1 in 10^3 or 1 in 10^2.

3.1.2 : $\gamma = 10$ and $\zeta_2 = 0 \cdot 1$

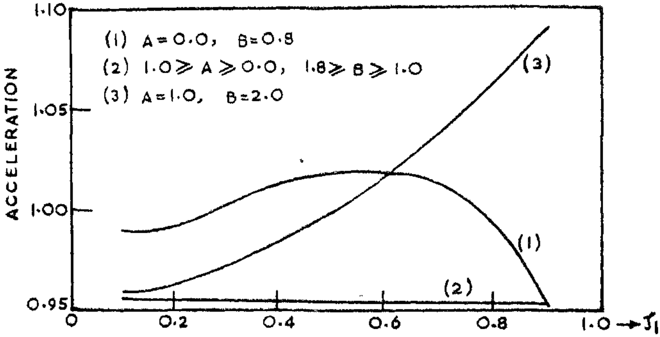

FIG. 3.8 ROUNDED STEP $\gamma = 50$, $\zeta_2 = 0.0$

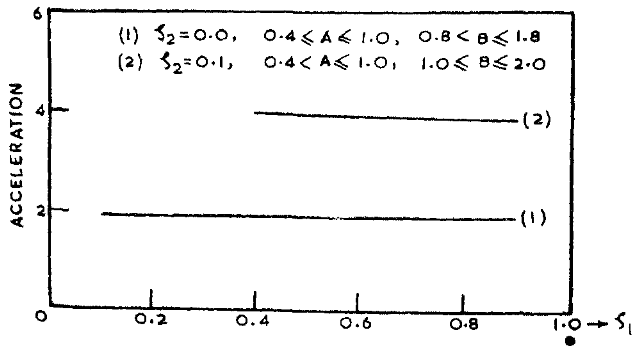

FIG. 3.9 ROUNDED STEP $\gamma = 50$

The peak acceleration (i) decreases with increase in ζ_1 for A equal to 0·2 and B less than or equal to 0·8. This decrement is very small (1 in 10^8); (ii) decreases and then increases with an increase in ζ_1 for the values of A, B lying between 1·0 to 0·2 and 1·2 to 0·6 respectively. This decrement is very small; (iii) increases with increase in ζ_1 for values of B and A greater

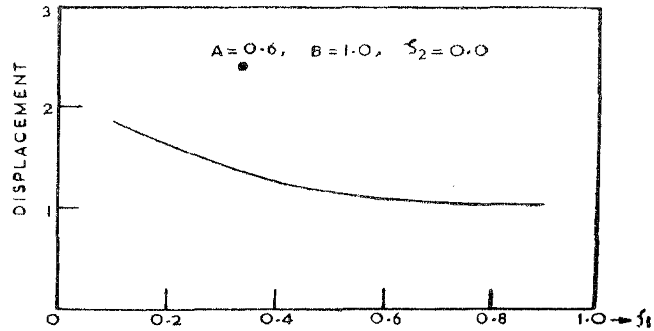

FIG. 3.10 ROUNDED STEP $\gamma = 50$

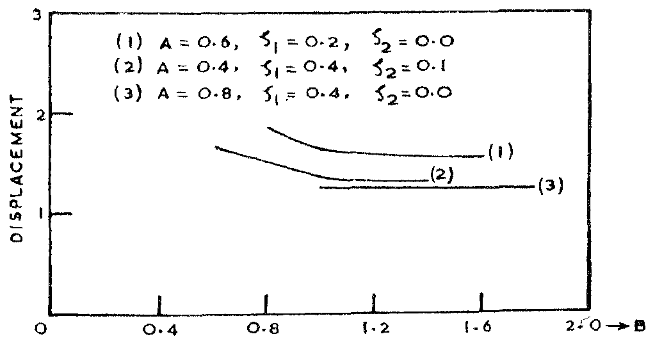

FIG. 3.11 ROUNDED STEP $\gamma = 50$

than 1·2 and 0·8 respectively (Fig. 3·6). The minimum of the peak acceleration obtained in case, (ii) is found to decrease with an increase in B and increase and then decrease with an increase in A (Fig. 3.7). The peak acceleration increases with increase in ζ_2 (Fig. 3. 14 a).

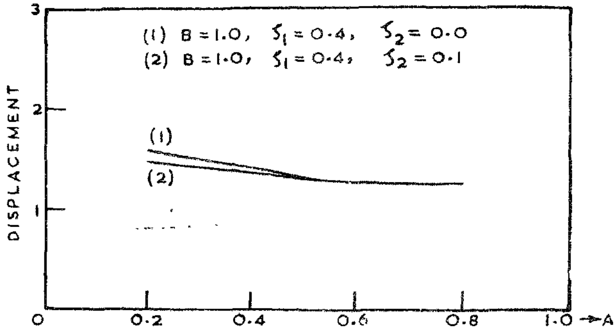

FIG. 3.12 ROUNDED STEP $\gamma = 50$

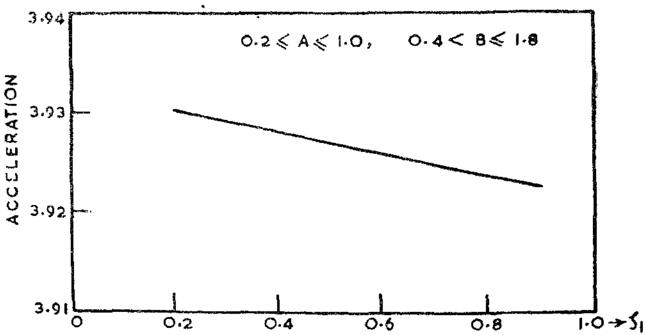

FIG. 3.13 ROUNDED STEP $\gamma = 50$, $\zeta_2 = 0.1$

The peak displacement (i) decreases with increase in ζ_1. This decrement is small for B less than 0·8 (Fig. 3.3); (ii) decreases with increase in B and A

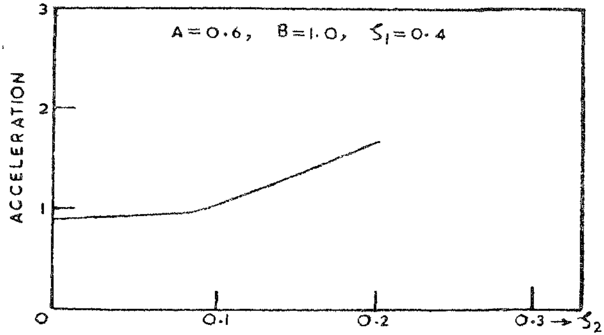

FIG. 3·14 a ROUNDED STEP $\gamma = 10$

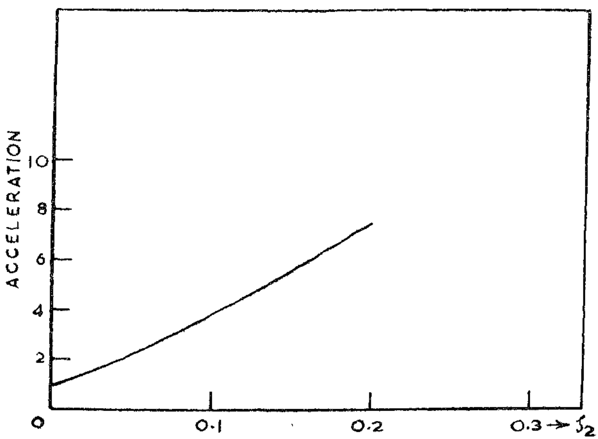

FIG. 3.14 b ROUNDED STEP $\gamma = 50$

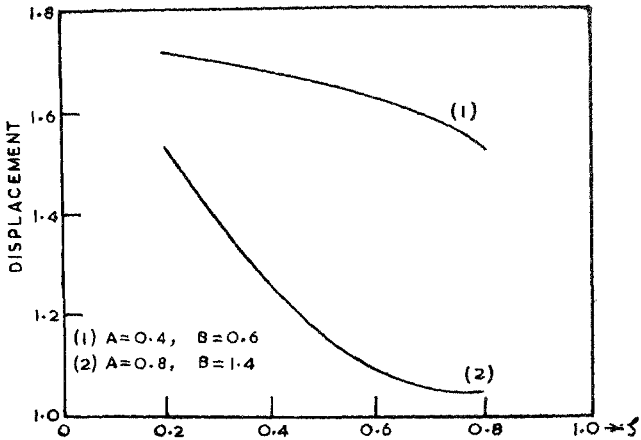

FIG. 3.15 ROUNDED STEP γ = 50, ζ_2 = 0.1

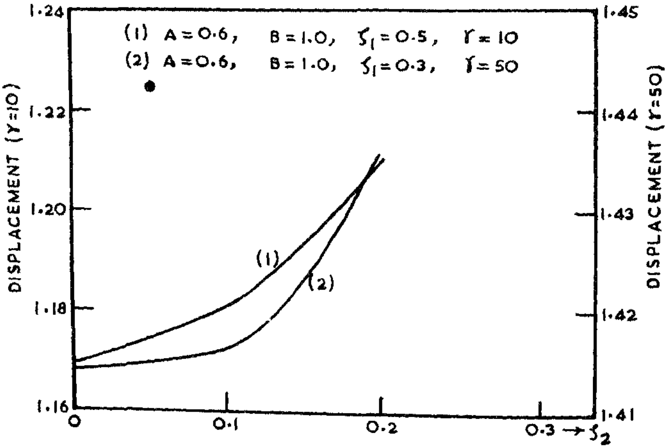

FIG. 3.16 ROUNDED STEP

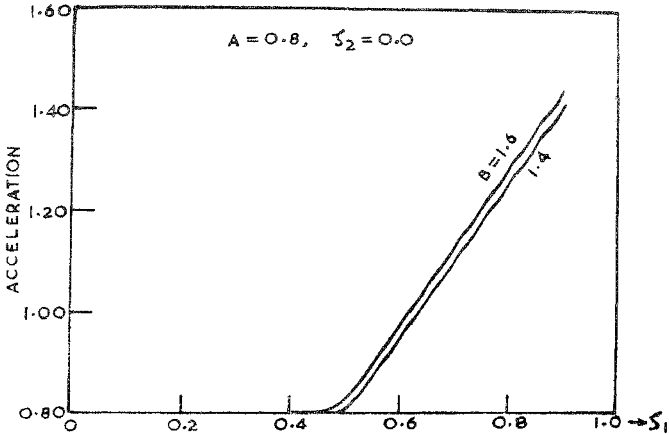

FIG. 3.17 ROUNDED PULSE γ: 10

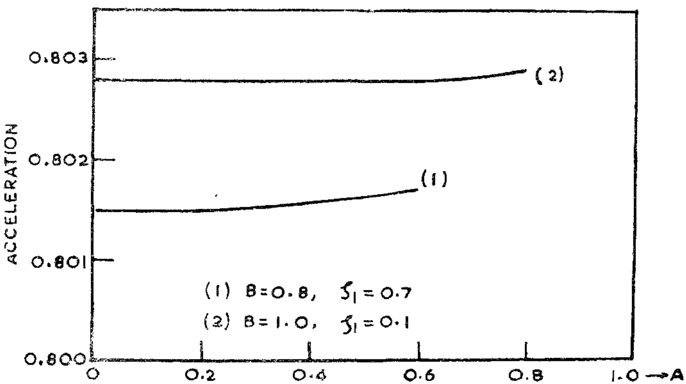

FIG. 3.18 ROUNDED PULSE $\gamma = 10$, $\zeta_2 = 0$

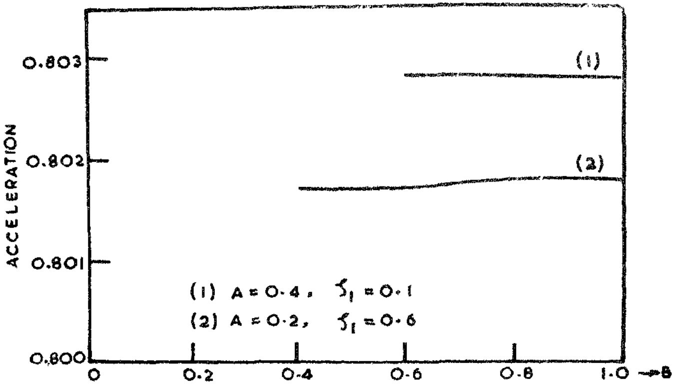

FIG. 3.19 ROUNDED PULSE $\gamma = 10$, $\zeta_2 = 0$

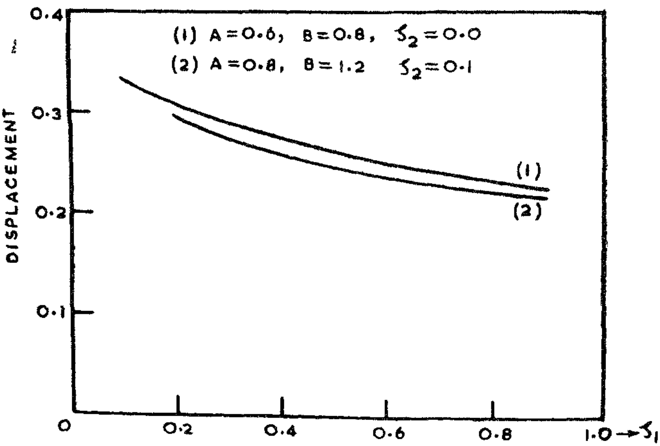

FIG. 3.20 ROUNDED PULSE $\gamma = 10$

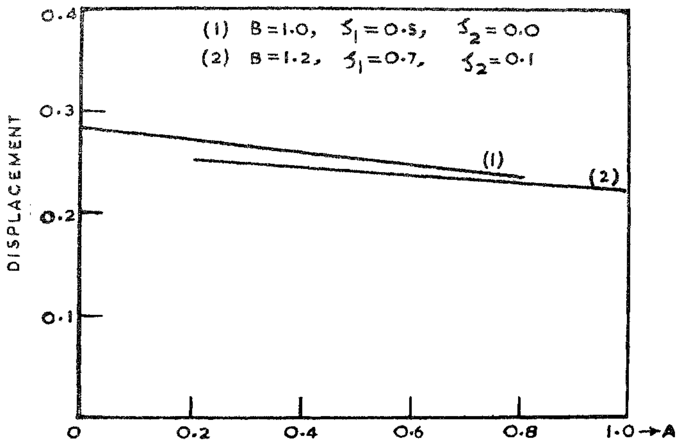

FIG. 3.21 ROUNDED PULSE $\gamma = 10$

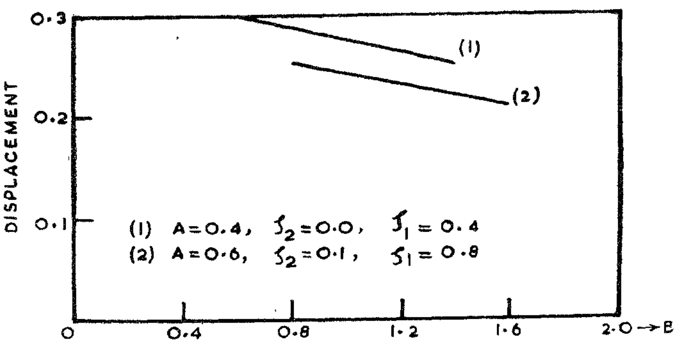

FIG. 3.22 ROUNDED PULSE $\gamma = 10$

(Figs. 3.4, 3.5). This is found to be violated for certain values of ζ_1. The peak displacement increases with increase in B and A (for A greater than or equal to 0·6) for ζ_1 greater than a certain value which varies from 0·3 to 0·5. This increment is very small.

3.1.3 : $\gamma = 50$ and $\zeta_2 = 0{\cdot}0$

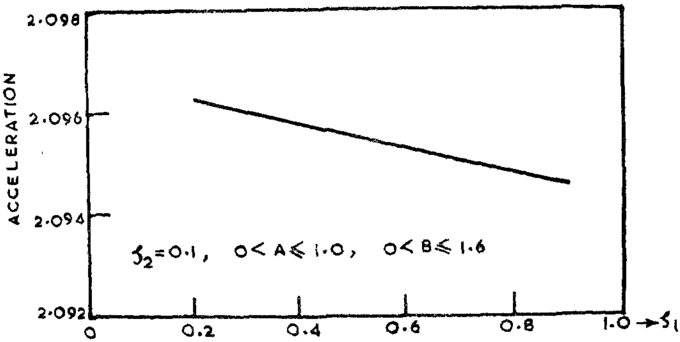

FIG. 3.23 ROUNDED PULSE γ = 10

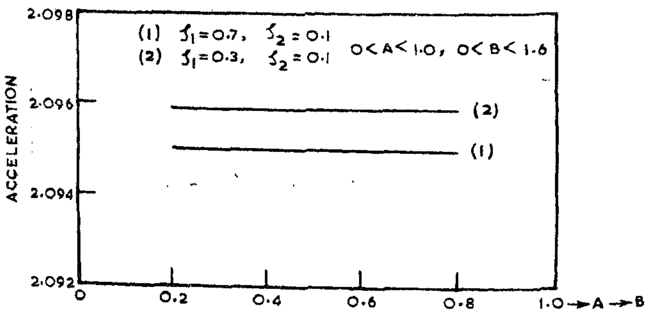

FIG. 3.24 ROUNDED PULSE γ = 10

The peak acceleration (i) increases and then decreases with increase in ζ_1 for A and B lying between 0·4 and 0·0, 0·8 and 0·2 respectively; (ii) decreases with increase in ζ_1 in the range where A lies between 1·0 to 0·0 and B between 1·8 to 1·0; (iii) increases with increase in ζ_1 for $A = 1·0$

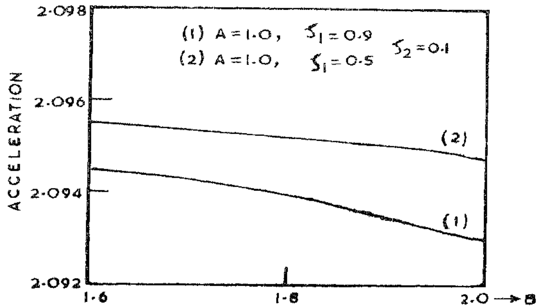

FIG. 3·25 ROUNDED PULSE $\Upsilon = 10$

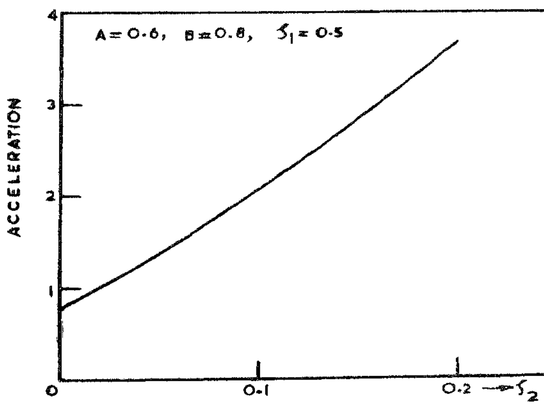

FIG. 3·26 ROUNDED PULSE $\Upsilon = 10$

and $B = 2 \cdot 0$ (Fig. 3·8); (iv) remains constant with A and B in the ranges given in case (ii) (Fig. 3.9).

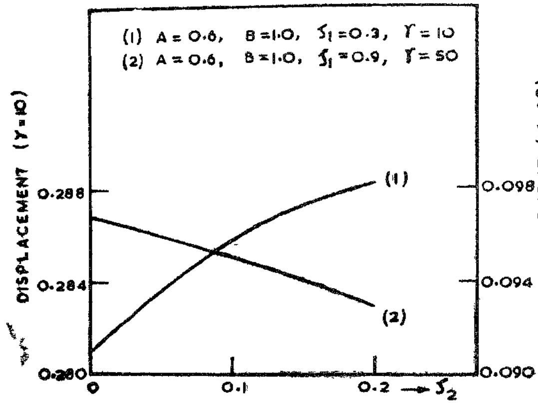

FIG. 3.27 ROUNDED PULSE

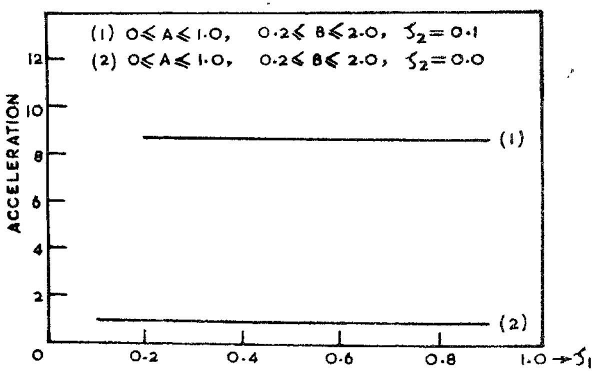

FIG. 3.28 ROUNDED PULSE $\gamma = 50$

The peak displacement (i) decreases with increase in ζ_1; (ii) decreases, with increase in B for A less than or equal to 0·6 and beyond this range it remains a constant; (iii) decreases with increase in A (Fig. 3.10, 3.11, 3.12).

3.1.4 : $\gamma = 50$ *and* $\zeta_2 = 0{\cdot}1$

The peak acceleration (i) decreases with increase in ζ_1 for A and B lying between 0·2 to 1·0 and 0·4 to 1·8 respectively; (ii) remains constant with A and B for a constant ζ_1 (Figs. 3.13, 3.9).

The peak acceleration increases with increase in ζ_2 (3.14 *b*).

The peak displacement (i) decreases with increase in ζ_1; (ii) decreases with increase in B for A less than or equal to 0·6 and remains constant with B for values of A greater than 0·6; (iii) decreases with increase in A (Figs. 3.15, 3.11, 3.12).

The peak displacement increases with increase in ζ_2 (Fig. 3.16).

3.2 *Input : Rounded Pulse*

3.2.1 : $\gamma = 10$ *and* $\zeta_2 = 0{\cdot}0$

The peak acceleration (i) decreases with increase in ζ_1 for ζ_1 less than or equal to 0·8 and A, B in the ranges 0 to 0·8 and 0 to 1·2 respectively. This decrement is very small, 1 in 10^3; (ii) decreases and then increases with increase in ζ_1 for A greater than or equal to 0·6 and B greater than or equal to 1·4; (iii) remains almost constant with A and B for the ranges 0 to 0·8 and 0 to 1·2 respectively (Figs. 3.17, 3.18, 3.19).

The peak displacement (i) decreases with increase in ζ_1; (ii) decreases with increase in A and B; (iii) cases (i) and (ii) hold good for $\zeta_2 = 0{\cdot}0$ and 0·1 (*See* Figs. 3.20, 3.21, 3.22).

3.2.2 : $\gamma = 10$ *and* $\zeta_2 = 0{\cdot}1$

The peak acceleration (i) decreases with increase in ζ_1 for A and B in the ranges 0 to 1·0 and 0 to 1·6 respectively; (ii) remains constant with A and B for the values of A and B lying between 0 to 1·0 and 0 to 1·6 respectively; (iii) decreases with increase in B for A greater than 0·8 and B greater than 1·6 (Figs. 3.23, 3.24, 3.25).

The peak acceleration increases with increase in ζ_2 (Fig. 3.26).

The peak displacement increases with increase in ζ_2 (Fig. 3.27).

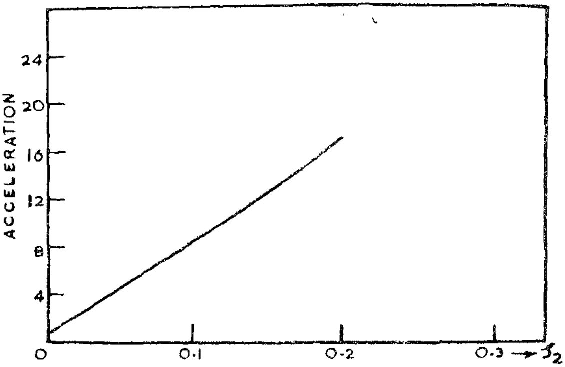

FIG. 3.29 ROUNDED PULSE $\gamma = 50$

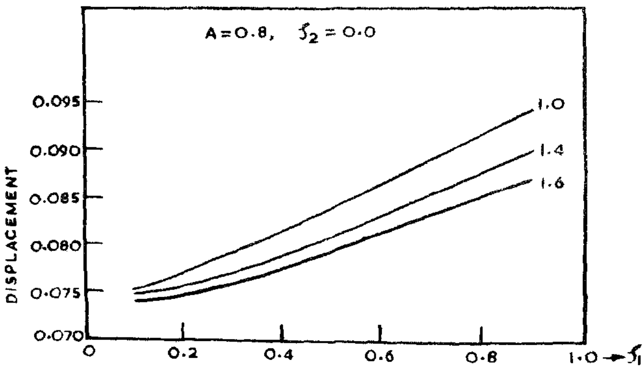

FIG. 3.30 ROUNDED PULSE $\gamma = 50$

3.2.3 : $\gamma = 50$ *and* $\zeta_2 = 0 \cdot 0,\ 0 \cdot 1$

The peak acceleration (i) decreases with increase in ζ_1. The decrement is very small; (ii) remains constant with A and B; (iii) increases with increase in ζ_2 (Figs. 3.28, 3.29).

The peak displacement (i) increases with increase in ζ_1; (ii) decreases with increase in A and B; (iii) decreases with increase in ζ_2 (Figs. 3.30, 3.31*, 3.32,* 3.27).

4. Illustration and Comparison

The values of the peak acceleration and displacement obtained with dual-phase damping are compared with that of constant damping. Examples are given.

4.1 *Rounded Step Displacement*

4.1.1 : $\gamma = 10$

The minimum value obtained for the peak acceleration is $0 \cdot 8631$. The values of the parameters are $A = 1 \cdot 0$, $B = 1 \cdot 4$, $\zeta_1 = 0 \cdot 4$ and $\zeta_2 = 0 \cdot 0$. The peak displacement for the above set is $1 \cdot 2636$. For $\zeta_1 \geqslant 0 \cdot 7$, the peak displacement in most cases is approximately $1 \cdot 0$. But the peak acceleration is different and greater than $0 \cdot 86$.

4.1.2 : $\gamma = 50$

The minimum peak acceleration got is $0 \cdot 9528$, with peak displacement approximately $1 \cdot 0$ for $\zeta_1 = 0 \cdot 9$, $\zeta_2 = 0 \cdot 0$ and A in the range 0 to $1 \cdot 0$ and B $1 \cdot 0$ to $1 \cdot 8$.

4.2 *Rounded Pulse*

4.2.1 : $\gamma = 10$

The minimum peak acceleration in this case is $0 \cdot 8010$ with a peak displacement $0 \cdot 3093$. The values of the parameters A, B, ζ_1, ζ_2 are $0 \cdot 2$, $0 \cdot 4$, $0 \cdot 9$ and $0 \cdot 0$ respectively.

The minimum peak displacement is $0 \cdot 1207$ with peak acceleration $1 \cdot 69$ for $A = 1 \cdot 0$, $B = 2 \cdot 0$, $\zeta_1 = 0 \cdot 9$ and $\zeta_2 = 0 \cdot 0$.

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* (A) indicates the scale marked for the curve with constant A; similarly for (B).

But, an optimum case could be chosen to be the one where the peak acceleration and displacement are 0·8017 and 0·1951 respectively. The values of the parameters are $A = 0{\cdot}8$, $B = 1{\cdot}0$, $\zeta_1 = 0{\cdot}9$ and $\zeta_2 = 0{\cdot}0$.

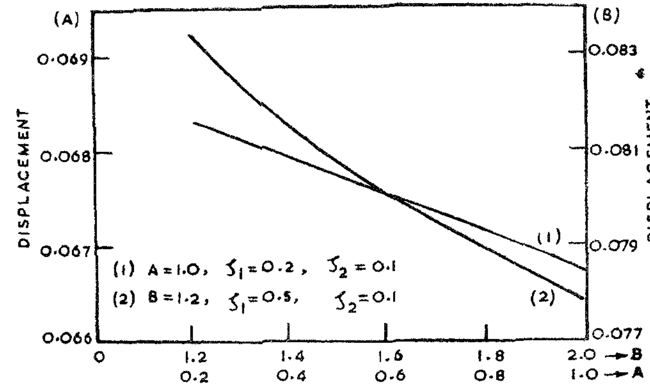

FIG. 3·31 ROUNDED PULSE γ = 50

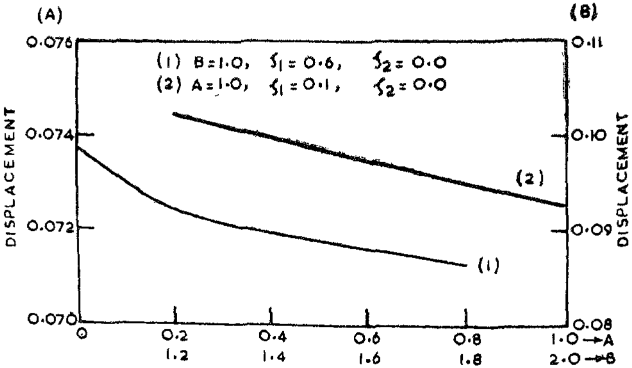

FIG. 3.32 ROUNDED PULSE γ = 50

4.2.2 : $\gamma = 50$

The minimum value of the peak acceleration obtained is $0 \cdot 9937$ and the peak displacement is about $0 \cdot 08$. The parameter values are $A = 1 \cdot 0$ $B = 2 \cdot 0$, $\zeta_1 = 0 \cdot 9$ and $\zeta_2 = 0 \cdot 0$.

4.3 *Comparison with Constant Damping*

Considering the first case (*i.e.*, $\gamma = 10$, step displacement) with constant damping $\zeta = 0 \cdot 4$, the peak acceleration of the mass M [2] is nearly $3 \cdot 0$ and the displacement is about $1 \cdot 5$. For low values of damping ($\zeta \leqslant 0 \cdot 05$) the peak acceleration is found to be less than $1 \cdot 0$ and the peak displacement is nearly $2 \cdot 0$. By using the dual-phase damping, it is possible to achieve low values of peak acceleration and displacement, *i.e.*, $0 \cdot 8631$ and $1 \cdot 2636$ respectively.

A similar behaviour is found in other cases also.

5. Conclusion

5.1. For all the cases considered here, the peak acceleration increases with increase in the value of ζ_2. The variation with ζ_1 is not much compared to that with ζ_2. It depends on the value of γ. If $\gamma = 50$, the variation is not much. If $\gamma = 10$, the peak acceleration decreases and then increases with increase in ζ_1, for many cases. This is observed for both inputs.

Irrespective of the type of input, the variation of peak acceleration with A and B is negligible for $\gamma = 50$.

5.2. The peak displacement increases with increase in ζ_2 for rounded step displacement for one set of A, B, ζ_1 and it decreases with increase in ζ_1 for a set of A, B, ζ_2.

In the case of rounded pulse displacement, for $\gamma = 10$, the peak displacement increases with increase in ζ_2 and for $\gamma = 50$, it decreases with increase in ζ_2.

The peak displacement, for the pulse case, decreases with increase in ζ_1 for $\gamma = 10$ and increases with increase in ζ_1 for $\gamma = 50$.

5.3. With dual-phase damping, the peak acceleration and displacement of the mass M could be reduced. By suitably selecting the parameters, the system could be optimised for either the peak acceleration or displacement keeping one or the other within bounds.

5.4. The effect of applying dual-phase damping to an aircraft landing gear is being investigated by the authors.

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