INFLUENCE OF FOUNDATION ROUGHNESS ON STRESSES AND DISPLACEMENTS UNDER FOUNDATIONS

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ABSTRACT

This investigation aims at evaluating the effect of foundation roughness on stresses and displacements inside the soil mass due to imposed axi-symmetrical loading with a nearly rigid foundation. The problem of rigid foundation has been simplified by assuming that the roughness at the foundation soil interface does not influence the distribution of normal contact stress. Using Hankel transforms, the mixed boundary problem has been reduced to the solution of duel integral equations. Numerical values for stresses and displacements have been obtained and compared with those of a smooth foundation. The effect of Poisson's ratio on stresses and displacements has been clearly demonstrated. It has been shown further that the surface settlement of the foundation, and the vertical stress inside the soil mass, are negligibly influenced by foundation roughness whereas the radial displacement and the radial stress are considerably influenced.

Key words : Contact problems, Elastic, Foundation engineering, Half-space, Rigid founda tion, Stresses and displacements.

INTRODUCTION

For the determination of stresses and displacements in a soil mass beneath foundations it is generally assumed that the contact between the foundation and the soil mass is frictionless. However, in most of the practical cases, foundations are rough rather than smooth. Galin [1] who investigated the special case of an indented half-plane with a variable coefficient of friction

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at the interface showed that a friction coefficient of 0·5 or more is sufficient to prevent any slippage at the interface. The actual value of the friction coefficient in case of foundations on granular media is close to 0·5 [2] and in cohesive soils, although the friction coefficient is low there is adhesion at the interface which prevents slippage. Hence, it is of interest to study the effect of foundation roughness on stresses and displacements in the soil mass and the special case when a rigid circular foundation is bonded to the surface has been considered herein.

The contact problems related to half-plane and half-space in the presence of adhesion have received much analytical attention in Russian literature [3, 4, 5, 6]. In all these works no numercial results concerning stresses and displacements have been presented. Conway *et al.* [7] have presented a numerical method for solving two and three dimensional contact problems.

In the present investigation stresses and displacements under a rigid circular foundation in the presence of perfect adhesion at the interface has been presented. This mixed boundary value problem gives rise to the solution of a set of simultaneous dual integral equations with two unknowns. Solution of this type of equations is available [8], but it is not suitable for numerical computations. Making a simplyfying assumption that the adhesion at the interface does not affect the distribution of normal contact stress [7], the mixed boundary problem now reduced to the solution of dual integral equations, which have been solved by the method of Erdely and Sneddon [8]. Numerical values of foundation settlement so obtained showed that the above assumption regarding normal stress distribution at the contact surface is quite reasonable. Numerical values of vertical and radial stresses as well as radial displacement have been obtained and these have been compared with the no roughness case. Further, the role of Poisson's ratio in the stress-displacement problem of a rough foundation has also been brought out.

MATHEMATICAL FORMULATION

Let a rigid foundation of radius a, with an equivalent uniformly distributed symmetrical normal load p, rest on the surfac of a semi-infinite medium (Fig. 1). The r and z axes of cylindrical co-ordinates lie along horizontal and vertical directions, the origin being at the centre of the contact surface Expressing the solution of Love's stress function in the form of Hankel integrals, the expressions for stresses and displacements become [9]

$$\sigma_z = \int_0^\infty \eta^3 \exp(-\eta z)\,[\eta A + (1 - 2\mu + \eta z)\,B]\; J_0(\eta r)\, d\eta \qquad (1)$$

$$\sigma_r = \int_0^\infty \eta^3 \exp(-\eta z)\,[-\eta A + (1 + 2\mu - \eta z)\,B]\; J_0(\eta r)\, d\eta$$

$$+ \frac{1}{r} \int_0^\infty \eta^2 \exp(-\eta z)\,[\eta A - (1 - \eta z)\,B]\; J_1(\eta r)\, d\eta \qquad (2)$$

$$\sigma_\theta = 2\mu \int_0^\infty B\eta^3 \exp(-\eta z)\, J_0(\eta r)\, d\eta$$

$$- \frac{1}{r} \int_0^\infty \eta^2 \exp(-\eta z)\,[\eta A - (1 - \eta z)\,B]\, J_1(\eta r)\, d\eta \qquad (3)$$

$$\tau_{rz} = \int_0^\infty \eta^3 \exp(-\eta z)\,[\eta A - (2\eta - \eta z)\,B]\; J_1(\eta r)\, d\eta \qquad (4)$$

$$w = \frac{1+\mu}{E} \int_0^\infty \eta^2 \exp(-\eta z)\,[\eta A + (2 - 4\mu + \eta z)\,B]\; J_0(\eta r)\, d\eta \qquad (5)$$

$$u = \frac{1+\mu}{E} \int_0^\infty \eta^2 \exp(-\eta z)\;[-\eta A + (1 - \eta z)\,B]\, J_1(\eta r)\, d\eta \qquad (6)$$

where, σ_r, σ_θ and σ_z are normal stresses along radial, tangential and vertical directions; τ_{rz} is the shear stress in any rz plane; u and w are radial and vertical deformations; A and B are integration constants; $J_n(x)$ is nth order Bessel function of the first kind with argument x : η is a dummy variable and E and μ are Young's modulus and Poisson's ratio of the medium.

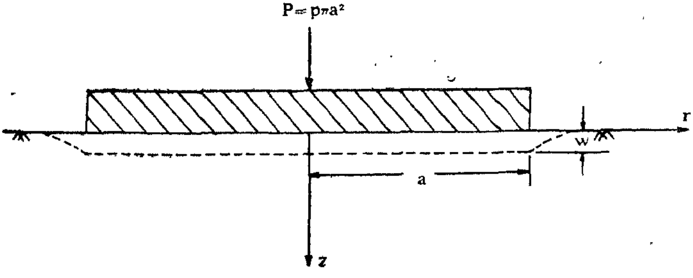

FIG. 1. Rigid Foundation on elastic half space.

Boundary conditons

With the assumption that adhesion at the interface does not influence the normal stress distribution at the interface. the boundary condition that is to be satisfied is

$$[\sigma z]_{z=0} = \begin{cases} -\dfrac{p}{2v\sqrt{1-\left(\dfrac{r}{a}\right)^2}}, & o < r < a \\ o, & r > a \end{cases} \tag{7}$$

For the condition of perfect adhesion between the foundation and the soil and remembering that beyond the loaded area the surface of the medium is shear stress free, we have

$$\text{(ii)}\ [u]_{z=0} = 0,\ o < r < a\ ;\ [\tau_{rz}]_{z=0} = o,\ r > a \tag{8}$$

SOLUTION

Using the above two boundary conditions Equations (1), (4) and (6) give rise to the following dual integral equations:

$$\int_0^\infty x^{-1} A(x) J_1(xR)\, dx = -\frac{pa^5}{4(1-\mu)} \frac{R}{1+\sqrt{1-R^2}},\ O < R < 1$$

$$\int_0^\infty A(x) J_1(xR)\, dx = -\mu pa^5 \frac{1}{R\sqrt{R^2-1}},\ R > 1 \tag{9}$$

where

$$R = \frac{r}{a},\ x = \eta a,\ A(x) = \eta^4 A \tag{10}$$

The solution of the above integral equation (8), obtained after performing several integrations and algebraic manipulations is

$$A(x) = -\frac{pa^5\, xg(x)}{\pi} \tag{11}$$

in which,

$$g(x) = \frac{x}{4(1-\mu)} g_1(x) + \mu g_2(x) \tag{12}$$

$$g_1(x) = \sum_{m=1}^{\infty} \frac{(-1)^{m+1}\left(1 + 1/3 + \cdots + \frac{1}{2m-1}\, x^2\right)^{(m-1)}}{m\,(2m-1)} \tag{13}$$

$$g_2(x) = 2\,[S_1(x)\sin(x) + S_2(x).x\cos(x)$$
$$+ \{\pi/2 - S_i(x)\}\sin(x)/x] \tag{14}$$

$$S_1(x) = \sum_{m=1}^{\infty} (-1)^{m+1}\, x^{2\,(m-1)} \sum_{n=1}^{\infty} \frac{(2n-1)\,!}{(2n+2m-1)\,!} \tag{15}$$

$$S_2(x) = \sum_{m=1}^{\infty} (-1)^{m+1}\, x^{2\,(m-1)} \sum_{n=1}^{\infty} \frac{(2n-2)\,!}{(2n+2m-1)\,!} \tag{16}$$

and $S_i(x)$ is the sinc integral defined as

$$S_i(x) = \int_0^1 \frac{\sin(xt)}{t}\, dt \tag{17}$$

For expressing stresses and displacements in non-dimensional form, the following additional notation is introduced:

$$Z = \frac{z}{a}\,. \tag{18}$$

With this, the expressions for stresses and displacements become

$$\sigma_z = \frac{p}{1-2\mu} \int_0^{\infty} [x^2 Z g(x)/\pi - (1 - 2\mu + xZ)\sin(x)/2]$$
$$\times \exp(-xZ)\, J_0(xR)\, dx, \tag{19}$$

$$\sigma_r = \frac{p}{(1-2\mu)} \int_0^{\infty} \Big\{[(2 - xZ)\, x g(x)/\pi - (1 + 2\mu - xZ)\sin(x)/2]$$
$$\times J_0(xR) - [(2 - 2\mu - xZ)\, g(x)/\pi - (1 - xZ)\sin(x)/2x]$$
$$\frac{J_1(xR)}{R}\Big\} \exp(-xZ)\, dx, \tag{20}$$

$$\sigma_\theta = \frac{p}{1-2\mu} \int_0^\infty \{[(2 - 2\mu - xZ)\, g(x)/\pi - (1 - xZ) \sin(x)/2x]$$

$$\times \frac{J_1(xR)}{R} + 2\mu\, [xg(x)/n - \sin(x)/2]\, J_0(xR) \exp(-xZ)\, dx, \tag{21}$$

$$\tau_{rz} = \frac{p}{1-2\mu} \int_0^\infty \{[\mu - xZ/2) \sin(x) - x(1 - xZ)\, g(x)/\pi]\}$$

$$J_1(xR) \exp(-xZ)\, dx, \tag{22}$$

$$w = \frac{1+\mu}{1-2\mu} \cdot \frac{pa}{E} \int_0^\infty [(2 - 4\mu + xZ) \sin(x)/2x -$$

$$(1 - 2\mu + xZ)\, g(x)/\pi]\, J_0(xR) \exp(-xZ)\, dx, \tag{23}$$

$$u = \frac{1+\mu}{1-2\mu} \cdot \frac{pa}{E} \int_0^\infty [(2 - 2\mu - xZ)\, g(x)/\pi - (1 - xZ)$$

$$\times \sin(x)/2x]\, J_1(xR) \exp(-xZ)\, dx. \tag{24}$$

In foundation engineering problems, a quantity which is of immense practical value is the settlement of the foundation, W (*i.e.*, w at $Z = 0$). Setting $Z = 0$ in Equation (23) and writing in the standard form as

$$W = \frac{2(1-\mu^2)}{E}\, pa\, I_w, \tag{25}$$

the expression for the influence coefficient for the foundation settlement I_w, becomes

$$I_w = \frac{1}{2(1-\mu)} [\pi/2 - \int_0^\infty \frac{g(x)}{\pi} J_0(xR)\, dx] \tag{26}$$

For comparing stresses and displacements under rough foundations with, those of smooth foundations, the expressions for the later case are used from Sneddon [9].

Computational Procedure

For evaluation of the infinite integrals appearing in the expressions for stresses and displacements, numerical integration by Gaussian quadrature (10) was used. The integrations were performed in steps with an increment of π Values of the functions $g_1(x)$ and $g_2(x)$ were calculated once for all for the values of x required by Gauss method using 32 points and stored in the computer memory.

Figure 2 shows the functions $g_1(x)$, $g_2(x)$ and $g(x)$. The functions are gradually decreasing alternating functions having properties almost similar to those of $J_1(x)$ and $J_0(x)$.

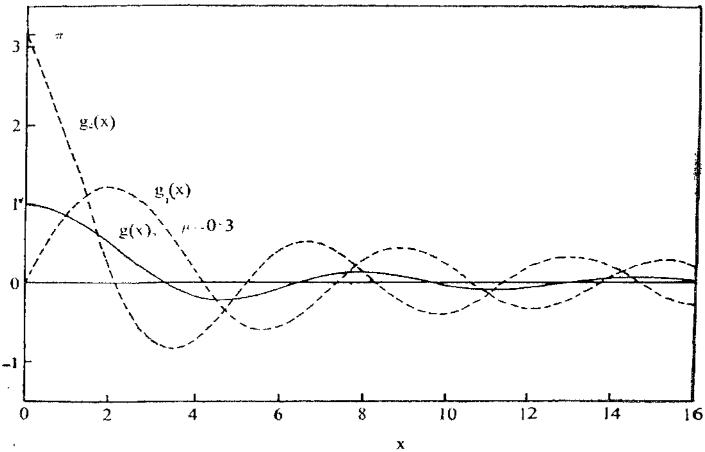

FIG. 2. Functions $g_1(x)$, $g_2(x)$, $g(x)$.

Evaluation of the function $g_2(x)$ poses a special problem as the convergence of the series $S_1(x)$ and $S_2(x)$ is extremely poor for the first few values of m. This difficulty was overcome by summing the n series for $m = 1$ to 4 by using Polygamma functions (10).

For numerical integration, values of the special functions $J_0(x)$, $J_1(x)$ and $S_0(x)$ are also necessary. The Bessel functions were calculated from their polynomial approximations (10) and the sine integral from its Chebyshev expansion (11).

NUMERICAL RESULTS AND DISCUSSION

Contrary to the case of a smooth foundation, the vertical stress and the influence coefficient for surface settlement of a rough foundation are influenced by the Poisson's ratio. The stresses at the contact surface are statically determinate and can be shown to be

$$\sigma_r = \sigma_\theta = \frac{\mu}{1-\mu}\sigma_z$$

giving $\sigma_r = \sigma_\theta = \sigma_z$ for $\mu = 0 \cdot 5$. Therefore for a Poisson's ratio of $0 \cdot 5$, the contact surface is in a state of hydrostatic stress and thus the shear stress at the interface vanishes. Hence, for $\mu = 0 \cdot 5$, the stresses and displacements under a rough foundation will be identical with those under a smooth foundation (7).

Again, from Saint-Venant's principle, the effect of foundation roughness would decrease as one moves away from the contact surface. Hence, it can be expected that for higher values of $\sqrt{R^2 + Z^2}$, the stresses and displacements for rough and smooth foundations will be almost identical.

Physically, adhesion at the interface means restraint of the soil mass in the lateral direction. This implies that below the loaded area, a rough foundation when compared to a smooth foundation, will have (*a*) lesser vertical deformation; and (*b*) lower values of vertical stress.

Foundation settlement

Figure 3 shows the influence values, I_w for a rough foundation for $\mu = 0$, $0 \cdot 3$ and $0 \cdot 5$. The figure clearly demonstrates that the adhesion at the interface has some influence on the normal contact stress as otherwise, for a particular value of μ, I_w would have been constant for all values of R, ranging from 0 to 1. In fact, I_w remains almost constant upto about $R = 0 \cdot 7$ and then increases. The deviation of I_w from a constant value, due to the assumed distribution of contact stress, is maximum at the edge. The deviation also decreases with μ and for $\mu = 0 \cdot 5$, I_w becomes constant with respect to R and has a value of $n/4$, which is a well known result 9) for a smooth foundation. However, even for $u = 0$ difference in I_w values between the edge and the centre is not very significant for practical purpose and is about 7 Per cent only.

It is known that the difference between the settlement of a rigid foundation and the average surface settlement of a area with uniformly distributed load is only 7·5 Per cent Therefore, in this case where assumed contact *stress distribution is almost correct (as otherwise there would have been* wider variation in I_w with *R*), the average settlement of the circular area with the assumed stress distribution will be quite close to the exact value of

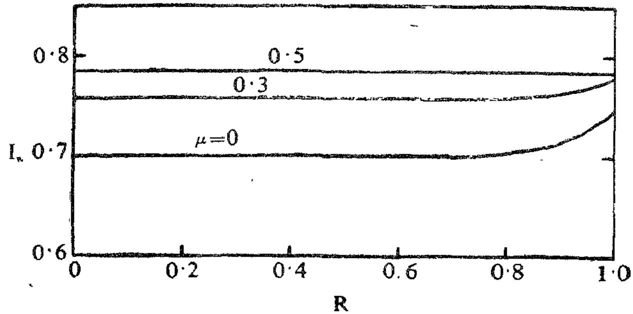

FIG. 3. Influence coefficients for surface settlement.

the settlement of a rigid foundation. The expression for the influence coefficient for average foundation settlement, I_{wo}, with the help of Equation [26] becomes

$$I_{wo} = \frac{1}{1-\mu}\left[\pi/4 - \int_0^\infty \frac{g(x)}{\pi x} J_1(x)\, dx\right] \tag{27}$$

The influence coefficients for average settlement so obtained have been shown in Fig. 4 for μ ranging from 0 to 0·5. The influence value is minimum for $\mu = 0$ and attains the maximum value of $\pi/4$ for $\mu = 0 \cdot 5$. The maximum variation of this influence value with μ is seen to be approximately 11 Per cent

The influence coefficient I_{wo} is useful for determining the immediate settlement of foundations. For saturated clays, the immediate settlement occurs without volume change and hence for foundations resting on saturated clays stresses and displacements are not at all affected by adhesion at the foundation base. For foundations resting on sands, for which the Poisson's ratio ranges from 0·15 to 0·35 [12] roughness has some influence on surface settlment. For an average Poisson's ratio of 0·3 the error in the surface settlement due to neglecting friction is only 3·5 Per cent which is negligible for all practical purposes. Hence, it can be concluded with reasonable confidence that in most of the cases, the effect of foundation roughness may be disregarded for computation of the immediate settlement of foundations.

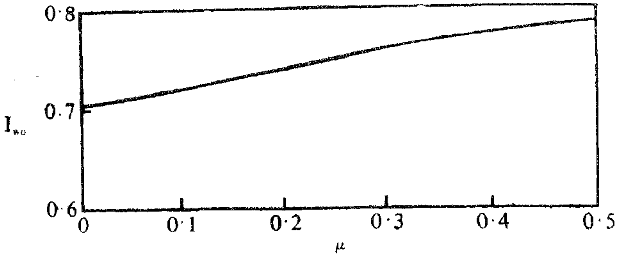

FIG. 4. Influence coefficients for foundation settlement (average surface settlement values.)

Verticul stress

For convenience, a non-dimensional parameter, I_z, which is the influence coefficient for vertical stress is introduced. It is defined as

$$I_z = -\sigma_z/p \tag{28}$$

In Fig. 5, I_z values have been plotted against Z for $\mu = 0, 0\cdot3$ and $0\cdot5$ taking $R = 0$. The influence values of $\mu = 0\cdot5$ are identical with those of a smooth foundation. It can be observed from the figure that Poisson's ratio plays a significant role in vertical stress distribution also. For a rough foundation, the vertical stress increases with the Poisson's ratio. Differences in influence coefficients between rough and smooth foundations decrease with depth as well as with μ. For example, at $Z = 2$ and 6, the percentage differences between the influence coefficients of smooth and rough foundations are 16 per cent and 5 per cent, respcetively. Figurc 5 enables one to estimate the vertical stress along the axis of a rough foundation, for any value of μ.

Radital deormaiion

The influence coefficient for radial deformation for a rough foundation is defined as

$$I_u = \frac{u}{E}pa \tag{29}$$

The same coefficient for a smooth foundation has been denoted by I_u. These influence coefficients, I_u and I_u have been shown in Fig. 6 for $R = 0 \cdot 5$, $1 \cdot 0$ and $3 \cdot 0$, taking $\mu = 0 \cdot 3$. Here also influence of roughness decreases with depth. The influence of roughness decreases also with R and for $R = 3 \cdot 0$, It can be seen that deviation of I_u from I'_u is almost negligible even at $Z = 0$, which can be explained by Saint-Venant's principle. Hence it may be pointed out that Poulos's remark [13] that radia l deformation is influenced by roughness is valid for small depths and small radial distances only.

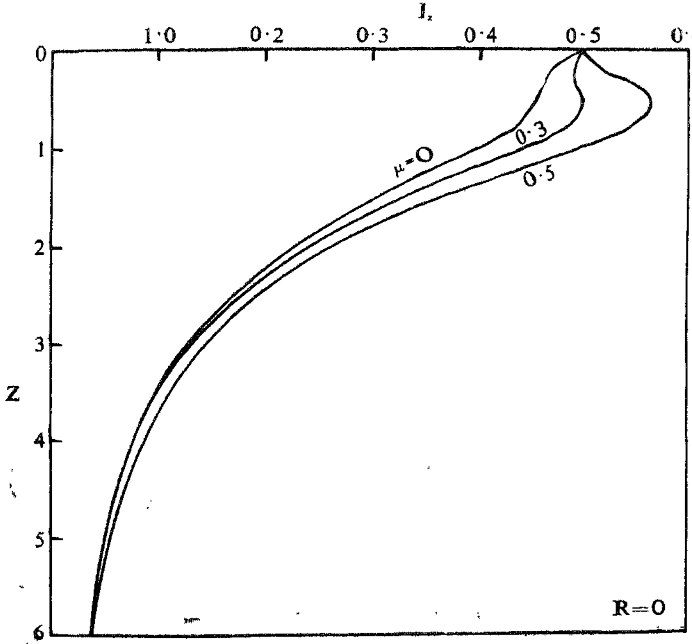

Fig. 5. Vertical stress distribution.

Radtal stress

Here again the influence coefficient for radial stress has been denoted by I_r and defined as

$$I_r = \sigma_r / p \tag{30}$$

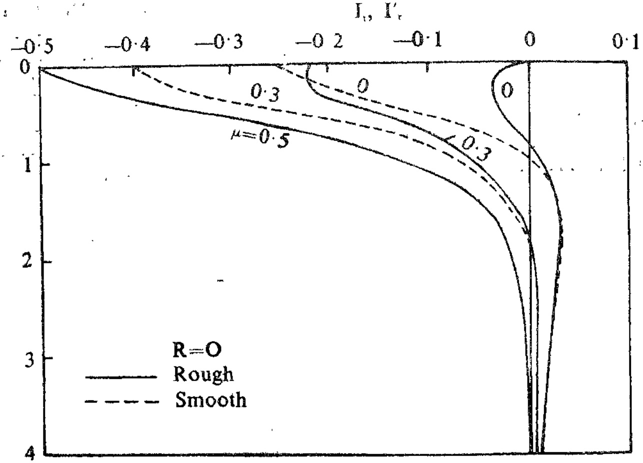

FIG. 6. Comparison of radial deformations under rough and smooth foundations.

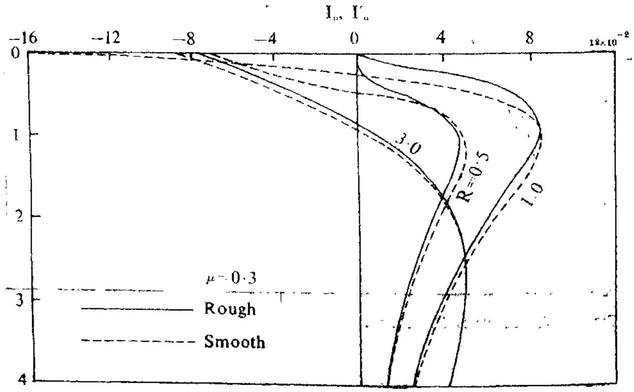

FIG. 7. Comparison of radial stresses under rough and smooth foundations.

and the influence coefficient for a smooth foundation has been denoted by I'_r. These influence coefficients have been presented in Fig. 7 for $\mu = 0$, $0 \cdot 3$ and $0 \cdot 5$ and for $R = 0$. It can be observed that the deviationof I_r, from I_r for any particular Z decreases with μ, finally vanishing at $\mu = 0 \cdot 5$. The deviation decreases also with depth. As in the case of vertical stress and radial deformation, here also roughness is seen to have influence for small depths only.

Concluding Remarks

Poisson's ratio plays an important part in the distribution of stresses and displacements under rough foundations. Deviation of stresses and displacements of a rough foundation from those of a smooth foundation decreases with μ and finally for $\mu = 0 \cdot 5$ (*i.e.*, ani ncompressible material) the stresses and displacements for the two types are identical. Deviations also decrease as one moves away from the contact surface. Friction or adhesion has more influence on stresses and displacements along horizontal directions than along vertical directions. For calculation of immediate settlement of foundations, friction or adhesion can be disregarded for all practical cases.

References

[1] Galin, L. A. . *Contact Problems in the Theory of Elasticity, N.C.* North Carolina State College Translation. Ed., I. N. Sneddon, 1961.

[2] Schultze, E. and Horn, A. The base friction for horizontally loaded footings in sand and gravel. 1967. *Geotechnique* 17, No 4, pp. 329–347.

[3] Mossakovskii, V. I. .. The fundamental mixed problem of the theory of elasticity for half-space with a circular line separating the boundary conditions, 1954, *PMM* 18, No 2, pp. 187–192.

[4] Ufliand, Ia. S. .. Contact problem in the theory of elasticity for a circular cylindrical punch in the presence of adhesion, 1956. *PMM* 20, No. 5., pp. 578–587.

[5] Ufli nd, Ic. S. .. Integral transforms in problems of theory of elasticity. *Izd-vo AN SSSR*, 1963.

[6] Abramian, B. L., Arutiunian, N. Kh. and Babloian, A. A. On symmetric pressure of a circular stamp on an elastic half-space in the presence of adhesion, 1966. *PMM* 30, No. 1 pp. 143–147.

[7] Conway, H. D., Vogel, S. M., Farnham, K. A. and So, S, Normal and shearing contact stresses in indented strips and, slabs, 1966. *Int. J. Engg. Sci.*, 4, No. 4, pp. 343–359.

[8] Sneddon, I. N. .. *Mixed Boundary Value Problems in Potential Theory*, John Wiley and Sons, New York, 1966, pp. 80–133.

[9] Sneddon, I. N .. *Fourier Transforms*, McGraw-Hill, New York, 1951, pp. 48–70, 450–510.

[10] Abramowitz, M. and Stegun, I. A. *Handbook of Mathematical Functions*, Dover Publications, New York, 1965, pp. 253–294, 355–434, 875–924.

[11] Luke, Y. L. .. *The special Functions and Their Approximations*, Vol. 2 Academic Press, New York, 1969, pp. 282–452.

[12] Harr, M. E. .. *Theoretical soil Mechanics*, Mc-Graw Hill, New York, 1966, Kogakusha, pp. 117–147.

[13] Poulos, H. G. .. Difficulties in prediction of horizontal deformations of foundations, 1972, *Proc. ASCE, JSMFE.*, 98, No. SM 8, pp 843–848.

NOTATIONS

A, B	= integration constants to be obtained from boundary conditions.
$A(x)$	= a function defined by Eq. (10).
a	= radius of the circular foundation.
E	= Young's modulus of soil.
$g(x)$, $g_1(x)$, $g_2(x)$	= functions defined by Eqs.(12), (13) and (14) respectively.
I_z, I_r	= influence coefficients for vertical and radial stresses, respectively.
I_w, u	= influence coefficients for vertical and radial displacements respectively.
I_{w0}	= influence coefficient for average settlement.
$J_n(x)$	= n th order Bessel function of the first kind.
m, n	= positive integers.
P	= total load on the foundation.
p	= equivalent uniformly distributed load on the contact area.
R, Z	= non-dimensional cylindrical coordinates $\left(R = \frac{r}{a},\ Z = \frac{z}{a}\right)$.
r, z	= cylindrical coordinates.
$S_1(x)$, $S_2(x)$	= series defined by Eqs. (15) and (16).
$S_i(x)$	= sine integral.
t	= a dummy variable.
u, w	= radial and vertical displacements, respectively.
W	= surface settlement.
x	= ηa.
η	= Poisson's ratio of soil.
μ	= a dummy variable.
σ_z, σ_r, σ_e	= normal stresses along vertical, radial and tangential directions.
τ_{rz}	= shear stress.