

ESTIMATION OF ERRORS IN IMAGE RESTORATION

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ABSTRACT

Spatial filtering for image restoration is prone to errors due to mismatch and misalignment of the filter. It is shown that in most of the practical cases, these errors can be reduced by spatial frequency scaling. A general methodology of estimating the errors of restoration due to mismatch of the filter is also developed.

Key words: Image restoration, Spatial filtering.

1. INTRODUCTION

Restoration of images degraded by relative disturbance between the photographic equipment and the scene being photographed has been investigated by many workers [1-4]. A spatial filter function used for restoration generally depends on the type and characteristics of the disturbance and it is therefore imperative to exactly match the filter with the disturbance for proper restoration. It is shown in this paper that in most of the practical cases, this match can be achieved by a simple frequency scaling. Further; some estimates of the sensitivity of restoration to matching are given.

We have discussed those disturbance functions which can be described by a single variable α in some cartesian coordinate system and by a single motion parameter A . We also assume that his function has the form $(1/A) \cdot h(\alpha/A)$ where h is an analytic function over the domain of α . As will be shown later, most of the disturbances that are of relevance to practical situations, are of this type.

It is wellknown that the blurring due to a disturbance can be treated as a convolution of the image and the point spread function of the disturbance and can therefore be effectively eliminated by spatial filtering [2-7]. The filter function F is related to the Fourier transform (FT) of the point spread function, H , through $F = 1/H$. The significant properties of this function are brought out in the next section.

2. Some properties of the filter function

As mentioned in sec. 1, the point spread function of the disturbance is assumed to have the form $(1/A) \cdot h(a/A)$. Its FT can therefore be put down as:

$$\begin{aligned} H &= (1/2\pi) \int_{-\infty}^{\infty} (1/A) \cdot h(a/A) \cdot \exp(-j\omega a) da \\ &= (1/2\pi) \int_{-\infty}^{\infty} (h) \cdot \lambda \exp(-j\lambda A\omega) d\lambda \end{aligned}$$

This shows that H is a function of $A\omega$. Consequently, the filter function F will also be a function of $A\omega$ and will henceforth be written as $F(A\omega)$. Since this function should asymptotically tend to unity as A tends to zero,

1. $F(0) = 1$ and
2. $F(A\omega)$ is analytic in some neighbourhood of $A\omega = 0$.
3. Finally since h is a real function, $H(A\omega) = H^*(-A\omega)$

where* denotes complex conjugate. We then get $F(A\omega) F(-A\omega)$ or $|F(A\omega)| = |F(-A\omega)|$. This absolute value of $F(A\omega)$ will be denoted by an even function $F_0(A\omega)$ and $F(A\omega)$ can then be represented as:

$$F(A\omega) = F_0(A) \cdot \exp\left(-j \sum_{m=0}^{\infty} k_m (A\omega)^{2m+1}\right) \quad (1)$$

Though this equation looks rather formidable in its general form, in most of the practical cases we have examined, only k_0 is nonzero.

3. Frequency scaling for filter matching

If the filter function has the form given in Eq. (1) then a change in A is equivalent to frequency scaling which can be achieved by a simple arrangement shown schematically in Fig. (1). This arrangement differs from a

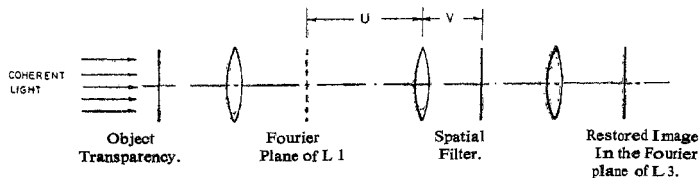


Fig. 1. Arrangement of optical components for spatial filtering.

conventional one [4] in that a lens L_2 is present between the Fourier plane of L_1 and the filter. The distances u and v are adjusted with the constraint.

$1/u + 1/v = 1/\text{focal length of } L_2$ to achieve proper magnification v/u of the frequency scale and equivalently match the filter parameter A with the parameter of the disturbance.

The question of how close this match should be can be answered only if the sensitivity of restoration to the motion parameter A is known. A method to estimate this sensitivity is given in the next two sections.

4. Sensitivity of restoration to the motion parameter

The original image $p_0(\alpha, \beta)$ is related to $F(A\omega)$ and the FT of the blurred image, $P(\omega, \sigma)$, through:

$$p_0(\alpha, \beta) = F^{-1}\{F(A\omega) \cdot P(\omega, \sigma)\} \quad (2)$$

where F^{-1} denotes the inverse FT operation. It is therefore obvious that the reconstruction of $p_0(\alpha, \beta)$ is sensitive to the motion parameter A . The error in restoration, dp_0 , can be related to the error in A , dA , as follows:

Differentiation of Eq. (2) gives

$$dp_0/dA = F^{-1}\{(\omega \cdot F'(A\omega)/F(A\omega))F(A\omega) \cdot P(\omega, \sigma)\} \quad (3)$$

Since $F(A\omega)$ is analytic in the neighbourhood of zero, so is $F'(A\omega)/F(A\omega)$. From Eq. (1) we get

$$F'(A\omega)/F(A\omega) = \sum_{n=0}^{\infty} a_n (A\omega)^{2n+1} - j \sum_{m=0}^{\infty} k_m (2m-1) (A\omega)^{2m} \quad (4)$$

where the first summation is the Taylor series expansion of the odd function $F'_0(A\omega)/F_0(A\omega)$ and is valid upto the first zero of $F_0(A)$. Multiplication by ω gives:

$$\begin{aligned} F'(A\omega)/F(A\omega) &= \sum_{n=0}^{\infty} (-1)^{n+1} a_n A^{2n+1} (j\omega)^{2n+2} \\ &+ \sum_{m=0}^{\infty} (-1)^{m+1} k_m (2m+1) A^{2m} (j\omega)^{2m+1} \end{aligned} \quad (5)$$

The solution of Eq. (3) using this expansion can be shown to be

$$\begin{aligned} dp_0/dA &= \sum_{n=0}^{\infty} (-1)^{n+1} a_n A^{2n+1} (d^{2n+2} p_0 / d\alpha^{2n+2}) \\ &+ \sum_{m=0}^{\infty} (-1)^{m+1} k_m (2m+1) A^{2m} (d^{2m+1} p_0 / d\alpha^{2m+1}) \end{aligned} \quad (6)$$

This equation explicitly relates the sensitivity of reconstruction (dp_0/dA) to the intensity variations of the original image.

5. Sensitivity of restoration to alignment

It is clear that Eq. (2) will be valid only if the filter axis coincides with the ω axis of $p(\omega, \sigma)$. If these two axes make an angle θ then the filter function acting on $P(\omega, \sigma)$ is $F(A(\omega \cos \theta + \sigma \sin \theta))$. For small values of θ , Eq. (2) gives

$$dp_0/d\theta = F^{-1} \{A\sigma F'(A\omega)/F(A\omega) \cdot F(A\omega) \cdot P(\omega, \sigma)\} \quad (7)$$

Using the expansion of $F'(A\omega)/F(A\omega)$ as in Eq. (4),

$$\begin{aligned} A\sigma F'(A\omega)/F(A\omega) = & \sum_{n=0}^{\infty} (-1)^{n+1} a_n A^{2n+2} (j\sigma) (j\omega)^{2n+1} \\ & + \sum_{m=0}^{\infty} (-1)^{m+1} k_m (2m+1) A^{2m+1} (j\sigma) (j\omega)^{2m} \end{aligned} \quad (8)$$

and finally from Eq. (7),

$$\begin{aligned} dp_0/d\theta = & \sum_{n=0}^{\infty} (-1)^{n+1} a_n A^{2n+2} (d^{2n+2} p_0/d\beta d\alpha^{2n+1}) \\ & + \sum_{m=0}^{\infty} (-1)^{m+1} k_m (2m+1) A^{2m+1} (d^{2m+1} p_0/d\beta d\alpha^{2m}) \end{aligned} \quad (9)$$

Using Eq. (9), the sensitivity of the restoration technique can be found out at various picture points.

6. Examples of disturbances

It has been shown by various workers that the filter functions for the restoration of linear motion and linear vibration degradations are: for linear motion [2, 5, 8, 9]: $F(A\omega) = (A\omega/2)/\sin(A\omega/2) \cdot \exp(jA\omega/2)$ where A is exposure time \times velocity in the image plane.

for vibration [6, 10]: $F(A\omega) = 1/J_0(A\omega)$

where A is the amplitude of vibration in the image plane.

Since these functions are of the form given in Eq. (1), method of filter matching described in sec. 3 can be applied.

For linear motion filter,

$$F_0'(A\omega)/F_0(A\omega) = \sum_{n=0}^{\infty} (|B_{2n+2}| / (2n+2)) (A\omega)^{2n+1}$$

where B_n is the Bernoulli's number

$$= (A\omega)/12 + (A\omega)^3/720 + (A\omega)^5/30240 + \dots$$

and for the vibration filter, in the range of interest 6,

$$F_0'(A\omega)/F_0(A\omega) = \cdot 476 (A\omega) + \cdot 1391 (A\omega)^3 + \cdot 05008 (A\omega)^5 \\ + \cdot 01628 (A\omega)^7 + \dots$$

It can be seen that both these series converge rapidly and therefore analysis of secs. (4) and (5) can be applied to obtain the sensitivities.

The filter function for restoration of images degraded by circular motion has the form [10, 9]:

$$1/J_0(2\pi((\omega_x A)^2 + (\omega_y A)^2)^{1/2})$$

where ω_x , ω_y are spatial frequencies in two perpendicular directions and A is the amplitude of vibration in the image plane. Even though this function cannot be described by Eq. (1), the filter matching method of sec. (3) is still applicable as both ω_x and ω_y scales are magnified by the same amount by lens $L2$.

For uniform accelerated motion with initial velocity u , acceleration h , total distance covered s and the exposure time T , the Fourier transform of the point spread function is given by [9]:

$$L(\omega) = \frac{\exp(jP_1)}{T(2h\omega)^{1/2}} \cdot \left\{ \sum_{n=0}^{\infty} J_{2n+1/2}(P_2) - J_{2n+1/2}(P_1) \right. \\ \left. - jJ_{2n+1+1/2}(P_2) + jJ_{2n+1+1/2}(P_1) \right\}$$

where

$$P_1 = \pi\omega u^2/h \quad \text{and} \quad P_2 = 2\pi\omega s + P_1 \quad (10)$$

If the two parameters are chosen to be (velocity)²/acceleration at the two extremums, then they are:

$$A_1 = u^2/h \quad \text{and} \quad A_2 = u^2/h + 2s \quad \text{and}$$

$$P_1 = \pi(\omega A_1), \quad P_2 = \pi(\omega A_2),$$

$$T(2h\omega)^{1/2} = 2(\omega A_1 + \omega A_2 - (\omega A_1(\omega A_1 + \omega A_2))^{1/2})^{1/2}$$

Thus even in this complicated case, the filter function $1/L(\omega)$ can be written as $F(\omega A_1, \omega A_2)$. The arrangement of Fig. 1 can be used for matching one of the two motion parameters at a time. The analysis of secs. 4 and 5 can give us the restoration sensitivities. $\delta P/\delta A_1$ and $\delta P/\delta A_2$. Total error of restoration is given by:

$$dP = (\delta P/\delta A_1) \cdot dA_1 + (\delta P/\delta A_2) \cdot dA_2$$

and therefore in this case, the restoration error can only be reduced but not eliminated completely. If the restoration is insensitive to one of the parameters, then matching the filter for the other parameter would minimise the error.

7. Conclusions

A serious drawback of the spatial filtering technique is that a filter which exactly matches the disturbance conditions must be used. It has however been observed that the spatial frequency and the parameter characterising the motion always occur in the product form in the filter function. Thus scaling in the frequency domain is equivalent to changing the motion parameter. Since frequency scaling only changes the scale and the absolute intensity of the reconstructed image, but leaves the relative intensities of different points unaltered, it does not degrade the image. It therefore can be gainfully employed for matching the filter to the motion parameter.

The sensitivity of reconstruction to the mismatch can be estimated if it is assumed that the image bandwidth is limited. Since the expressions for this sensitivity involve the differential coefficients of image intensity w.r.t. the spatial coordinates, it can be concluded that restoration error will be localised in nature and will be present mainly near the sharp boundaries. These expressions also indicate that the error is linearly dependant upon the mismatch and is a function of higher powers of the magnitude of motion parameter. This analysis however is only an approximate one since degradation of higher frequency components is not taken into account. It can however serve as a first estimate of restoration errors due to filter mismatch as analysis of a real picture shows that bulk of the picture energy is present in the low frequency terms.

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